

Estimating the reproduction number and forecasting the impact of COVID-19 in Kuwait using a modified compartmental epidemiological model

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Title: Estimating the reproduction number and forecasting the impact of COVID-19 in Kuwait using a modified compartmental epidemiological model

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Abstract

A modified compartmental epidemic model was developed to simulate the state of Kuwait protocol in fighting COVID-19 pandemic. The next generation matrix method was used to derive an expression for the basic reproduction number, R_0 . Basic and effective reproduction numbers were calculated using data from the intrinsic growth rate of the confirmed COVID-19 cases. R_0 was found to be 2.18. Three scenarios that varied by effective reproduction number were used to estimate the future course of the disease: a high value of $R = 1.98$, a middle value of $R = 1.62$, and a low value of $R = 1.2$. The maximum number of beds required in general hospitals in each scenario were estimated at 141 184, 85 341, and 16 412, respectively. For intensive care units, the estimated numbers of beds required were 16 461, 9 645, and 1788. Maximum deaths also varied and were estimated to be 29 202, 23 973, and 11 565. For the maximum value of R , it is estimated to peak on August 27, 2020. For the middle value of R , it is estimated to peak on September 20, 2020. For the minimum value of R , it is estimated to peak on December 21, 2020.

1. Introduction

The compartmental model in epidemiology helps health authorities make intelligent decisions and has been proven to be effective in predicting the spread of pathogens since the time it was developed by W. O. Kermack and A. G. McKendrick [1–3]. Since then, other models have been developed to address specific needs for comprehending the spread of emerging or established pathogens [4–6]. The recent emergence of the SARS-CoV-2 virus and COVID-19 disease has paralyzed the world. Research has focused on the virus since it surfaced in Wuhan, China, in December 2019 [7]. The characterization of virus characteristics has been rapid as a result of the intense focus. Understanding the dynamics of disease spread is critical to help the policymakers prepare for what is to come and plan accordingly. In each country, health authorities establish procedures to delay the emergence of disease and to curb its spread. In this study, a compartmental epidemiological model was developed to estimate the spread of SARS-CoV-2 and COVID-19 in Kuwait.

Kuwait acted swiftly to control the spread of the disease; all arrivals from countries with reported COVID-19 cases were institutionally quarantined beginning February 24, 2020; schools and universities were closed on February 27; non-essential government employees were ordered to stay home on March 12; passengers flights were suspended, prayer services in Mosques were banned, and public parks were closed on March 13; stricter measures, such as closing non-essential retail shops, were instituted on March 14. Finally, on March 22, 2020, a partial lockdown was imposed. Despite these precautions, Kuwait reported its first untraced domestic verified COVID-19 case on March 18. Since then, the daily reported cases have followed an increasing trajectory [8]. When the government imposed a total lockdown on May 11, the daily number of confirmed COVID-19 cases reached 920 cases with a cumulative total of 9701 cases. Although the total lockdown was partially lifted on June 1, the disease continues to spread, with daily new cases in the hundreds (Figure 1).

2. Method

The health authority in Kuwait has established a protocol for dealing with newly discovered cases of COVID-19, which was outlined during a press conference [8]. When a case is discovered, the patient is sent into institutionalized quarantine and will remain there until recovered. If the case requires medical attention, then the patient will be transferred to a hospital, and if required, transferred to an intensive care unit. Some infected individuals are asymptomatic and would remain undetected. We assume that no deaths will arise from these cases.

A compartmental epidemiological model was developed to simulate this protocol (Figure 2). The total population (N) was divided into susceptible (S), exposed (E), hidden (H), infected (I), first-level quarantine (QA), second-level quarantine (QB), first-level hospitalization (HOA), second-level hospitalization (HOB), intensive care (IC), recovery (HOC), death (D), and recovered (R) compartments (Table 1). The S population includes individuals who can get

infected but have not yet contracted the virus. The E compartment includes individuals who are infected but not yet infectious. It was established that the E individuals would be infectious two days before the onset of the symptoms[9]. Therefore, an adjustment term ϵ_E was applied to individuals in the E state to account for the latent period of infection with a mean incubation period of $1/\kappa$ days[5]. The H compartment includes infected individuals who are not showing symptoms; therefore, these individuals are undetected but will contribute to the spread of the virus during their infectious period. This compartment accounts for $\eta = 17.9\% - 30.8\%$ of the infected pool[10,11]. Individuals in the H compartment have a mean infectious period of $1/\gamma_1$, after which it is assumed that they recover[9,12]. The I compartment includes infected individuals who are showing symptoms. It is expected that individuals in this state will be detected after a mean period of $1/\gamma_2$ days from the onset of the symptoms. This mean rate depends on how fast the health authorities identify and isolate the infected individuals. The mean period was defined after the model was compared to the daily counts of new cases reported in Kuwait. After identifying infected individuals, health authorities institutionally quarantine the patients; therefore, they cease being infectious to the general population. The three compartments E , H , and I are infectious and could transmit the virus from being in contact with the S compartment at a daily contact rate of β . It was estimated by the WHO[13] that 80% of infected individuals with symptoms do not need medical attention. To model this, individuals in quarantine were classified into two levels. Initially, a patient is classified as QA , immediately after being detected and removed from the I compartment. While in the QA compartment, the condition of a patient is monitored for a mean period of $1/\tau_1$. A QA patient will be either classified as QB if mild symptoms appear or HOA if medical attention is required. The fraction of cases that become classified as QB is f_1 . Patients will stay in the QB compartment for a mean period of $1/\tau_2$ until they cease to be infectious. When a patient ceases to be a threat to the susceptible population, that individual will be moved into the R compartment. The health condition of patients in the HOA compartment will either deteriorate or become stable after a mean time of $1/\tau_3$. A fraction f_2 of the patients will move into the IC compartment because of their need for ventilators. The remainder will be moved to the HOB compartment. Patients in the HOB compartment will recover at a mean period of $1/\tau_4$. Patients in the IC compartment will remain for a mean period of $1/\tau_5$, at which time a fraction f_3 of IC patients will die and the rest will be moved to the HOC compartment. Subsequently, these patients will be moved at a rate of $1/\tau_6$ to the R compartment. Vital dynamics were ignored in our model. The value of the mean periods for each compartment and percentages are presented in Table 2.

The equations for the model explained above are listed below.

$$\frac{dS}{dt} = -\frac{S}{N}\beta(\epsilon_E E + H + I) \quad (1)$$

$$\frac{dE}{dt} = \frac{S}{N}\beta(\epsilon_E E + H + I) - \kappa E \quad (2)$$

$$\frac{dH}{dt} = \eta\kappa E - \gamma_1 H \quad (3)$$

$$\frac{dI}{dt} = (1 - \eta)\kappa E - \gamma_2 I \quad (4)$$

$$\frac{dQA}{dt} = \gamma_2 I - \tau_1 QA \quad (5)$$

$$\frac{dQB}{dt} = f_1 \tau_1 QA - \tau_2 QB \quad (6)$$

$$\frac{dHOA}{dt} = (1 - f_1) \tau_1 QA - \tau_3 HOA \quad (7)$$

$$\frac{dHOB}{dt} = (1 - f_2) \tau_3 HOA - \tau_4 HOB \quad (8)$$

$$\frac{dIC}{dt} = f_2 \tau_3 HOA - \tau_5 IC \quad (9)$$

$$\frac{dHOC}{dt} = (1 - f_3) \tau_5 IC - \tau_6 HOC \quad (10)$$

$$\frac{dD}{dt} = f_3 \tau_5 IC \quad (11)$$

$$\frac{dR}{dt} = \gamma_1 H + \tau_2 QB + \tau_4 HOB + \tau_6 HOC \quad (12)$$

At the initial state the total population N was assumed to be constant and equal to 4.7 million[14], which is the total population of the country. It is also the sum of all the compartments.

$$N = S + E + H + I + QA + QB + HOA + HOA + IC + HOC + R + D \quad (13)$$

Over time, deaths will be subtracted from the total population N at a rate of

$$\frac{dN}{dt} = -f_3 \tau_5 IC. \quad (14)$$

2.1. The reproduction number R and the contact rate β calculated from the infection data

The basic reproduction number, R_0 , is critical for determining whether a disease will die out or spread. It can be calculated from the infection data and the initial rate at which the number of infected individuals increases. In the model Equations (1-14), the next generation method, N.G.M, [15–17] (appendix) was used to calculate an expression for R_0 as follows:

$$R_0 = \beta \left(\frac{\varepsilon_E}{\kappa} + \frac{\eta}{\gamma_1} + \frac{(1-\eta)}{\gamma_2} \right). \quad (15)$$

