

# Drone-Drone Collisions Over New York City

Edward Walbridge (✉ [edwalbridge@outlook.com](mailto:edwalbridge@outlook.com))

Ergon Institute LLC <https://orcid.org/0000-0002-1521-5077>

---

## Article

**Keywords:** Delivery drones, drone system safety, drone collisions, air traffic management, drone reliability, Unmanned Aerial Vehicle (UAV), Unmanned Aircraft System (UAS), Unmanned Traffic Management (UTM).

**Posted Date:** August 5th, 2020

**DOI:** <https://doi.org/10.21203/rs.3.rs-50860/v1>

**License:**  This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

---

## **Title: Drone-Drone Collisions Over New York City**

**Author:** Edward Walbridge<sup>1\*</sup>

**Affiliation:** <sup>1</sup>Ergon Institute LLC.

5      \*Correspondence to: [\*\*edwalbridge@outlook.com\*\*](mailto:edwalbridge@outlook.com), Ergon Institute LLC, 1440 N. Lake Shore Drive, #26G, Chicago, IL 60610

**Keywords:** Delivery drones, drone system safety, drone collisions, air traffic management, drone reliability, Unmanned Aerial Vehicle (UAV), Unmanned Aircraft System (UAS), Unmanned Traffic Management (UTM).

10      **Abstract:** Delivery of packages by drones to homes and workplaces is faster and cheaper than delivery by van and hence has significant potential. That potential can only be realized if delivery by drones is safe. In particular, such drones must not pose a threat from in-air drone-drone collisions. To determine how serious the drone collision problem might be, we: 1) focus on a situation where they would  
15      be maximized, New York City, and 2) investigate the “do nothing” alternative, in which the drones do nothing to avoid collisions. Such drones, lacking a collision-avoidance capability, are “dumb” drones, and will have a much higher collision rate than “smart” drones, which do have that capability and will ultimately be

deployed. The do nothing alternative is useful for determining the extent to which a collision-avoidance capability is needed. The dumb drone collision rate is found to be unacceptably high. Actual smart drones to be ultimately deployed must therefore have a highly effective collision-avoidance capability to bring collision rates close to zero. The needed effectiveness is quantified.

## Introduction

Because drones can deliver packages faster and at lower cost (especially for one vehicle-trip per package) than street-level vehicles, the nascent drone delivery industry holds much promise.<sup>1,2,3</sup> Companies developing and testing drone delivery systems include Amazon, Wing (a subsidiary of Alphabet), UPS, DHL, FedEx, and Uber Eats.<sup>4,5,6,7,8,9,10,11,12,13,14,15,16</sup> Gartner, a consultancy, predicts that “in 2026, more than one million drones will be carrying out retail deliveries”<sup>1</sup>. For that same year, “FedEx expects the U.S. parcel market alone to double in size to more than 100 million packages per day”, referring to not just drone deliveries, but vehicle deliveries as well.<sup>16</sup> Of these 90% could be deliveries by drone.<sup>5,7</sup>

Conclusion: the drone delivery industry has huge potential.

That potential will only be reached, however, if delivery by drones, is safe. That is, drone debris must not fall out of the sky due to collisions of delivery drones with buildings, towers, wires, flying taxis, passenger drones, helicopters,

winged aircraft, or other delivery drones. Our focus herein is on collisions between delivery drones. For such drones, what is needed is a collision-avoidance capability that will reduce the number of collisions/day from the relatively high level that will occur sans that capability to zero or near-zero. The collision-avoidance capability would consist of a traffic management system that would keep drones from getting too close to each other and, should that system fail, a capacity for detecting imminent collisions and swerving to avoid them. The case of no collision-avoidance capability is a “do-nothing to avoid collisions” alternative, whose drones we refer to as “dumb drones”, in contrast to “smart drones” which do have that capability. We seek an estimate of collisions/day for the do-nothing/dumb-drone alternative in order to quantify the challenge of reducing that collision rate to an acceptable near-zero level. To this end we consider where drone-drone collisions would be maximized, viz., over a dense urban area. We choose New York City.

