# No or Diffuse Phase-Transition With Temperature in One-dimensional Ising Model?

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# No or diffuse phase-transition with temperature in one-dimensional Ising model?

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## Abstract

For nearly a century since Ising model was proposed in 1925, it is agreed that there is no phase-transition with temperature in the one-dimensional based on no global-spontaneous-magnetization in whole temperature region. In this letter, the exact calculation of local-spontaneous-magnetization shows that a diffuse phase-transition with temperature occurs in one-dimensional Ising model. In addition, although diffuse phase-transition phenomenon is common in the systems of heterogeneous-components and small-sizes etc., there is no accurate prediction of corresponding theoretical models so far, so the present works lay the theoretical foundation of this kind of phase-transition.

In the nearly 100 years since Ising model (IM)[1], one of the most important microscopic models of phase-transition[2-8], was proposed in 1925, it is agreed that there is no temperature dependent phase-transition in the one-dimensional. This is because the global-spontaneous-magnetization of the model system is zero in whole temperature range, i.e. there is no global-spontaneous-magnetization[1,2] as shown in the Supplementary Information (SI).

However, the absence of global-spontaneous-magnetization does not deny the existence of short-range local-spontaneous-magnetization in one-dimensional-IM (1D-IM). In this paper, the local-spontaneous-magnetization with temperature and size in the model is calculated accurately, and the results show that 1D-IM has a diffuse phase-transition with temperature[9-13]. In other words, our conclusion subverts the century consensus in this model.

## Results

The Hamiltonian ($H_{1\mathrm{D-IM}}$) of one-dimensional Ising model (1D-IM) is,

$$H_{1\mathrm{D-IM}} = \lim_{N\to\infty}\left[-J\sum_{i=1}^{N-1}\sigma_i\sigma_{i+1}\right] \quad (1)$$

in which $\sigma_i$ is the ith spin and $\sigma_i = \pm 1$, $J$ the interaction energy constant between the nearest-neighbor spins, and $N$ the total number of spins in the model system (Fig. S1).

In the model, the local magnetic moment ($s_l^r$) including $l$ nearest-neighbor spins is,

$$s_l^r \equiv \mu\sum_{i=0}^{l-1}\sigma_{r+i} \quad (2)$$

where $\mu$ is the magnetic moment of a spin, and $r$ expresses an arbitrary reference site.

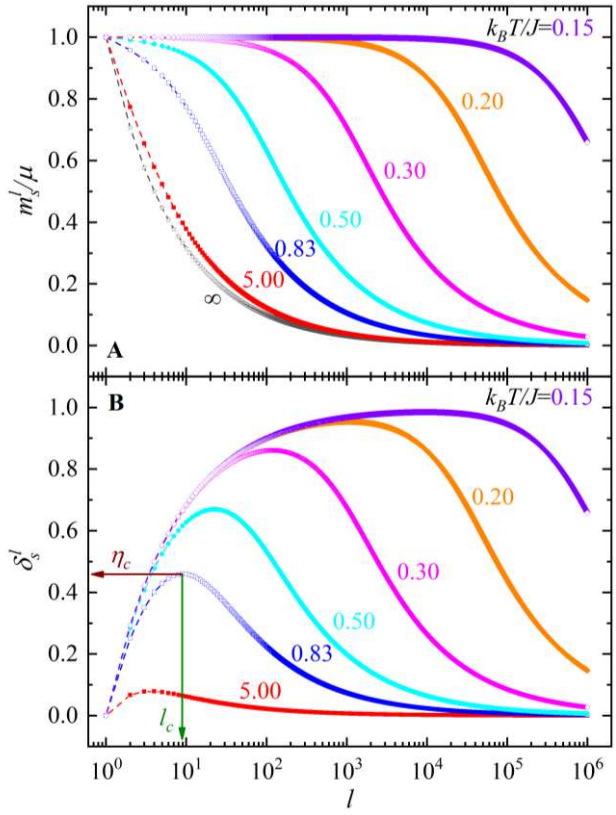


Fig. 1. For series $T$, (A) local-spontaneous-magnetization ($m_s^l$) and (B) relative local-spontaneous-magnetization ($\delta_s^l$) vs local spin number ($l$) in one-dimensional Ising model.