R_0 is important for identifying the equilibrium state of the disease[16]. A disease will die out when $R_0 < 1$, or it will persist and become endemic when $R_0 = 1$, or it will increase epidemically when $R_0 > 1$. There are two ways to control such a spread, regardless of location. The first approach is to decrease the daily contact rate β , and this can be achieved by social distancing measures and lockdowns. The second approach is to implement an aggressive locate and isolate campaign, where individuals in the infectious E , H , and I compartments are located and isolated, thereby minimizing their exposure to the susceptible S compartment. In other words, eliminate ε_E , $1/\gamma_1$, and $1/\gamma_2$, each of which reduces R_0 . We relied on the recorded data of the confirmed number of infected individuals, which was provided by the ministry of health[8], to calculate the effective reproduction number in Kuwait. After linearizing the model for the infectious compartments in Equations (2), (3), and (4), the eigenvalue equation for the disease free linear Jacobean matrix of the model (appendix) was found to be

$$\beta\kappa(1-\eta)(\gamma_1 + \lambda) - (\gamma_2 + \lambda)(\kappa\gamma_1 - \beta\kappa\eta - \beta\gamma_1\varepsilon_E + \kappa\lambda + \gamma_1\lambda - \beta\varepsilon_E\lambda + \lambda^2) = 0. \quad (16)$$

In the equation above γ_1 , γ_2 , and ε_E values are tabulated in Table 2. β and the leading eigenvalue, λ , are unknowns to be determined. λ of Equation (16) is the intrinsic growth rate of the epidemic[18]. The growth rate was calculated using the following expression[19]:

$$\lambda = \frac{LN \left(\frac{n_2}{n_1} \right)}{t_2 - t_1}, \quad (17)$$

For R_0 , n_1 is the total number of cases on March 18 and n_2 is the total number of confirmed cases on June 10. $t_2 - t_1$ is the number of days from March 18th until June 10. For the effective reproduction number, R , n_1 and n_2 are the seven-day rolling averages of the reported cases. $t_2 - t_1$ was taking to be seven days. COVID-19 cases due to travel were excluded from the calculation because we assumed that they were located and isolated at the airport and did not contribute to the local spread of the disease. The daily contact rate β was then calculated using Equations (17) and (16). Once β was found, then R_0 was calculated from equation (15). The seven-day rolling average of the daily effective reproduction number, which was used in the calculation until its last recorded date, is shown in Figure 3. It ranges from a maximum value of 5.79 on March 31, 2020, to a minimum value of 0.59 on June 10, 2020. The R_0 from the beginning of the epidemic on March 18 until June 10 is 2.18. Three scenarios were used for predicting the epidemiology of cases: a high value equal to the average of all the effective reproduction numbers ($R = 1.98$), a middle value that was calculated 7 days after the partial lockdown on April 7 ($R = 1.62$), and a low value that was calculated 7 days after the total lockdown on May 11 ($R = 1.20$). The values were calculated

until June 10, 2020, which was the last recorded data available to include in this study. The high value represents a return to normal life with some restrictions, the middle value represents a partial lockdown in which some businesses return to normal operation, and the low value represents a partial lockdown in which only essential businesses are open. The 7-day period was considered to be a good estimate for the measures taken by the authorities to be reflected in the recorded data.

3. Results

The recorded data for the number of confirmed COVID-19 cases and deaths count and rate were compared with the model calculations until June 10 to determine the accuracy of the model. The recorded data were almost identical to the model, as shown in Figures 4 and 5. The number of confirmed infected cases reported by the health authority followed the pattern of the model with a slight deviation that began on May 14 and reached its maximum on June 3. There was a small peak on May 19 for the model results, but the peak from the actual cases appeared two days later. The number of confirmed cases and the model results began to decrease after that. The number of confirmed cases peaked because of the total lockdown that was imposed on May 11. However, a perfect opportunity to control the epidemic was missed by not extending the lockdown for another three weeks. According to the model, the number of cases will start to rise again and reach a peak value on August 11 for the high value of R , on September 6 for the middle value of R , and on December 5 for the low value of R . The number of cases per day estimated for these three peaks are 60 000, 32 589, and 5652, respectively. The number of cases per day is estimated to decrease after these dates until they reach single-digit values on November 18, 2020, January 2, 2021, and July 18, 2021, respectively (Figure 5).

The hospitalization rate is predicted to peak on August 27, 2020, September 20, and December 21 at values of 141 184, 85 341, and 16 412, respectively, for the three values of R (Figure 6). The peak number of hospitalized patients occurs two weeks after the peaks of infected cases. Hospitalized cases represent the combined number of patients in the *HOA*, *HOB*, and *HOC* compartments. The most crucial data that all countries are attempting to quantify are the number of patients who will need to be admitted into an intensive care unit. These are the patients who require mechanical ventilators. Not having enough ventilators could be a life or death situation, and a shortage would cause the percentage of deaths among infected patients to increase. The maximum number of patients in intensive care units is estimated to be 16 461, 9645, and 1788 patients for the three different values of R . The peak numbers for the *IC* compartment are predicted to occur on August 27, September 22, and December 22, respectively. These are the number of beds in ICU wards for which the authorities should prepare (Figure 7). Total deaths also vary with R . In the scenario with the high value of $R=1.98$, the death rate is estimated to peak on August 28 at 535 deaths per day. The cumulative number of deaths is estimated to be 29 202 on January 3, 2021 (the predicted end of the pandemic for this scenario). In the scenario with the middle value of $R= 1.62$, the

death rate peaks at 313 deaths per day on September 21, 2020, with 23 973 cumulative deaths on March 6, 2021. In the lowest and most optimistic scenario of $R = 1.20$, the death rate peaks at 58 deaths per day on December 13, 2020, and 11 565 cumulative deaths at the end of the pandemic on September 21, 2021. A summary of these results is shown in Table 3.

4. Discussion

Knowing the reproduction number is crucial to understanding how the epidemic will develop. To halt or slow the spread of disease, R must be reduced. In Kuwait, the health authority is waging a campaign to stop the spread of the disease, but to save more lives, they needed to impose a total lockdown for at least six weeks combined with an engaged trace and locate strategy. When the number of the new cases per day decreases to double digits, and the locations of outbreaks are pinpointed, opening up in stages can begin with partial lockdowns in the areas that are still experiencing outbreaks. Such a strategy should reduce the number of new cases, but it will not eradicate the disease. Therefore, any return to normal activities will revive the spread of the virus. The course of the disease will likely go through waves. Whenever an easing of restrictions occurs, the number of cases will start to increase. Authorities would likely re-introduce restrictive measures to help lower the cases again. This cycle could be repeated until herd immunity is achieved. Achieving herd immunity when $R = 2.18$ means that 54.1% of the population of Kuwait needs to have been infected[20]. This percentage needs to be achieved in the absence of any infected individuals (i.e., $I = 0$). This slow process is estimated to take two years to reach, but in the absence of a vaccine or effective treatments, it would be the best approach to contain the virus and reduce the number of deaths. If no such measures are implemented, then the health authority and government of Kuwait must prepare for the worst-case scenario and increase the capacity of hospital beds to 142 000 and intensive care unit beds to 16 461.

5. Declarations of interest

none

6. Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

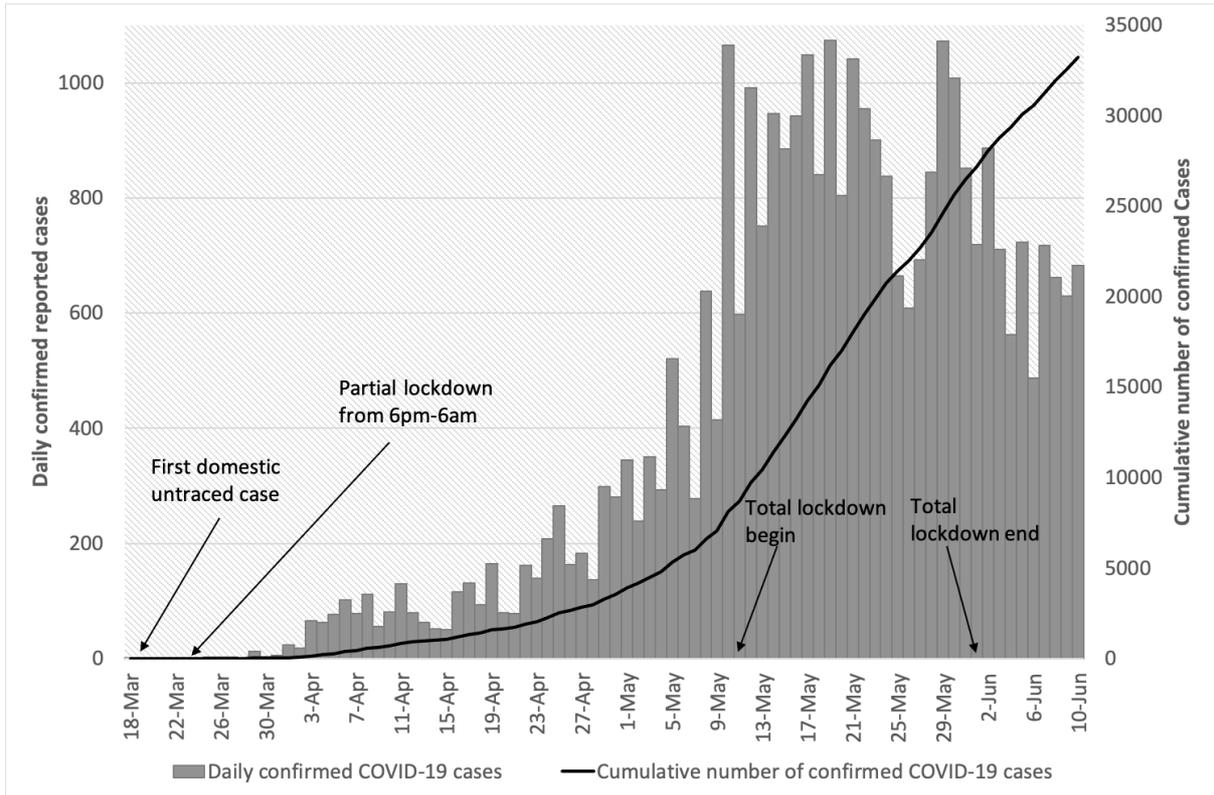


Figure 1: Daily counts and the cumulative number of confirmed COVID-19 cases (March 18–June 10, 2020).

