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The optimization solution of the vertical rolling force by using Unified Yield criterion

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Abstract: Vertical rolling is an important technique to control the width of continuous casting slab in hot rolling field. Accurate prediction of vertical rolling force is a core point to maintain rolling mill equipment. Due to the limitation of the algorithm, the prediction accuracy of most vertical rolling force models based on the energy method can only reach more than 10%. Therefore, it is challenging to optimize the rolling force model to improve the prediction accuracy. This paper presents an innovative approach for optimizing the calculation of vertical rolling force with a unified yield criterion. Firstly, the maximal width of dog bone region is determined by the slip-line method and described the dog bone shape via sine function model. Furthermore, proposed velocity and corresponding strain rate fields satisfy kinematically admissible condition is used to calculate the total power. Finally, the analytical solution of the rolling force and dog bone shape model is obtained by repeatedly optimizing the weighted coefficient b of intermediate principal shear stress on the yield criterion. And the effectiveness of the proposed mechanical model is verified by measured data in strip hot rolling field and other models' results. The results shows that the prediction accuracy of vertical rolling force model can be improved by optimizing the value of b . Then, the impacts of reduction rate, initial thickness and friction factor on dog-bone shape size and vertical rolling force are discussed.

Keywords: vertical rolling force; slip-line method; Dog-bone shape; Unified Yield criterion.

Nomenclature

W_0, W_E	Half of the initial and final slab width
W_x	Half of the slab width
ΔW	Unilateral reduction, $\Delta W = W_0 - W_E$
ΔW_x	Unilateral reduction in deformation zone, $\Delta W_x = W_0 - W_x$
h_0	Half of the initial slab thickness
h_I, h_{II}	Half of the slab thickness in zone I and II
h_p	Peak height of deformation zone
h_r	Edge height of deformation zone
R	Radius of edge roll
l	Projected length of contact arc, $l = \sqrt{2R\Delta W}$
v_0	Entrance velocity
v_R	Peripheral velocity of edge roll
θ	Bite angle, $\theta = \sin^{-1}(l/R)$
φ	Contact angle
d_0	The maximum width of dog bone region
d_φ	Deformation zone's width during edge rolling
d_E	Deformation zone's width after edge rolling
β	Undetermined parameters
b	Yield criterion parameter
v_x, v_y, v_z	Components of velocity vector

J^*	Total power
$\dot{W}_i$	Internal plastic deformation power
$\dot{W}_f$	Friction power
$\dot{W}_s$	Shear power
σ_s	Material yield stress
k	Yield shear stress, $k = \sigma_s / \sqrt{3}$
m	Friction factor
$J^*_{\min}$	Minimum value of total power
τ_f	Friction stress
M	Rolling torque
ω	Roll angular velocity
$\bar{P}$	Rolling force
n_σ	Stress state coefficient
χ	Arm factor
x, y, z	The directions of length, width and thickness

1 Introduction

With the development of rolling technology, continuous casting and rolling have been extensively applied in many iron and steel enterprises. And the width accuracy of hot rolled strip steel products is an extremely important technical indicator in continuous casting and rolling process. At present, the task of controlling the width of continuously cast slabs is mainly undertaken by vertical rolling and Sizing Press (SP), and the reduction of the vertical rolling is much smaller than that of the SP process. Due to small reduction and high width to thickness ratio, plastic deformation is concentrated on a tiny edge zone of the slab and presents obvious dog bone shape after vertical rolling[1]. The dog bone shape has a significant influence on the width spread of subsequent flat rolling process, and can not be measured on-line, so the accurate prediction of dog-bone shape and rolling force is of enormous significance to the automatic control.

Okado et al.[2], Toaze et al. [3] and Shibahara et al.[4] have investigated the dog-bone shape by simulating edge rolling process with pure lead and obtained a series of empirical formulas. Ginzburg[5] modified Toaze's formulas and drew the conclusion that the relative dog bone peak height increased with the increase of reduction and decreased with the increase of the initial thickness of the slab. Xiong et al.[6,7] obtained an experiential model of dog bone shape after vertical rolling through physical simulation experiment on laboratorial rolling mill. However these formulas can only describe the shape of the exit deformation zone, and the prediction accuracy is considerably affected by different rolling conditions.

