Self-Delayed Feedback Based Car Following Control With Velocity Uncertainty of Preceding Vehicle On Gradient Roads

Cong Zhai (✉ 957083516@qq.com)

South China Normal University

Weitiao Wu

South China University of Technology <https://orcid.org/0000-0001-9047-0812>

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Self-delayed feedback based car following control with velocity uncertainty of preceding vehicle on gradient roads

Cong Zhai^{1,2} ; Weitiao Wu^{1*}

(1. School of Civil Engineering and Transportation, South China University of Technology, Guangzhou 510641 Guangdong China

2. School of Transportation and Civil Engineering and Architecture, Foshan University, Foshan 528000 Guangdong China)

Abstract: Uphill and downhill roads are prevalent in mountainous areas and freeways. Despite the advancement of vehicle-to-vehicle (V2V) communication technology, the driving field of vision is still largely limited under such a complex road environment, which hinders the sensors accurately perceiving the speed of the front vehicle. As such, a fundamental question for autonomous traffic management is how to control traffic flow associated with the velocity uncertainty of preceding vehicles? This paper seeks to develop a controlling framework for corporative car following control under such complex road environment. To this end, we first propose a traffic flow model accounting for the uncertainty effect of preceding vehicles velocity on gradient road. We further design a new self-delayed feedback controller based on the velocity and headway difference between the current time step and historical time step, in an aim to enhance the robustness of traffic flow. The sufficient condition where traffic jam does not occur is derived from the perspective of the frequency domain via Hurwitz criteria and H_∞ norm of transfer functions. The bode diagram reveals that the robustness of closed-loop traffic flow model has been significantly enhanced. We also conduct simulations to verify the theoretical analysis.

Keywords: Car following model; Self-delayed feedback controller; Gradient road; Frequency domain; Stability

1. Introduction

Over the past decades, the rapid development of urban economy has spurred the growth of car ownership and traffic congestion. Along with this, a variety of approaches and countermeasures have been proposed to alleviate traffic congestion by making use of emerging technology. On one hand, numerous traffic management and planning approaches have emerged, such as bus route optimization tactics [1-2] and online electric vehicle charge scheduling [3]. On the other hand, another stream of research direction mainly concentrated on the development of traffic flow models to reproduce traffic congestion and explore the underlying mechanism. Motivated by the advancement in V2V communication technology, this paper devises a self-delayed feedback control for autonomous traffic considering uncertain velocity of preceding vehicle on gradient road. The autonomous traffic is modeled as a car-following model with a designed controller.

According to the research subjects, current traffic flow theoretical models can generally be divided into following two branches: macroscopic and microscopic traffic flow models. The former mainly includes the continuous models [4-8] and lattice hydrodynamic models [9-15], while the latter includes the car-following models [16-18] and the cellular automata models

¹ Corresponding Author: ctwtwu@scut.edu.cn

[19-20]. In the former category, a number of en-route vehicles are modeled as compressible continuous fluid with the time- and location-dependent vehicle density and average speed. By constructing the partial differential equations of local density and local velocity, the dynamic behavior of traffic flow can be revealed by solving the equations. Although such type of models is less computationally expensive, it cannot analyze the microscopic driving behavior of the vehicle. In contrast, the latter takes individual vehicles as the research subject, and establishes differential equations based on the speed, headway and acceleration information of vehicles to study the vehicle motion behavior. In 1995, a seminal work of car-following model were put forward by Bando, termed optimal velocity (OV) model^[21]. The optimal velocity function has an inflection point and elegant mathematical and physical properties, which facilitates linear and nonlinear theoretical derivation. Since then, the OV model has attracted wide attentions, yielding a number of extended and more realistic traffic flow models. The representative works can be referred to Table 1.

