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Magnetohydrodynamics Mixed Convective Couple Stress Hybrid Nanofluid Darcy-Forchheimer Flow through a Rotating Porous Space

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Abstract

In this study, we consider the magnetohydrodynamics mixed convective couple stress hybrid nanofluid Darcy-Forchheimer flow through a rotating porous space with velocity slip condition. The nonlinear thermal stratification and thermal radiation of Magnetohydrodynamics (MHD) are discussed in detail. For relative analysis, we have taken the nanoparticles samples of Aluminum oxides (Al_2O_3) and Titanium dioxide (TiO_2). The rotation in the disk produces for the generation of the flow in the system. Furthermore, the variable permeability and porosity of porous space is regarded as Darcy-Forchheimer expression. The resulting nonlinear system of ODE's are solved by Homotopy Analysis Method (HAM). The governing of several sundry parameters i.e. "Couple Stress, coefficient of inertia, radiation parameter, magnetic parameter, Prandtl number, heat source or sink parameter" are presented both graphically as well as in numerical tables. The behavior of the flow predicted that the increase of both mixed convection and couple stress parameters cause increase in the momentum profile. Temperature of the system rises for higher values of radiation parameter and magnetic parameter. The higher local heat transfer rate of Aluminum oxides (Al_2O_3) and Titanium oxide (TiO_2) or water is examined as compared to hybrid nanofluid.

Keywords:

Darcy-Forchheimer porous; Titanium oxide (TiO_2); Aluminum oxide (Al_2O_3); water; Hybrid nanofluid; variable porosity and permittivity; Couple stress fluid; Velocity slip

conditions; Nonlinear thermal radiation; Thermal stratification; Heat absorption/generation, Magnetohydrodynamics (MHD).

Nomenclature:

Symbol	Description	Symbol	Description
(u, v, w)	velocity components (m/s)	Ω	constant angular velocity
M	Magnetic field parameter	T_w	Surface temperature (K)
ρ_{hnf}	Density of hybrid nanofluid (kgm^{-3})	C_b	drag coefficient
$(\rho c_p)_{hnf}$	volumetric heat capacity of hybrid nanofluid ($m^2/s^2 K$)	d	variable permeability
ϕ	nanoparticle volume fraction	d^*	variable porosity
θ	Dimensionless temperature (-)	T_0	Reference temperature (K)
k_f	Thermal conductivity ($Wm^{-1}K^{-1}$)	σ^*	Stefan Boltzmann constant
G_r	Grashof number	K_∞	dimensional permeability
q_r	Rosseland approximation	ε_∞	dimensional porosity
Pr	Prandtl number	k	mean absorption coefficient
C_f, C_g	Skin friction coefficient	T_∞	Ambient temperature (K)
K	coupled stress parameter	μ_{hnf}	dynamic viscosity of hybrid nanofluid (kg / ms)
Nu	Nusselt number	g	acceleration due to gravity (m / s^2)
λ	mixed convection or buoyancy parameter	Q	volumetric rate of heat generation/absorption
ρ_f	Density (kgm^{-3})	ν_f	Kinematic viscosity ($m^2 s^{-1}$)
f'	Dimensionless velocity (-)	μ_f	Dynamic viscosity ($kgm^{-1}s^{-1}$)
ϕ_1, ϕ_2	volume fraction of first and second nanoparticles	w_1, w_2	the first nanoparticle, the second nanoparticle
θ_w	Temperature ratio parameter	Re_r	Reynolds number
$(\rho c)_p$	Effective heat capacity of nanoparticles ($m^2/s^2 K$)	ξ	Similarity variable
k_1	Non-dimensional porosity parameter	T	Temperature of the fluid (K)
α	heat source parameter	γ^*	velocity slip parameter
S_t	thermal stratification parameter		

Introduction:

The study of nanofluids has gained a valuable attraction due to its vast applications in different fields of industry. In order to increase the thermal conductivity of base fluid, nanoparticles have been introduced. Because the simplest base fluids do not meet the required cooling in the industries. For this purpose, the nanoparticles of very small size (1nm to 100nm) can be included in both Newtonian and non-Newtonian models. The various forms of chemically stable nanoparticles are diamond, graphite, carbon nanotubes (CNTs) and fullerene. In the oxides of metals, it consists of zirconia, alumina, titanium and silica. In metals, it includes gold (Au) and copper (Cu). AlN and SiN are metal nitrides, SiC is a metal carbide and functionalized nanoparticles. The two different types of hybrid nanoparticles can increase the thermal conductivity of a single nanoparticle by hybridization, known as hybrid nano-fluids. It is the next generation of nano-fluid having innovative thermophysical characteristics. The first mathematician was Choi [1] who introduced the nanoparticles in base fluid. During his experimental work, he found that nanoparticles in base fluid increase efficient thermal conductivity of the common fluid. The literature review reveals that there have been numerous investigations into the boundary layer flow of Nano fluids on a stretch surface. Kuznetsov et al. [2] described the simultaneous solutions over a vertical plate of viscous nanofluids. The pioneer work of Khan et al. [3] represented his work on the topics concerning the nanofluids flow on stretching surfaces. The analytical solutions of steady laminar nanoliquids flow for suction or injection effects by Nadeem et al. [4]. Mehboob et al. [5] explored numerically the impact of heat transfer on boundary layer flow of nanofluid over a nonlinearly elongating sheet. Exponentially flow of the nanofluid with the condition of convective boundary layer stream was explored by Mustafa et al. [6]. It has been observed that the geometry of particles strongly disturbs the nanoparticles thermal conductivity. Das and Tiwari et al. [7], providing numerical solutions of mixed convective flow of nanofluids. The improvement in heat transfer in the past on stretch surface flow of nanofluid has been analyzed by Vajravelu [8]. The stream of nanofluid in 3D under the condition of convective boundary layer over a sheet elongating in two directions was explored by Khan [9]. The flow of mixed convective nanofluid composed of SiO_2-H_2O past a spinning circular cylinder was explored by Selimfedigil [10]. Hayat and Nadeem et al. [11] investigated the heat transfer enhancement in chemically reactive flow of silver and copper oxide ($Ag-CuO$ /water) hybrid nanofluid. Rashad [12] presented in his work the natural convective stream of