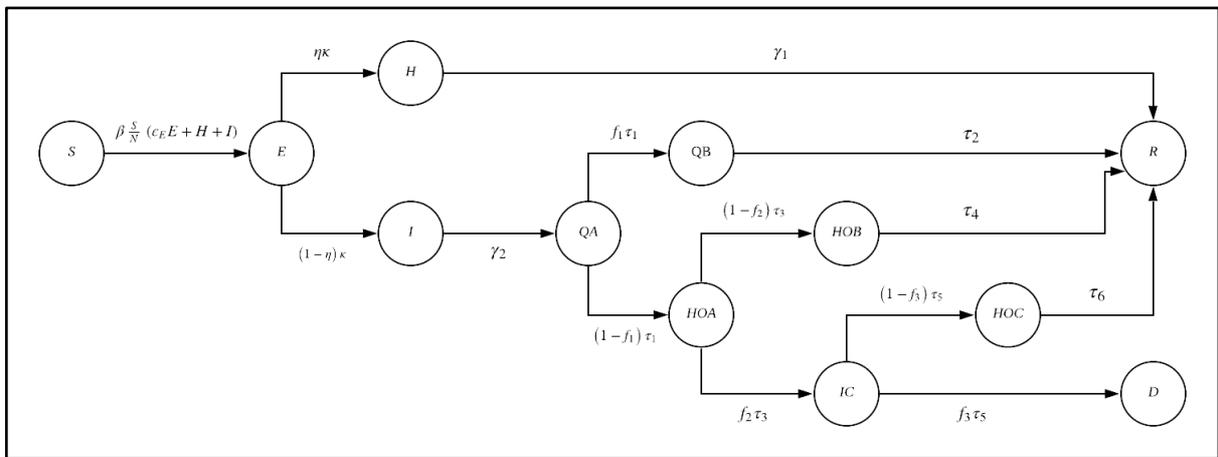


Figure 2: Flow diagram of the compartmental epidemic model for COVID-19 in Kuwait.

S	Susceptible: individuals who can become infected
E	Exposed: individuals who are infected but not yet infectious
H	Hidden: individuals who are infected but have not developed symptoms or been detected by the health authorities
I	Infected: individuals who are infected, are showing symptoms, and have been detected by the health authorities
QA	First-level quarantine: infected individuals under quarantine before showing symptoms
QB	Second-level quarantine: infected individuals who do not require hospitalization.
HOA	First-level hospitalization: infected patients who need medical treatment
HOB	Second-level hospitalization: infected patients who need medical treatment but do not require the intensive care unit
IC	Intensive care: patients who are admitted to the intensive care unit
HOC	Recovery: patients who recover in the hospital after leaving the ICU
D	Death: deceased patients
R	Recovered: recovered patients

Table 1: Definitions of variables used in the model

Notation	Definition	Value	Reference
β	Daily contact rate	Variable; Equations 16 and 17; Figure 3	
ε_E	Adjustment to the E compartment to account for the latent period, which is assumed to be two days before the onset of the symptoms	2/5.1	[9]
$1/\kappa$	Mean incubation period	5.1 days	[21]
η	Percentage of the hidden cases	17.9%–30.8%	[10,11]
$1/\gamma_1$	Mean infectious period of H compartment	9 days	[9]
$1/\gamma_2$	Mean period for detecting I individuals	1 day	Data fit
$1/\tau_1$	Mean period for monitoring patients in the QA compartment	6 days	[22]
f_1	The fraction of the cases in QA that are sent to QB	80%	[13]
$1/\tau_2$	Mean period for patients in QB to recover and become non-infectious	7 days	[22,23]
$1/\tau_3$	Mean period for determining whether a patient in the HOA compartment will be sent to the HOB or IC compartment.	3.5 days	[22]
f_2	The fraction of the patients sent to the IC	20%	[22]
$1/\tau_4$	Mean period of recovery for patients in HOB compartment	11.5 days	[24]
$1/\tau_5$	Mean period that patients remain in the IC compartment	8 days	[25]
$1/\tau_6$	Mean duration of hospital stay after leaving the IC compartment	3.5 days	[26]
f_3	The fraction of patients in the IC compartment who die	26%	[27]

Table 2: Parameters used in the model

Scenario	1	2	3
Reproduction number, R	1.2	1.62	1.98
Total population	4 700 000	4 700 000	4 700 000
Total number of patients that are infected and showing symptoms, I	1 113 779	2 305 961	2 808 481
Total number of recovered patients	1 459 000	3 024 200	3 683 300
Maximum hospital beds	16 412	85 341	141 184
Maximum Intensive care unit beds	1788	9645	16 461
Total deaths	11 565	23 973	29 202
Peak date when maximum hospital beds are required	Dec. 21, 2020	Sep. 20, 2020	Aug. 27, 2020
The end date of the pandemic	Sep. 21, 2021	Mar. 6, 2021	Jan. 3, 2021

Table 3: Summary of results for each of the three scenarios tested

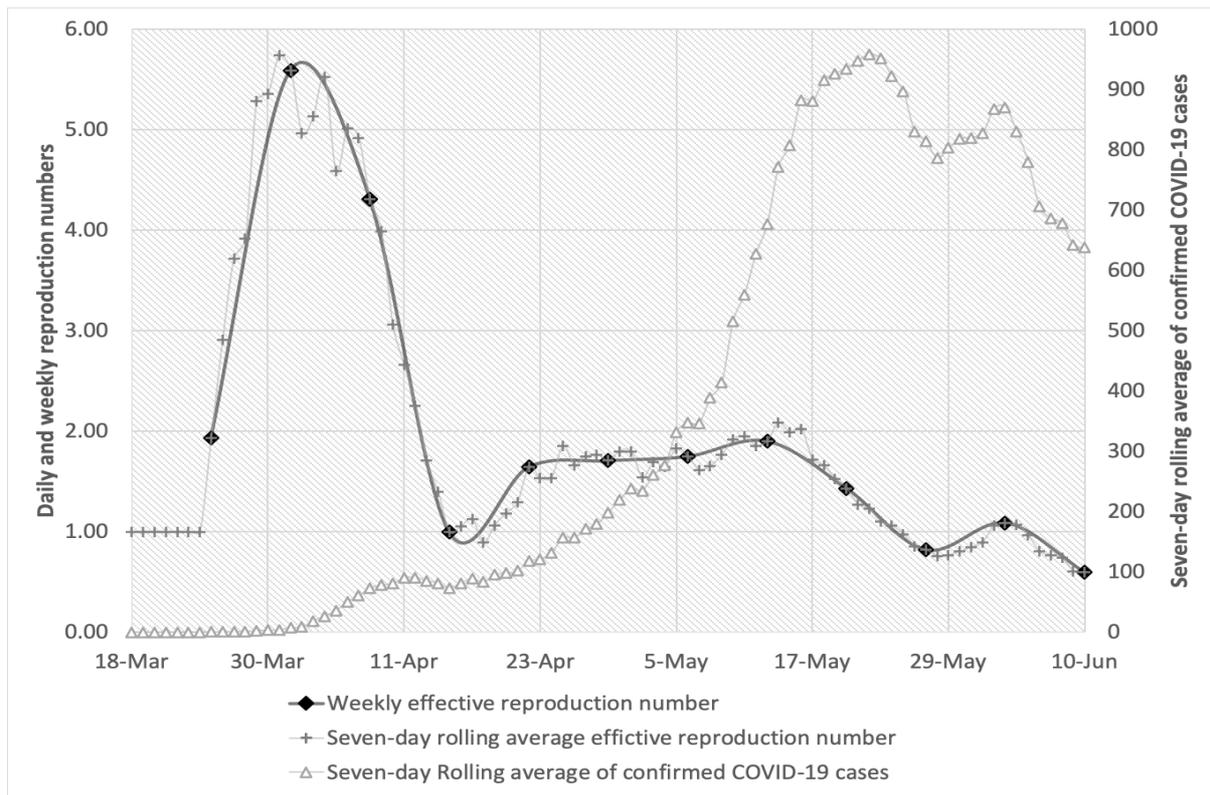


Figure 3: Seven-day rolling average and weekly effective reproduction numbers (March 18–June 10, 2020).