The finite element method (FEM) has been one of the best ways to analyze vertical rolling because it is suitable for solving intricate geometric deformation. Huisman and Huetink[8] investigated the effect of different vertical roll radius on the dog bone shape after edge rolling, and verified the FEM results by using plasticine as experimental material. Chuang et al.[9] considered the geometric and material nonlinearities, and established the three-dimensional vertical rolling and subsequent flat rolling models based on the explicit dynamic equations by using dynamic relaxation method, and obtained the shape parameters data of dog-bone after edge rolling. Through ABAQUS Explicit Solver, Forouzan et al.[10] proposed a three-dimensional elastic-viscoplastic finite element model for edge rolling, studied the characteristics of geometric shape change and deformation law, and compared it with Sizing Process. Ruan et al.[11] set up a 3-D rigid-plastic FEM model with DEFORM to discuss the displacement velocity field of the deformation zone and the distribution of dog bone shape under different technological rolling parameters. Nevertheless, although the results obtained by the FEM have high accuracy, that can not be used for on-line automatic control in practical production due to the large amount of computation time.

In relation to the development of the analytical solution of vertical rolling force and dog bone shape model, Yun et al.[12] proposed a mathematical model consisting of exponential function and quartic function, but the parameters of dog bone shape and edge rolling force were obtained by fitting FEM simulation data. Since the model did not offer the expression of deformation power functional, Yun did not get the analytical solution of edge rolling process. The author previously proposed a method for calculating the vertical rolling force and dog bone shape by combining slip-line and exponent velocity field[13]. Unfortunately, although the paper makes an accurate prediction of the maximum width of the dog bone region, the calculation error of edge rolling force is slightly larger

due to the selection of upper bound method, and the proposed dog bone shape model is somewhat rough.

Unified Yield(UY) criterion was proposed by YU after noticing of non-linearity of Mises's yield criterion, which is not a single yield criterion but a series of continuously variable linear yield criteria[14]. Then Zhao deduced the linear plastic work rate per unit volume of UY criterion by flow rule[15]. Therefore, for the purpose of improving the prediction accuracy of edge rolling force, a new mathematical model is proposed by energy method with Unified Yield(UY) criterion, the calculated accuracy of vertical rolling force is further improved by optimizing the weight coefficient in UY criterion. Finally, the calculated shape and mechanical parameters are verified, and the variation law of stress state coefficient under different rolling conditions is discussed.

2 Optimization of sine function dog bone shape model

As shown in Fig. 1, the maximum depth of dog bone region has been determined by the geometric characteristics of edge rolling slip-line field in our previous study[16], and is expressed as follows:

$$d_0 = B'D' = BD = AB = 2h_0 \quad (1)$$

Fig. 1 Slip-line field and hodograph for edge rolling.

Based on equation (1), the author optimized Liu's sine function dog bone shape model[17], as shown in Fig. 2, the mathematical expressions of half thickness $h(x, y)$ are as follows:

Zone I: $(0 < y < W_E - d_E)$; half thickness $h_1 = h_1(x, y)$ is

$$h_1 = h_0 \quad (2)$$

Zone II: ($W_E - d_E < y < W_x$); half thickness $h_{II} = h_{II}(x, y)$ is

$$h_{II} = h_0 + \beta \Delta W_x - \beta \Delta W_x \sin \left[\frac{3\pi(y - W_x)}{2d_\varphi} \right] \quad (3)$$

Where β is undetermined parameter, and on the basis of the incompressibility condition

$$\beta = \frac{3\pi h_0}{d_\varphi(2+3\pi)}. \quad W_x \text{ is half of the slab width, } W_x = R + W_E - \sqrt{R^2 - (l-x)^2}.$$

The peak height of deformation zone is

$$h_p = h_{II} \left(l, W_E - \frac{d_E}{3} \right) = h_0 + \frac{6\pi h_0 \Delta W}{d_E(2+3\pi)} \quad (4)$$

The edge height of deformation zone is

$$h_r = h_{II}(l, W_E) = h_0 + \frac{3\pi h_0 \Delta W}{d_E(2+3\pi)} \quad (5)$$

Where d_E is the width of dog bone zone after edge rolling, $d_E = d_0 - \Delta W = 2h_0 - \Delta W$.

