# The lifting force of an airplane wing when flying horizontally at high speeds. Explanation of the vortex trail.

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**The lifting force of an airplane wing when flying horizontally at high speeds. Explanation of the vortex trail.**

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**Abstract**

In this paper, an explanation is given of the lift force of an airplane during horizontal flight. It is shown that during a flight, five vertical forces act on the airplane: gravity; pressure gradient with a minus sign; Archimedes force; potential force and the vortex force obtained from the action minimum. The first three forces were known before. The potential force was also known from the Bernoulli equation, but its effect on the airfoil from the air had not previously been taken into account. The vortex force obtained from the minimum action in the application to a continuous medium was not taken into account in aerodynamics. In horizontal flight the vortex force is directed upwards, it compensates for the gravity of the airplane at high speed commensurate with the speed of sound. The paper provides an explanation of the vortex trail behind the airplane, mentioned in the Millennium problem Navier-Stokes equation.

**Keywords:** lifting force; airfoil; vortex trail.

## 1. Introduction.

Every time when we fly in a plane, my wife says" I can't understand why the plane is flying and not falling". I explain that at high speed in the air there is a change in the pressure that keeps the plane in flight, because the air pressure under the wing of the airplane is greater than the air pressure over the wing of the airplane. Actually, it is written in all textbooks on aerodynamics. Meanwhile it never occurred to me to doubt this simple logic.

As it turned out, this is completely erroneous reasoning. It all started with an article: "No one can explain why planes stay in the air" [1], which my friend sent me on May 10, 2020 after reading my popular article about cyclones and anticyclones and the problem of the Millennium of turbulence [2].

The paper Scientific American [1] told about the problem of describing the horizontal flight of an airplane which is more than a hundred years old. A hundred years is certainly not a thousand. On the other hand, this task could not have occurred before the first planes appeared. In addition, Albert Einstein himself had a hand in it. In 1916, he tried to solve this problem, but could not, as it is written in the paper [1]. In general, it is quite a worthy problem, with a history.

I read an article [1] and I can't understand what the problem is. Air blows the wings of the plane. The Bernoulli equations which are almost three hundred years old are fulfilled for the air. It is claimed that upon the wing of the airplane air speed is more than the speed under the wing, so the pressure under the wing more and it keeps the plane in horizontal flight.

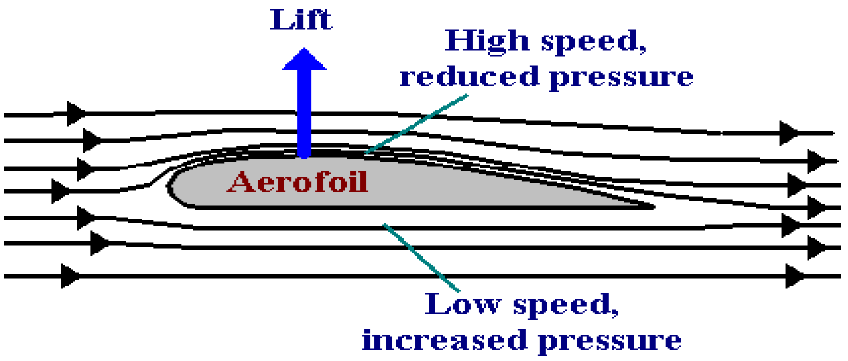


Figure 1. Movement of air around the wing of the airplane.
Incorrect distribution of air speed and pressure around the airfoil.

The author of the article writes “Here is only one problem” [1]. Air can come off the wing from above, so there is a kind of "vacuum or discharge". This is because the wing of the airplane is designed so that it reflects the air from above when flying horizontally.

Why then is the air speed over the wing of the airplane greater? After all, if the wing of an airplane chops off some of the air from above, then how can the speed of the air flowing around the wing from above be greater? Look, all the wings of airplanes have a thickening in front, just like the wings of birds in nature. We are considering horizontal flight. The plane moves evenly and rectilinearly, with the lower surface of the airplane wing lying in a horizontal plane. There is no angle of attack on the wing. The plane has already gained altitude, passengers have unbuckled their seat belts and are calmly watching TV or sleeping. Meanwhile, the plane's wing reflects the air from above, just as a hill reflects the wind in an open field.