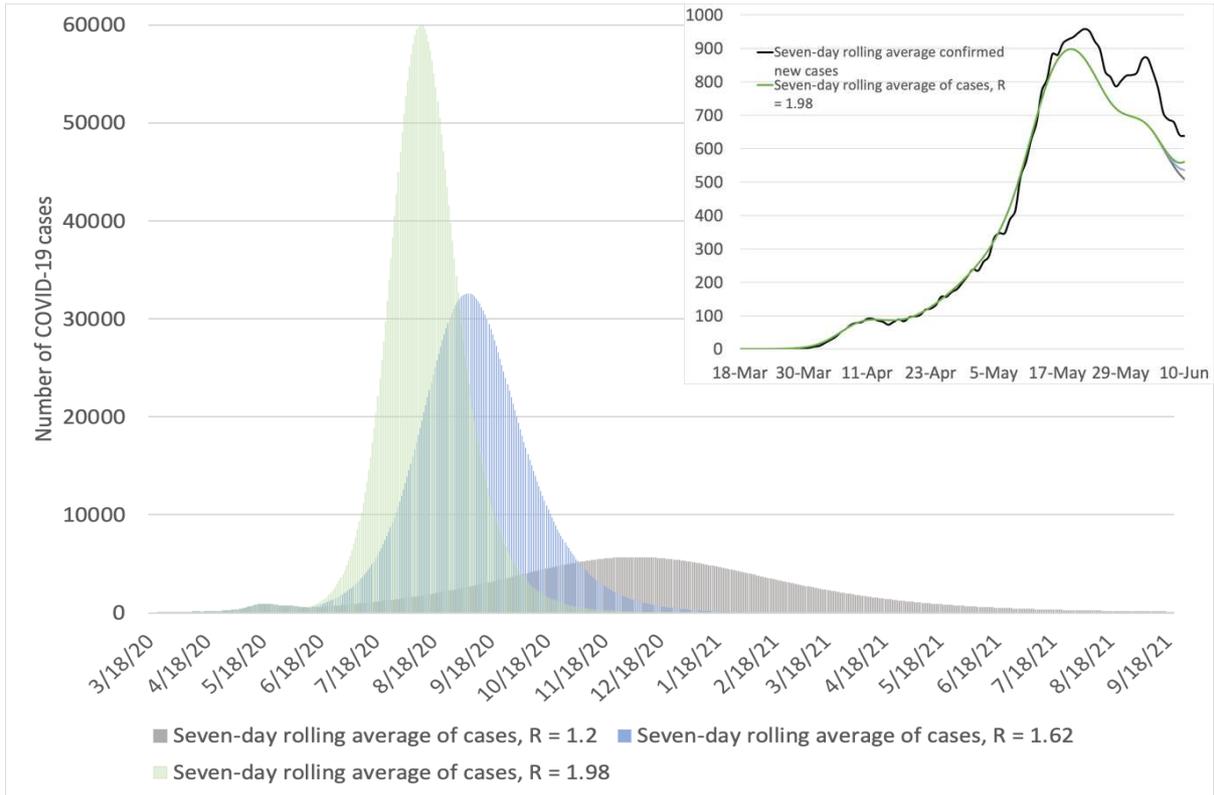


Figure 5: Figure 6: Seven-day rolling average of COVID-19 cases estimated by the model for the three scenarios. Inset: Seven-day rolling average of the model prediction versus actual confirmed COVID-19 cases (March 18–June 10, 2020).

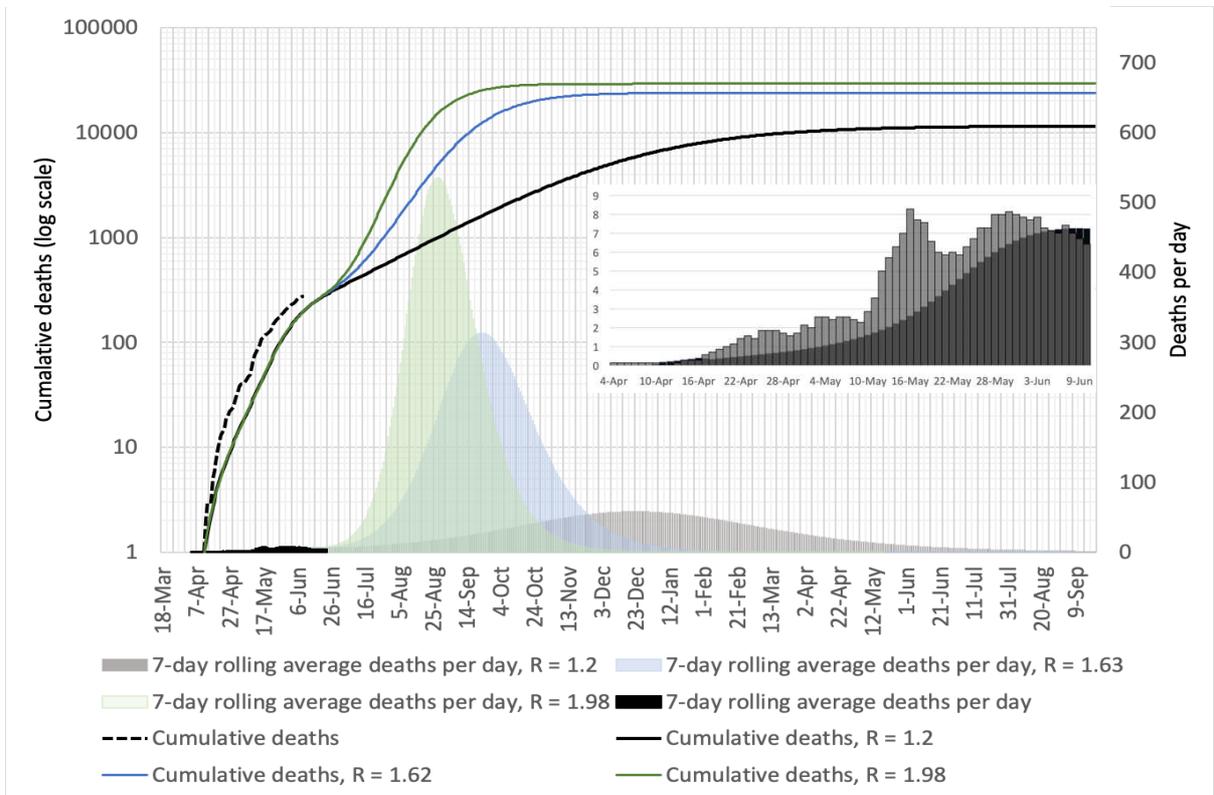


Figure 4: Seven-day rolling average of deaths per day and the cumulative number of deaths estimated by the model for the three scenarios. The dotted line is the total actual confirmed deaths. Inset: Comparison of seven-day rolling average of deaths per day between actual data and the model prediction (April 4–June 10, 2020). The first death occurred on April 4, 2020.

