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Bistable and coexisting attractors in current modulated edge emitting semiconductor laser: Control and microcontroller-based design

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Abstract

This paper reports on the numerical analysis, control of coexisting attractors and microcontroller-based design of current modulated edge emitting semiconductor laser (CMEESL). The stability of equilibrium points of solitary edge emitting semiconductor laser found is investigated. By varying the amplitude of modulation current density, CMEESL displays periodic behaviors, period-doubling to chaotic behavior, bistability and coexistence between limit cycle and chaotic attractors. The coexistence between chaotic and limit cycle attractors is destroyed and controlled to a desired monostable trajectory by means of the linear augmentation method. In addition, a microcontroller-based circuit is also designed to indicate that CMEESL can be used in real applications. Microcontroller-based circuit outputs and numerical analysis results confirm each other.

Keywords: Edge emitting semiconductor laser, modulation current density, bistable periodic attractors, coexisting attractors, linear augmentation method, microcontroller-based design.

I. Introduction

Bistability is the coexistence of twin identical dynamical behaviors for example twin periodic attractors or chaotic attractors for fixed set of system parameters and by varying only the initial conditions. The bistability phenomenon has been found in many dynamical systems such as semiconductor lasers, linearly forced isotropic turbulence, financial systems and many other chaotic and hyperchaotic oscillators with some special features such as line equilibrium, unstable node, quintic nonlinearity [1-6]. Lee et *al.* theoretically analyzed the dynamical behaviors in injection-locked semiconductor lasers and identified a period-doubling bifurcation route to chaos and bistability [1]. The first experimental observation of monostable, bistable and multistable states in a single-longitudinal mode semiconductor ring laser has been reported in [2]. Heinrich et *al.* experimentally and theoretically studied a two-color Fabry-Pérot laser subject to dual injection and demonstrated a bistability between locked states [3]. Bistability in a single-mode distributed feedback semiconductor laser with positive optoelectronic feedback has been observed experimentally in [4]. The bistability phenomenon can be useful in several areas such as communications, economic dominance in financial markets, mechanical, electrical and all-optical systems [7-9].

Multistability known as coexistence of more than one stable attractor for fixed set of system parameters and by varying only the initial conditions has been found in various areas of physics, chemistry, biology, economy and in nature [10, 11]. Multistability lead to the randomness features of a dynamical system and therefore it can be useful for limited engineering applications as image processing and random bits generation [12-14]. Usually, multistability gives rise to inconveniences in dynamical systems which abet to reduce considerably the performances of such dynamical systems. In a laser system with intracavity second harmonic generation, Baer demonstrated that the multistability creates the green problem [15]. Many other examples explaining the inconveniences of multistability in dynamical systems have been reported in [11]. This study examines both analytically and numerically a CMEESL. It allows us not only to identify bistable and coexisting attractors but also to control the coexisting attractors by using the linear augmentation method.

Laser dynamical systems can show different chaotic dynamics such as chaotic [16, 17], fractional-order chaotic [18] and multi-scroll [19]. Such chaotic laser systems can be used in different

engineering design applications. Chaotic laser systems have been mainly used in communication [20-22], encryption [18] and random number generation [23] applications. These applications are generally studied as simulations. For use in real applications, the hardware implementation of chaotic systems must also be designed. For this purpose, the design of laser system made with microcontroller and field programmable gate array (FPGA) is rarely encountered in the literature [24, 25]. Microcontroller-based design is much low-cost and more practical than FPGA design. In this study, a microcontroller-based circuit is also designed to indicate that CMEESL systems can be used in real applications.

The rest of the study is organized as follows. The next section presents the numerical analysis and control of coexisting attractors in CMEESL. Section 3 deals with the microcontroller implementation of CMEESL and the control CMEESL. Section 4 concludes this study.