Everyone knows that the wind is much weaker over the hill. According to the Bernoulli equation, the pressure is greater where the wind is weaker, and less where the wind is stronger. It is obvious that when flying horizontally under the wing of an airplane  the air speed is higher, so the pressure under the wing should be less. So it turns out that passengers are calmly flying and sleeping, and the air pressure is higher over the wing of the airplane and, nevertheless, the plane does not fall, but calmly flies.

There is no doubt that the air speed is lower above the airfoil. This is exactly the case where no measurements are needed. The fact that the wing of the airplane repels the air flow from above is also drawn in pictures in all textbooks on aerodynamics [3, 4].

In addition, the air really presses on the wing from above, based on the design of the airfoil, since the tangent to the airfoil has an acute angle with the horizon, and from below the air passes parallel to the airfoil and does not exert additional pressure (Fig. 2).

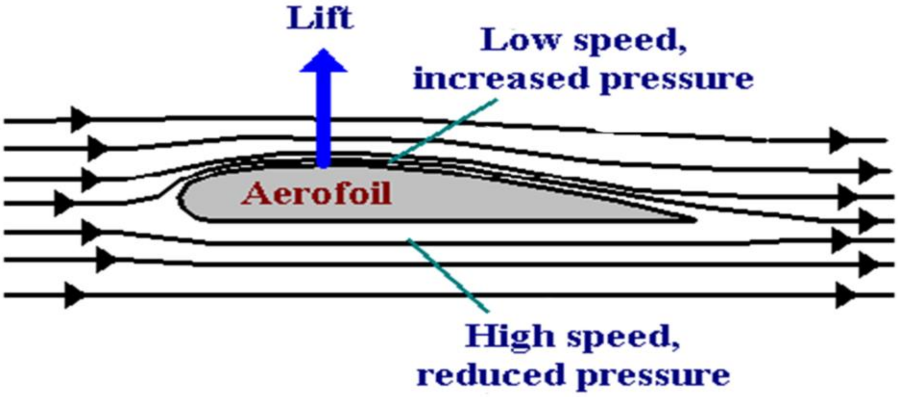


Figure 2. Distribution of air speed and pressure around the aerofoil of the airplane.

The acute angle between the tangent to the aerofoil and the horizon line at the top of the airplane wing, in fact, means that the aerofoil creates an angle of attack from above for oncoming air flow. Therefore, the pressure at the horizontal position of the airplane wing (Fig. 2) is always greater over the wing of the airplane.

Then it is not clear why the authors of the article [1] and textbooks on aerodynamics [3, 4] decided that the air pressure is greater under the wing of the airplane? Perhaps to reassure students of aviation institutes or they will refuse to fly on planes. It turns out that the difference in air pressure presses on the wing of the airplane from above (Fig. 2). Then why does the airplane not fall?

Firstly, I want to immediately reassure readers that even if the air pressure is greater over the wing of the airplane, this does not mean that the plane is affected by a downward force as a result of potential air movement around the wing of the airplane. The fact is that textbooks on aerodynamics [3,4] use an incorrect interpretation of the Bernoulli equation.

The Bernoulli equation describes a stationary potential air movement in which the sum of all forces acting on a unit volume of air is zero. Therefore, the sum of the forces acting from this volume of air is also zero. This is indicated by Euler's equations of hydrodynamics for stationary potential air movement.

Indeed, the sum of the forces acting on a unit volume of air at a steady stationary potential motion is zero, so the pressure gradient with a minus sign $-grad(P)$ is equal to the velocity gradient squared multiplied by ½ of the density $\frac{\rho}{2}grad(\upsilon^2)$, and as a result, these two forces compensate for each other: $-grad(P)-\frac{\rho}{2}grad(\upsilon^2)=0$. This equation is in many textbooks on aerodynamics, it follows from the Euler equation for mechanics of continuum medium in 1752.

The expression $-\frac{\rho}{2}grad(\upsilon^2)$ is nothing more than the potential force or gradient of the kinetic energy of air with a minus sign. However, in textbooks on continuum mechanics, this force is called the potential Euler term: $-\frac{1}{2}grad(\upsilon^2)$, multiplied by the density $\rho$.

However, in [5] the force $-\frac{\rho}{2} grad(\upsilon^2)$ was obtained from the minimum of the action, it has the physical dimension Newton, so it is a force. In the article [5], it was called a potential force, as Euler firstly recorded it. It is the potential force that describes the correct movement of cyclones and anticyclones in the air, as shown in [2, 6].