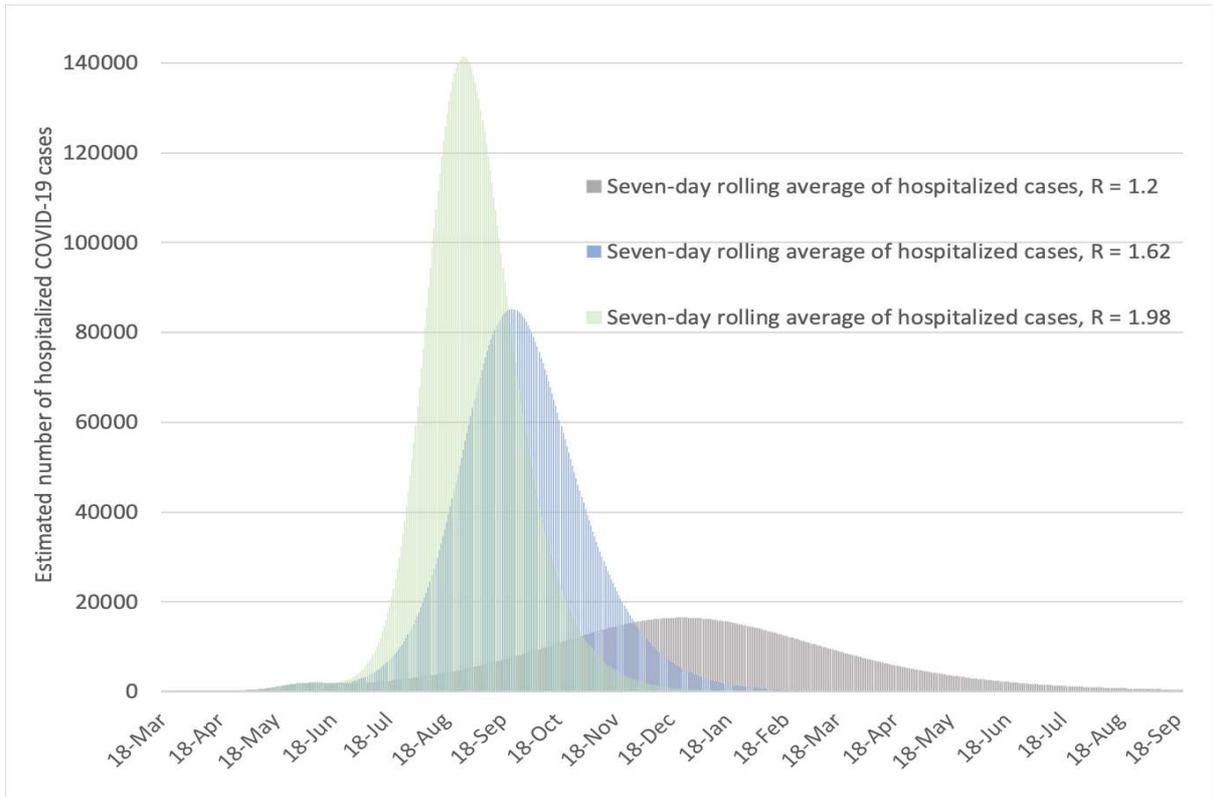


Figure 7: Estimated number of patients who require hospitalization due to COVID-19.

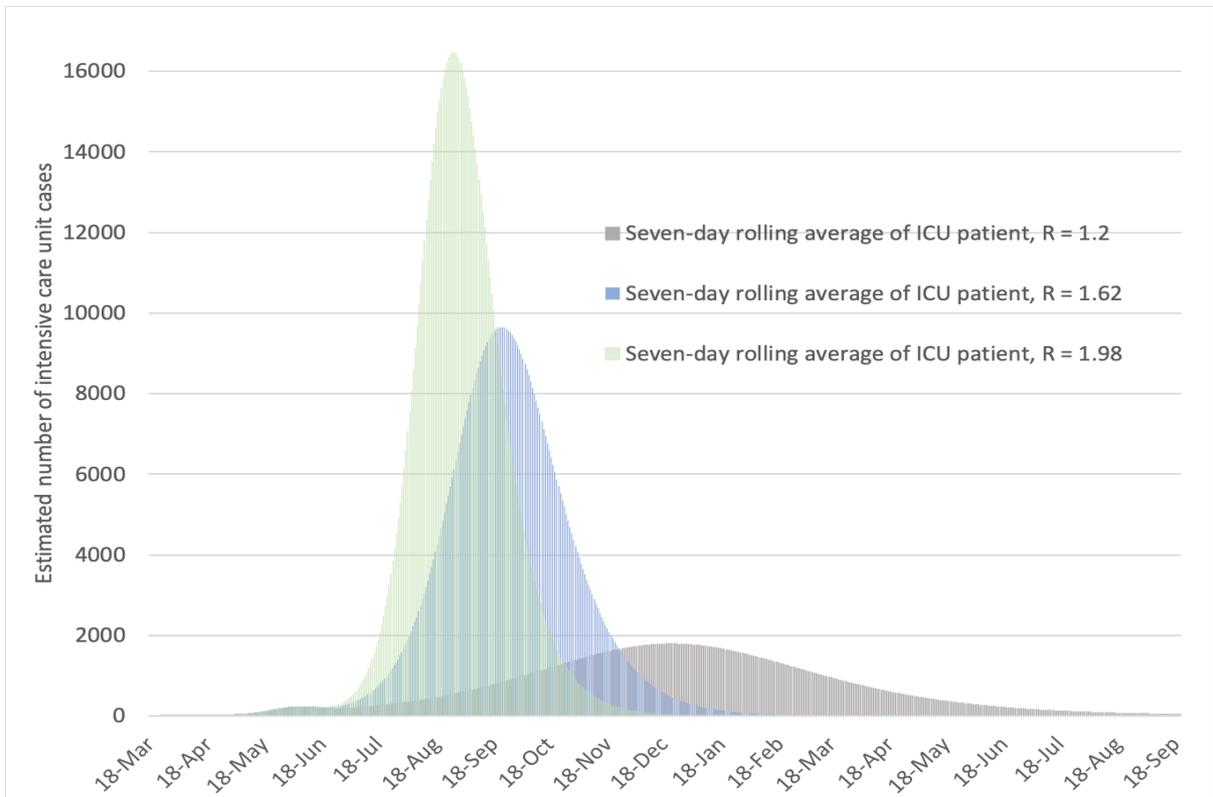


Figure 8: Estimated number of intensive care unit beds needed to treat patients who require mechanical ventilators.

Appendix [15–18]

The disease free equilibrium, DFE:

$$(S^*, E^*, H^*, I^*, QA^*, QB^*, HOA^*, HOB^*, IC^*, HOC^*, D^*, R^*) = (N, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \quad (1A)$$

The infectious compartment are

$$\frac{dE}{dt} = \frac{S}{N} \beta (\epsilon_E E + H + I) - \kappa E, \quad (2A)$$

$$\frac{dH}{dt} = \eta \kappa E - \gamma_1 H, \quad (3A)$$

$$\frac{dI}{dt} = (1 - \eta) \kappa E - \gamma_2 I. \quad (4A)$$

Write the subset Equations (2A-4A) in the form of

$$\frac{dx}{dt} = \mathcal{F}(x) - \mathcal{V}(x). \quad (5A)$$

The above results in

$$\begin{bmatrix} \frac{dE}{dt} \\ \frac{dH}{dt} \\ \frac{dI}{dt} \end{bmatrix} = \begin{bmatrix} \frac{S}{N} \beta (\epsilon_E E + H + I) \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \kappa E \\ \gamma_1 H - \eta \kappa E \\ \gamma_2 I - (1 - \eta) \kappa E \end{bmatrix}. \quad (6A)$$

Finding the Jacobian matrix of the model

$$J = \begin{bmatrix} \frac{\partial E}{\partial E} & \frac{\partial E}{\partial H} & \frac{\partial E}{\partial I} \\ \frac{\partial H}{\partial E} & \frac{\partial H}{\partial H} & \frac{\partial H}{\partial I} \\ \frac{\partial I}{\partial E} & \frac{\partial I}{\partial H} & \frac{\partial I}{\partial I} \end{bmatrix} = \begin{bmatrix} \frac{S}{N} \beta \epsilon_E - \kappa & \frac{S}{N} \beta & \frac{S}{N} \beta \\ \eta \kappa & -\gamma_1 & 0 \\ (1 - \eta) \kappa & 0 & -\gamma_2 \end{bmatrix} \quad (7A)$$

Linearized the Jacobian matrix around the DFE,

$$J^* = \begin{bmatrix} \beta \epsilon_E - \kappa & \beta & \beta \\ \eta \kappa & -\gamma_1 & 0 \\ (1 - \eta) \kappa & 0 & -\gamma_2 \end{bmatrix}_{DFE}. \quad (8A)$$

Factor the linear Jacobian matrix, J^* into

$$J^* = F - V \quad (9A)$$

Where

$$F = \frac{\partial \mathcal{F}_i(\mathbf{x})}{\partial x_j} \quad (10A)$$

and

$$V = \frac{\partial \mathcal{V}_i(\mathbf{x})}{\partial x_j}. \quad (11A)$$

Arrange the Equations in the form of

$$\frac{d\mathbf{x}}{dt} = (F - V)\mathbf{x} \quad (12A)$$

To yield,

$$\begin{bmatrix} \frac{dE}{dt} \\ \frac{dH}{dt} \\ \frac{dI}{dt} \end{bmatrix} = \left(\begin{bmatrix} \beta\epsilon_E & \beta & \beta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} \kappa & 0 & 0 \\ -\eta\kappa & \gamma_1 & 0 \\ -(1-\eta)\kappa & 0 & \gamma_2 \end{bmatrix} \right) \begin{bmatrix} E \\ H \\ I \end{bmatrix}. \quad (13A)$$

The next generation matrix, NGM

$$FV^{-1} = \begin{bmatrix} \beta\epsilon_E & \beta & \beta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \kappa & 0 & 0 \\ -\eta\kappa & \gamma_1 & 0 \\ -(1-\eta)\kappa & 0 & \gamma_2 \end{bmatrix}^{-1} = \begin{bmatrix} \beta \left(\frac{\epsilon_E}{\kappa} + \frac{\eta}{\gamma_1} + \frac{(1-\eta)}{\gamma_2} \right) & \frac{\beta}{\gamma_1} & \frac{\beta}{\gamma_2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (14A)$$

The basic reproduction number, R_0 , is spectral radius of the NGM,

$$R_0 = \rho(FV^{-1}). \quad (15A)$$

Then,

$$R_0 = \beta \left(\frac{\epsilon_E}{\kappa} + \frac{\eta}{\gamma_1} + \frac{(1-\eta)}{\gamma_2} \right). \quad (16A)$$

The leading eigenvalue equation for the linear Jacobean matrix Equation (8A),

$$\beta\kappa(1-\eta)(\gamma_1 + \lambda) - (\gamma_2 + \lambda)(\kappa\gamma_1 - \beta\kappa\eta - \beta\gamma_1\epsilon_E + \kappa\lambda + \gamma_1\lambda - \beta\epsilon_E\lambda + \lambda^2) = 0. \quad (17A)$$

λ in Equation (17A) is the intrinsic growth rate of the epidemic.