If there are many drone flights over a city, but only one drone at a time is in the air, there will be no collisions. If many are in the air at one time, but all travel on radial paths from a central point, the outgoing drones at one altitude and the incoming at a lower one, there will be no collisions, assuming care is taken to ensure that at the central point rising and descending drones do not conflict.

However, if Amazon drones travel radially out and in from one central point and Wing drones travel from a different central point, the Amazon and Wing drones, flying at the same altitude, could collide. We assume many such central points, roughly uniformly distributed over the area of New York City.

5           Molecules of a gas do not have a collision-avoidance capability; they just bump into each other. The dumb drones of our model are essentially molecules – very large molecules: we assume a drone molecule radius of one meter, the dimension of Amazon’s Prime Air drone.<sup>4,5,6,7</sup> We treat the cloud of dumb molecule drones as a gas and apply the Kinetic Theory of Gases and Queueing  
10 Theory to estimate the number of collisions per day of dumb drones over New York City.

## Results

From kinetic theory the Mean Free Path (MFP) of a molecule in a gas is  
15 given by<sup>17</sup>:

$$\text{MFP} = 1/(4\pi a^2 N) \tag{1}$$

where  $a$  = radius of the molecule and  $N$  = volume density of the molecules, i.e., number of molecules per unit volume. The challenge is to obtain an estimate for

*N.* We have that:

$$NV = P \tag{2}$$

Where  $V$  is the volume of the cloud of dumb drones buzzing over (in this case) New York City and  $P$  is the total number of drones in the cloud, i.e., its population.

5 In the US, commercial drones cannot be flown above an altitude of 400 ft (122 m) without special permission from the FAA<sup>18</sup>. We assume a vertical height of 100 m (328 feet) for the permissible drone fly zone: 22 m to 122 m. This is more generous than allowed by a bill introduced in the US Senate on October 16, 2019.<sup>19</sup> That bill would give a property owner control of the airspace up to 200 ft (61 m) over his/her property, resulting in a commercial drone fly zone extending from 61 m to 122 m, as opposed to the 22 m to 122 m vertical range assumed here. At 100 m thickness the commercial drone fly zone is still a very thin slice of air (punctured by tall buildings) sandwiched between the zone above, the domain of commercial aircraft, light aircraft, air taxis, and helicopters, and the zone below.

10

15 New York City<sup>20</sup> has an area of 785 km<sup>2</sup>. With a thickness of 100 m the drone cloud over the city has volume,  $V = 78.5 \text{ km}^3$ .

The drone delivery system over New York City will be a queueing system<sup>21</sup> in which drones are the “calling units” that are “serviced” by being sent to and from the delivery point (i.e., zero time spent in a waiting queue waiting to be

serviced; all drones immediately start being serviced upon launch). Assuming a steady state, the queueing equation for this situation is:

$$P = \lambda W_s \tag{3}$$

where  $\lambda$  = average number of drones/hour launched into the drone cloud and

5  $W_s$  = average service time in hours.

Of the 1.5 million packages delivered per day in NYC in 2019<sup>22</sup>, we assume that 10%, or 150,000, could be readily delivered by drones. 10% is conservative, as can be seen from the remarks of Jeff Wilke, Amazon's CEO Consumer Worldwide at Amazon's June 5, 2019 re:MARS Conference in Las Vegas. He  
10 there unveiled the company's latest Prime Air drone, a delivery drone that can fly up to 15 miles (24 km) round trip to deliver, within 30 minutes, packages of less than 5 pounds<sup>5,7,8</sup>. Wilke stated: "And while 5 pounds may not sound like a lot, it represents between 75 and 90 percent of the packages that Amazon delivers to its customers today." The 150,000 drone deliveries/day over New York City (based  
15 on Ref. 22) converts to 8,333/hour assuming no deliveries between midnight and 6 AM, i.e.,  $\lambda = 8,333$  drone launches/hour. The Amazon drone can deliver packages within 30 minutes of an order being placed. Assume a delivery time of 20 minutes of which 10 minutes are devoted to preparing the package and loading

it into the drone and 10 minutes are spent in flight to the destination. The round-trip flight time is then 20 minutes: 10 minutes to and 10 minutes back from the destination point. In this case  $W_s = 20$  minutes. Taking  $a = 1$  m, using the parameter values mentioned, and combining equations 1 - 3 we find:

5 
$$\text{MFP} = V/(4\pi a^2 \lambda W_s) = 2,250 \text{ km} \quad (4)$$

A drone flying out for 10 minutes and back for 10 minutes will, at a speed of 30 mph (48 kmph), travel 10 mi (16 km) round trip, i.e., much less than the MFP. So unlikely to have a collision with another drone in one delivery flight.

The probability of a collision in one flight out and back, round-trip distance  $d$ , is given by:

10

$$1 - e^{-(d/\text{MFP})}$$

For  $d = 16$  km and  $\text{MFP} = 2250$  km,  $d/\text{MFP} = 0.007$ . For  $d/\text{MFP} \ll 1$ , as in this case, expanding the exponential leads to:

$$1 - e^{-(d/\text{MFP})} = d/\text{MFP}$$

15 for the probability of a collision in one round-trip drone flight of distance  $d$ .

While a collision is unlikely in one drone round-trip, an individual drone can make multiple round trips in one day, and there are many drones making trips each day. With our assumed 150,000 drone deliveries per day over New York City, at

$d = 16$  km per delivery, the total distance traveled by the drone fleet per day will be  $2.4 \times 10^6$  km, or 1,070 MFPs, that is, 1,070 collisions/day at one collision per MFP. Actually, not quite! The 1,070 collisions/day count the collision of the  $i$ 'th drone with the  $j$ 'th, and the  $j$ 'th with the  $i$ 'th, as two distinct collisions, whereas there is just one. Dividing by 2 to correct for this double counting yields an estimate of 535 drone-drone collisions/day over New York City. This is a rough number given all our assumptions and variable inputs, so we will say approximately 500 collisions/day. In any case, too many collisions to be acceptable! 500 collisions/day is the case for dumb drones, i.e., drones without collision-avoidance capability. Actual smart drones, having such a capability, will have to do much better. How much better? Let  $CPD_{acc}$  = Acceptable (average) number of Collisions/Day.  $CPD_{acc}$  has to be specified by municipal authorities or the FAA. For now, we arbitrarily take 10 collisions/year =  $2.74 \times 10^{-2}$  collisions/day =  $CPD_{acc}$ . 500 collisions/day is too large by a factor of  $2 \times 10^4$ , so the collision-avoidance capability of smart drones has a daunting task: to reduce the collision rate by at least that factor. One could say that smart drones will be tested 500 times per day and must avoid a collision (almost) every time.

Complex systems subject to occasional failures can be evaluated by their Reliability,  $r$ , defined in this case as:

$$r = \text{DPD}/\text{LPD} = (\text{LPD} - 2\text{CPD})/\text{LPD} = 1 - 2(\text{CPD}/\text{LPD}),$$

where:

DPD = the number of drone deliveries per day that are successful in the sense that not only is the delivery consummated but the drone successfully returns to its base, and

LPD = the total number of drone launches per day, and

CPD = the number of drone-drone collisions per day.

The factor of 2 enters because for every encounter/collision 2 drones are lost. (We ignore drone failures not due to drone-drone collisions.)  $r$  is the probability that a single launch will result in a successful delivery. Note that  $0 \leq r \leq 1$ , with  $r = 1$  corresponding to zero encounters and zero collisions: every launched drone makes its delivery and returns to its base. DPD = 150,000 assumes  $r = 1$ . More precisely, we take LPD = 150,000 with DPD somewhat less, since, actually  $r < 1$ . LPD = 150,000 and CPD<sub>acc</sub> =  $2.74 \times 10^{-2}$  collisions/day, requires a Reliability of  $r = 0.9999996$ , six nines. For comparison, ISO standard 26262 implies a Reliability of seven nines for future self-driving cars driven one 30-mile round trip commute in the US, using the complex but familiar system of cars and highways.<sup>23,24,25</sup>