Therefore, there is no difference for potential air movement, where the pressure is higher: below the wing of the airplane or above the wing of the airplane. The pressure gradient is compensated by the kinetic energy gradient of the air, as the change in internal energy of air is equal to the change in kinetic energy of air, and the sum is a constant. This is indicated by the Bernoulli equation of 1738 for stationary potential motion of a continuous medium.

The sum of the forces acting on the wing of the airplane from the air, associated with its stationary potential horizontal flow, is zero. Why it is not written in textbooks on aerodynamics is a rhetorical question. How then to explain to students pilots, "why planes remain in the air" when flying horizontally [1]. And what to teach in the course of aerodynamics about the lift of the wing of an airplane when moving horizontally if the sum of the known forces acting on the wing of the airplane from the air is zero according to the Bernoulli equations.

You can't write in a textbook on aerodynamics that we don't know why the plane doesn't crash during the flight. Therefore, in textbooks on aerodynamics, they reason from the opposite: the plane flies and does not fall, so the air pressure is greater from below, therefore, the air speed must be greater from above the wing of the airplane according to Bernoulli's law [1]. Agree, "it is iron logic". Although in fact, everything is the opposite, in terms of the direction of the pressure gradient in the air (Fig. 2).

Note that when solving this problem, you need to separate two concepts: air pressure and the force acting on the airplane from the air. There is pressure in the air, and there is pressure of forces on the wing of the airplane from the air – these are different things. The plane flies horizontally and does not fall as a result of the equality of gravity and the sum of all the forces that act on the plane from the air. It is necessary to prove that the resulting force from the air is directed upwards and is equal to the weight of the airplane. And the second question: where the air pressure is higher, above or below the wing of the airplane. After all, air pressure is a characteristic of air, not an airplane.

The pressure on the wings of the resulting force acting from the air, of course, is directed upwards. However, this is not the air pressure gradient with the minus sign: $-grad(P)$ which is directed downward (Fig. 2). It is also not a potential force $-\frac{\rho}{2} grad(\upsilon^2)$ directed upwards, since the sum of the potential forces acting on a solid from the stationary potential flow of air is zero. This is well known from the time of Bernoulli (1738) and Euler (1752). This was also known to Einstein in 1916. This is the question, "why do planes stay in the air?" [1]. Then what force keeps the plane in the air, when flying horizontally? After all, it is clear to everyone that miracles do not happen.

As is known [3,4,7], Zhukovsky used the fact that the air flow velocity is higher over the wing of the airplane when deducing the lift force (Fig. 1). Therefore, he received a lifting force. However, if we take into account the real situation that the speed of air flow under the airfoil is greater than the speed over the  airfoil in horizontal flight, then the Zhukovsky force is directed downward, along with the air pressure that presses on the wing of the airplane from above (Fig.2).

The viscosity in the air is low, it obviously will not help to keep the airplane in the air. Prandtl took into account the air viscosity in the boundary layer when describing the lift force of an airplane [7]. But he also assumed that the air velocity is greater over the wing of the airplane (Fig. 1), since the strength of the viscosity depends on the speed. Therefore, if we now recalculate the forces received by Prandtl in accordance with (Fig. 2), they will also be directed downward.

Thus, the viscosity and similarity principle, which is used in describing the motion of a viscous fluid [7], will not help us to explain the lift force of an airplane wing. The airfoil sets the pressure gradient and speed gradient along the vertical axis. Therefore, if Zhukovsky and Prandtl obtained upward forces based on the velocity distribution in Fig. 1, then now, when recalculated in accordance with Fig. 2, these forces will be directed downward.

Let's remember all the forces that operate in a continuous medium. Here, as they say, the drowning man grasps at straws. Remember the heroic case when a pilot landed a plane on the Hudson river in 2009. The depth of the Hudson river at the point where the plane landed was thirteen meters, and the plane was kept on the water by the power of Archimedes.

As it is known the force of Archimedes in water is equal to the weight of the displaced liquid. In air, Archimedes ' force is equal to the weight of the displaced air. It is due to this force that balloons fly, since warm air is lighter than cold, so the air inside the balloon is heated with a gas burner.

An airplane flying in the air is also affected by the Archimedean force equal to the weight of the displaced air. However, the weight of the displaced air is less than the weight of the same volume of displaced water by a thousand times. Obviously, the power of Archimedes in the air will not save us and will not be able to explain why the plane does not fall when flying horizontally, but it must be taken into account.