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Figures

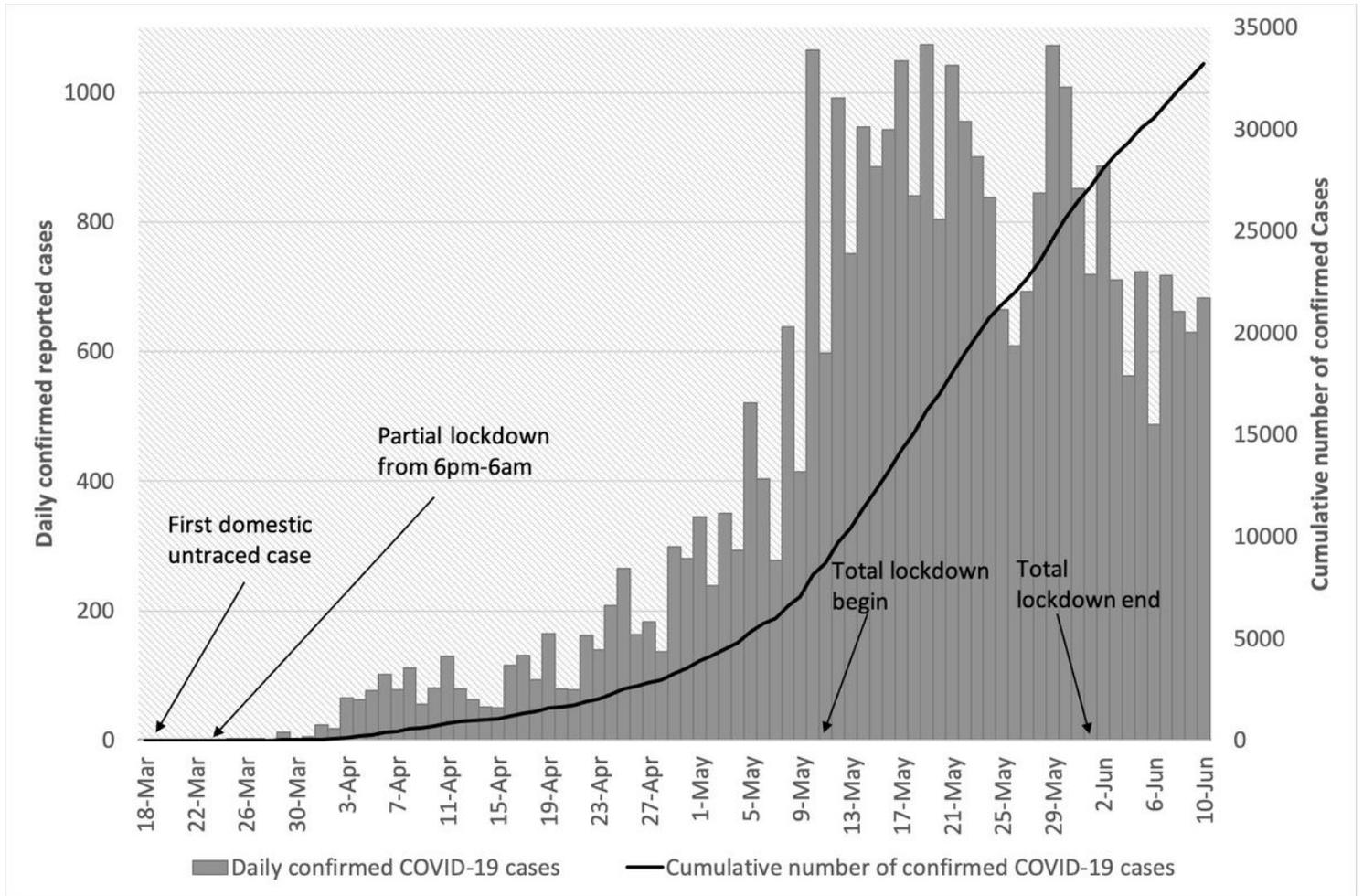


Figure 1

Daily counts and the cumulative number of confirmed COVID-19 cases (March 18–June 10, 2020).

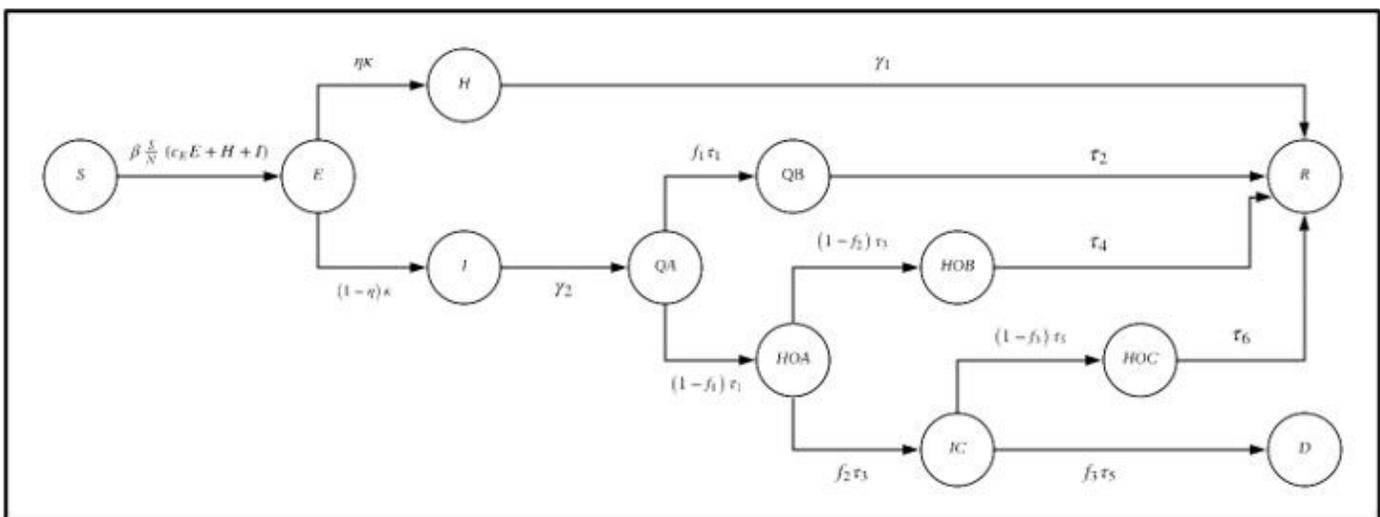


Figure 2

Flow diagram of the compartmental epidemic model for COVID-19 in Kuwait.