Table 1 An summary of OV model research in recent years

Author	Characteristics	Remark
Jiao	Collision sensitivity	literature [22]
Yu, Zhao	Acceleration	literatures [23-24]
Xu	Asymmetric optimal velocity	literature [25]
Sun, Jin, Sun	Average speed	literatures [26-28]
Ma, Ma, Chen	Backward looking	literatures [29-32]
Li, Sun, Li, Yan	Electronic throttle angle	literatures [33-36]
Hou, Zhou, Zheng	Visual angle	literatures [37-39]
Li, Li, Jin, Li	Non-lane-discipline	literatures [40-43]
Tang	Driver's bounded rationality	literatures [44-45]
Zhai, Song, Liu	Traffic jerk	literatures [46-48]
Tang, Zhang	Traffic interruption probability	literatures [49-50]
Li, Kuang	Multiple effect	literatures [51-52]
Tang, Kuang	Honk effect	literatures [53-54]
Zhai, Sun	Road condition	literatures [55-56]
Cao, Yu, Wang	Memory effect	literatures [57-59]
Tang, Wang, Zhou	Anticipation effect	literatures [60-62]
Yu, Li	Delay effect	literatures [63-64]
Peng, Tang	Driver's characteristic	literatures [65-66]
Wang, Sun	Driver's desire	literatures [67-68]

In practice, the gradient roads are not uncommon in the economically underdeveloped areas or mountainous freeways, as shown in Fig.1. The road slope may exert great influence on road traffic jam [69-70]. Despite the development of Internet of Vehicles, the driving field of vision is still limited in a traffic environment with gradient roads, which hinders the sensors perceiving the speed of the front vehicle accurately. Therefore, the velocity uncertainty of preceding vehicles should not be neglected in such complex road environment. However, to date, the existing studies on traffic flow assume that the current vehicle can accurately receive the headway or velocity information of preceding vehicles. Motivated by this fact, there is an increasing need to devise systematic traffic flow control algorithm for such traffic environment. To this end, we first propose an extended traffic flow model accounting for the effect of velocity uncertainty of preceding vehicle on the gradient road. It proceeds to design a new self-delayed feedback controller without relying on any extra traffic information. The stability criterion of closed-loop system is also derived.

The structure of this paper is as follows: In Section 2, a modified car following model taking into account the effect of velocity uncertainty of preceding vehicle on gradient road is presented. In Section 3, the stability condition of closed-loop car following model is obtained via Hurwitz criterion and H_∞ norm of transfer functions. In Section 4, some numerical examples are conducted out to verify the conclusions of the above theoretical analysis, finally, the conclusions and follow-up research works of this paper are given in Section 5.

Fig.1 Common gradient road in urban and mountain area, (a) Hebei 66th highway; (b) the 318th highway in Guizhou Province; (c) Continuous downhill sign along the highway; (d) Continuous uphill sign along the highway

2. Model

In 1995, Bando [21] proposed an OV model to study the car following characteristic of vehicles on a single lane where the overtaking is forbidden, and the dynamic equation is expressed as:

$$\frac{dv_n(t)}{dt} = a[V(y_n(t)) - v_n(t)] \quad (1)$$

$$\frac{dy_n(t)}{dt} = v_{n+1}(t) - v_n(t) \quad (2)$$

where $v_n(t)$ and $v_{n+1}(t)$ respectively represent the instantaneous velocity information of current vehicle n and preceding vehicle $n+1$ at time t . a represents the sensitivity of the driver. $y_n(t)$ represents the instantaneous headway of the target vehicle, $V(g)$ represents the optimal velocity function, which is uniquely determined by the parameter $y_n(t)$, specifically

$$V(y) = \frac{v_{\max}}{2} [\tanh(y - y_c) + \tanh(y_c)] \quad (3)$$

where $v_{\max}$ and y_c respectively represent the maximum speed and safe distance.