## 2. Equations of air movement and equations of equilibrium of the airplane in the air.

To understand what force holds the plane in the air, we will write down all the forces that act on the plane in the air. Let's use the equation of motion (1) in [2]. From Newton's second law for air, taking into account gravity, we get:

$$\rho\frac{\partial\vec{\upsilon}}{\partial t}=-grad(P)-\frac{\rho}{2}grad(\upsilon^2)-\frac{\vec{\upsilon}}{c^2}\frac{\partial P}{\partial t}-\frac{\vec{\upsilon}}{c^2}(\vec{\upsilon}grad(P))+\frac{\upsilon^2}{c^2}grad(P)+\rho\vec{g}\ . \qquad (1)$$

Here on the right are the forces acting on a single volume of air.

$-grad(P)$ - this is the pressure gradient with a minus sign, the one that everyone was hoping for and which turned out to be directed downward (Fig. 2).

$-\frac{\rho}{2}grad(\upsilon^2)$ - the potential force that saved us when describing cyclones and anticyclones [2,6] and which is equal to and opposite to the pressure gradient with a minus sign, with a non-vortex potential movement of air.

$-\dfrac{\vec{\upsilon}}{c^2}\dfrac{\partial P}{\partial t}$ - part of the centrally symmetric force $\vec{f}_C = -\dfrac{\rho}{2} grad(\upsilon^2) - \dfrac{\vec{\upsilon}}{c^2}\dfrac{\partial P}{\partial t}$ obtained from the minimum action [5], proportional to the time derivative of the pressure, which is equal to zero when the airplane is moving at a constant speed, since the pressure in the air does not depend on time (here $c$ – the speed of sound).

$\vec{f}_V = -\dfrac{\vec{\upsilon}}{c^2}(\vec{\upsilon} grad(P)) + \dfrac{\upsilon^2}{c^2} grad(P)$ - vortex force in the air, obtained from the minimum action [5].

$\rho\vec{g}$ – gravitational force acting on the air and directed downward.

Equation (1) is initially written in this form:

$$\rho\frac{\partial \vec{\upsilon}}{\partial t} = -grad(P) + \vec{f}_C + \vec{f}_V + \rho\vec{g}\,. \qquad (1')$$

Here the forces obtained from the minimum of action are recorded $\vec{f}_C$, $\vec{f}_V$ [5] in physics, all forces are obtained from the minimum of action, and not from the differentiation of the velocity field, as Euler did in 1752. It is obvious that differentiating the velocity field $\vec{\upsilon}(\vec{x},t)$ as a complex function, it is impossible to obtain force. Therefore, the expression $-\dfrac{\rho}{2} grad(\upsilon^2)$ is still not called a force in the physical literature, although its dimension is Newton.

Let's consider the movement of a single volume of air. We will not now take into account turbulence we will assume that the plane does not shake during the flight. Note that the vortex force $\vec{f}_V$ is zero when the velocity direction coincides with the direction of the pressure gradient. In this case, the scalar multiplication of vectors is equal to the product of their magnitudes and $\vec{f}_V = -\dfrac{\vec{\upsilon}}{c^2}(\vec{\upsilon} grad(P)) + \dfrac{\upsilon^2}{c^2} grad(P) = 0$ is a property of a one-dimensional potential flow. Then, for a stationary one-dimensional air movement, it follows from (1) that the force $-grad(P)$ is equal to the force $-\dfrac{\rho}{2} grad(\upsilon^2)$ and the potential Bernoulli motion is fulfilled.

In fact, the fact that the air around the airplane moves potentially in accordance with Bernoulli's law, as written in article [1] – this is a question, since at high speed the air is strongly compressed when it collides with the airplane, heats up and flies in different directions. Therefore, it is quite possible that the pressure and air speed will be higher under the wing of the airplane. This is what happens on takeoff, at a non-zero angle of attack. However, in the future, for the sake of simplicity, we will adhere to the generally accepted position that in horizontal flight with a zero angle of attack in the first approximation for the air near the wing of the airplane, the Bernoulli equations are fulfilled and the sum of potential forces is zero.

Let us now consider the forces acting on the plane.
When the plane is moving horizontally, the air resistance force acts on it. It is compensated by the work of the airplane's engines, in fact, this is what fuel is spent in flight.