Fig. 2 Definition sketch of the bite zone and sine function dog bone profile in edge rolling.

3 Velocity and strain rate fields

$dc - dC$ is the lateral variation of an infinitesimal segment in the direction of length, and

$w = w(x, y)$ is the lateral displacement as shown in Fig. 2.

$$\frac{dw}{dy} = \frac{dc - dC}{dC} \quad (6)$$

Half thickness is $h = h(x, y)$, considering the incompressibility condition, the velocity of rolling direction is

$$v_x = \frac{v_0 h_0}{h} \frac{dC}{dc} \quad (7)$$

Substituting Eq. (7) into Eq. (6) and noticed that $\frac{dw}{dy} = 1$ and $\frac{dw/dy}{1 + dw/dy} \approx \frac{dw}{dy}$, then

$$v_x = \frac{v_0 h_0}{h} \left(1 - \frac{dw}{dy} \right) \quad (8)$$

According to the properties of stream function

$$\frac{v_y}{v_x} = \frac{dw}{dx} \quad (9)$$

Substituting Eq. (8) into Eq. (9), the metal flow velocity of width direction is derived as follows

$$v_y = \frac{v_0 h_0}{h} \left(1 - \frac{\partial w}{\partial y} \right) \frac{\partial w}{\partial x} \quad (10)$$

Based on plane deformation assumption

$$v_x = v_0 \quad (11)$$

Then in accordance with the incompressibility condition

$$\begin{aligned} \mathcal{E}_z &= -(\mathcal{E}_x + \mathcal{E}_y) = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) \\ &= v_0 h_0 \left\{ \frac{\partial}{\partial y} \left(\frac{1}{h} \right) \left[\frac{\partial w}{\partial x} \left(\frac{\partial w}{\partial y} - 1 \right) \right] + \frac{1}{h} \frac{\partial^2 w}{\partial x \partial y} \left(\frac{\partial w}{\partial y} - 1 \right) + \frac{1}{h} \frac{\partial}{\partial w} \frac{\partial^2 w}{\partial y^2} \right\} \end{aligned} \quad (12)$$

Noticing when $z=0$, $v_z=0$, the metal flow velocity of vertical direction v_z can be obtained by integrating $\mathcal{E}_z$

$$\begin{aligned} v_z &= \int \mathcal{E}_z dy \\ &= v_0 h_0 z \left\{ \frac{\partial}{\partial y} \left(\frac{1}{h} \right) \left[\frac{\partial w}{\partial x} \left(\frac{\partial w}{\partial y} - 1 \right) \right] + \frac{1}{h} \frac{\partial^2 w}{\partial x \partial y} \left(\frac{\partial w}{\partial y} - 1 \right) + \frac{1}{h} \frac{\partial}{\partial w} \frac{\partial^2 w}{\partial y^2} \right\} \end{aligned} \quad (13)$$

From Eqs. (8) and (11)

$$w = w(x, y) = \int_0^y (1 - h/h_0) dy \quad (14)$$

From Eqs. (2), (3), (14) and boundary conditions, the lateral displacement of each region can be obtained as follows

Zone I:

$$w_I = \int_0^{W_E - d_E} (1 - h_I/h_0) dy = 0 \quad (15)$$

Zone II:

$$w_{II} = -\frac{3\pi\Delta W_x}{(2+3\pi)d_\phi} \left[y - (W_x - d_\phi) \right] - \frac{2\Delta W_x}{2+3\pi} \cos \left[\frac{3\pi(y - W_x)}{2d_\phi} \right] \quad (16)$$

It can be obtained from Eq. (16), $w_{II}(x, W_x) = -\Delta W_x$, and it satisfies boundary condition.