Worldwide airline fatal accident rate is one accident per 2,520,000 flights.<sup>26</sup>

For comparison, the number of dumb drone launches (= flights) per collision is 300 for LPD = 150,000. Of course, one fatal airline accident typically involves multiple fatalities and so is far more serious than a drone-drone collision that merely (one hopes) drops debris on a city rooftop. Further, such a collision is really two drone accidents; launches per drone accident is actually 150.

Continuing the case of LPD = 150,000 over New York City (now distinguishing between LPD and DPD), one readily calculates that the drone density  $N = 35.4$  drones/km<sup>3</sup> and that the average separation between drones is 530 m, large compared to the 2 m drone diameter. The second from leftmost column of Table 1 summarizes the numbers for the LPD = 150,000 case.

The 500 collisions per day in that table is based on the assumption that, of New York City's 1.5 million packages being delivered per day<sup>22</sup>, 10% are attempted via drone launch. Amazon's Jeff Wilke stated that far more than 10%, up to 90%, could be drone delivered<sup>5,7,8</sup>. Not restricting ourselves to 10%, we have:

$$\text{LPD} = (\lambda \text{ deliveries/hour}) \times 18 \text{ hours/day, and} \quad (5)$$

$$\text{CPD} = (\text{LPD}) \times d / 2\text{MFP} = (36\pi d a^2 W_s \lambda^2) / V \quad (6)$$

where the 2 in the denominator of Equation 6 eliminates double counting.

Note that CPD varies as  $\lambda^2$ , so that for 50% of packages drone-delivered, CPD will be 25 times greater than for 10%, i.e.,  $25 \times 500 = 12,500$  collisions per day. With increasing  $\lambda$  the cumulative km traveled increases and the MFP decreases; hence  $\lambda$  to the second power.

5            A drone delivery system that delivers only 10% of the packages needing delivery seems hardly worth the investment. 50% makes more sense. The third from the leftmost column in Table 1 shows entries for 50%, the fourth from the leftmost entries for 90%. Note that one launch equals one attempted delivery.

10          Table 1. Case of 1.5 million deliveries/day in New York City (Ref. 22), including both street-level and non-street-level deliveries, the latter by dumb drones. Deliveries by drone are attempted deliveries, i.e., launches; not all attempted deliveries are successful.

% Deliveries/day by drone	10	50	90
# Launches/day = LPD	150,000	750,000	1,350,000
Drone collisions/day	500	12,500	40,500
Improvement factor required to bring collision rate down to $CPD_{acc.} = 2.74 \times 10^{-2}$ collisions/day	$2 \times 10^4$	$4.6 \times 10^5$	$1.5 \times 10^6$
Reliability, $r$ , required to meet $2.74 \times 10^{-2}$ collisions/day	0.9999996	0.99999993	0.99999996

Drone launches per collision	300	60	33
Drones/km <sup>2</sup> of ground surface	3.5	18	32
Average separation between drones (m)	530	240	180

What about the future? “FedEx expects the U.S. parcel market alone to double in size to more than 100 million packages per day by 2026, with e-commerce a significant driver of accelerating volumes.”<sup>16</sup>. New York City’s population is 2.6% of the U.S. population, so can be expected to have roughly 2.6 million deliveries/day (vs. the 1.5 million from Ref. 22) by 2026. Assuming again 10% of these NYC deliveries are by drone, yields 1,600 drone collisions/day in 2026; 40,000 at 50% drone delivery, and 130,000 at 90% drone delivery. See Table 2, which shows the same variables as Table 1, but for 2.6 million deliveries per day in New York City.

Table 2. Case of 2.6 Million Deliveries/Day in New York City, including both street-level and non-street-level deliveries, the latter by dumb drones. Deliveries by drone are attempted deliveries, i.e., launches; not all attempted deliveries are successful.