Let us now consider the vertical forces acting on the airplane. First of all, it is the gravity, which is equal to $M\vec{g}$. In the sum of the Archimedean force it is equal $(M-\rho V)\vec{g}$, here $V$ – the volume of the airplane. Gravity is compensated by forces acting on the wings of the airplane from the air. Here we only take into account the area of the wings, since above and below the fuselage there is almost no difference in air pressure and speed due to symmetry, so the area of the fuselage can be ignored.

Note that usually the wings are combined into one common area, taking into account the area under the fuselage, since on some airplanes they cannot be divided. Therefore, we will assume that the small area under the fuselage adjacent to the wings is also included in the area of the wings.

In the equation of air movement (1), the volumetric forces acting on a unit of air volume are recorded. To get the force acting on the wing of the airplane, it is necessary to sum up the volumetric forces.

Forces $-grad(P) - \frac{\rho}{2} grad(\upsilon^2)$ compensate each other in each volume of air according to the Bernoulli's equation, so they do not affect the airplane. If the airplane were at rest and there was a pressure difference on the wing, then you can multiply the pressure difference by the area of the wings of the airplane and get the lifting force. And since the force $-grad(P)$ is equal to the force $\frac{\rho}{2} grad(\upsilon^2)$ and both these forces exist together as a result of the movement of the airplane, they are equal to each other according to Bernoulli's law in each unit volume of air. They can be summed, but the sum of the zeros is zero.

So, to compensate gravity, we only have the vortex force $\vec{f}_V = -\frac{\vec{\upsilon}}{c^2}(\vec{\upsilon} grad(P)) + \frac{\upsilon^2}{c^2} grad(P)$.

The first term $-\frac{\vec{\upsilon}}{c^2}(\vec{\upsilon} grad(P))$ of the vortex force is zero, since the pressure gradient vector $grad(P)$ is directed vertically, and the air velocity $\vec{\upsilon}$ is horizontal, near the wing of the airplane. Therefore, the scalar multiplication $(\vec{\upsilon} grad(P))$ is zero, like the scalar multiplication of perpendicular vectors.

The second term of the vortex volume force: $\frac{\upsilon^2}{c^2} grad(P)$ – not equal to zero. This force keeps the plane in horizontal flight at high speed. Let's prove this statement.

Please note that the force $\frac{\upsilon^2}{c^2} grad(P)$, directed upwards, since the pressure of air is more above the wing of the airplane because of the design of the wing profile (Fig. 2). To obtain the force acting on the wing of the plane it is necessary to sum the forces from all unit volumes of air over and under the wing of the plane.

Sum the expression $\frac{\upsilon^2}{c^2} grad(P)$ along the vertical axis. We assume that the speed does not change much, then get the force acting on the unit area of the wing in the form: $\frac{\upsilon^2}{c^2}(P - P_0)$, where $P_0$ is the pressure under the wing of the airplane , and $P$ is the pressure over the wing of the plane. The pressure $P_0$ coincides with the air pressure at infinity $P_\infty$, since under the wing of the airplane the pressure does not change at zero angle of attack. Now we need to multiply this expression by the area of the wings $2S$, and we get the lifting force of the airplane in the form $2S\frac{\upsilon^2}{c^2}(P-P_0)$ when moving horizontally in a straight line at a speed of $\upsilon$. Above the wing, the pressure is greater, so this force is directed upwards, Q.E.D.

Thus, the sum of forces acting on the airplane in the projection on the vertical axis, in the first approximation, has the form:

$$-(M-\rho V)g+2S\frac{\upsilon^2}{c^2}(P-P_0)=0\,. \tag{2}$$

Now it becomes clear why you need the thickening on top of the wing of the airplane (Fig. 2). Thickening of the wing of the plane, in fact, creates the angle of attack from above, it is arranged to have more air pressure above the wing of the plane and that the plane could fly horizontally at high speed and not fall.

## 3. The lifting force of the airplane. Explanation and Conclusions.

We will arrange, as they say in such cases, a debriefing. What happened, how did it happen that the vortex force saved the plane from falling during horizontal flight, and how does the resulting lifting force $2S\frac{\upsilon^2}{c^2}(P-P_0)$ differ from the lifting force recorded in textbooks on aerodynamics: $2S(P_0-P)$ [3, 4].