Substituting Eq. (15) into Eqs. (10) and (13), the velocity field of zone I is derived as follows

$$v_{xI} = v_0, v_{yI} = v_{zI} = 0 \quad (17)$$

Then the axial strain rate field in zone I can be deduced on the basis of Cauchy equation

$$\dot{\epsilon}_x = \dot{\epsilon}_y = \dot{\epsilon}_z = 0 \quad (18)$$

Placing Eq. (16) into Eqs. (10) and (13) receives the velocity field of zone II is

$$\begin{aligned} v_{xII} &= v_0 \\ v_{yII} &= \frac{3\pi v_0 (y - W_x + d_\phi)}{(2+3\pi)d_\phi^2} \left[2h_0 - \Delta W_x \sin \left(\frac{3\pi(y - W_x)}{2d_\phi} \right) \right] \frac{dW_x}{dx} + \frac{2v_0}{2+3\pi} \cos \left(\frac{3\pi(y - W_x)}{2d_\phi} \right) \frac{dW_x}{dx} \\ v_{zII} &= -\frac{\pi v_0 z}{(2+3\pi)} \left\{ \frac{6h_0}{d_\phi^2} \left[1 - \sin \left(\frac{3\pi(y - W_x)}{2d_\phi} \right) \right] - \frac{9\pi(y - W_x + d_\phi)\Delta W_x}{2d_\phi^3} \cos \left(\frac{3\pi(y - W_x)}{2d_\phi} \right) \right\} \frac{dW_x}{dx} \end{aligned} \quad (19)$$

From Eqs. (17) and (19), $v_{zI}|_{x=0} = v_{zII}|_{x=0} = 0$, $v_{yII}|_{x=l} = 0$, $v_{zI}|_{x=l} = v_{zII}|_{x=l} = 0$,

$v_{zI}|_{x=l} = v_{zII}|_{x=l} = 0$, $v_{xI}|_{y=W_E - d_E} = v_{xII}|_{y=W_E - d_E} = v_0$, $v_{yI}|_{y=W_E - d_E} = v_{yII}|_{y=W_E - d_E} = 0$,

$v_{zI}|_{y=W_E - d_E} = v_{zII}|_{y=W_E - d_E} = 0$. The equations above satisfy the boundary conditions.

Based on Cauchy equation, the axial strain rate field in zone II is determined as follows

$$\dot{\epsilon}_{xII} = 0$$

$$\begin{aligned}\dot{\epsilon}_{yII} &= \frac{\pi v_0}{(2+3\pi)} \left\{ \frac{6h_0}{d_\phi^2} \left[1 - \sin\left(\frac{3\pi(y-W_x)}{2d_\phi}\right) \right] - \frac{9\pi(y-W_x+d_\phi)\Delta w_x}{2d_\phi^3} \cos\left(\frac{3\pi(y-W_x)}{2d_\phi}\right) \right\} \frac{dW_x}{dx} \\ \dot{\epsilon}_{zII} &= -\frac{\pi v_0}{(2+3\pi)} \left\{ \frac{6h_0}{d_\phi^2} \left[1 - \sin\left(\frac{3\pi(y-W_x)}{2d_\phi}\right) \right] - \frac{9\pi(y-W_x+d_\phi)\Delta w_x}{2d_\phi^3} \cos\left(\frac{3\pi(y-W_x)}{2d_\phi}\right) \right\} \frac{dW_x}{dx}\end{aligned}\quad (20)$$

From Eqs. (18) and (20), $\dot{\epsilon}_{xI} + \dot{\epsilon}_{yI} + \dot{\epsilon}_{zI} = 0$, $\dot{\epsilon}_{xII} + \dot{\epsilon}_{yII} + \dot{\epsilon}_{zII} = 0$.

So, they are kinematically admissible velocity and strain rate field.

4 Total power functional

4.1 UY criterion

Unified Yield (UY) criterion is a unified expression of various linear yield criteria in the error triangle between Tresca's and twin shear stress yield loci on the π -plane[14], as shown in Fig. 3.

The UY criterion is usually presented by

$$\begin{cases} \sigma_1 - \frac{b\sigma_2}{1+b} - \frac{\sigma_3}{1+b} = \sigma_s & \text{if } \sigma_2 \leq \frac{\sigma_1 + \sigma_3}{2} \\ \frac{\sigma_1}{1+b} + \frac{b}{1+b}\sigma_2 - \sigma_3 = \sigma_s & \text{if } \sigma_2 \geq \frac{\sigma_1 + \sigma_3}{2} \end{cases} \quad (21)$$

Where b is the yield criterion parameter, which represents the effect of the intermediate principal shear stress on the yield of materials and $0 \leq b \leq 1$. σ_1 , σ_2 and σ_3 are principal stresses.

