Gravitational constant of Electron

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Research Article

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Gravitational constant of Electron

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Abstract:

In this paper, the unique relationship between Gravitational and Electrostatic attraction is comprehended which provide us reason and derivation of known modern physics of charged sub-atomic particle and its problem using Newtonian Mechanics. Here, the Plasma frequency of cold electron is used for deriving the attraction constant for charged–mass objects in terms of gravitational constant i.e. $G = 3n \cdot \frac{V_o}{V_n} \times \frac{e^2}{4\pi\epsilon_0 m_e m_p}$ and by setting the value obtained result in Newton's gravitational formula it gives us Coulomb's force. The equation for charged particles placed at a certain distance with some conditions. Furthermore, a short derivation of the classical radius of electron and Bohr's Radius is shown by using the obtained attraction constant with the justifications.

Introduction:

We know that at subatomic levels the particles don't show any effective gravitational effect instead strong nuclear force holds the composition of the particles and electrostatic force governs the interaction between other charged particles hence dominating the gravitational force of that particle – consider any pair of elementary particles say electron pair, the gravitational coupling constant of electron pair is $\alpha_G \approx 1.7518 \times 10^{-45}$ and fine structure constant of the pair $\alpha \approx 7.29735 \times 10^{-3}$. We can clearly see that gravitational attraction between these pair is far way smaller than the electrostatic repulsion, this is one of the several manner to conclude that gravity is far weaker force than electromagnetic interaction and gravity of neutral mass object is determined in direct proportion of its mass and inverse proportion to square of its radius with a proportionality constant – Newtons Gravitational constant. This constant is universal and applied to every neutral charged-mass body to determine its gravity, but in the case of uniform charged-mass object we use coulomb's force formula to determine the electrostatic force between them. Here, we will find relation between both types of forces which will grant us an “attraction constant”.

This constant may further allow us to derive result for pair of charged particle using Newtonian Mechanics and open a field for new discovery.

1. Method and Conditions:

Plasma Oscillation is the organized motion of electrons or ions in plasma. Each particle in plasma assumes a position such that the total force resulting from all the particles is zero, thus producing a uniform state with a net charge of zero.⁴

The plasma oscillation of cold electron⁵ is considered here because it doesn't include any thermal energy to influence the rate of oscillation, and completely depends upon the electrostatic force between electrons and protons. In this case, frequency is expressed by

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}.$$

On the other hand, Orbital frequency is the number of rotation per unit time of the given astronomical object around another massive body. The frequency of the object is influenced by the gravitational attraction by the other massive object. Not only astronomical body, the entire object which has mass and a system imitating the revolution around another mass have the frequency influence by their gravitational interaction. Such as, electron-proton pair taken from the hydrogen atom, if we only consider the orbital period of the

electron around proton caused by the influence of gravitational interaction between them neglecting electrostatic disturbance would be $\frac{T^2}{R^3} = \frac{4\pi^2}{G \times m_p}$. Where G is the Newton's Gravitational Constant, m_p is the mass of the proton, R is the orbital radius of the electron and T is the orbital period.⁶

As the frequency we are dealing in the plasma is longitudinal. So, in order to compare it we have to define the situation which showcases some similarity in both of the concepts. If we consider the sideward projection of electron revolving around proton then we will get the back and forth motion on a plane, which simulates the same oscillation of electron's motion in plasma, without affecting its actual orbital frequency. Through, this we will find the value of the Attraction constant in terms of Gravitational constant between electrons and protons. And via this result I will find out the Bohr's radius.

I compare the gravitational orbital frequency and the plasma oscillation frequency of electron to find out the Gravitational constant in terms of charges. By using the result I derive the result for the radius of electron.

2. Single Electron Oscillation

Let us consider a finite isolated system consisting of plasma gas with number density n_e of electrons. Take the formula of plasma oscillation for cold electrons (in S.I. Unit):

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}} \quad (1)$$

Where, e is the charge of electron, ϵ_0 is the permittivity in the vacuum and m_e is the mass of the electron.