$Cu - AlO_3$ /water hybrid under the impacts of MHD and joule heating. Some developments in the study of hybrid nanofluids are consulate in refs: [13-15]. In industries Mixed convection boundary layers flows having the large number of real life uses such as electronic devices, heat exchanger, nuclear reactor, central solar receiver et al. [16]. In light of these numerous applications it is opening a new door for the researchers, to investigate the consequences of mixed convection boundary layer flow analytically as experimentally. Hayat [17] have discussed analytically viscoelastic mixed convective nanofluid through stretched cylinder. The transfer of heat through time dependent convective flow over an elongating belt was discussed by Turkyilmazoglu [18]. The heat transfer through time dependent convective boundary layer over shrinking/stretching surface by the spectral relaxation technique was analyzed by Haroun [19]. Behazadmehr [20] designed and studied convective flow of nanoliquids in tube. The heat-transfer properties of convective nano-fluids subjected to stretched sheet was investigated by Mahdi [21]. The branch of magnetohydrodynamics shortly known as MHD has incorporates ascending phenomenon when attractive fields is smeared to electrically engaged liquids. The research in the field of magnetohydrodynamics (MHD) has confirmed reckless in modern research “Davidson [22]”. There are applications in the method of accumulation in the exploration of the stream and heat-transfer of electrically conductive liquids in the vision of an attractive field over a hot surface. For example: cooling a metal plate, polymer explosion and atomic reactors etc. MHD of two-layer electro-osmotic flow with entropy generation over micro-parallel stations was considered by Xie and Jian [23]. Khan [24] studied the behavior of steady state flow in 3D with MHD for Powell–Eyring nanofluids through convective and nanoparticles mass flux situations. Ellahi [25] examined Heat transmission in a limit-layer stream with entropy generation and MHD impact. The MHD mixed convective flow of nanofluid composed of copper and water ($Cu - H_2O$) due to rotation of the circular cylinder across a trapezoidal vessel saturated by spongy medium was analyzed by Ali et al [26]. In this study they shown that the perpendicular magnetic field disparages the stream function by a large amount compared to inclined and horizontal. Thermal stability mentions the formation or accumulation of layers. It happens when thermal Stimulation of density gradient $>>0$ is not upset by convective flow. Because of this situation the system remains in stable balance. Thermal stratified fluid flow of nanofluids with MHD under solar radiation was explored numerically by Kadasamy et al. [27]. Zaib and Shafi et al. [28] studied electrically conducting and chemically

reactive fluid flow of viscous fluid with the generation of heat or thermally stratification and absorption. The doubly stratified flow of nanoliquid over a vertical plate was studied by Srinivasacharya and Surender et al. [29]. Sheremet [30] has explored the free convective stream of nanofluid by keeping the impact of thermal stratification inside the cavity with square shape. The purpose of this article to investigate the combined effect of Magnetohydrodynamics MHD numerically, the non-linear thermal radiation of Darcy-Forchheimer fluid flow of mixed convective couple stress of hybrid nanofluids using Titanium oxide (TiO_2), Aluminum oxide (Al_2O_3) nanoparticles along with thermal stratification. The velocity slip condition is also analyzed. The flow for this problem was generated by rotating disk. The governing system of nonlinear equations is solved by Homotopy Analysis Method HAM. The different emerging variables are examined graphically. In addition for all physical quantities the numeric data is interpreted and tabularized. The graphs of for different governing variables are plotted and their results are discussed.

Problem Formulation:

Three dimensional i.e. 3D steady flow of mixed convective couple stress hybrid-nanofluid is produced by a rotating disk with magnetohydrodynamics MHD. Velocity slip condition for the flow is illustrated in **Figure 1**. The angular velocity Ω of the rotation of the disk is considered uniformly. The x, y and z component of velocity (u, v, w) in the increasing direction (r, ψ, z) which results the equation of the flow is satisfied.