It is obvious that these forces differ in direction and the relative multiplier $\upsilon^2/c^2$. Force $2S\frac{\upsilon^2}{c^2}(P-P_0)$ it is lifting force and is directed upwards (Fig. 2), and the force $2S(P_0-P)$ from textbooks on aerodynamics can not be called lifting, since it is directed down.

Let's explore the lifting force $2S\frac{\upsilon^2}{c^2}(P-P_0)$ and test it at different speeds.

Let's assume that the speed of an airplane is ten times less than the speed of sound $300\,m/c$, i.e., less than $30\,m/c$ (or $108\,km/h$). Then the relative coefficient $\upsilon^2/c^2$ is less than one hundredth. In this case, the lifting force is very small and the plane will not even be able to take off at this speed (meaning a large airplane, not a light glider).

Let's now consider another extreme case, when a large airliner flies in horizontal flight at a speed close to the speed of sound (about $1000\,km/h$), but slightly less than the speed of sound. In the first approximation, you can put $\upsilon=c$. Then the pressure gradient in the equations of motion (1) is annulated since: $-grad(P)+\frac{\upsilon^2}{c^2}grad(P)=0$ at $\upsilon=c$, and the plane is flying, in fact, only at the potential force given by the speed gradient: $-\frac{\rho}{2}grad(\upsilon^2)$.

So, where the speed should be higher for the airfoil in horizontal flight – of course, under the wing of the airplane (Fig. 2), then the force $-\frac{\rho}{2} grad(\upsilon^2)$ will be directed upwards – from a higher speed to a lower speed.

This solution for speeds close to the speed of sound was tested on May 11, 2020, as soon as the article [1] was received, so it was immediately clear why the thickening of the upper wing of the airplane was made.

After all, knowing that the plane is affected by the vortex force: $\frac{\upsilon^2}{c^2} grad(P)$, it is not difficult to calculate and understand that the speed is greater under the wing of the plane. Since at speed equal to the speed of sound, the pressure gradient $grad(P)$ is annulated in (1) and the airplane in flight at a speed equal to the speed of sound is held only by the potential force: $-\frac{\rho}{2} grad(\upsilon^2)$.

Note that this result is exact, it does not depend on the assumption that the movement of air around the airplane satisfies the potential Bernoulli's equation. When the speed of an airplane is equal to the speed of sound, the pressure gradient falls out of the equations of motion (1) for air with a horizontal movement of the airplane. Therefore, the wings of all planes are arranged so that when flying horizontally, the air speed is greater under the wing of the airplane, otherwise the plane will simply fall when flying horizontally at high speed.

But it's time to tell you where the vortex force $\vec{f}_V$ appeared in the equations of motion (1), and why it is responsible for the lift of the airplane wing. In June 2020, an article was published in the journal PTEP [5], which describes eddy movements in the atmosphere that are directly related to the solution of the Millennium problem – the description of turbulence.

In [5], we derive the forces acting in a continuous medium $\vec{f}_C$ and $\vec{f}_V$ (1'), which were obtained from the action minimum. In this work, the lifting force in tornadoes was also obtained, which is equal to $\frac{\upsilon_\perp^2}{c^2}\frac{\partial P}{\partial z}$, where $\upsilon_\perp$ is the horizontal vortex velocity in the tornado, and the pressure gradient $\frac{\partial P}{\partial z}$ is directed along the vertical axis upwards, towards the expansion of the tornado funnel.

Thus, the lift in a tornado is the same as the lift in an airplane. The only difference is that in a tornado, the air moves in a circle in a horizontal plane, and the airplane flies straight and itself creates forces acting on it from the air. Therefore, it was not even thought that a straight-flying plane could somehow be useful to the vortex force, until the problem of horizontal flight was not told in [1].

However, in a tornado, everything is less obvious than with an airplane, since it is difficult to measure anything in a tornado. This is the main problem associated with studying tornadoes, since the tornado breaks all measuring devices before they have time to measure anything.

There is a scale of intensity of tornadoes, it is called The Fujita scale [8]. The Fujita scale clearly shows that the speed $270km/h$ at which a large passenger plane takes off and the wind speed in a F3 tornado $254km/h$, at which it lifts large cars, are of the same order. From this it follows that the lifting force in tornadoes is of the same nature as the lifting force in an airplane.