And the specific plastic work rate per unit volume of UY criterion is derived by Zhao[15]

$$D(\dot{\epsilon}_{ij}) = \frac{1+b}{2+b} \sigma_s (\dot{\epsilon}_{\max} - \dot{\epsilon}_{\min}) \quad (22)$$

Respectively, $\dot{\epsilon}_{\max}$ and $\dot{\epsilon}_{\min}$ are maximum and minimum strain rate in deformation, $\dot{\epsilon}_{\max} = \dot{\epsilon}_z$ and $\dot{\epsilon}_{\min} = \dot{\epsilon}_y$. Zhao's research has made considerable progress in the physical linearization of forming energy rate functional integral, which provides a necessary basis for the first application of UY criterion in this paper to improve the accuracy of vertical rolling force calculation.

Fig. 3 Yield loci in π -plane.

4.2 Inter-deformation power

Substituting Eqs. (18) and (20) into Eq. (22), the inter-deformation power can be calculated as follows

$$\begin{aligned}
 W_i^{\&} &= \frac{4(1+b)}{2+b} \int_0^{W_x} \int_{W_E-d_E}^{W_x} \int_0^{h_1} \sigma_s (\&_{\max} - \&_{\min}) dz dy dx \\
 &= \frac{4(1+b)\sigma_s}{2+b} \int_0^{W_x} \int_{W_E-d_E}^{W_x} \int_0^{h_1} \left(\frac{-2\pi v_0}{(2+3\pi)} \left\{ \frac{6h_0}{d_\varphi^2} \left[1 - \sin \left(\frac{3\pi(y-W_x)}{2d_\varphi} \right) \right] - \frac{9\pi(y-W_x+d_\varphi)\Delta W_x}{2d_\varphi^3} \cos \left(\frac{3\pi(y-W_x)}{2d_\varphi} \right) \right\} \frac{dW_x}{dx} \right) dz dy dx \\
 &= \frac{-4(1+b)\sigma_s}{2+b} \frac{\pi v_0 h_0}{(2+3\pi)} \int_{W_0}^{W_E} \left[\frac{(4+6\pi)h_0}{\pi d_\varphi} + \frac{(8+9\pi)h_0}{2+3\pi} \frac{3\Delta W_x}{d_\varphi^2} - \frac{(8+3\pi)}{4(2+3\pi)} \frac{3\Delta W_x^2}{d_\varphi^2} - \frac{2}{\pi} \frac{\Delta W_x}{d_\varphi} \right] dW_x \\
 &= \frac{1+b}{2+b} \frac{3\sigma_s v_0 h_0}{(2+3\pi)^2 (2h_0 - \Delta W)} \left[16\pi(2h_0 - \Delta W)h_0 \ln \left(\frac{2h_0}{2h_0 - \Delta W} \right) + \frac{4(2h_0 - \Delta W)}{3} (4+6\pi+9\pi^2)\Delta W + (8+15\pi)\pi\Delta W^2 \right]
 \end{aligned} \tag{23}$$

4.3 Friction power

Friction acts on the contact surface between slab and vertical roll ($y = W_x$), and the tangential velocity discontinuity is

$$\Delta v_t = v_R - \frac{v_0}{\cos \varphi} \tag{24}$$

The velocity discontinuity is

$$\Delta v_f = \sqrt{\left(v_{zII}|_{y=W_x}\right)^2 + \Delta v_t^2} \quad (25)$$

It is noted that friction stress $\tau_f = mk = m\sigma_s/\sqrt{3}$ and velocity discontinuity Δv_f are always in the same direction, then the friction power is deduced by using collinear vector inner product

$$\begin{aligned} \dot{W}_f &= 4 \int_0^l \int_0^{h_{rx}} |\tau_f| |\Delta v_f| \cos(\Delta v_f, \tau_f) ds \\ &= 4 \int_0^l \int_0^{h_{rx}} \tau_f |\Delta v_f| ds = 4mk \int_0^l \int_0^{h_{rx}} \sqrt{\left(v_{zII}|_{y=W_x}\right)^2 + \Delta v_t^2} \frac{dzdx}{\cos \varphi} \end{aligned} \quad (26)$$