$$\frac{\partial}{\partial r} u + \frac{1}{r} u + \frac{\partial}{\partial z} w = 0, \quad (1)$$

$$\begin{aligned} u \frac{\partial u}{\partial r} - \frac{v^2}{r} + w \frac{\partial u}{\partial z} = \nu_{hnf} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{\nu_{hnf} \mathcal{E}(z)}{K(z)} u - \frac{\sigma_{hnf} \beta_0^2}{\rho_{hnf}} u \\ - \frac{\eta_0}{\rho_{hnf}} \frac{\partial^4 u}{\partial y^4} - \frac{C_b \mathcal{E}^2(z)}{\sqrt{K(z)}} u \sqrt{u^2 + v^2} + \frac{\left[(1-\phi_2) \{ (1-\phi_1) \rho_f \beta_f + \phi_1 \rho_1 \beta_1 \} \right] + \phi_2 \rho_2 \beta_2}{\rho_{hnf}} g (T - T_\infty), \end{aligned} \quad (2)$$

$$\begin{aligned}
u \frac{\partial v}{\partial r} + \frac{1}{r} uv^2 + \frac{w \partial u}{\partial z} = \nu_{hnf} \left(\frac{\partial^2 v}{\partial r^2} + \frac{\partial v}{r \partial r} - \frac{1}{r^2} v + \frac{\partial^2 v}{\partial z^2} \right) - \frac{\nu_{hnf} \varepsilon(z)}{K(z)} v - \frac{\sigma_{hnf} \beta_0^2}{\rho_{hnf}} v \\
- \frac{\eta_0}{\rho_{hnf}} \frac{\partial^4 v}{\partial y^4} - \frac{C_b \varepsilon^2(z)}{\sqrt{K(z)}} v \sqrt{u^2 + v^2} + \frac{[(1-\phi_2)\{(1-\phi_1)\rho_f \beta_f + \phi_1 \rho_1 \beta_1\}] + \phi_2 \rho_2 \beta_2}{\rho_{hnf}} g(T - T_\infty),
\end{aligned} \quad (3)$$

$$\begin{aligned}
u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = \nu_{hnf} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{\nu_{hnf} \varepsilon(z)}{K(z)} w - \frac{\sigma_{hnf} \beta_0^2}{\rho_{hnf}} w \\
- \frac{\eta_0}{\rho_{hnf}} \frac{\partial^4 w}{\partial y^4} - \frac{C_b \varepsilon^2(z)}{\sqrt{K(z)}} w \sqrt{u^2 + v^2} + \frac{[(1-\phi_2)\{(1-\phi_1)\rho_f \beta_f + \phi_1 \rho_1 \beta_1\}] + \phi_2 \rho_2 \beta_2}{\rho_{hnf}} g(T - T_\infty),
\end{aligned} \quad (4)$$

$$u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \alpha_{hnf} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) - \frac{1}{(\rho c_p)_{hnf}} \frac{\partial q_r}{\partial z} + \frac{Q}{(\rho c_p)_{hnf}} (T - T_\infty), \quad (5)$$

Figure: 1 Schematic diagram of the 3D problem

$$\left\{ \begin{array}{l} \frac{u}{L} = \frac{\partial u}{\partial z}, v = r \left(\Omega + \frac{L}{r} \frac{\partial}{\partial z} u \right), w = 0, T = T_w = T_0 + Ar, \text{ at } z = 0 \\ u \rightarrow 0, v \rightarrow 0, T \rightarrow T_\infty = T_0 + Br, \text{ at } z = \infty \end{array} \right\} \quad (6)$$

Where

$$K_\infty \left(1 + d e^{\frac{z}{r}} \right) = K, \quad \text{Where } K \text{ is function of } z \quad (7)$$

$$\varepsilon_\infty \left(1 + d^* e^{\frac{z}{r}} \right) = \varepsilon(z), \quad (8)$$

$$qr = -\frac{4\sigma^*}{3k} \frac{\partial T^4}{\partial z} = -\frac{16\sigma^* T^3}{3k} \left(\frac{\partial T}{\partial z} \right), \quad (9)$$

Thus the energy equation becomes:

$$\begin{aligned} u \left(\frac{\partial}{\partial r} T \right) + w \left(\frac{\partial}{\partial z} T \right) &= \alpha_{hnf} \left(\frac{\partial^2}{\partial r^2} T + \frac{1}{r} \frac{\partial}{\partial r} T + \frac{\partial^2}{\partial z^2} T \right) + \frac{1}{(\rho c_p)_{hnf}} \frac{16\sigma^*}{3k} \frac{\partial}{\partial z} \left(T^3 \frac{\partial T}{\partial z} \right) \\ &+ \frac{Q}{(\rho c_p)_{hnf}} (T - T_\infty), \end{aligned} \quad (10)$$

The hybrid nanofluid Theoretical model is [13]

$$\begin{aligned} \mu_{hnf} &= \frac{\mu_f}{(1 - \phi_1 - \phi_2)^{2.5}}, \nu_{hnf} = \frac{\mu_{hnf}}{\rho_{hnf}}, \rho_{hnf} = (1 - \phi_1 - \phi_2)(\rho)_f + \phi_1(\rho)_1 + \phi_2(\rho)_2, \\ \alpha_{hnf} &= \frac{k_{hnf}}{(\rho c_p)_{hnf}}, (\rho c_p)_{hnf} = (1 - \phi_1 - \phi_2)(\rho c_p)_f + \phi_1(\rho c_p)_1 + \phi_2(\rho c_p)_2, \\ \frac{k_{hnf}}{k_f} &= \frac{k_1\phi_1 + k_2\phi_2 + 2\phi k_f + 2\phi(\phi_1 k_1 + \phi_2 k_2) - 2(\phi_1 + \phi_2)^2 k_f}{k_1\phi_1 + k_2\phi_2 + 2\phi k_f + \phi(\phi_1 k_1 + \phi_2 k_2) + (\phi_1 + \phi_2)^2 k_f}. \end{aligned} \quad (11)$$

ρ_1 and ρ_2 are densities of Titanium oxide (TiO_2) and Aluminum oxide Al_2O_3 respectively. k_1 , k_2 and k_f are thermal conductivities of Titanium oxide (TiO_2), Aluminum oxide Al_2O_3 and base fluids respectively.