As, there is no natural frequency for these oscillations in an infinite medium, as they get discharge to the surroundings. So, we have to define volume of the isolated system, let us take it ' V_n ', then the number density of the electron will be: $n_e = n/V_n$. Where, n is the number of electrons.

$$\omega_{pe} = \sqrt{\frac{n}{V_n} \frac{e^2}{\epsilon_0 m_e}} \quad (2)$$

Writing it in terms of frequency and simplifying it:

$$\nu_{pe} = \sqrt{\frac{n}{V_n} \times \frac{e^2}{4\pi^2 \epsilon_0 m_e}} \quad (3)$$

Now, we have the oscillation frequency of n electron in cold plasma.

Now, let us assume an electron-proton pair having r distance between them, so the orbital period ' T ' of electron around proton due to its gravity is given by (Considering the orbit circular and neglecting electrostatic force of attraction):

$$T = \sqrt{\frac{3\pi V_0}{G M_p}} \quad (4)$$

Where G , M_e , and V_0 are Gravitational Constant, central mass (in this case its proton's mass) and Volume inside the radius of the orbit respectively.

Simplifying eq. (4) in terms of frequency ν ,

$$\nu = \sqrt{\frac{G M_p}{3\pi V_0}} \quad (5)$$

Though, as I have stated the reason in previous section, we can compare eq. (3) and eq. (5) by equating, we get G equivalent to:

$$G \equiv 3n \cdot \frac{V_0}{V_n} \times \frac{e^2}{4\pi \epsilon_0 m_e m_p} \quad (6)$$

Where, the above result is an attraction constant between two charged-mass object (which in this case is electron and proton).

Setting the equivalent value of G in Newton's Gravitational Force formula between proton and electron situated at a distance r .

$$F_g = \frac{G m_e m_p}{r^2}$$

$$\rightarrow F_g = \left[3n \cdot \frac{V_0}{V_n} \times \frac{e^2}{4\pi \epsilon_0 m_e m_p} \right] \times \frac{m_e m_p}{r^2}$$

$$\rightarrow F_g = (3 \times (1) \cdot \frac{V_0}{V_n}) \times \frac{1}{4\pi \epsilon_0} \times \frac{e^2}{r^2} \quad (7)$$

(Here, $n = 1 \because$ a single electron-proton pair is considered.)

Or
$$F_g = 3 \cdot \frac{V_o}{V_n} \times F_e$$

Where, $F_e = \frac{1}{4\pi\epsilon_o} \times \frac{e^2}{r^2}$ is the formula of electrostatic force between a pair of electron and proton.

We have obtained a result consisting of electrostatic force between an electron and a proton with a constant which provides a condition to make this formula work in accordance to Coulomb's force of attraction i.e. for electron-proton pair separated r distance from each other the volume inside the orbit should be one-third of volume of the isolated space in which those charge particles lies or $3 \cdot \frac{V_o}{V_n} = 1$.

V_n is the volume of the isolated space in which system is lying or say under which an observer is observing.

This may suggests that in this case it depends on observer or observing equipment or detector the fate of magnitude of attraction between electron and proton.

3. Result and Discussion:

The attraction constant between n electrons and protons is $G = 3n \cdot \frac{V_o}{V_n} \times \frac{e^2}{4\pi\epsilon_o m_e m_p}$. Now, with help of this constant, I will be deriving the new classical radius of electron.