II. Bistability, coexisting attractors and its control in CMEESL

The rate equations describing CMEESL are given by the following dimensionless form [26]:

$$\frac{dP}{dt} = [(1+2n)(1-\sigma P)-1]P, \quad (1a)$$

$$\frac{dn}{dt} = \varepsilon_0 \left\{ i_{dc} \left[1 + m \sin(2\pi f_m \tau_p t) \right] - n - (1+2n)(1-\sigma P)P \right\}, \quad (1b)$$

where $\sigma = 2s/(g\tau_s)$, $\varepsilon_0 = \tau_p/\tau_s$, $I_{th} = [1/(g\tau_p) + N_0]/\tau_s$, $i_{dc} = (I_{dc} - I_{th})/I_{th}$ and $m = I_m/(I_{dc} - I_{th})$.

The parameters t , P , n , s , g , N_0 , I_{dc} , I_m , I_{th} , f_m , τ_s , τ_p are the time, photon density, carrier density, nonlinear gain suppression factor gain coefficient, carrier density required for transparency, dc bias current density, amplitude of modulation current density, threshold current density, modulation frequency, carrier and photon lifetimes, respectively. The values of the parameters used in this paper are the same as in [26]:

$g = 8.4 \times 10^{-13} m^3 s^{-1}$, $s = 5.0 \times 10^{-27} m^3$, $N_0 = 1.435 \times 10^{24} m^{-3}$, $\tau_s = 1.025 ns$, $\tau_p = 2.041 ps$ and

$I_{th} = 1.969 \times 10^{33} m^{-3} s^{-1}$. The equilibrium points of the solitary edge emitting semiconductor laser

can be derived by setting $\frac{dP}{dt} = \frac{dn}{dt} = 0$ with $m = 0$. Then by solving the obtained system of equations, the equilibrium points are given by:

$$P^* = 0 \text{ or } P^* = i_{dc} - n^*, \quad (2a)$$

$$n^* = i_{dc} \text{ or } 2\sigma(n^*)^2 + (2 + \sigma - 2\sigma i_{dc})(n^*) - \sigma i_{dc} = 0. \quad (2b)$$

Since the discriminant $\Delta = (2 + \sigma - 2\sigma i_{dc})^2 + 8\sigma^2 i_{dc} > 0$ of Eq. (2b) is positive, Eq. (2b) has two roots: $n_1^* = -[(2 + \sigma - 2\sigma i_{dc}) + \sqrt{\Delta}]/(2\sigma)$ and $n_2^* = [-(2 + \sigma - 2\sigma i_{dc}) + \sqrt{\Delta}]/(2\sigma)$. Therefore, solitary edge emitting semiconductor laser can have two or three equilibrium points if $P^* > 0$. The characteristic equation of CMEESL evaluated at the equilibrium point $E = (P^*, n^*)$ is:

$$\lambda^2 + \left[-2\varepsilon_0\sigma(P^*)^2 + 4\sigma P^* n^* + 2\varepsilon_0 P^* + 2\sigma P^* + \varepsilon_0 - 2n^* \right] \lambda + \left[\begin{array}{l} -2\varepsilon_0\sigma(P^*)^2 + 4\varepsilon_0\sigma P^* n^* + \\ 2\varepsilon_0\sigma P^* + 2\varepsilon_0 P^* - 2\varepsilon_0 n^* \end{array} \right] = 0. \quad (3)$$

According to the Routh-Hurwitz criterion, the real parts of all the roots λ of Eq. (3) are negative if and only if

$$-2\varepsilon_0\sigma(P^*)^2 + 4\sigma P^* n^* + 2\varepsilon_0 P^* + 2\sigma P^* + \varepsilon_0 - 2n^* > 0, \quad (4a)$$

$$-2\varepsilon_0\sigma(P^*)^2 + 4\varepsilon_0\sigma P^* n^* + 2\varepsilon_0\sigma P^* + 2\varepsilon_0 P^* - 2\varepsilon_0 n^* > 0. \quad (4b)$$

The stability analysis of equilibrium points $E = (P^*, n^*)$ as function of the parameter i_{dc} is depicted in Fig. 1.