% Deliveries/day by drone	10%	50%	90%
# Launches/day = LPD	260,000	1,300,000	2,340,000

Drone collisions/day	1,600	40,000	130,000
Improvement factor required to bring collision rate down to $CPD_{acc.} = 2.74 \times 10^{-2}$ collisions/day	$6 \times 10^4$	$1.5 \times 10^6$	$4.7 \times 10^6$
Reliability, $r$ , required to meet $2.74 \times 10^{-2}$ collisions/day	0.9999998	0.99999996	0.99999998
Drones launches per collision	163	33	18
Drones/km <sup>2</sup> of ground surface	6	31	55
Average separation between drones (m)	400	185	140

Tables 1 and 2 list estimates of the average separation between drones. These estimates depend only on the spatial density of drones, not on their collision rate.

Thus, even for zero collisions/day the average separation will be 140 m in the

5 Table 2, 90% case. Migrating geese watch out!

## Discussion

For a pandemic like the coronavirus, it is useful to consider the alternative of doing nothing to counter its spread in order to craft a proportionate response. And

so it is with drone collisions. We have investigated the do-nothing-to-avoid-collisions case of “dumb” drones – drones with no collision-avoidance capability, like large molecules – and found that that alternative yields collision rates that are far too high to be acceptable, rates ranging from 300 to 130,000 collisions per day for plausible scenarios. For the future potential of delivery drones to be realized it is imperative that the actual drones to be deployed be “smart”, so as to bring the collision rate down to zero or near zero. How to do this? By planning, by design for fail-safe operation, by testing, by doing simulations, by Reliability Analyses, including Failure Mode Effects and Criticality Analysis (FMECA) and Fault Tree Analysis (FTA), and finally, by Software Reliability Assessment (SRA). The FAA must insist that these measures be applied with sufficient intensity that long-term safe drone delivery operation is assured.

Hardware faults, software bugs, and cyberattacks could lead to multiple collision incidents, perhaps even system-wide failure. The drone industry has to be very careful because even a few collisions could arouse the public against drones, leading to limits on their use. It is clear from the dumb drone collision catastrophe we have predicted that the FAA must insist great rigor and thoroughness in the development of safe, smart delivery drones.

## Methods

Our method was to use the Kinetic Theory of Gases and Queueing Theory to estimate the drone-drone collision frequency in an extreme case: over New York City with no collision-avoidance capability implemented.

## 5 Data availability

All data used can be found from the References below.

## Code availability

No code was used other than functions in Excel.

## References

- 10 1. Goasduff, L. Why flying drones could disrupt mobility and transportation beyond COVID-19. *Smarter With Gartner*, May 19 (2020)  
[https://www.gartner.com/smarterwithgartner/why-flying-drones-could-disrupt-mobility-and-transportation-beyond-covid-19/?utm\\_medium=social&utm\\_source=twitter&utm\\_campaign=SM\\_GB\\_YOY\\_GTR\\_SOC\\_SF1\\_SM-SWG-CV&utm\\_content=&sf234174535=1](https://www.gartner.com/smarterwithgartner/why-flying-drones-could-disrupt-mobility-and-transportation-beyond-covid-19/?utm_medium=social&utm_source=twitter&utm_campaign=SM_GB_YOY_GTR_SOC_SF1_SM-SWG-CV&utm_content=&sf234174535=1)  
15
2. Shoolman, A. Drone delivery – Is it a good idea? *Drones monthly magazine*.  
<https://www.dronesmonthly.com/drone-delivery-is-it-a-good-idea/>

3. Stolaroff, J. K., Samaras, C., O'Neill, E. R., Lubers, A., Mitchell, A. S. & Ceperley, D. Energy use and life cycle greenhouse gas emissions of drones for commercial package delivery. *Nature Communications* **9**, Article Number: 409 (2018). <https://www.nature.com/articles/s41467-017-02411-5#Tab2>.