New Classical radius of electron

The radius of orbital R of an object revolving around a body of mass M at a speed of v is given by, $R = \frac{GM}{v^2}$. Assuming, an electron revolving around a proton in an isolated space (of known volume V_n) about near the speed of light, then $R_p = \frac{Gm_p}{c^2}$. At this point, electron must be very near to the proton, and inserting the value of attraction constant in the equation we get:-

$$R_p = 3n \cdot \frac{V_o}{V_n} \times \frac{e^2}{4\pi\epsilon_o m_e c^2} \quad (8)$$

Assuming the constant $3n \cdot \frac{V_o}{V_n} = 1$, the radius of orbital of electron around proton is $\frac{e^2}{4\pi\epsilon_o m_e c^2} \approx 2.817940288 \times 10^{-15} m$ which seems

similar to the charged radius of an electron (but in this case we are finding the charge radius of proton using the modified radius of the orbital formula so, it's merely a coincident or it may be a correction). As well, radius of orbital is minimum when the velocity of the particle in orbital is maximum; here it's near the speed of light which gives us the near value of the radius of the proton. So, we can take the result obtained as a reference to determine the radius of the proton. Although, we know the experimental radius of the proton and it somehow is uncertain⁷.

Similarly, with electrons we can find its radius of orbital, just by observing proton with respect to electron then in this point of view proton is spinning around electron. Then replacing mass of electron m_e from mass of proton m_p in eq.(8) and putting the constant $3n \cdot \frac{V_o}{V_n} = 1$, the radius of orbital of proton revolving at a speed of light around electron is $\frac{e^2}{4\pi\epsilon_o m_p c^2} \approx 1.534698248 \times 10^{-18} m$. As it is the minimum radius of the orbital then the real value of radius of an electron must be less than the obtained value.

These values of radii of proton and an electron are in favor of the expected radius.

Bohr's Radius:

Radius of orbital of electron in S1 orbital around nucleus in hydrogen atom moving at speed

$$mv_e r = \frac{nh}{2\pi} \rightarrow v_e = \frac{h}{2\pi r_b m_e} \rightarrow$$

$v_e = 2.187691264 \times 10^6 m/s$ (Bohr's velocity of electron in S1 orbital), where r_b is Bohr's radius of electron in S1 orbital.

$$R_p = 3n \cdot \frac{V_o}{V_n} \times \frac{e^2}{4\pi\epsilon_o m_e v_e^2} \quad (9)$$

$$R_p = 3n \cdot \frac{V_o}{V_n} \times 5.291772109 \times 10^{-11} m$$

If we assume the constant value to one then we are getting the Bohr's radius⁸ of hydrogen atom in S1 orbital and after comparing the above equation with

Bohr's radius of hydrogen atom in n^{th} orbital, then in this case value of constant is coming equal to:

$$3n \cdot \frac{V_o}{V_n} = \frac{n_q^2}{Z}$$

Where n_q is quantum number and Z is the atomic number of the respective atom. The constant here is associated with quantum number of electron and the atomic number of atom. And it also suggests that the radius of the orbital of an electron is affected by the virtue of our observation; volume of the isolated space V_n which is consisting of whole system within the limit of it we can observe the whole system with accuracy.

Relation between Gravitational Coupling Constant² and Fine Structure Constant

Gravitational attraction between electron-electron pair is given by

$$\alpha = \frac{Gm_e^2}{\hbar c}$$

Substituting the value of the G for attraction between two electrons in the above equation:

$$\alpha = 3n \cdot \frac{V_o}{V_n} \times \frac{e^2}{4\pi\epsilon_o m_e \cdot m_e} \times \frac{m_e^2}{\hbar c} = \left[3n \cdot \frac{V_o}{V_n} \right] \times \frac{e^2}{4\pi\epsilon_o \hbar c}$$

Putting constant value in square bracket to one, then we get the expression for the fine-structure constant³ of two electrons.

4. Conclusion:

It is shown that using Newtonian mechanics we can determine the characteristics of charged particles having mass (represented with the help of proton and electron). Here we have derived the minimum radius of orbital of a proton and an electron, which give us reference below which the real charged radius of these particles lies. And most precisely the Bohr's radius is found when the attraction constant between electron-proton pair is inserted in the formula for orbital radius between two massive objects.

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