In order to eliminate the phenomenon of the excessive acceleration and unreasonable deceleration in the traditional optimal velocity model, Helbing and Tilch [71] presented an improved generalized force (GF) model expressed as follows:

$$\frac{dv_n(t)}{dt} = a[V(y_n(t)) - v_n(t)] + \lambda H(-\Delta v_n(t)) \Delta v_n(t) \quad (4)$$

where $\Delta v_n(t)$ represents the velocity difference between current vehicle n and preceding vehicle $n+1$, and $\Delta v_n(t) = v_{n+1}(t) - v_n(t)$. λ is the corresponding weight coefficient, $H(g)$ represents the Heaviside function, and

$$H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases} \quad (5)$$

Later, in order to overcome the low starting wave speed in above-mentioned generalized force model, Jiang et al. [72] presented a full velocity difference (FVD) model, which is written as follows:

$$\frac{dv_n(t)}{dt} = a[V(y_n(t)) - v_n(t)] + \lambda \Delta v_n(t) \quad (6)$$

Since the FVD model, extensive derivative models are developed. considering that uphill and downhill roads are prevalent in mountainous areas and freeways, the literature [69] proposed a traffic flow model considering road slope information to study the impact of road slope information on traffic jam, and the kinetic equation is as follows

$$\frac{dv_n(t)}{dt} = a[V^{op}(y_n(t)) - v_n(t)] + \lambda \Delta v_n(t) \quad (7)$$

where $V^{op}(g)$ represents the modified optimal velocity function constrained by the slope information, and the function is taken as follows:

$$V^{op}(y) = \frac{v_{f,\max} - v_{g,\max}(\theta)}{2} [\tanh(y - y_c(\theta)) + \tanh(y_c(\theta))] \quad (8)$$

where $v_{g,\max}(\theta)$ and $y_c(\theta)$ respectively represent the speed adjustment and safety distance item affected by the acceleration of gravity. $v_{f,\max}$ is the maximum velocity without any slope. Fig. 2 shows a schematic diagram of the force of the vehicle on the uphill and downhill scenarios. The specific value of $v_{\max}(\theta)$ of can be determined by

$$v_{\max}(\theta) = \frac{mg \sin \theta}{\mu} \quad (9)$$

where m and g respectively represent the mass of the vehicle and the acceleration of gravity, i.e. $g = 9.8m/s^2$. μ is the friction coefficient, to simplify the analysis we set $\mu = mg$. Since the safety distance differs in the uphill ($\theta > 0$) and downhill ($\theta < 0$) scenarios [69], we have that

$$y_c(\theta) = y_c(1 - \eta \sin \theta) \quad (10)$$

where h_c is the safety distance of vehicles on the general road without any slope information, ξ is constant, which is taken as $\xi = 1$ to simplify the analysis. As a result, the corresponding optimal speed function can be rewritten as

$$V^{op}(\rho) = \frac{v_{f,\max} - \sin \theta}{2} V_0(\rho) \quad (11)$$

where

$$V_0(\rho) = \tanh(y - y_c(\theta)) + \tanh(y_c(\theta)) \quad (12)$$

Fig.2 Schematic diagram of vehicle forces on different scenarios; (a) uphill; (b) downhill

To date, the existing studies on traffic flow assume that the current vehicle can accurately receive the traffic information of preceding vehicle. However, as discussed previously, the driving field of vision will be still limited in a traffic environment with gradient roads even though the V2V technology is in use, such that the sensors are difficult to accurately perceive the speed of front vehicle. In other words, certain errors of perceived speed may exist between the vehicle ahead and the real data. As a result, there is an imminent need to incorporate the velocity uncertainty of preceding vehicles in the design of car following control under such complex road environment. In view of this, we introduce the perception error term of preceding vehicle on Eq. (7), yielding a new traffic flow model as follows:

$$\frac{dv_n(t)}{dt} = a[V^{op}(y_n(t)) - v_n(t)] + a\lambda[(1 + \varepsilon)v_{n+1}(t) - v_n(t)] + u_n(t) \quad (13)$$

where $u_n(t)$ is the designed control input. ε is the error coefficient representing the difference between the perceived data and real data; if $\varepsilon > 0$, then the error is positive; otherwise, the error is negative; here we assume that $\|\varepsilon\| < 1$. When $\varepsilon = 0$, the proposed model is consistent with the literature [69]. Therefore, proposed new traffic flow model can be regarded as a generalized form of previous research.