5

4. [Wilke, J. Keynote talk about drone delivery to homes, at re:MARS 2019, June 6, 2019](#)

5. Wilke, J. A drone program taking flight. *The Amazon blog: dayone*, June 5, 2019 <https://blog.aboutamazon.com/transportation/a-drone-program-taking-flight>

10

6. Lardinois, F. A first look at Amazon's new delivery drone. *Tech Crunch*, June 5, 2019. <https://techcrunch.com/2019/06/05/a-first-look-at-amazons-new-delivery-drone/>

7. Vincent, J. & Gartenberg, C. Here's Amazon new transforming Prime Air delivery drone. *The Verge*, June 5, 2019. <https://www.theverge.com/2019/6/5/18654044/amazon-prime-air-delivery-drone-new-design-safety-transforming-flight-video>

15

8. Alphabet's Wing brings door-to-door drone delivery to the U.S. *Inside Unmanned Systems*, October 25, 2019.  
<https://insideunmannedsystems.com/alphabets-wing-brings-door-to-door-drone-delivery-to-the-u-s/>
- 5 9. Laris, M. Wing, FedEx and Walgreens to launch free drone delivery pilot program in Virginia. *The Washington Post*, September 19, 2019.  
[https://www.washingtonpost.com/local/trafficandcommuting/wing-fedex-and-walgreens-to-launch-free-drone-delivery-pilot-next-month-in-virginia/2019/09/19/d3f5e62a-da46-11e9-a688-303693fb4b0b\\_story.html](https://www.washingtonpost.com/local/trafficandcommuting/wing-fedex-and-walgreens-to-launch-free-drone-delivery-pilot-next-month-in-virginia/2019/09/19/d3f5e62a-da46-11e9-a688-303693fb4b0b_story.html)
- 10 10. Walgreens will be first retailer in U.S. to test on-demand drone delivery service with Wing. *Business Wire*, September 19, 2019.  
<https://seekingalpha.com/pr/17638273-walgreens-will-first-retailer-u-s-test-demand-drone-delivery-service-wing>.
- 15 11. Zaveri, M. Wing, owned by Google's Parent Company, gets first approval for drone deliveries in U.S. *The New York Times*, April 23, 2019

<https://www.nytimes.com/2019/04/23/technology/drone-deliveries-google-wing.html?searchResultPosition=2>

12. Kim, A. Delivery drones may finally become a reality in the US skies.

*DOGOnews*, October 30, 2019.

5

<https://www.dogonews.com/2019/10/30/delivery-drones-may-finally-become-a-reality-in-the-us-skies>

13. McNabb, M. UPS announces FAA certification to operate drone airline:

“History in the Making”. *DroneLife*, October 1, 2019.

10

<https://dronelife.com/2019/10/01/ups-announces-faa-certification-to-operate-drone-airline-history-in-the-making/>

14. Fraser, B. Press Release – U.S. Transportation Secretary Elaine L. Chao Announces

FAA Certification of UPS Flight Forward as an Air Carrier. *FAA*, October 1, 2019.

[https://www.faa.gov/news/press\\_releases/news\\_story.cfm?newsId=24277](https://www.faa.gov/news/press_releases/news_story.cfm?newsId=24277)

15. Saunders, Mark. Uber Eats to start San Diego drone deliveries in 2020.

15

*ABC 10 News, San Diego*, October 30, 2019.

<https://www.10news.com/news/local-news/uber-eats-to-start-san-diego->

[drone-deliveries-in-2020](#)

16. Frantz, G. Parcel carriers evolve as e-commerce explodes. *DCVELOCITY*, June 10, 2019. <https://www.dcvelocity.com/articles/20190610-parcel-carriers-evolve-as-e-commerce-explodes/>.

5 17. Sears, F. W. *Thermodynamics, The Kinetic Theory of Gases, and Statistical Mechanics* (Addison-Wesley Publishing, 1955).