To describe the temperature dependence of the amplitude of $s_l^r$ (excluding the orientations corresponding to its signs) in this paper, the local-spontaneous-magnetization ($m_s^l$) is defined as (see SI and METHODS),

$$m_s^l \equiv \frac{1}{l}\sqrt{\lim_{n\to\infty}\frac{1}{Z_n}\sum_{\sigma_i=\pm1,\cdots\sigma_n=\pm1}(s_l^r)^2\exp\left[\frac{J}{k_BT}\sum_{i=1}^{n-1}\sigma_i\sigma_{i+1}\right]} = \frac{\mu}{l}\sqrt{2\left[\frac{l-\gamma^l}{1-\gamma}-\frac{\gamma(1-\gamma^{l-1})}{(1-\gamma)^2}\right]-l} \quad (3)$$

here $k_B$ is Boltzmann constant, $T$ the temperature of the heat bath in which the one-dimensional spin chain is located, $Z_n$ the partition function of the spin orientation ensemble of 1D-IM (SI), and $n$ the spin number of the subsystems in the ensemble (Fig. S1), and $\gamma \equiv \tanh\left(\frac{J}{k_BT}\right)$.

Obviously, $m_s^\infty$ is the global stochastic magnetization when $l \to \infty$. From Eq. 3, we can get that $m_s^\infty = 0$ except $T \to 0$, which is consistent with the past result[1,2]。

Fig. 1A shows $m_s^l$ vs $l$ for series $T$, and it can be seen that: (i) At high temperature (e.g. $T = 5.00J/k_B$), $m_s^l$ decreases rapidly with increasing $l$, which indicates that the spatial scale

of local-spontaneous-magnetization is small; and (ii) At low temperature (e.g. $T = 0.15J/k_B$), $m_s^l \to \mu$ in a large range of $l$, which states clearly that the local-spontaneous-magnetization regions not only has a large spatial scale, but also almost all the spins in the regions have the same orientation.

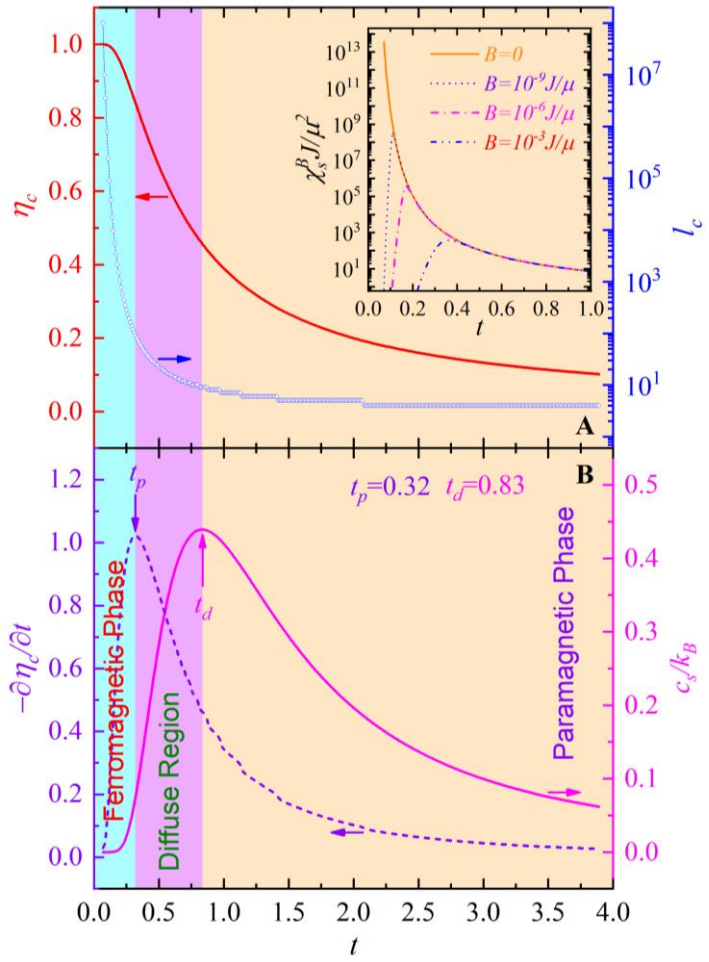


Fig. 2. (A) characteristic spontaneous magnetization ($\eta_c$) and its characteristic spatial size ($l_c$), as well as (B) $-\partial\eta_c/\partial t$ and heat capacity per spin ($c_s$) in 1D-IM vs reduced temperature ($t \equiv k_B T/J$). Inset of Fig. 2A shows static susceptibility ($\chi_s^B$) per spin vs $t$ for a series of external magnetic fields ($B$).