3. Control scheme for the car-following model

In order to improve road traffic efficiency, we propose a new self-delayed feedback control for the car-following model. This control scheme only relies on the information of velocity and headway difference between the current time and historical time, specifically

$$u_j(t) = k_1(y_j(t) - y_j(t - \tau_1)) + k_2(v_j(t) - v_j(t - \tau_1)) \quad (14)$$

where τ_1 is delay time. k_1 and k_2 are the weight coefficients corresponding to the headway and the speed difference, respectively.

Remark 1: The traditional linear feedback controller, with the vehicle speed difference or vehicle spacing difference of successive vehicles as the control input, requires accurate road traffic information. In the current underdeveloped traffic perception environment, the speed accuracy of preceding vehicle may be limited, such that the designed controller based on such information will greatly affect the control performance. On the contrary, the self-feedback controller Eq. (14) only relies on the speed and headway difference between the current time and the previous historical time, without additional external traffic information. As such, it can handle the issue of velocity uncertainty effect. In other words, the proposed controller Eq. (14) is particularly suitable for the

current underdeveloped Internet of Vehicles environment.

To obtain the range of control gain coefficients k_1 and k_2 , we perform stability analysis on the closed-loop traffic flow model under the feedback controller (14) based on the traditional control theory. Firstly, the steady state of the model is

$$\begin{bmatrix} y_j, v_j \end{bmatrix}^T = \begin{bmatrix} y^*, v^* \end{bmatrix}^T \quad (15)$$

where $y^* = h$, $v^* = \frac{V^{op}(h)}{1 - \lambda \varepsilon}$.

Let $\delta v_j = v_j - v^*$ and $\delta y_j = y_j - y^*$, substituting the above items into Eq. (13) and linearizing this equation yields the following equations:

$$\begin{cases} \frac{d\delta v_n(t)}{dt} = a[\Lambda \delta y_n(t) - \delta v_n(t)] + a\lambda[(1 + \varepsilon)\delta v_{n+1}(t) - \delta v_n(t)] + \\ k_1(\delta y_j(t) - \delta y_j(t - \tau_1)) + k_2(\delta v_j(t) - \delta v_j(t - \tau_1)) \\ \frac{d\delta y_n(t)}{dt} = \delta v_{n+1}(t) - \delta v_n(t) \end{cases} \quad (16)$$

where $\Lambda = \left(\frac{v_{\max} - \sin \theta}{2} \right) V_0'$, $V_0' = \frac{dV_0(\rho)}{d\rho} \Big|_{\rho=\rho_0}$, $\delta y_{j+1}(t - \tau_1) = y_{j+1}(t - \tau_1) - y_j^*$,

$\delta v_{j+1}(t - \tau_1) = v_{j+1}(t - \tau_1) - v_j^*$.

Applying the Laplace transform to the above formula, we have that

$$\begin{cases} sV_n(s) - V_n(0) = a[\Lambda Y_n(s) - V_n(s)] + a\lambda[(1 + \varepsilon)V_{n+1}(s) - V_n(s)] + \\ k_1(1 - e^{-s\tau_1})Y_n(s) + k_2(1 - e^{-s\tau_1})V_n(s) \\ sY_n(s) - Y_n(0) = V_{n+1}(s) - V_n(s) \end{cases} \quad (17)$$

where $P_j(s) = L(\rho_j)$ and $Q_j(s) = L(q_j)$. $L(\mathfrak{g})$ represents the Laplace transform. s is the complex variable.