In Table 1 thermo-physical properties of nanoparticles and water are discussed and given as:

Table 1: Thermo-physical properties of nanoparticles characteristics and water.

Physical properties	Base fluid	Nanoparticles	
		TiO_2	Al_2O_3
$\rho (kg / m^3)$	9.97×10^2	4.23×10^3	4.0×10^3
$k (W / mk)$	6.13×10^{-1}	8.4	40
$c_p (J / kgK)$	4.179×10^3	6.92×10^2	7.73×10^2

Considering

$$\begin{aligned}
 u &= r\Omega f'(\xi), \\
 v &= \Omega g(\xi), \\
 w &= -\sqrt{2\Omega\nu_f} f(\xi), \\
 \theta(\xi) &= \frac{T - T_\infty}{T_w - T_0}, \\
 \xi &= \left(\frac{2\Omega}{\nu_f} \right)^{1/2} z,
 \end{aligned} \tag{12}$$

And

$$T = (T_w - T_0)\theta(\xi) + T_\infty$$

Then Equation (1) is satisfied and Equations. (2 to 11) gives

$$\begin{aligned}
& \frac{1}{(1-\phi_1-\phi_2)\left(1-\phi_1-\phi_2+\frac{\rho_1}{\rho_f}\phi_1+\frac{\rho_2}{\rho_f}\phi_2\right)} \left(2f''' - \frac{1}{2k_1 \text{Re}_r} \left(\frac{1+d^*e^{-\xi}}{1+de^{-\xi}} \right) f' \right) \\
& -F_r \left(\frac{1+d^*e^{-\xi}}{\sqrt{1+de^{-\xi}}} \right) (f'^2 + (0.5)g^2) - (f'^2) + (g^2) + 2ff'' - \frac{\frac{\sigma_{hmf}}{\rho_{hmf}}}{\rho_f} Mf' + 4 \frac{\rho_f}{\rho_{hmf}} Kf^v + A_3\lambda\theta = 0,
\end{aligned} \tag{13}$$

$$\begin{aligned}
& \frac{1}{(1-\phi_1-\phi_2)\left(1-\phi_1-\phi_2+\frac{\rho_1}{\rho_f}\phi_1+\frac{\rho_2}{\rho_f}\phi_2\right)} \left(2g'' - \frac{1}{2k_1 \text{Re}_r} \left(\frac{1+d^*e^{-\xi}}{1+de^{-\xi}} \right) g \right) \\
& -F_r \left(\frac{1+d^*e^{-\xi}}{\sqrt{1+de^{-\xi}}} \right) (g^2 + (0.5)f'^2) - f'g + fg' - \frac{\frac{\sigma_{hmf}}{\rho_{hmf}}}{\rho_f} Mg + 4 \frac{\rho_f}{\rho_{hmf}} Kg^{iv} + A_3\lambda\theta = 0,
\end{aligned} \tag{14}$$

$$\begin{aligned}
& \frac{1}{(1-\phi_1-\phi_2)\left(1-\phi_1-\phi_2+\frac{(\rho_{cp})_1}{(\rho_{cp})_f}\phi_1+\frac{(\rho_{cp})_2}{(\rho_{cp})_f}\phi_2\right)} \left[\left(\frac{k_{hmf}}{k_f} + \frac{4}{3} R \left[\left(\frac{1}{\theta_w + s_t} \right) \theta + 1 \right]^3 \right) \theta' \right] + \text{Pr} f \theta' \\
& + \frac{k_f}{k_{hmf}} \alpha \theta = 0,
\end{aligned} \tag{15}$$

$$\begin{aligned}
& f = 0, f'' = \frac{f'}{\gamma}, 1 - g + \gamma g' = 0, \theta = 1 - S_t \text{ at } \xi = 0 \\
& \theta \rightarrow 0, g \rightarrow 0, f' \rightarrow 0 \quad \text{at } \xi \rightarrow \infty
\end{aligned} \tag{16}$$

In the above equations

γ^* = Slip velocity parameter,

k_1 = Porosity parameter,

Re_r = Local Reynolds number,

F_r = Local inertia parameter,

Pe_r = Peclet number,

θ_w = Variable for temperature,

M = Magnetic field parameter,

λ = Mixed convection i.e. buoyancy parameter,

S_t = Thermal stratification parameter, R = Radiation parameter,

P_r = Prandtl number, and heat source parameter (α)