## Discussion

According to Landau theory[14], the order parameter is the essential to phase-transition, which characterizes the relative change of the low to high temperature phase. Therefore, in order to describe the relative variation of $m_s^l$ to $m_s^l(T \to \infty)$, the relative local-spontaneous-magnetization ($\delta_s^l$) is introduced here,

$$\delta_s^l \equiv \frac{m_s^l - m_s^l(T \to \infty)}{\mu} \tag{4}$$

In fact, $m_s^l(T \to \infty)$ represents the autocorrelation ( ${\sigma_i}^2$ ) of a spin (Eq. 3 and M5 in METHODS), and of course has nothing to do with the phase transition process.

At series $T$, $\delta_s^l$ vs $l$ is shown in Fig. 1B, which indicates that for all temperatures, $\delta_s^l$ has a single diffuse peak as a function of $l$. In this paper, the maximum value of $\delta_s^l$ is expressed as

$\eta_c$, and the corresponding value of $l$ as $l_c$. Obviously, $\eta_c$ and $l_c$ can be used as the characteristic parameters to describe the local-spontaneous-magnetization and its spatial size, so here $\eta_c$ is called the characteristic spontaneous magnetization and $l_c$ the characteristic spatial size of $\eta_c$ in 1D-IM. Moreover, the dispersion of the $\delta_s^l$ peak shows that both the size of the local-spontaneous-magnetization regions and its internal magnetization have obvious distributions.

$\eta_c$ and $l_c$ vs $T$ (Fig. 2A) show that with decreasing $T$: (i) $\eta_c$ first increases slowly, then rapidly, and slowly again. It should be noted that at high temperature, $\eta_c$ is still not zero (e.g. $\eta_c = 0.10$ when $T = 3.82J/k_B$). In addition, $\eta_c \to 1$ at nonzero low temperature (e.g. $\eta_c = 0.9999$ for $T = 0.07J/k_B$); and (ii) $l_c$ first increases slowly and then rapidly. For example, $l_c = 4$ when $T = 3.82J/k_B$, and $l_c = 10^8$ for $T = 0.07J/k_B$ (if the lattice parameter of 1D-IM is assumed to be 0.5 nanometers, the characteristic size of the local-spontaneous-magnetization region will reach the macroscopic ~ 5 centimeters). These results state clearly that in 1D-IM, there exists a diffuse transition between the nanoscale regions of small spontaneous magnetization at high temperature and the macroscopic domains of large spontaneous magnetization at low temperature.

By comparing the order parameters[10-13], heat capacity[15-17], and domain structure evolution[18-20] of existing diffuse phase-transition, with the diffuse variation of $\eta_c$, the diffuse peak[1,2] of heat capacity ($c_s$) per spin (SI and Fig. 2B), and the diffuse transition between the nanoscale regions of local-spontaneous-magnetization to the macroscopic domains as a function of $T$, it can be concluded that a diffuse phase-transition with temperature occurs in 1D-IM.

According to the method of reference[21], the temperature corresponding to the maximum of $-\frac{\partial \eta_c}{\partial T}$ is defined as the characteristic temperature ($T_p$) of diffuse phase-transition in 1D-IM, and it is obtained $T_p = 0.32J/k_B$ (Fig. 2B). It should be pointed out that $T_p$ is lower than the peak temperature ($T_d = 0.83J/k_B$) of $c_s$. For second-order or continuous phase-transition, the peak temperatures of heat capacity and the negative of the differential of order parameter to $T$ are equal to each other[14,22,23], so the authors thinks that the difference between $T_p$ and $T_d$ just reflects the dispersion of the diffuse phase-transition. In order to describe this dispersion, the dispersion degree ($\varphi$) of the diffuse phase-transition is proposed as,

$$\varphi \equiv \frac{T_d - T_p}{T_d + T_p} = 0.44 \tag{5}$$

It is worth noting that the static susceptibility ($\chi_s$) per spin in 1D-IM (inset of Fig. 2A and SI) always increases rapidly with decreasing $T$[1,2], instead of the $\lambda$-type peak of second-order phase-transition[14], which is thought to be one of the key evidences that no phase-transition with temperature exist in this model.

In order to further explore the micro mechanism of the above characteristic of $\chi_s$, the static susceptibility ($\chi_s^B$) per spin in 1D-IM as a function of $T$ in a fixed external magnetic field ($B$) is calculated (SI), as shown in the inset of Fig. 2A. We can see that for finite small $B$, there is a single diffuse peak of $\chi_s^B$ with $T$, and the peak temperature moves to high temperature with the increase of $B$. This is due to that very small $B$ can saturate the magnetization of 1D-IM at low temperature (as shown in Fig. S2, the saturation $B \sim 10^{-9} J/\mu$ for $T = 0.10J/k_B$), while the saturated magnetization leads to a smaller value $\chi_s^B$. Because the saturation magnetization corresponds to the single domain state of the model, the increase of $\chi_s$ at low temperature is caused by the movement of domain walls[24,25].