By transforming Eq. (17) to eliminate the intermediate variable $Y_n(s)$, then the transition function of is

$$G(s) = \frac{a\Lambda + a\lambda(1 + \varepsilon)s + k_1(1 - e^{-s\tau_1})}{d(s)} \quad (18)$$

where $d(s)$ represents the characteristic polynomial, and

$$d(s) = s^2 + a(1 + \lambda)s + a\Lambda + k_1(1 - e^{-s\tau_1}) - k_2(1 - e^{-s\tau_1})s.$$

Lemma 1^[52]: According to the classical control theory, the traffic flow is stable and the traffic jam phenomenon will disappear if the following conditions are established simultaneously:

I) $d(s)$ is stable;

II) $\|G(s)\|_{\infty} \leq 1$;

For condition (I), to sort out the characteristic polynomial $d(s)$ we have

$$d(s) \equiv P(s) + Q(s)e^{-s\tau_1} \quad (19)$$

where $P(s) = s^2 + (a + a\lambda - k_2)s + a\Lambda + k_1$, $Q(s) = k_2s - k_1$.

Furthermore, the formula (19) is written as follows:

$$\begin{aligned}
F(w) &= [\operatorname{Re} P(iw)]^2 + [\operatorname{Im} P(iw)]^2 - [\operatorname{Re} Q(iw)]^2 - [\operatorname{Im} Q(iw)]^2 \\
&= (-w^2 + a\Lambda + k_1)^2 + (a + a\lambda - k_2)^2 w^2 - k_1^2 - k_2^2 w^2 \\
&= w^4 + m_1 w^2 + m_2
\end{aligned} \tag{20}$$

where $m_1 = (a + a\lambda)^2 - 2a(1 + \lambda)k_2 - 2(a\Lambda + k_1)$, $m_2 = a^2\Lambda^2 + 2a\Lambda k_1$.

Based on the literature [1], if the following conditions are met, then the critical function $F(w)$ has no real roots, specifically

$$m_1 \geq 0, m_2 \geq 0 \text{ or } m_1 < 0, m_1^2 - 4m_2 < 0. \tag{21}$$

Substituting m_1 and m_2 into the formula (21), we have that

$$a(1 + \lambda)k_2 + k_1 \leq \frac{(a + a\lambda)^2}{2} - a\left(\frac{v_{\max} - \sin \theta}{2}\right)V_0', \quad k_1 \geq \frac{-a(v_{\max} - \sin \theta)V_0'}{4} \tag{22}$$

or

$$\begin{aligned}
a(1 + \lambda)k_2 + k_1 &> \frac{(a + a\lambda)^2}{2} - a\left(\frac{v_{\max} - \sin \theta}{2}\right)V_0', \\
\left[(a + a\lambda)^2 - 2a(1 + \lambda)k_2\right]^2 + 4k_1^2 &< 4\left[(a + a\lambda)^2 - 2a(1 + \lambda)k_2\right]\left[a\left(\frac{v_{\max} - \sin \theta}{2}\right)V_0' + k_1\right]
\end{aligned} \tag{23}$$

In summary, if condition (22) or (23) is hold, then the characteristic polynomial $d(s)$ satisfy stable. Figs. 3 and 4 show the delay-independent stable region of $d(s)$ in phase diagram (k_1, k_2) under downhill and uphill scenarios. The shaded region represents the set of parameters combination where characteristic polynomial $d(s)$ is guaranteed stable.

For condition (II), we have that

$$\left\{ \begin{aligned} \|G(s)\|_{\infty} &= \sup_{w \in [0, \infty)} |G(iw)| \leq 1 \\ |G(iw)| &= \sqrt{G(iw)G(-iw)} \\ &= \sqrt{\frac{[a\Lambda + k_1(1 - \cos(w\tau_1))]^2 + [a\lambda(1 + \varepsilon)w + k_1 \sin(w\tau_1)]^2}{[-w^2 + a\Lambda + k_1(1 - \cos(w\tau_1)) + k_2 w \sin(w\tau_1)]^2 + [a(1 + \lambda)w + k_1 \sin(w\tau_1) - k_2 w(1 - \cos(w\tau_1))]^2}} \leq 1 \end{aligned} \right. \tag{24}$$