18. Dorr, L. Fact Sheet – Small Unmanned Aircraft Regulations (Part 107). *FAA*, July 23, 2018. [https://www.faa.gov/news/fact\\_sheets/news\\_story.cfm?newsId=22615/](https://www.faa.gov/news/fact_sheets/news_story.cfm?newsId=22615/)

10 19. New drone bill would let states, not federal government, set drone rules in the USA. *Unmanned Airspace*, October 17, 2019. <https://www.unmannedairspace.info/emerging-regulations/new-drone-bill-would-let-states-not-federal-government-set-drone-rules-in-the-usa/>

20. *The Physics Textbook/ Area of New York City*. *U.S. Census Bureau*, May

27, 2002. <https://hypertextbook.com/facts/2002/JordanLevine1.shtml>

21. Hillier, F. S. & Lieberman, G. J. *Introduction to Operations Research* (Holden-Day, Inc., 1967).

22. Haag, M. & Hu, W. 1.5 Million Packages a Day: The Internet Brings  
5 Chaos to N. Y. Streets. *The New York Times*, page A1, October 27, 2019.  
<https://www.nytimes.com/2019/10/27/nyregion/nyc-amazon-delivery.html?searchResultPosition=1>.

23. 50+ Car Accident Statistics in the U.S. & Worldwide. *The Wandering RV*.  
10 October 17, 2019. <https://www.thewanderingrv.com/car-accident-statistics/>

24. U.S. Dept. of Transportation. U.S. Vehicle Miles. *Bureau of Transportation Statistics*. January 30, 2020.  
<https://www.bts.gov/content/us-vehicle-miles>

25. Gupta, M. Self-Driving Cars: Reliability Challenges, Solutions, and Social

Adoption. *DesignNews*, June 22, 2018.

<https://www.designnews.com/electronics-test/self-driving-cars-reliability-challenges-solutions-and-social-adoption/87842508158921>.

26. Ranter, H. Aviation Safety Network releases 2018 airliner accident

5

statistics. *Aviation Safety Network*, January 1, 2019. <https://news.aviation-safety.net/2019/01/01/aviation-safety-network-releases-2018-airliner-accident-statistics/>

10

**ACKNOWLEDGEMENTS:** For helpful discussions I thank Dr. Gastone Celisia, Wayne Davidson, and Frank Straka.

This research was supported by Ergon Institute LLC.

15

## **SUPPLEMENTARY INFORMATION**

Tables 1 and 2 include values for the “Average separation between drones”.

We present here the procedure by which those estimates were calculated:

A. The  $m^2$  of ground surface per drone was obtained by taking the inverse of the “Drones/ $km^2$  of ground surface” values in Tables 1 and 2, then converting to  $m^2$ . We define a vertical cell extending from the bottom of the drone fly zone (22 m above ground) to the top, 100 m higher, and having a square horizontal cross section of area  $L^2 =$  the  $m^2$  of ground surface per drone,  $L$  being the length of each side of the square.  $L$  is also the horizontal distance between vertical cell centerlines.

B. Taking the single drone in each cell to be on average located on the cell centerline, and the cells to be non-overlapping (for greatest separation) we have, for the average separation,  $Z_{av}$ :

$$Z_{av} = (1/H)^2 \int_{x=0}^{x=100} \int_{y=0}^{y=100} [L^2 + (y - x)^2]^{1/2} dy dx,$$

where  $H = 100$  m,  $x =$  vertical distance from cell base to first drone, and  $y =$  vertical distance from base of adjacent cell to drone in that cell.

C. Using the transformation,  $(y - x) = L \tan \theta$ , and integrating over  $y$ , results in:

$$Z_{av} = (1/2)(L/H)^2 \int_{x=0}^{x=100} \left[ \frac{\sin \theta}{\cos^2 \theta} + \ln \left| \tan \left( \frac{\theta}{2} + \frac{\pi}{4} \right) \right| \right]_{\theta=\arctan\{-\frac{x}{L}\}}^{\theta=\arctan\{\frac{100-x}{L}\}} dx$$

The integral over  $x$  was done numerically, yielding the average separation between drone numbers shown in Tables 1 and 2.

## Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [SUPPLEMENTARYINFORMATION4B.docx](#)