In particular, because the measurement magnetic field used in experiments is always finite, the susceptibility peak (inset of Fig. 2A) will appear in 1D-IM as long as the experimental measurement is carried out. In other words, the continuously increasing characteristic of the theoretically predicted $\chi_s$ ($\chi_s^{B=0}$) with decreasing $T$ cannot be measured directly, which is only ideal value.

Although diffuse phase-transition is common in component-heterogeneous[10-13,26-29] and small-size[30-32] systems, there is no accurate calculation of the corresponding theoretical model so far[21]. Therefore, the exact results in this paper lay the theoretical foundation of this kind of phase-transition.

Weiss mean-field theory[33] is always thought to be not suitable for 1D-IM because it predicts that there is a phase-transition with $T$ in this system. The present and relevant[2,22,23] results show that this theory is valid for IM of all dimensions in judging whether there exists phase-transition, although its predicting phase-transition behaviors, such as the transition temperature and the subtle characteristics of order parameter and heat capacity, are different from the exact solutions[2,22,23]. It also shows that Weiss-type mean-field approximation[21] is an effective and feasible method for Ising-category models of phase-transition which are too complex and difficult to get their exact solutions.

Traditionally, the description to 1st and 2nd-order phase-transitions is based on global-order-parameter (such as global-spontaneous-magnetization and global-spontaneous-polarization etc.)[1,2,14,33], but it cannot describe the spatially heterogeneous behavior of the order-parameter corresponding to the diffuse-phase-transition in component-heterogeneous[10-13,26-29] and small-size[30-32] systems. Obviously, as the scale of the local-order-parameter proposed in Ref.21 and this paper tends to infinity, it becomes the global-order-parameter. So, local-order-parameter can provide a unified description of 1st and 2nd-order phase-transitions as well as diffuse-phase-transition.

## Methods

The method to calculate the local-spontaneous-magnetization ($m_s^l$) in 1D-IM is as the follows.

The Hamiltonian ($H_n$) of the subsystem (Fig. S1) without external magnetic field is,

$$H_n = -J\sum_{i=1}^{n-1}\sigma_i\sigma_{i+1} \tag{M1}$$

and the partition function ($Z_n$) of the spin orientation ensemble (Fig. S1) is,

$$Z_n \equiv \sum_{\sigma_i=\pm1,\cdots\sigma_n=\pm1}\exp\left[v\sum_{i=1}^{n-1}\sigma_i^1\sigma_{i+1}^1\right] = \cosh^{n-1}(v)Q_n \tag{M2}$$

In which,

$$Q_n \equiv \sum_{\sigma_i=\pm1,\cdots\sigma_n=\pm1}\prod_{i=1}^{n-1}(1+\gamma\sigma_i\sigma_{i+1}) = 2^n \tag{M3}$$

Let,

$$X_l \equiv \lim_{n\to\infty}\frac{1}{Z_n}\sum_{\sigma_i=\pm1,\cdots\sigma_n=\pm1}(s_l^r)^2\exp\left[v\sum_{i=1}^{n-1}\sigma_i\sigma_{i+1}\right] \tag{M4}$$

and according to,

$$(s_l^r)^2 = \mu^2 \left\{ l + 2 \left[ \sum_{i=1}^{l-1} \sigma_{r+i-1}^1 \sigma_{r+i}^1 + \sum_{i=2}^{l-1} \sigma_{r+i-2}^1 \sigma_{r+i}^1 + \cdots \sum_{i=l-1}^{l-1} \sigma_r^1 \sigma_{r+l-1}^1 \right] \right\} \quad \text{(M5)}$$

we obtain,

$$X_l = \mu^2 \left[ l + 2 \sum_{k=1}^{l-1} (l-k) \zeta_k \right] \quad \text{(M6)}$$

where $\zeta_k$ is the correlation function between $\sigma_r$ and $\sigma_{r+k}$, i.e.

$$\begin{aligned} \zeta_k &\equiv \lim_{n\to\infty} \frac{1}{Z_n} \sum_{\sigma_i=\pm1,\cdots\sigma_n=\pm1} \sigma_r \sigma_{r+k} \exp\left[ v \sum_{i=1}^{n-1} \sigma_i \sigma_{i+1} \right] \\ &= \lim_{n\to\infty} \frac{1}{Q_n} \sum_{\sigma_i=\pm1,\cdots\sigma_n=\pm1} \prod_{i=1}^{n-1} (1 + \gamma \sigma_i \sigma_{i+1})\, \sigma_r \sigma_{r+k} \end{aligned} \quad \text{(M7)}$$