These are defined by:

$$\gamma^* = L \left(\frac{2\Omega}{v_f} \right)^{1/2}, k_1 = \frac{K_\infty}{r^2 \varepsilon_\infty}, \text{Re}_r = \frac{U_w r}{v_f}, F_r = \frac{C_b \varepsilon_\infty^2}{\sqrt{K}}, M = \frac{\sigma \beta_0^2}{\rho r \Omega},$$

$$\theta_w = \frac{T_0}{Ar}, S_t = \frac{B}{A}, R = \frac{4\sigma^* T_\infty^3}{kk_f}, \text{Pr} = \frac{v_f}{\alpha_f}, Pe_r = \text{Re}_r \text{Pr}, \alpha = \frac{Qv_f}{ck_f}, \lambda = \frac{Gr_r}{\text{Re}_r^2}.$$
(17)

$$A_1 = A_2 \times \left(1 - \frac{\rho_2 w_1 + \rho_1 w_2}{\rho_1 \rho_2} \left(\frac{1}{\frac{\rho_2 \rho_f w_1 + \rho_1 \rho_f w_2 + \rho_1 \rho_2 w_3}{\rho_1 \rho_2 \rho_f}} + \frac{1}{\frac{\rho_2 \rho_f w_1 + \rho_1 \rho_f w_2 + \rho_1 \rho_2 w_3}{\rho_1 \rho_2 \rho_f}} \frac{\rho_s}{\rho_f} \right) \right),$$

$$A_2 = \left(1 - \frac{\frac{\rho_2 w_1 + \rho_1 w_2}{\rho_1 \rho_2}}{\frac{\rho_2 \rho_f w_1 + \rho_1 \rho_f w_2 + \rho_1 \rho_2 w_3}{\rho_1 \rho_2 \rho_f}} \right)$$

$$A_3 = A_2 \left[\frac{\rho_1 \rho_f w_2}{\rho_2 \rho_f w_1 + \rho_1 \rho_f w_2 + \rho_1 \rho_2 w_f} \left\{ \frac{\left(1 - \frac{\rho_2 \rho_f w_1}{\rho_2 \rho_f w_1 + \rho_1 \rho_f w_2 + \rho_1 \rho_2 w_f} \right) + \frac{\rho_2 \rho_f w_1}{\rho_2 \rho_f w_1 + \rho_1 \rho_f w_2 + \rho_1 \rho_2 w_f} \frac{(\rho\beta)_1}{(\rho\beta)_f}}{\frac{\rho_1 \rho_f w_2}{\rho_2 \rho_f w_1 + \rho_1 \rho_f w_2 + \rho_1 \rho_2 w_f} \frac{(\rho\beta)_2}{(\rho\beta)_f}} \right\} + \frac{\rho_1 \rho_f w_2}{\rho_2 \rho_f w_1 + \rho_1 \rho_f w_2 + \rho_1 \rho_2 w_f} \frac{(\rho\beta)_2}{(\rho\beta)_f} \right]$$
(18)

The Nusselt number and Skin friction coefficient for the problem is defined as :

$$\left. \begin{aligned} [Re_r]^{1/2} C_f &= \frac{1}{(1 - \phi_1 - \phi_2)^{2.5}} f''(0), \\ [Re_r]^{1/2} C_g &= \frac{1}{(1 - \phi_1 - \phi_2)^{2.5}} g'(0), \\ \frac{1}{2} [Re_r]^{-1/2} Nu &= - \left(\frac{k_{hmf}}{k_f} + \frac{4}{3} R \left(\frac{1}{\theta_w + S_t} + 1 \right)^3 \right) \theta'(0), \end{aligned} \right\}$$
(19)

Solution by HAM

The solution of the problem in Equation (13 to 15) was found by using HAM under the boundary conditions in Equation (16).

$$\dot{f}^v = L_f(\dot{f}), \dot{g}^{iv} = L_g(\dot{g}), \dot{\theta}'' = L_\theta(\dot{\theta}), \quad (20)$$

The linear operators for the problem are:

$$\begin{aligned} L_f(e_1 + \eta(e_2 + e_3\eta + e_4\eta^2 + e_5\eta^3)) &= 0, \\ L_g(e_6 + \eta(e_7 + e_8\eta + e_9\eta^2)) &= 0, \\ L_\theta(e_{10} + e_{11}\eta) &= 0, \end{aligned} \quad (21)$$

The non-linear operator is chosen as N_f , N_g and N_θ identify in system:

$$\begin{aligned} N_f(\dot{f}(\xi; \zeta), \dot{g}(\xi; \zeta), \dot{\theta}(\xi; \zeta)) &= \frac{1}{(1 - \phi_1 - \phi_2) \left(1 - \phi_1 - \phi_2 + \frac{\rho_1}{\rho_f} \phi_1 + \frac{\rho_2}{\rho_f} \phi_2 \right)} \\ &\left(2\dot{f}_{\xi\xi\xi} - \frac{1}{2k_1 \text{Re}_r} \left(\frac{1 + d^* e^{-\xi}}{1 + d e^{-\xi}} \right) \dot{f}_\xi \right) - F_r \left(\frac{1 + d^* e^{-\xi}}{\sqrt{1 + d e^{-\xi}}} \right) \left(\dot{f}_\xi^2 + \frac{1}{2} \dot{g}^2 \right) - \dot{f}_\xi^2 + 2\dot{f}_{\xi\xi\xi} \\ &+ \dot{g}^2 - \frac{\sigma_{hmf}}{\rho_{hmf}} M \dot{f}_\xi + 4 \frac{\rho_f}{\rho_{hmf}} K \dot{f}_{\xi\xi\xi\xi\xi} + A_3 \lambda \dot{\theta}, \end{aligned} \quad (22)$$