For the purpose of obtaining the analytical solution of the friction power, the mean value theorem of integral is used to calculate the average value of Δv_t , h_r and v_{zII}

$$\Delta \bar{v}_t = \frac{\int_0^\theta \left(v_R - \frac{v_0}{\cos \varphi}\right) d\varphi}{-\theta} = v_R - \frac{v_0}{2\theta} \ln \left(\frac{R+l}{R-l}\right) \quad (27)$$

$$\bar{h}_r = \frac{h_{rl} + h_0}{2} = h_0 + \frac{3\pi h_0 \Delta W}{2(2+3\pi)(2h_0 - \Delta W)} \quad (28)$$

$$\bar{v}_{zII} = \frac{1}{l\bar{h}_r} \int_0^l \int_0^{\bar{h}_r} \left(v_{zII}|_{y=W_x}\right) dzdx = \frac{\pi v_0 \bar{h}_r}{4(2+3\pi)l} \left[9\pi \ln \left(\frac{2h_0}{2h_0 - \Delta W}\right) + \frac{3(3\pi-2)\Delta W}{2h_0 - \Delta W} \right] \quad (29)$$

Substituting $ds = \frac{dx dy}{\cos \varphi}$ and $dx = -R \cos \varphi d\varphi$ into Eq. (26), it becomes

$$\begin{aligned} \dot{W}_f &= 4 \int_0^l \int_0^{h_{rx}} \tau_f |\Delta v_f| ds = 4 \int_0^l \int_0^{h_{rx}} |\tau_f| |\Delta v_f| \cos(\Delta v_f, \tau_f) ds = 4 \int_0^l \int_0^{h_{rx}} \Delta v_f \tau_f ds \\ &= \frac{4m\sigma_s \bar{h}_r R \theta}{\sqrt{3}} \left\{ \Delta \bar{v}_t \left[1 + (\bar{v}_{zII}/\Delta \bar{v}_t)^2 \right]^{-1/2} + \bar{v}_{zII} \left[1 + (\Delta \bar{v}_t/\bar{v}_{zII})^2 \right]^{-1/2} \right\} \end{aligned} \quad (30)$$

4.4 Shear power

On the basis of the mean value theorem, the velocity field in the transverse region of the entrance becomes

Zone I:

$$\bar{v}_{yI} = \bar{v}_{zI} = 0 \quad (31)$$

Zone II:

$$\begin{aligned}\bar{v}_{yII} &= -\frac{3v_0 \tan \theta}{2(2+3\pi)h_0} \frac{\int_{W_0-2h_0}^{W_0} \left[\pi(y-W_0+2h_0) + \frac{4}{3}h_0 \cos\left(\frac{3\pi(y-W_0)}{4h_0}\right) \right] dy}{2h_0} = -\frac{\tan \theta v_0 (9\pi^2 - 8)}{6\pi(2+3\pi)} \\ \bar{v}_{zII} &= \frac{3\pi \tan \theta v_0 z}{2(2+3\pi)h_0} \frac{\int_{W_0-2h_0}^{W_0} \left[1 - \sin\left(\frac{3\pi(y-W_0)}{4h_0}\right) \right] dy}{2h_0} = \frac{\tan \theta v_0 z}{2h_0}\end{aligned}\quad (32)$$

Then the shear power can be expressed as

$$\begin{aligned}W_s^* &= 4k \int_{W_0-2h_0}^{W_0} \int_0^{h_0} \sqrt{(\bar{v}_{yII})^2 + (\bar{v}_{zII})^2} dz dy = \frac{4k \tan \theta v_0}{2h_0} \int_{W_0-2h_0}^{W_0} \int_0^{h_0} \sqrt{z^2 + \left[\frac{9\pi^2 - 8}{3\pi(2+3\pi)} h_0 \right]^2} dz dy \\ &= \frac{2\sigma_s \tan \theta v_0 h_0^2}{\sqrt{3}} \left[\sqrt{1 + \left[\frac{9\pi^2 - 8}{3\pi(2+3\pi)} \right]^2} + \left[\frac{9\pi^2 - 8}{3\pi(2+3\pi)} \right]^2 \ln \frac{1 + \sqrt{1 + \left[(9\pi^2 - 8)/3\pi(2+3\pi) \right]^2}}{(9\pi^2 - 8)/3\pi(2+3\pi)} \right]\end{aligned}\quad (33)$$