Based on,

$$\begin{aligned} I_0 &\equiv \sum_{\sigma_r=\pm1} (1 + \gamma \sigma_{r-1} \sigma_r)(1 + \gamma \sigma_r \sigma_{r+1})\, \sigma_r \\ &= 2\gamma(\sigma_{r-1} + \sigma_{r+1}) \end{aligned} \quad \text{(M8)}$$

$$\begin{aligned} I_1 &\equiv \sum_{\sigma_{r+1}=\pm1} I_1 (1 + \gamma \sigma_{r+1} \sigma_{r+2}) \\ &= 2^2 \gamma(\sigma_{r-1} + \gamma \sigma_{r+2}) \end{aligned} \quad \text{(M9)}$$

$$\cdots$$

$$\begin{aligned} I_k &\equiv \sum_{\sigma_{r+k}=\pm1} I_{k-1} (1 + \gamma \sigma_{r+k} \sigma_{r+k+1}) \sigma_{r+k} \\ &= 2^{k+1} \gamma(\gamma^{k-1} + \gamma \sigma_{r-1} \sigma_{r+k+1}) \end{aligned} \quad \text{(M10)}$$

we get,

$$\begin{aligned} \zeta_k &= \lim_{n\to\infty} \frac{1}{Q_n} \sum_{\substack{\sigma_i=\pm1,\cdots\sigma_{r-1}=\pm1 \\ \sigma_{r+k+1}=\pm1,\cdots\sigma_n=\pm1}} I_k \prod_{i=1}^{r-2} (1 + \gamma \sigma_i \sigma_{i+1}) \prod_{i=r+k+1}^{n-1} (1 + \gamma \sigma_i \sigma_{i+1}) \\ &= \gamma^k \end{aligned} \quad \text{(M11)}$$

From Eq.M6 and M11, we obtain,

$$X_l = \mu^2 \left[ l + 2 \sum_{k=1}^{l-1} (l-k) \gamma^k \right] \quad \text{(M12)}$$

Here, $\sum_{k=1}^{l-1} (l-k)\gamma^k$ is the well-known arithmetic-geometric series, and,

$$X_l = \mu^2 \left\{ 2 \left[ \frac{l - \gamma^l}{1-\gamma} - \frac{\gamma(1-\gamma^{l-1})}{(1-\gamma)^2} \right] - l \right\} \quad \text{(M13)}$$

Therefore,

$$m_s^l = \frac{\mu}{l} \sqrt{2 \left[ \frac{l - \gamma^l}{1-\gamma} - \frac{\gamma(1-\gamma^{l-1})}{(1-\gamma)^2} \right] - l} \quad \text{(M14)}$$

and obviously,

$$m_s^l (T \to \infty) = \frac{\mu}{l^{1/2}} \quad \text{(M15)}$$

## Data availability

The data that support the plots and other findings of this paper are available from the corresponding author upon a reasonable request.

## Code availability

The code is available from the corresponding author upon a reasonable request.