$$\begin{aligned} N_g[\dot{f}(\xi; \zeta), \dot{g}(\xi; \zeta), \dot{\theta}(\xi; \zeta)] &= \frac{1}{(1 - \phi_1 - \phi_2) \left(1 - \phi_1 - \phi_2 + \frac{\rho_1}{\rho_f} \phi_1 + \frac{\rho_2}{\rho_f} \phi_2 \right)} \\ &\left(2\dot{g}_{\xi\xi} - \frac{1}{2k_1 \text{Re}_r} \left(\frac{1 + d^* e^{-\xi}}{1 + d e^{-\xi}} \right) \dot{g} \right) - F_r \left(\frac{1 + d^* e^{-\xi}}{\sqrt{1 + d e^{-\xi}}} \right) \left(\dot{g}^2 + \frac{1}{2} \dot{f}_\xi^2 \right) - \dot{f}_\xi \dot{g} \\ &+ \dot{f}_\xi \dot{g} - \frac{\sigma_{hmf}}{\rho_{hmf}} M \dot{g} + 4 \frac{\rho_f}{\rho_{hmf}} K \dot{g}_{\xi\xi\xi\xi\xi} + A_3 \lambda \dot{\theta}, \end{aligned} \quad (23)$$

$$N_{\theta}[\theta(\xi; \zeta), f(\xi; \zeta)] = \frac{1}{(1 - \phi_1 - \phi_2) \left(1 - \phi_1 - \phi_2 + \frac{(\rho_{cp})_1}{(\rho_{cp})_f} \phi_1 + \frac{(\rho_{cp})_2}{(\rho_{cp})_f} \phi_2 \right)} \quad (24)$$

$$\left[\left(\frac{k_{hnf}}{k_f} + \frac{4}{3} R \left[\left(\frac{1}{\theta_w + s_t} \right) \theta + 1 \right]^3 \right) \theta_{\xi} \right]_{\xi} + \Pr f \theta_{\xi} + \frac{k_f}{k_{hnf}} \alpha \theta,$$

While BCs are

$$\left. \frac{\partial f(\xi; \zeta)}{\partial \xi} \right|_{\xi=0} = \gamma \left. \frac{\partial^2 f(\xi; \zeta)}{\partial \xi^2} \right|_{\xi=0}, \quad f(\xi; \zeta)|_{\xi=0} = 0, \quad g(\xi; \zeta)|_{\eta=0} = 1 + \gamma \left. \frac{\partial g(\xi; \zeta)}{\partial \eta} \right|_{\xi=0}, \quad (25)$$

$$\theta(\xi; \zeta)|_{\xi=0} = 1 - S_t, \quad \left. \frac{\partial f(\eta; \zeta)}{\partial \xi} \right|_{\xi=\infty} = 0, \quad g(\xi; \zeta)|_{\xi=\infty} = 1, \quad \theta(\xi; \zeta)|_{\xi=\infty} = 0.$$

Here, ζ is the embedding parameter $\zeta \in [0, 1]$, to standardize the convergence of the solution of

h_f, h_g and h_{θ} is used. By choosing $\zeta = 0$ and $\zeta = 1$ we have:

$$\begin{aligned} f(\xi) &= f(\xi; 1), \\ g(\xi) &= g(\xi; 1), \\ \theta(\xi) &= \theta(\xi; 1). \end{aligned} \quad (26)$$

Develop the Taylor's series for $f(\xi; \zeta), g(\xi; \zeta)$ and $\theta(\xi; \zeta)$ about the point $\zeta = 0$

$$\begin{aligned} f(\xi; \zeta) &= f_0(\xi) + \sum_{n=1}^{\infty} f_n(\xi) \zeta^n \\ g(\xi; \zeta) &= g_0(\xi) + \sum_{n=1}^{\infty} g_n(\xi) \zeta^n \\ \theta(\xi; \zeta) &= \theta_0(\xi) + \sum_{n=1}^{\infty} \theta_n(\xi) \zeta^n \end{aligned} \quad (27)$$

$$\begin{aligned}
f_n(\xi) &= \frac{1}{n!} \frac{\partial^n f(\xi; \zeta)}{\partial \xi^n} \Big|_{p=0}, \\
g_n(\xi) &= \frac{1}{n!} \frac{\partial^n g(\xi; \zeta)}{\partial \xi^n} \Big|_{p=0}, \\
\theta_n(\xi) &= \frac{1}{n!} \frac{\partial^n \theta(\xi; \zeta)}{\partial \xi^n} \Big|_{p=0},
\end{aligned} \tag{28}$$

While BCs are:

$$\begin{aligned}
f'(0) &= 0, f''(0) = \gamma f''', g'(0) = 1 + \gamma g''(0), \theta'(0) = 1 - S_t, \\
f'(\infty) &\rightarrow 0, g(\infty) \rightarrow 1, \theta(\infty) \rightarrow 0.
\end{aligned} \tag{29}$$

Results and Discussion:

This section describes the behavior of emerging flow variables i.e. the porosity parameter k_1 , local inertial parameter F_r , R_e local Reynolds number, the variable parameter for temperature θ_w , magnetic parameter M , λ = mixed convection or buoyancy parameter, stratification parameter S_t , R radiation parameter, and p_r Prandtl number and heat source parameter which is represented by α . The impact on velocity profile and temperature of these parameters is shown in figures 2-19. the impact of magnetic parameter (M) on $f'(\xi), g(\xi)$ is shown in figures 2 and 3. From the figures it is obvious that magnetic parameter is inversely proportional to the velocity profile ($f'(\xi), g(\xi)$). Velocity profile ($f'(\xi), g(\xi)$) decreases while the magnetic field increases. The Lorentz force increases for larger value of magnetic parameter these forces resist the fluid flow. Thus ($f'(\xi), g(\xi)$) corresponding to the boundary layer decreases showing quite opposite behaviors. Figures 4 and 5 show the impact of F_r local inertial parameter on velocity profiles $f'(\xi), g(\xi)$. It is noticed that the stronger physical resistance is offered to the flow of the base fluid cause reduction in the characteristic of $f'(\xi), g(\xi)$, which reduces the corresponding boundary layer. Figures 6 and 7 demonstrate that increasing values of porosity

factor enhance extra resistance to the system. Consequently the stronger frictional force results decay in $f'(\xi), g(\xi)$ physically. Figures 8 and 9 show that enhancement in the mixed convection parameter insensibly yields to enhance dimensionless velocity distribution. It also gives sharp rise in the velocity profile $(f'(\xi), g(\xi))$ close to stretching wall. One the important feature of mixed convection parameter is that “in heat transfer mode in mixed convection for the problem the dimensionless flow field becomes coupled with the dimensionless temperature field. Figures 10 and 11 are showing the impact of Local Reynolds number (Re_r) on the velocity profile $f'(\xi), g(\xi)$. It is obvious from these figures that the velocity profile is -improved for greater estimation of Local Reynolds number Re_r . Figures 12 and 13 also denotes the outcome of couple-stress parameter K on the $f'(\xi), g(\xi)$. It can be comprehended from these figures that the $f'(\xi), g(\xi)$ increases while increasing couple-stress parameter K . However, the increase in velocity profiles will be not significant for large values of K . That is to say that large values of K will lead to pure viscous fluid. Figure 14 displays the possessions of p_r verses $\theta(\xi)$ temperature profiles. These figure show that the temperature profile showing decrease when increasing the value of Prandtl number p_r . It is obvious from the figure the increment in Prandtl number p_r cause decrease in the thermal diffusion. Figure 15 discloses that the temperature for the nanofluid is raised due to the influence of R (thermal radiation parameter). Enhancement in radioactive heat flux fosters the molecular passage inside the base fluid, and thus numerous collisions between molecules translate in the form of thermal energy. Consequently, higher value of temperature distribution has been observed as a result of a rise in the value of thermal radiation parameter R . It is remarkable to remember that being extremely viscous in kind; hyperbolic tangent nanofluids produce more heat than Newtonian nanofluid. The effect of heat sink for a very small i.e. $\alpha < 0$ or for a very large value i.e. $\alpha > 0$ parameter in the dimensionless temperature are illustrated in Figure 16 and 17. With increase in the strength of heat source $\alpha > 0$ fluid, temperature increases while increasing the thickness of thermal boundary layer. On the other hand increase in heat sink strength $\alpha < 0$, falls the temperature due to decrease in the thermal boundary layer thickness. Figure 18 shows relation between $\theta(\xi)$

(temperature) and (θ_w) . It is clear from the figure that $\theta(\xi)$ (temperature) is a decreasing function of (θ_w) for titanium oxide (TiO_2) and Aluminum oxide (Al_2O_3)/water. Figure 19 is plotted between (S_t) and temperature $\theta(\xi)$. It describes that $\theta(\xi)$ reduces for larger value of (S_t) for nanofluid and hybrid nanofluid.

Figure 2: Influence of M and $f'(\xi)$ when $K = 0.7, k_1 = 0.3, F_r = 0.5, \lambda = 0.3, Re_r = 1.3$.

Figure 3: Influence of M on $g(\xi)$ when $K = 0.7, k_1 = 0.3, F_r = 0.5, \lambda = 0.3, Re_r = 1.3$.

Figure 4: Influence of F_r on $f'(\xi)$ when $K = 0.7, k_1 = 0.3, M = 1.0, \lambda = 0.3, \text{Re}_r = 1.3$.

Figure 5: Influence of F_r on $g(\xi)$ when $K = 0.7, k_1 = 0.3, M = 1.0, \lambda = 0.3, \text{Re}_r = 1.3$.

Figure 6: Influence of k_1 on $f'(\xi)$ when $K = 0.7, F_r = 0.5, M = 1.0, \lambda = 0.3, \text{Re}_r = 1.3$.

Figure 7: Influence of k_1 on $g(\xi)$ when $K = 0.7, F_r = 0.5, M = 1.0, \lambda = 0.3, \text{Re}_r = 1.3$.

Figure 8: Influence of λ on $f'(\xi)$ when $k_1 = 0.3, K = 0.7, F_r = 0.5, M = 1.0, \text{Re}_r = 1.3$.

Figure 9: Influence of λ on $g(\xi)$ when $k_1 = 0.3, K = 0.7, F_r = 0.5, M = 1.0, \text{Re}_r = 1.3$.

Figure 10: Influence of Re_r on $f'(\xi)$ when $k_1 = 0.3, K = 0.7, F_r = 0.5, M = 1.0, \lambda = 0.3$.