4.5 Total power and its optimization

Substituting Eqs. (23), (30) and (33) into $J^* = W_i^* + W_f^* + W_s^*$ yields the analytical solution of total power functional

$$\begin{aligned}J^* &= \frac{1+b}{2+b} \frac{3\sigma_s v_R h_0 \cos \theta}{(2+3\pi)^2 (2h_0 - \Delta W)} \left[16\pi(2h_0 - \Delta W) h_0 \ln \left(\frac{2h_0}{2h_0 - \Delta W} \right) + \frac{4(2h_0 - \Delta W)}{3} (4 + 6\pi + 9\pi^2) \Delta W + (8 + 15\pi) \pi \Delta W^2 \right] \\ &+ \frac{4m\sigma_s \bar{h}_r R \theta}{\sqrt{3}} \left\{ \Delta \bar{v}_i \left[1 + (\bar{v}_{zII}/\Delta \bar{v}_i)^2 \right]^{-1/2} + \bar{v}_{zII} \left[1 + (\Delta \bar{v}_i/\bar{v}_{zII})^2 \right]^{-1/2} \right\} \\ &+ \frac{2\sigma_s \tan \theta v_0 h_0^2}{\sqrt{3}} \left[\sqrt{1 + \left[\frac{9\pi^2 - 8}{3\pi(2+3\pi)} \right]^2} + \left[\frac{9\pi^2 - 8}{3\pi(2+3\pi)} \right]^2 \ln \frac{1 + \sqrt{1 + \left[(9\pi^2 - 8)/3\pi(2+3\pi) \right]^2}}{(9\pi^2 - 8)/3\pi(2+3\pi)} \right]\end{aligned}\quad (34)$$

J^* is only relevant to the yield criterion parameter b when deformation resistance σ_s , reduction ΔW , radius of edge roll R , peripheral velocities v_R , initial thickness of slabs $2h_0$ and friction factor of edge roll-slab arc m are given. Therefore, the value of b can be repeated optimized between 0 and 1 according to the measured vertical rolling force data to improve the accuracy of Eq. (34). Then, the corresponding values of rolling torque M , rolling force $\bar{P}$ and stress state coefficient n_σ can be achieved separately as [18]

$$M = \frac{J^*}{2\omega}, \quad \bar{P} = \frac{M}{\chi l}, \quad n_\sigma = \frac{\bar{P}}{\sigma_s} \quad (34)$$

The arm factor $\chi = 0.3 \sim 0.6$ was researched in hot rolling process[19], and $\chi=0.5$ is selected in this paper under equipment and process parameters.

5 Results

5.1 Dog-bone shape

To verify the accuracy of the dog bone shape model proposed in this paper, the ratio of dog-bone peak height to width h_p/W_0 was calculated by Eq.(4) under different engineering strain $\Delta W/W_0$, initial thickness h_0 and edge roll radius R . And the comparison between optimized sine function model's results and those of other models are shown in Figs. 4-5. The comparison results demonstrate that the deviation between Eq.(4) and Ginzburg's model is within 5.7%, and that between Eq.(4) and Xiong's model is within 8%. As can be seen from Fig. 4 the deflections between the present model and other two models will increase with the increase of the reduction ΔW . When the value of $\Delta W/W_0$ less than 0.015, the comparative deflections were reduced to 1.5% and 5%. Because the present model assumed that the edge rolling process is a plane deformation process, it is reasonable that the value of h_p/W_0 is higher than that of Xiong's model and Ginzburg's model.

Fig. 4 Influence of $\Delta W/W_0$ on h_p/W_0 .

Fig. 5 Influence of h_0/W_0 on h_p/W_0 .

5.2 Edge rolling force

Fig. 6 Comparison between calculated rolling force under different b and actual measured data.

To verify the accuracy of the edge rolling force model proposed in this paper, the values of roll force under different roll conditions were calculated by Eqs.(33) and (34). By optimizing the value of b over and over again, the error between the calculated value of vertical rolling force and actual measured data is reduced to less than 5% when $b=0.613$. In addition, twin shear stress($b=1$) and Tresca yield criterion ($b=0$) were used to calculate vertical rolling force, and the maximum error between the

calculated value and measured data are within 12% and 15%, as shown in Fig. 6. This indicates that the application of UY criterion can effectively improve the accuracy of vertical rolling force calculation by the energy method.