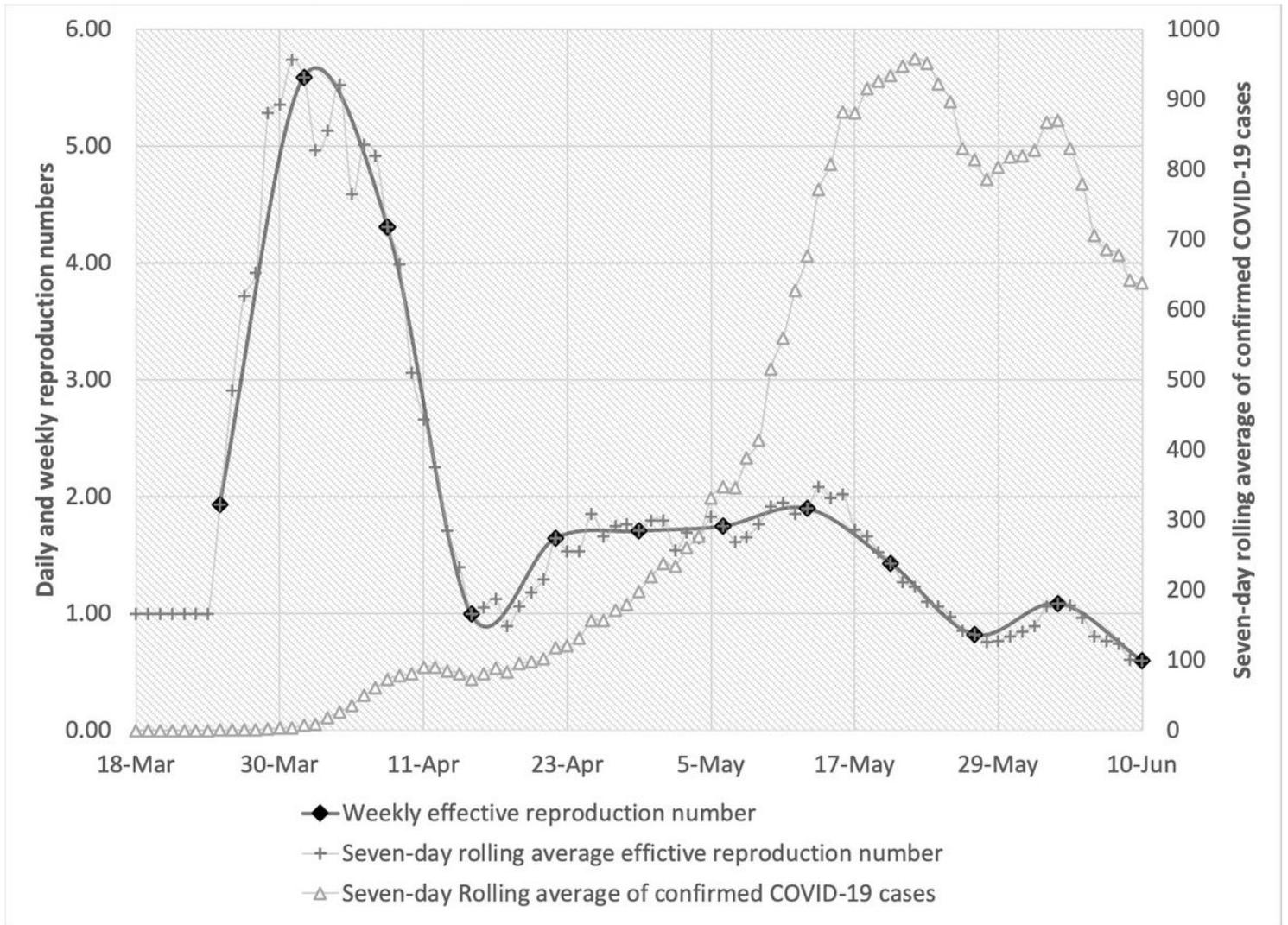


Figure 3

Seven-day rolling average and weekly effective reproduction numbers (March 18–June 10, 2020).

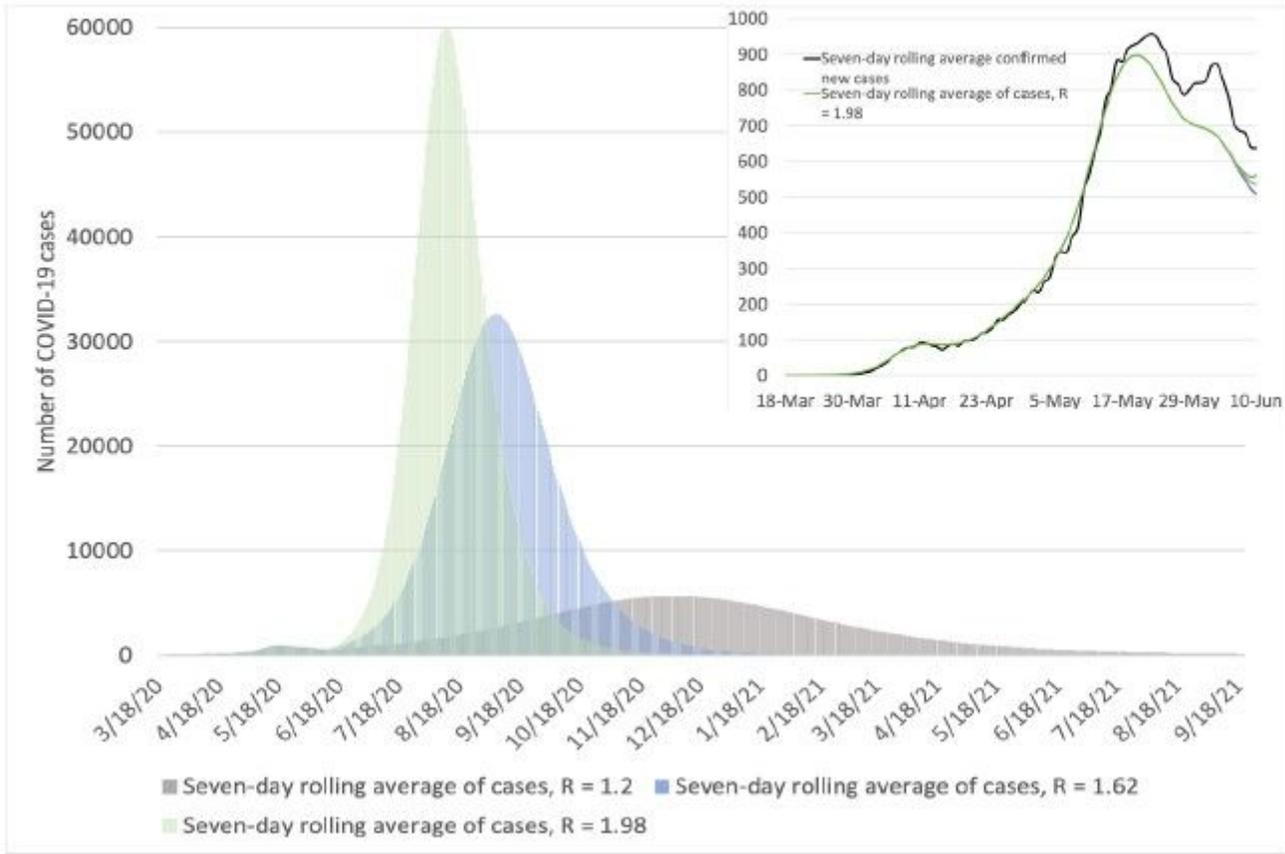


Figure 4

Seven-day rolling average of COVID-19 cases estimated by the model for the three scenarios. Inset: Seven-day rolling average of the model prediction versus actual confirmed COVID-19 cases (March 18–June 10, 2020).

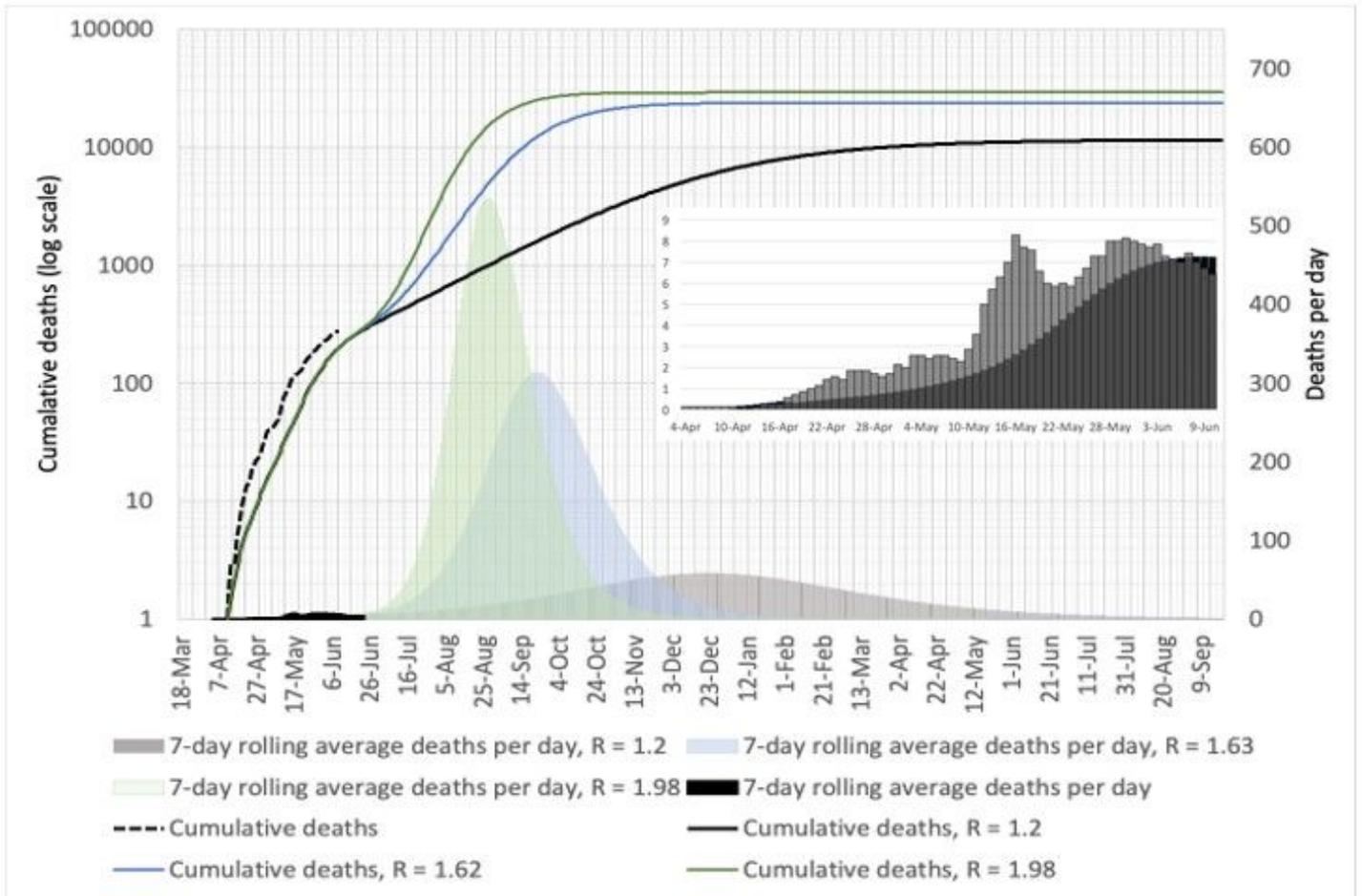


Figure 5

Seven-day rolling average of deaths per day and the cumulative number of deaths estimated by the model for the three scenarios. The dotted line is the total actual confirmed deaths. Inset: Comparison of seven-day rolling average of deaths per day between actual data and the model prediction (April 4– June 10, 2020). The first death occurred on April 4, 2020.