Figure 1: Stability diagram of equilibrium points $E = (P^*, n^*)$ versus the parameter i_{dc} . Black lines indicate stable branches and grey lines represent unstable branches.

Figure 1 reveals that by varying parameter i_{dc} from 0 to 3, the solitary edge emitting semiconductor laser has one stable equilibrium point and two unstable equilibrium points. For $i_{dc} = 0.1$, the different dynamical behaviors of CMEESL versus the modulation parameters m and f_m is presented in Fig. 2.

Figure 2: Two-parameters $(m, f_m [GHz])$ depicting various dynamical behaviors of CMEESL for $i_{dc} = 0.1$.

In Figure 2, Periodic behaviors are in black color and chaotic behaviors are in cyan color. So, CMEESL can display periodic and chaotic behaviors. By fixing the parameter $f_m = 600 MHz$, the bifurcation diagrams of photon density P and corresponding largest Lyapunov (LLE) versus the parameter m are plotted in Fig. 3.

Figure 3: Bifurcation diagrams depicting local maxima of photon density P (a) and corresponding LLE (b) as function of the parameter m for $i_{dc} = 0.1$ and $f_m = 600 \text{ MHz}$. Bifurcation diagrams are obtained by scanning the parameter m upwards (black shaded areas) and downwards (red shaded areas).

Figure 3 reveals that CMEESL can display not only periodic behaviors, bistable periodic behaviors and coexisting behaviors but also period-doubling to chaotic behaviors. The LLE of Fig. 3 (b) confirms the results found in Fig. 3 (a). Figure 4 shows the phase portrait in the plane (n, P) which illustrates bistable and coexisting attractors for specific values of parameter m and initial conditions.

Figure 4: Bistable and coexisting attractors for specific values of parameter m and initial conditions: (a) $m = 0.8255$ and (b) $m = 1.2$. The curves in left panel are obtained by using the initial conditions $(P(0), n(0)) = (0.1, 0.001)$ whereas curves in right panel are obtained by using the initial $(P(0), n(0)) = (3.1, 0.001)$. The remaining parameters are $i_{dc} = 0.1$ and $f_m = 600 \text{ MHz}$.

For $i_m = 0.8255$, system (1) exhibits bistable (asymmetric) period-2-oscillations for two different initial conditions as shown in left and right panels of Fig. 4(a). When $m = 1.2$, system (1) displays limit cycle and chaotic attractors for two different initial conditions as seen in left and right panels of Fig. 4(b). The basin of attraction of system (1) in the plane (n, P) is presented in Fig. 5 for $m = 1.2$.

Figure 5: Cross section of the basin of attraction of CMEESL in the plane $(n(0), P(0))$ for $m=1.2$, $i_{dc}=0.1$ and $f_m=600\text{MHz}$.

From Fig. 5, it is noticed that CMEESL can display chaotic (cyan color) or periodic behaviors (black color) depending on the initial conditions.

Due to the inconveniences of coexisting attractors in most nonlinear systems [11] and in lasers as well, the control of coexisting attractors found in CMEESL is investigated by using a linear augmentation control scheme [27-30]. To destroy the coexistence between limit cycle and chaotic attractors, system (1) is coupled with a linear system u as follows:

$$\frac{dP}{dt} = [(1+2n)(1-\sigma P)-1]P, \quad (2a)$$

$$\frac{dn}{dt} = \varepsilon_0 \{ i_{dc} [1+m \sin(\omega t)] - n - (1+2n)(1-\sigma P)P \} + \alpha u, \quad (2b)$$

$$\frac{du}{dt} = -ku - \alpha(n-\gamma), \quad (2c)$$

where α is the coupling strength, γ is the control parameter which serves to locate the position of equilibrium point and k is the decay parameter of the linear system u . The bifurcation diagram depicting local extrema of controlled system (2) and the LLE as function of coupling strength α are shown in Fig. 6.