Obviously, the lift effect is related to the relative multiplier $\upsilon^2/c^2$ before the pressure gradient in the vortex force $\frac{\upsilon^2}{c^2}grad(P)$ (1). Of course, more lift can be obtained by increasing the pressure gradient, but if the speed is small, then one pressure gradient will not help. So here are both important multipliers: the pressure gradient in air $grad(P)$, which is associated with the device profile of an airplane wing, and the relative multiplier $\upsilon^2/c^2$ that is associated with the ratio of velocity of airplane to velocity of sound in air.

The relative multiplier $\upsilon^2/c^2$ in the vortex force $\vec{f}_V$ clearly indicates the field-theoretic nature of the wing lift. Its origin is the same as in electrodynamics. As you know, the relativistic multiplier $\upsilon^2/c^2$ is associated with the names of Lorentz, Einstein, and Pauli, who built the theory of relativity and used it in field theory. In article [5], the forces $\vec{f}_C$, $\vec{f}_V$ obtained from the minimum action in the application to a continuous medium were calculated.

These forces were first recorded by Kadich-Edelen in 1983 from an analogy with electrodynamics [9]. They were derived in 2015 from the action minimum for the minimal interaction induced by the translation subgroup, as well as the Coulomb and Lorentz forces. In a continuous medium, these forces have the form $\vec{f}_C$, $\vec{f}_V$ (1) [5,6].

We can’t get the forces $\vec{f}_C$, $\vec{f}_V$ which contain a relative multiplier using mechanistic reasoning from the molecular kinetic theory of Newton. In other words, you can't explain the lift of an airplane wing using the idea of gas as a set of moving molecules, which they are still trying to do in aerodynamics.

As shown by further research and stated in [5], the forces $\vec{f}_C$, $\vec{f}_V$ are the forces of a strong fundamental interaction obtained in an application to a continuous medium. These forces are relativistic in nature, as are the Coulomb and Lorentz forces.

Thus, of the main three forces acting on the plane's wing in horizontal flight:
$-grad(P) - \frac{\rho}{2}grad(\upsilon^2) + \frac{\upsilon^2}{c^2}grad(P)$ - the pressure gradient with a minus sign $-grad(P)$, the potential force $-\frac{\rho}{2}grad(\upsilon^2)$ and the vortex force components $\frac{\upsilon^2}{c^2}grad(P)$, in modern aerodynamics [3,4] only the pressure gradient with a minus sign is used: $-grad(P)$.

In this case, the wing of the airplane is arranged so (Fig. 2) that the force $-grad(P)$ is directed down, when flying horizontally, and the forces $-\frac{\rho}{2}grad(\upsilon^2) + \frac{\upsilon^2}{c^2}grad(P)$ are directed up.

Therefore, the existing theory of aerodynamics, which only takes into account the downward force $-grad(P)$ (Fig. 2), does not help, but only hinders designers from designing airplane. Obviously, if you do not take into account all the forces acting in a continuous medium, you can not describe the dynamics of the continuous medium and the aerodynamics of the airplane.

We must pay tribute to the fact that thanks to the efforts of Zhukovsky and Prandtl, the lifting force was written from dimensional considerations in the form $F = 2C\rho\upsilon^2 S$, where $C$ is a dimensionless coefficient [3,4,7]. As noted above, this force was derived from the incorrect assumption that the speed is greater over the wing of the airplane (Fig. 1).

However, it was guessed almost correctly. Indeed, if we put that $P = \rho c^2$, then, in the first approximation, the lifting force in (2) can be written as:

$$F = 2(\rho - \rho_\infty)\upsilon^2 S. \tag{3}$$

Thus, it becomes clear that it is the force (3) Zhukovsky and Prandtl tried to obtain. However, they acted out of misconceptions about the movement of air around the wing of the airplane (Fig. 1). Therefore, even if the relativistic force (3) was guessed correctly, it does not give a complete understanding of the movement of the airplane in the air, since only horizontal flight is considered here. To describe the movement of the airplane it is necessary to take into account all the forces acting in the air (1), obtained from the minimum action.

Knowing the correct equations of motion of the continuous medium (1) it is possible to explain the horizontal flight of the airplane at high speed and the lift of an airplane wing (2). Therefore, now it will be possible to simulate airplane using a computer, without having to resort to expensive and not always possible experiments using wind tunnels and test pilots.

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