Therefore, if the inequality $|G(iw)| \leq 1$ is hold for any $w \in [0, +\infty)$, then the following condition is met, specifically,

$$\begin{aligned}
&[-w^2 + a\Lambda + k_1(1 - \cos(w\tau_1)) + k_2 w \sin(w\tau_1)]^2 + [a(1 + \lambda)w + k_1 \sin(w\tau_1) - k_2 w(1 - \cos(w\tau_1))]^2 - \\
&[a\Lambda + k_1(1 - \cos(w\tau_1))]^2 - [a\lambda(1 + \varepsilon)w + k_1 \sin(w\tau_1)]^2 \geq 0
\end{aligned} \tag{25}$$

Theorem 1: Based on the traffic flow model (13), for given parameters λ , k_1 , k_2 and τ_1 , if the conditions (22) (or (23)) and (25) are met, then traffic congestion can be effectively suppressed under the delay feedback controller (14).

Figs. 5 and 6 present the amplitude change of the transfer function under different key parameters k_1 , k_2 and τ_1 for the designed controller (14), where Fig. 5 and Fig. 6 correspond to downhill and uphill scenario, respectively. As shown in Fig. 5(a), the amplitude of the transfer function $|G|$ is effectively reduced as the parameters k_1 , k_2 increase; when $k_1=2.5$ and $k_2=2.5$, the theorem 1 is satisfied, the amplitude vertex of the transition function is less than 1. Therefore, the

designed controller (14) is conducive to enhance the robustness of traffic flow. Fig. 5(b) compares the amplitude of the transfer function $|G|$ under different delay time τ_1 . When $\tau_1=0.5$, the amplitude of the bode curve is greater than 0, and the traffic disturbance will exacerbate over time and induce traffic jam phenomenon. As the parameter τ_1 continues to increase, the amplitude of bode curve keep decreases. When $\tau_1=2.5$, the amplitude of the bode curve is approximately less than 1, which means that the traffic flow is stable. Therefore, it can be concluded that a higher value of parameter τ_1 is conducive to improving the robustness of traffic flow.

Fig.3 Delay-independent stable region of characteristic polynomial $d(s)$ on phase diagram (k_1, k_2) with $a=2$, $\theta=-5^\circ$, $\lambda=-0.1$; (a) condition (22); (b) condition (23).

Fig.4 Delay-independent stable region of characteristic polynomial $d(s)$ on phase diagram (k_1, k_2) with $a=2$, $\theta=5^\circ$, $\lambda=-0.1$; (a) condition (22); (b) condition (23).

Fig.5. Bode-plot for different parameters (a) k_1, k_2 , (b) τ_1 ($\theta = -6^\circ$)

Fig.6. Bode-plot for different parameters (a) k_1, k_2 , (b) τ_1 ($\theta = 6^\circ$)

4. Simulation example

In this section, we will perform numerical simulations under periodic boundary conditions to verify the effect of designed self-delay feedback controller on suppressing traffic congestion and reducing energy consumption. The initial position information is selected as follows:

$$x_n(0) = \begin{cases} 4, n \neq \frac{N}{2}, \frac{N}{2} + 1 \\ 4 - \delta, n = \frac{N}{2} \\ 4 + \delta, n = \frac{N}{2} + 1 \end{cases} \quad (26)$$

where the number of vehicles $N=100$, δ is the disturbance term, i.e., $\delta=0.1$, the other default parameters are selected as follows:

$$v_{\max} = 2, y_c = 4, a = 1.2, \lambda = -0.1, \varepsilon = 0.1.$$

Fig. 7 shows the spatiotemporal evolution of the headways with different controller gains coefficient k_1, k_2 under the downhill scenario. Fig. 7(a) corresponds to the traffic flow without designed controller ($k_1, k_2=0$). It shows that the headways on the road oscillate frequently with stop-and-go waves, which indicates that the traffic jam prone to occur. With the increasing of control gain coefficients k_1, k_2 , the fluctuating frequency and amplitude of density waves have been effectively reduced. Typically, when $k_1=2, k_2=2$, the initial disturbance has been completely suppressed, and the stop-and-go phenomenon disappears. Therefore, higher values of parameter k_1 ,

k_2 contributes to suppressing traffic congestion.