Figure 6: Bifurcation diagrams depicting the local maxima (black shaded areas) and local minima (grey shaded areas) of P (a) and corresponding LLE (b) versus the coupling strength α for $k = 0.7$, $\gamma = 0.5$, $m = 1.2$, $f_m = 600 \text{ MHz}$ and $i_{dc} = 0.1$. Bifurcation diagrams obtained by scanning the coupling strength α upwards and downwards are presented in (a1) and (a2), respectively.

By increasing the coupling strength α , the bifurcation diagram of controlled system (2) in Figs. 6 (a1) and (a2) displays coexistence between chaotic and limit cycle attractors up to $\alpha \approx 0.0045$ where a monostable chaotic attractor occurs followed by the coexistence between period-2- and period-3- attractors. By further increasing the coupling strength α , the controlled system (2) exhibits monostable period-2-attractor and limit cycle attractor, respectively. The LLE of Fig. 6 (b) confirms the dynamical behaviors found in Figs. 6 (a1) and (a2). Therefore, the controlled system (2) transforms the multistable attractors to desired monostable attractors. In order to illustrate the

effects of the control of coexisting attractors, the phase portraits of controlled system (2) for specific value of the coupling strength α are plotted in Fig. 7.

Figure 7: Phase portraits of system (2) in the plane (n, P) for specific values of the coupling strength α : (a) $\alpha = 0.003$, (b) $\alpha = 0.0054$, (c) $\alpha = 0.009$ and (d) $\alpha = 0.04$. The others parameters are $k = 0.7$, $\gamma = 0.5$, $m = 1.2$, $f_m = 600 \text{ MHz}$ and $i_{dc} = 0.1$. The curves in black line are obtained by using the initial conditions $(P(0), n(0), u(0)) = (0.1, 0.001, 0.0)$ whereas curves in red line are obtained by using the initial conditions $(P(0), n(0), u(0)) = (3.1, 0.001, 0.0)$.

For $\alpha = 0.003$, coexistence between chaotic and limit cycle attractors for two different initial conditions is presented in Fig. 7 (a). By increasing the coupling strength to $\alpha = 0.0054$, the controlled system (2) displays monostable chaotic attractor as shown in Fig. 7 (b). Coexistence between period-2- and period-3-attractors is depicted in Fig. 7 (c) for $\alpha = 0.009$. By further

increasing the coupling strength α , the controlled system (2) exhibits monostable limit cycle attractor as shown in Fig. 7 (d).

III- Microcontroller-based design of the CMEESL

In this section, a microcontroller-based hardware design circuit of the CMEESL systems of which numerical simulations is given is realized. The designed circuit is given in Fig. 8.

Figure 8: Microcontroller-based circuit design of CMEESL systems.

Reduced instruction set computer based ATmega328p microcontroller which have 32 KB program memory, 2 KB data memory and an UART (Universal Asynchronous Receiver Transmitter) unit is used in the circuit of Fig. 8. In this circuit, 4-way dip switch is used to obtain all the states of the CMEESL systems (1) and (2). The dip-switch values from which different states of the systems can be selected as given in Table 1. Thus, the user can obtain the outputs of the desired state of the CMEESL systems on the same circuit.

Table 1: Dip-switch values for system and state selection.