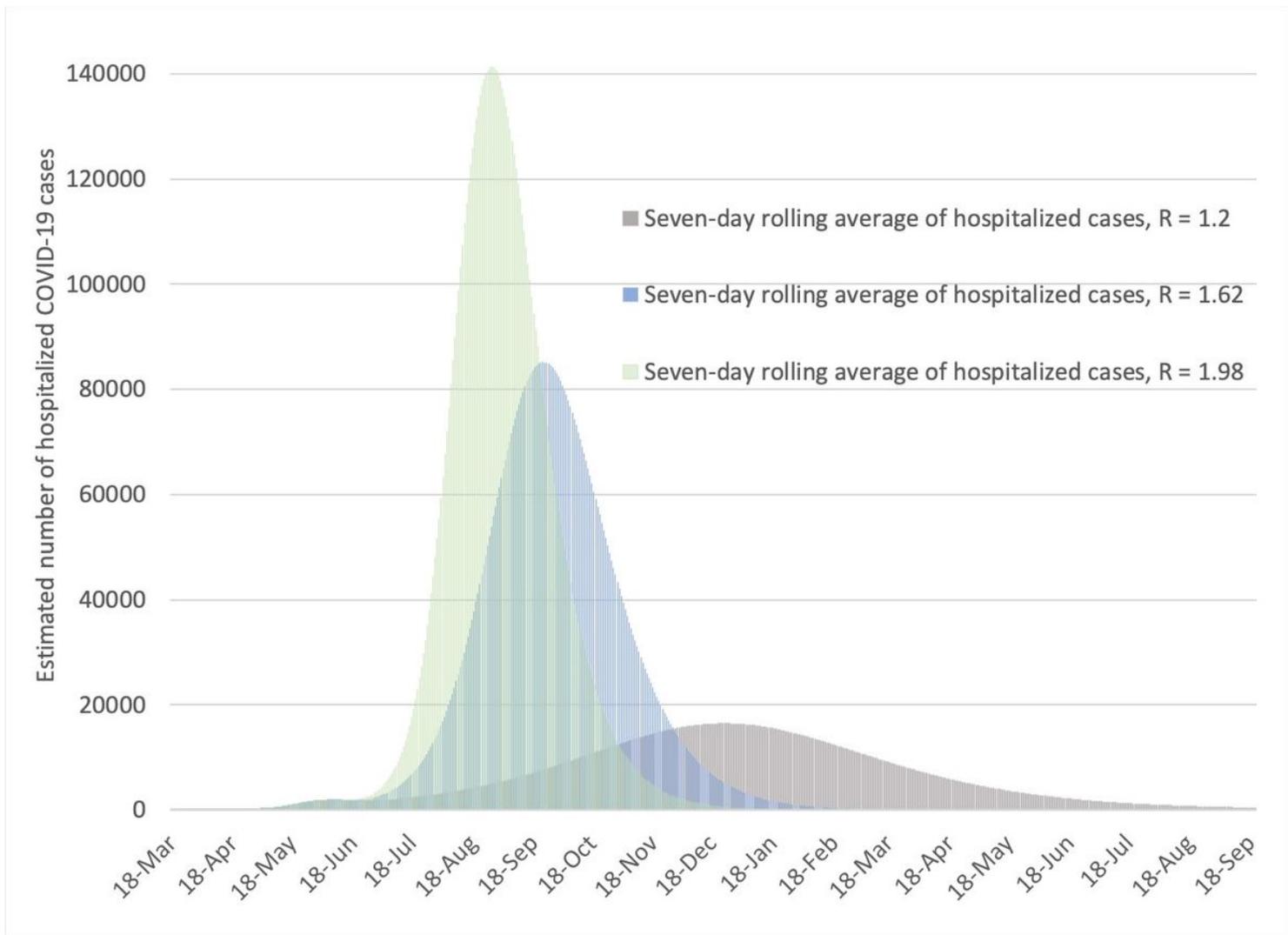


Figure 6

Estimated number of patients who require hospitalization due to COVID-19.

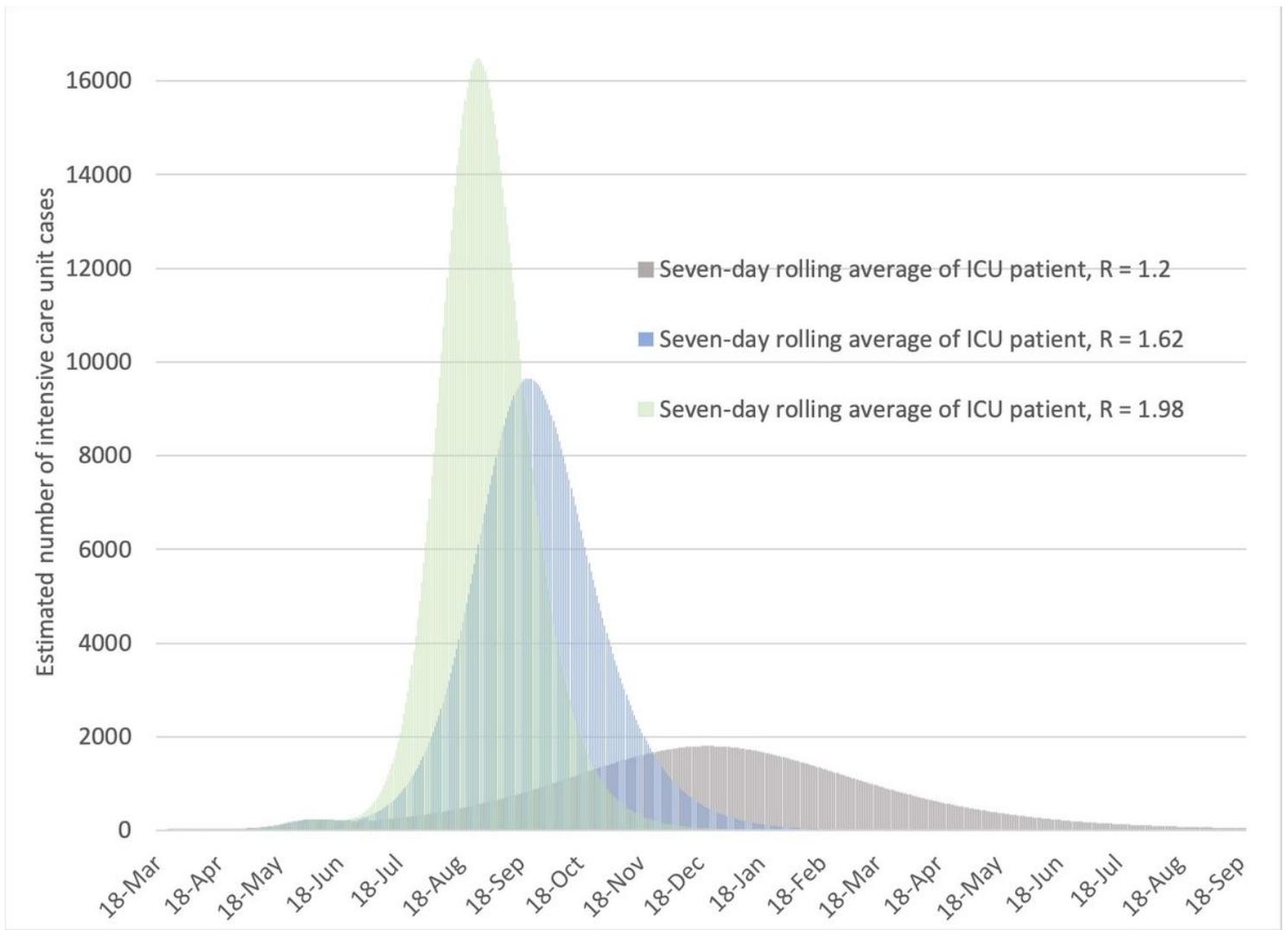


Figure 7

Estimated number of intensive care unit beds needed to treat patients who require mechanical ventilators.