Fig. 8 shows the instantaneous headway distribution of all vehicles corresponding to Fig.7 at $t=10^4$ s. As we can see, the fluctuating amplitude of headways without control in Fig.8 (a) is the largest, while the fluctuating amplitude is approximately equal to 0 in Fig. 14(d) indicating that the traffic flow is stable.

Fig. 9 describes the evolution of the headway between vehicles on the road under different values of parameter τ_1 . One can see that the headway fluctuation gradually decreases over time with the increase of parameter τ_1 , which suggests that a higher value of parameter τ_1 is conducive to improving the anti-interference ability of traffic flow.

Fig. 10 shows the instantaneous headway distribution of all vehicles corresponding to Fig.9 at $t=10^4$ s. We can see that the fluctuating amplitude of headways has been effectively reduced with the increase of parameter τ_1 , and the fluctuating amplitude between vehicles can be neglected in Fig.10(d). In other words, the initial disturbance has been completely diluted when the designed controller (14) is applied.

Figs.11-14 analyze the relationship between control gain coefficients k_1 , k_2 , delay time τ_1 and traffic flow stability for the uphill scenario. With the increase of the gain coefficients k_1 , k_2 or delay time, τ_1 , the robustness of traffic flow model can be enhanced. Therefore, the designed self-delayed feedback controller is effective in suppressing traffic congestion and enhancing the anti-interference ability for both the uphill or downhill scenarios.

Fig.7. The spatio-temporal evolutions of headways under different values of parameter k_1 and k_2 after $t=10^4$ s, where (a) $k_1=0, k_2=0$; (b) $k_1=0.5, k_2=0.5$; (c) $k_1=1, k_2=1$; (d) $k_1=2, k_2=2$. ($\tau_1 = 0.1, \theta = -6^\circ$)

Fig.8. The instantaneous headway distribution between vehicles along the road corresponding to Fig.7

Fig.9. The spatio-temporal evolutions of headway under different values of parameter τ_1 after $t=10^4$ s, where (a)

Fig.10. The instantaneous headway distribution between vehicles along the road corresponding to Fig.9

Fig.11. The spatio-temporal evolutions of headway under different values of parameter k_1 and k_2 after $t=10^4$ s, where (a) $k_1=0, k_2=0$; (b) $k_1=0.5, k_2=0.5$; (c) $k_1=1, k_2=1$; (d) $k_1=2, k_2=2$. ($\tau_1 = 0.1, \theta = 6^\circ$)

Fig.12. The instantaneous headway distribution between vehicles along the road corresponding to Fig.11

Fig.13. The spatio-temporal evolutions of headway under different values of parameter τ_1 after $t=10^4$ s, where (a)

$\tau_1 = 0.1$; (b) $\tau_1 = 0.2$; (c) $\tau_1 = 0.3$; (d) $\tau_1 = 0.4$. ($k_1=0.6, k_2=0.5, \theta = 6^\circ$)

Fig.14. The instantaneous headway distribution corresponding to Fig.13

5. Conclusion

This paper proposes a new self-delayed feedback controller for the car-following model with velocity uncertainty of preceding vehicle on gradient road. The stability condition of closed-loop traffic flow model is obtained via Hurwitz criteria and H_∞ norm of transfer functions. We investigate the influence of parameters k_1 , k_2 and τ_1 on traffic jam from the uphill and downhill scenarios. Results show that as parameters of k_1 , k_2 increases, the traffic congestion can be effectively suppressed. A higher value of delay time τ_1 will decrease the fluctuation amplitude of headway, which indicates that the robustness of the traffic flow model is enhanced.

Despite the newly developed autonomous traffic flow model, there are a few strong assumptions in the current research. For example, the delay time term in designed feedback controller is constant, whereas the parameter uncertainties may be more realistic in the complex traffic environment[73]. In addition, we only verify the feasibility of designed control scheme without using real traffic data. These issues will be addressed in the follow-up research.

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