Figure 11: Influence of Re_r on $g(\xi)$ when $k_1 = 0.3, K = 0.7, F_r = 0.5, M = 1.0, \lambda = 0.3$.

Figure 12: Influence of K on $f'(\xi)$ when $k_1 = 0.3, F_r = 0.5, M = 1.0, \lambda = 0.3, \text{Re}_r = 1.3$.

Figure 13: Influence of K on $f'(\xi)$ when $k_1 = 0.3, F_r = 0.5, M = 1.0, \lambda = 0.3, \text{Re}_r = 1.3$.

Figure 14: Influence of Pr on $\theta(\xi)$ when $\theta_w = 1.1, S_t = 0.5, \alpha = 1.0, R = 0.8$.

Figure 15: Influence of R on $\theta(\xi)$ when $P_r = 7.0, \theta_w = 1.1, S_t = 0.5, R = 0.8$.

Figure 16: Influence of $\alpha > 0$ on $\theta(\xi)$ when $P_r = 7.0, \theta_w = 1.1, S_t = 0.5, \alpha = -1.0, R = 0.8$.

Figure 17: Influence of $\alpha < 0$ on $\theta(\xi)$ when $P_r = 7.0, \theta_w = 1.1, S_t = 0.5, \alpha = 1.0, R = 0.8$.

Figure 18: Influence of θ_w on $\theta(\xi)$ when $P_r = 7.0, S_t = 0.5, \alpha = 1.0, R = 0.8$.

Figure 19: Influence of S_t on $\theta(\xi)$ when $P_r = 7.0, \theta_w = 1.1, \alpha = 1.0, R = 0.8$.

Table discussion:

Table 2 shows that C_f and C_g are increased when the values of F_r, k_1, M are increased. They C_f and C_g are decreased, when the values of K, Re_r and λ is increased. Table 3 shows that Nu is increased when the values when the values of R, α, θ_w, S_t are increased. The Nu is decreased, when the values of p_r is increased.

Table 2: Impact of different physical parameters over Skin friction

$$[Re_r]^{\frac{1}{2}} C_f = \frac{1}{(1-\phi_1-\phi_2)^{2.5}} f''(0), [Re_r]^{\frac{1}{2}} C_g = \frac{1}{(1-\phi_1-\phi_2)^{2.5}} g'(0),$$

F_r	k_1	M	K	Re_r	λ	$\frac{1}{(1-\phi_1-\phi_2)^{2.5}} f''(0)$	$\frac{1}{(1-\phi_1-\phi_2)^{2.5}} g'(0)$
0.1	0.5	0.5	0.4	1.0	0.6	1.2347805	1.4207392
0.3						1.3437309	1.5349731
0.5						1.5768929	1.7236894
	0.5					1.1423802	2.0451896
	0.7					1.2816925	2.1239842
	0.9					1.3469237	2.2058903
		0.5				1.2487189	1.5243892
		1.0				1.4586316	1.7598134
		1.5				1.6236892	1.9209678
			0.4			2.4108219	1.2348123
			0.8			2.3904527	1.1607837
			1.0			2.1486349	1.1038031

1.0	2.7283981	2.5379341
1.3	2.5361693	2.3963703
1.6	2.4643643	2.1361342
0.6	3.0185372	2.0138735
1.0	2.6433934	1.8123834
1.5	2.4376192	1.6794374

Table 3: Impact of various physical parameters over Nusselt number

$$\frac{1}{2} [Re_r]^{\frac{1}{2}} Nu = - \left(\frac{k_{hmf}}{k_f} + \frac{4}{3} R \left(\frac{1}{\theta_w + S_t} + 1 \right)^3 \right) \theta'(0),$$

α	Pr	R	S_t	θ_w	$-\left(\frac{k_{hmf}}{k_f} + \frac{4}{3} R \left(\frac{1}{\theta_w + S_t} + 1 \right)^3 \right) \theta'(0)$
0.5	7.0	0.2	0.2	1.0	0.9216734
1.0					0.6328432
1.5					0.4637381
	7.0				0.5370358
	7.5				0.7195309
	8.0				0.9063276
		0.2			0.7314907
		0.4			0.8021434
		0.8			0.8956873
			0.2		0.3984305
			0.5		0.3769196
			1.0		0.3415809
				1.0	0.8714792
				1.5	0.6438503
				1.8	0.3274239

Conclusions:

The Darcy-Forchheimer flow of mixed convective couple stress hybrid nanofluid with Magnetohydrodynamics (MHD), nonlinear thermal radiation and thermal stratification is investigated. The major outcomes are listed as follows:

- There is a decreasing in $f'(\xi)$, $g(\xi)$ for magnetic parameter (M) , (F_r) and (k_l) .
- There is an increasing in $f'(\xi)$, $g(\xi)$ for couple-stress parameter K and mixed convection parameter.
- Augmentation in velocity profiles are analyzed through Re_r .

- Increase in Pr decreases the temperature profile, while increase in R increases the temperature profile.
- Temperature profile exhibits opposite behavior for of heat sink ($\alpha < 0$) or source ($\alpha > 0$) parameter.
- Temperature of the nanofluid is a decreasing function of (S_t) and (θ_w) .

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Figures

Figure 1

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