## References


1. Ising, E. Report on the theory of ferromagnetism. *Z. Phys.* **31**, 253-258 (1925).
2. McCoy, B. M. & Wu, T. T. *The two-dimensional Ising model*. (Dover Publications, Inc., Mineola, New York, 2014).
3. Li, Y. Ising model for strings. *Nat. Phys.* **16**, 1006 (2020).
4. Young, J. T., Gorshkov, A. V., Foss-Feig, M. & Maghrebi, M. F. Nonequilibrium Fixed Points of Coupled Ising Models. *Phys. Rev. X*. **10**, (2020).
5. Christiansen, H., Majumder, S., Henkel, M. & Janke, W. Aging in the Long-Range Ising Model. *Phys. Rev. Lett.* **125**, (2020).
6. Walker, N., Tam, K. & Jarrell, M. Deep learning on the 2-dimensional Ising model to extract the crossover region with a variational autoencoder. *Sci. Rep.* **10**, (2020).
7. Wang, B., Hu, F., Yao, H. & Wang, C. Prime factorization algorithm based on parameter optimization of Ising model. *Sci. Rep.* **10**, (2020).
8. Imanaka, Y., Anazawa, T., Kumasaka, F. & Jippo, H. Optimization of the composition in a composite material for microelectronics application using the Ising model. *Sci, Rep.* **11**, (2021).
9. Smolenskii, G. A., Isupov, V. A., Agranovskaya, A. I. & Krainik, N. N. New ferroelectrics of complex composition. *Sov. Phys. Sol. Stat.* **2**, 2651-2654 (1961).
10. Kimura, T., Tomioka, Y., Kumai, R., Okimoto, Y. & Tokura, Y. Diffuse phase transition and phase separation in Cr-doped $Nd_{1/2}Ca_{1/2}MnO_3$: A relaxor ferromagnet. *Phys. Rev. Lett.* **83**, 3940-3943 (1999).
11. Granzow, T., Woike, T., Wohlecke, M., Imlau, M. & Kleemann, W. Change from 3D-Ising to random field-Ising-model criticality in a uniaxial relaxor ferroelectric. *Phys. Rev. Lett.* **92**, 65701 (2004).
12. Gehring, P. M. et al. Reassessment of the Burns temperature and its relationship to the diffuse scattering, lattice dynamics, and thermal expansion in relaxor $Pb(Mg_{1/3}Nb_{2/3})O_3$. *Phys. Rev. B*. **79**, 224109 (2009).
13. Stock, C. et al. Interplay between static and dynamic polar correlations in relaxor $Pb(Mg_{1/3}Nb_{2/3})O_3$. *Phys. Rev. B*. **81**, 144127 (2010).
14. Landau, L. Zur Theorie der Phasenumwandlungen II. *Phys. Z. Soviet Union*. **545**, 26-35 (1937).
15. Moriya, Y., Kawaji, H., Tojo, T. & Atake, T. Specific-heat anomaly caused by ferroelectric nanoregions in $Pb(Mg_{1/3}Nb_{2/3})O_3$ and $Pb(Mg_{1/3}Ta_{2/3})O_3$ relaxors. *Phys. Rev. Lett.* **90**, 205901 (2003).
16. Kleemann, W., Dec, J., Shvartsman, V. V., Kutnjak, Z. & Braun, T. Two-dimensional Ising model criticality in a three-dimensional uniaxial relaxor ferroelectric with frozen polar nanoregions. *Phys. Rev. Lett.* **97**, 65702 (2006).
17. Tachibana, M., Sasame, K., Kawaji, H., Atake, T. & Takayama-Muromachi, E. Thermal signatures of nanoscale inhomogeneities and ferroelectric order in $[PbZn_{1/3}Nb_{2/3}O_3]_{1-x}[PbTiO_3]_x$. *Phys. Rev. B*. **80**, 94115 (2009).
18. Burns, G. & Scott, B. A. Index of refraction in dirty displacive ferroelectrics. *Sol. State Commun.* **13**, 423-426 (1973).
19. Shvartsman, V. V. & Lupascu, D. C. Lead-free relaxor ferroelectrics. *J. Am. Cera. Soc.* **95**, 1-26 (2012).
20. Shvartsman, V. V., Dkhil, B. & Kholkin, A. L. Mesoscale domains and nature of the relaxor state by piezoresponse force microscopy. *Annu. Rev. Mater. Res.* **43**, 423-449 (2013).
21. Zhang, L. L. & Huang, Y. N. Theory of relaxor-ferroelectricity. *Sci. Rep.* **10**, 50601 (2020).
22. Onsager, L. Crystal statistics I: A two-dimensional model with an order-disorder transition. *Phys. Rev.* **65**, 117-149 (1944).
23. Yang, C. N. The spontaneous magnetization of a 2-dimensional Ising model. *Phys. Rev.* **85**, 808-815 (1952).
24. Huang, Y. N., Wang, Y. N. & Shen, H. M. Internal-friction and dielectric loss related to domain-walls. *Phys. Rev. B*. **46**, 3290-3295 (1992).
25. Huang, Y. N. et al. Domain freezing in potassium dihydrogen phosphate, triglycine sulfate, and CuAlZnNi. *Phys. Rev. B*. **55**, 16159-16167 (1997).
26. Cross, L. E. Relaxor ferroelectrics. *Ferroelectrics*. **76**, 241-267 (1987).
27. Kumar, A. et al. Atomic-resolution electron microscopy of nanoscale local structure in lead-based relaxor ferroelectrics. *Nat. Mat.* **20**, (2021).