Dip-switch values (1 - On, 0 - Off)	CMEESL system	State of the systems
0000	System (1)	Bistable (asymmetric) period 2 ($m = 0.8255$, $p(0) = 0.1$, $n(0) = 0.001$)
0001	System (1)	Bistable (asymmetric) period 2 ($m = 0.8255$, $p(0) = 0.1$, $n(0) = 0.001$)
0010	System (1)	Limit cycle attractor ($m = 1.2$, $p(0) = 3.1$, $n(0) = 0.001$)
0011	System (1)	Chaotic attractor ($m = 1.2$, $p(0) = 0.1$, $n(0) = 0.001$)
1000	System (2)	Chaotic attractor ($\alpha = 0.003$, $p(0) = 0.1$, $n(0) = 0.001$, $u(0) = 0.0$)
1001	System (2)	Limit cycle attractor ($\alpha = 0.003$, $p(0) = 3.1$, $n(0) = 0.001$, $u(0) = 0.0$)
1010	System (2)	Monostable chaotic attractor ($\alpha = 0.0054$, $p(0) = 0.1$, $n(0) = 0.001$, $u(0) = 0.0$)
1011	System (2)	Period-2 attractor ($\alpha = 0.009$, $p(0) = 0.1$, $n(0) = 0.001$, $u(0) = 0.0$)
1100	System (2)	Period-3 attractor ($\alpha = 0.003$, $p(0) = 3.1$, $n(0) = 0.001$, $u(0) = 0.0$)
1101	System (2)	Monostable limit cycle attractor ($\alpha = 0.004$, $p(0) = 0.1$, $n(0) = 0.001$, $u(0) = 0.0$)

The microcontroller outputs obtained for the selected system and state are transmitted to the computer via the serial communication unit (UART). In order for the system to produce output for the desired CMEESL system and status, the specified dip-switch value is selected and then the reset button is pressed and released. In this way, data is transferred to the computer (PC) via serial communication as shown in Fig. 9.

Figure 9: Microcontroller-based circuit design data outputs for (a) system (1) and (b) system (2).

The phase portraits extracted from the data obtained from the microcontroller-based circuit for systems (1) and (2) are given in Figs. 10 and 11, respectively.

Figure 10: Phase portraits of CMEESL obtained from microcontroller-based circuit outputs (a) Bistable (asymmetric) period-2 for $m = 0.8255$ and $(p(0) = 0.1, n(0) = 0.001)$, (b) Bistable (asymmetric) period-2 for $m = 0.8255$ and $(p(0) = 3.1, n(0) = 0.001)$, (c) Chaotic attractor for $m = 1.2$ and $(p(0) = 0.1, n(0) = 0.001)$ and (d) limit cycle for $m = 1.2$ and $(p(0) = 3.1, n(0) = 0.001)$.

Figure 11: Phase portraits of system (2) obtained from microcontroller-based circuit outputs: (a) Chaotic attractor (blue color) for $\alpha = 0.003$, and $(p(0) = 0.1, n(0) = 0.001, u(0) = 0.0)$, limit cycle (red color) for $\alpha = 0.003$ and $(p(0) = 3.1, n(0) = 0.001, u(0) = 0.0)$, (b) monostable chaotic attractor for $\alpha = 0.0054$ and $(p(0) = 0.1, n(0) = 0.001, u(0) = 0.0)$, (c) period-2 attractor (blue color) for $\alpha = 0.009$ and $(p(0) = 0.1, n(0) = 0.001, u(0) = 0.0)$, period-3 attractor (red color) for $\alpha = 0.009$ and $(p(0) = 3.1, n(0) = 0.001, u(0) = 0.0)$ and (d) monostable limit cycle for $\alpha = 0.04$ and $(p(0) = 0.1, n(0) = 0.001, u(0) = 0.0)$.

The microcontroller-based circuit outputs of Figs. 10 and 11 and numerical simulations results of Figs. 4 and 7 confirm each other.

IV- Conclusion

The research reported in this paper demonstrated the existence of bistable attractors, coexisting attractors and its control in current modulated edge emitting semiconductor laser. Thanks to Routh–Hurwitz stability criteria, it was found that solitary edge emitting semiconductor laser has one stable equilibrium point and two unstable equilibrium points. By using a linear augmentation method, it was possible to control to a desired monostable trajectory the coexisting attractors found in current modulated edge emitting semiconductor laser. In addition to the study, the microcontroller-based circuit of the CMEESL systems is also designed. It was proved that the systems can be used in real applications according to the microcontroller-based circuit outputs.

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Figures

Figure 1

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Figure 2

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Figure 11

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