28. Li, F. et al. Giant piezoelectricity of Sm-doped Pb(Mg1/3Nb2/3)O-3-PbTiO3 single crystals. *Science*. **364**, 264 (2019).
29. Krogstad, M. J. et al. The relation of local order to material properties in relaxor ferroelectrics. *Nat. Mater.* **17**, 718-724 (2018).
30. Chattopadhyay, S., Ayyub, P., Palkar, V. R. & Multani, M. Size-induced diffuse phase-transition in the nanocrystalline ferroelectric $PbTiO_3$. *Phys. Rev. B*. **52**, 13177-13183 (1995).
31. Muthuselvam, I. P. & Bhowmik, R. N. Grain size dependent magnetization, electrical resistivity and magnetoresistance in mechanically milled $La_{0.67}Sr_{0.33}MnO_3$. *J. Alloys Comp.* **511**, 22-30 (2012).
32. Li, J. et al. Scale-invariant magnetic textures in the strongly correlated oxide NdNiO3. *Nat. Commun.* **10**, (2019).
33. Weiss, P. Molecular field and ferro-magnetism. *Phys. Z.* **9**, 358-367 (1908).

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## Author contributions

Y-N.H. and L-L.Z. conceived this article together. Y-N.H. calculated the local-spontaneous-magnetization. L.Z. did the numerical calculations and made plots.

## Competing interests

Authors declare that they have no competing interests.

## Additional information

All data are available in the main text or the supplementary information.

# Figures

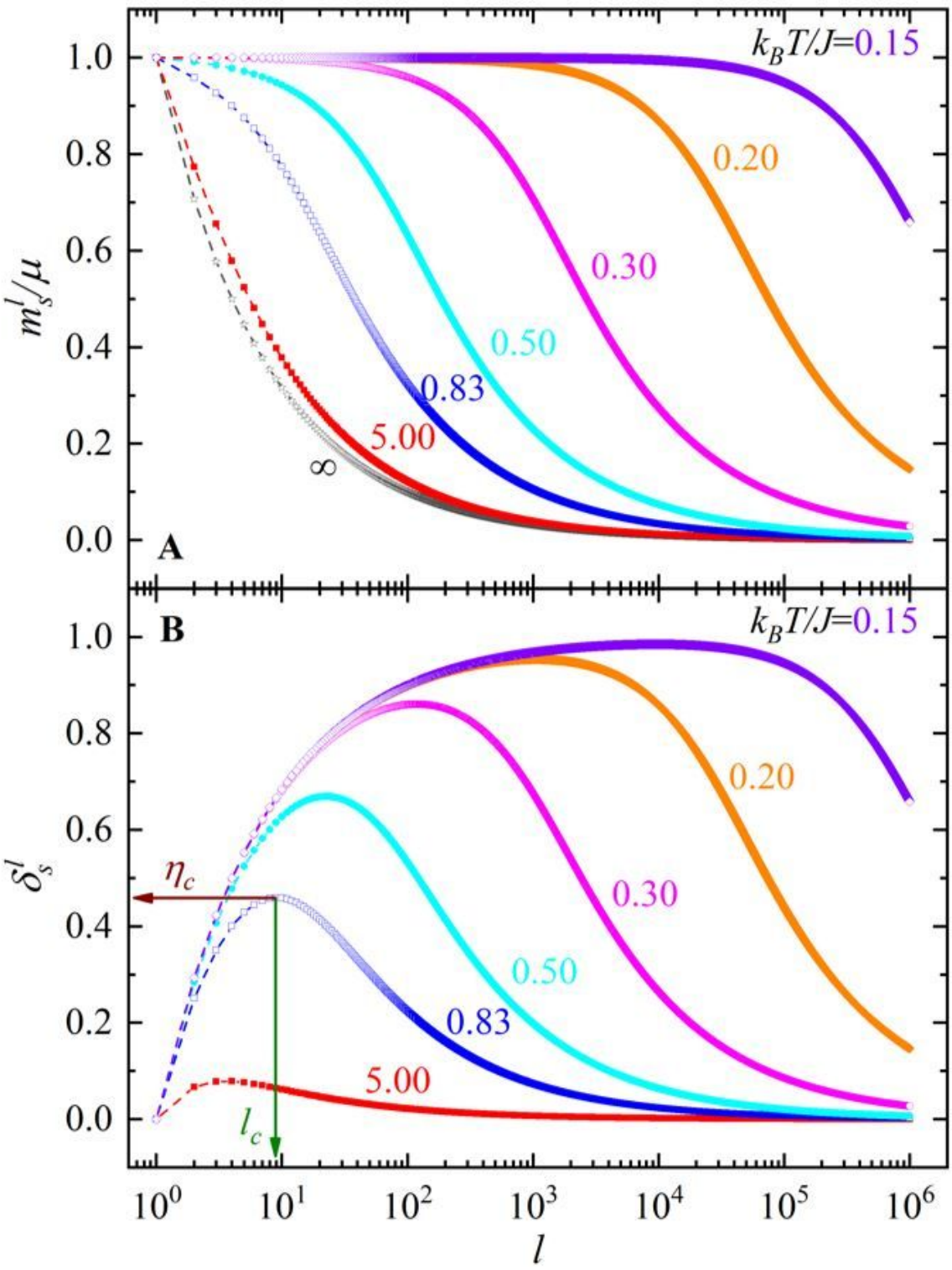


**Figure 1**

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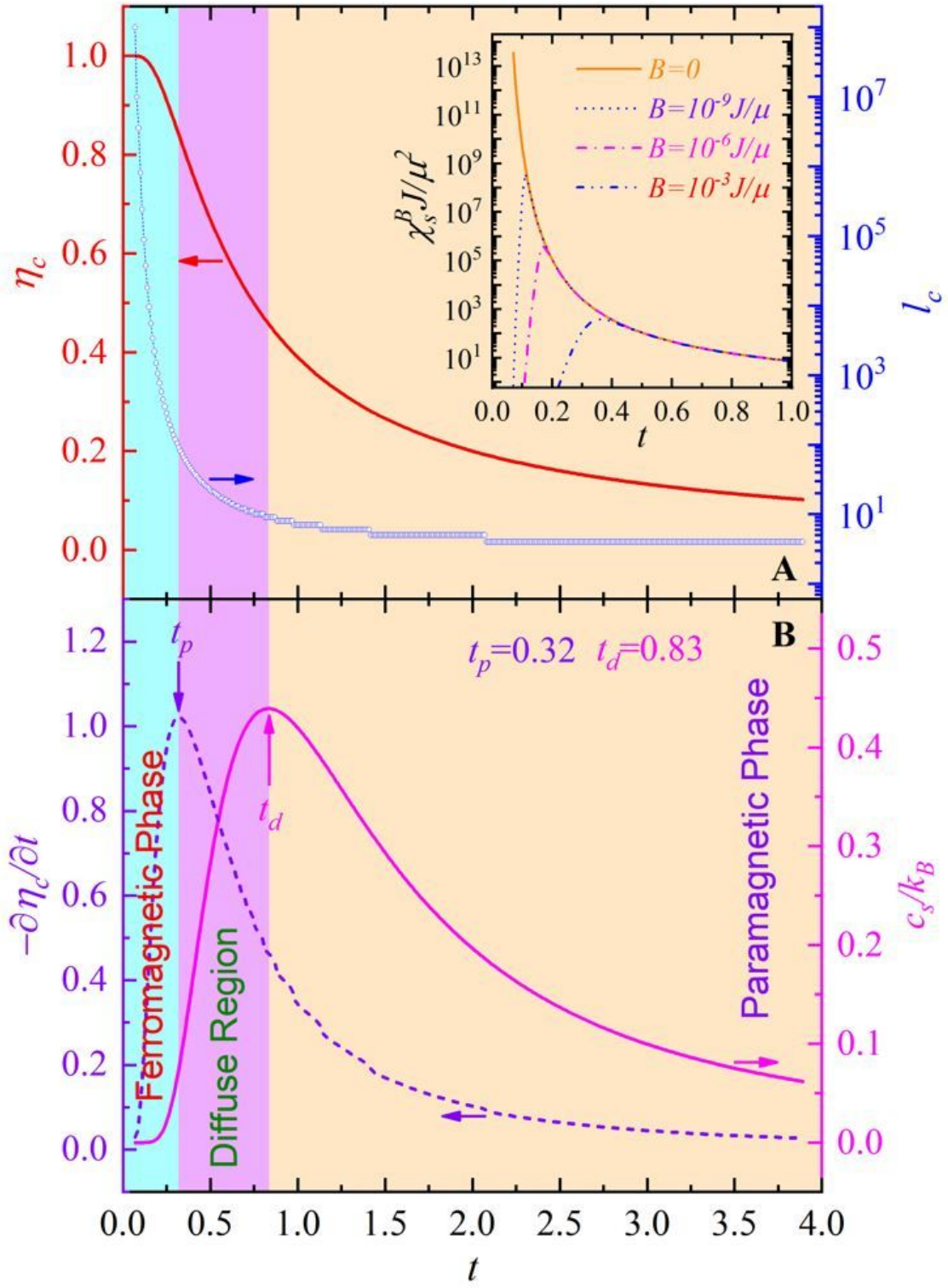


**Figure 2**

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