6 Discussion

Based on the dog-bone shape model Eq.(4) and vertical rolling force model Eqs.(33)-(34) proposed in present paper, the influence of different rolling process parameters on dog-bone peak height h_p and vertical rolling force is studied, the results are shown in Figs. 4-5 and 7-8.

Figure 4 reflects the significant increases in dog-bone peak height during vertical rolling process with the increase in the engineering strain $\Delta W/W_0$ increases. This is caused by the length of the contact arc and the volume of the deformed metal increases, while ΔW increases. Since the plastic deformation is concentrated on the edge of the slab, h_p increases obviously. In Figure 5, the dog-bone peak height h_p shows a linear growth trend with the increase of the slab's initial thickness h_0 , which is consistent with Xiong's and Ginzburg's models. This is because when h_0 increases only, the contact arc length between the slab and the vertical roll remains unchanged, but the contact surface of roll slab becomes lager, which increases the volume of metal involved in plastic deformation, resulting in an increase in value of h_p . Furthermore, although h_p increases with h_0 , the increase in h_p is much smaller than the increase in h_0 , which is why the curves in Fig. 5 are linear.

Fig. 7 Influence of $\Delta W/W_0$ and h_0/W_0 on stress state coefficient n_σ .

Fig. 8 Influence of $\Delta W / W_0$ and friction factor m on stress state coefficient n_σ .

The vertical rolling force is another key parameter studied in present paper, according to the calculation results in Fig. 6, it can be seen that the variation of the yield criterion parameter b has a drastic positive effect on the calculated value of vertical rolling force, because the inter-deformation power accounts for a large proportion of total power in vertical rolling process. After optimizing vertical rolling force model by adjusting the value of yield criterion parameter b , the impact of slab thickness h_0 and reduction ΔW on the edge rolling force is studied, and the results are presented in Fig. 7. Figure 7 reveals that the stress state coefficient n_σ increase significantly with the increase of h_0 / W_0 and engineering strain $\Delta W / W_0$, and the increasing trend of n_σ is from sharp to gentle as the h_0 / W_0 and $\Delta W / W_0$ increase. In addition, it is evident from Figure 7 that the influence of engineering strain on the stress state coefficient is much greater than that of the initial thickness, and it is the main factor affecting the change of rolling force.

Figure 8 reflects the influence law of friction coefficient m and $\Delta W / W_0$ on the influence coefficient of the stress state. When the engineering strain is small, the friction coefficient has little influence on the rolling force, but as the engineering strain becomes larger, the friction coefficient has a rapid increase in the influence of the rolling force.

6 Conclusions

1. The UY criterion is firstly proposed establishing the vertical rolling force model, the yield

criterion parameter b is converted into an optimal parameter of vertical rolling force. Then, the value of b is repeatedly optimized, and it is determined that the mechanical model in this paper achieved the highest accuracy when $b=0.613$, and the error between the optimized model calculation results and the field measured data is within 5%.

2. The vertical rolling forces calculated by UY, TSS and Tresca yield criteria are compared with actual measured data. It is noted that the prediction accuracy of the rolling force is effectively improved by optimizing the yield parameter b . This proves that it is feasible and effective to use UY criterion to optimize the solution accuracy of mechanical parameters.
3. The sine function dog bone shape model is optimized by using the slip line field to determine the maximum depth of the dog bone area, which ensures the prediction accuracy and simplifies the calculation steps, which can improve the prediction efficiency of the slab shape after vertical rolling.
4. Engineering strain is the primary factor that affects the height of the bone peak of the dog bone shape. The increase of the initial thickness of the slab also increases the height of the bone peak, but it is far from the obvious influence of the engineering strain on the height of the bone peak. And the increase of engineering strain, initial slab thickness and friction coefficient will cause different degrees of increase in rolling force. The co-increasing of engineering strain and friction coefficient will make the rolling force present the most obvious increasing trend.

Declarations

Ethical Approval: Not applicable.

Consent to Participate: Not applicable.

Consent to Publish: Not applicable.

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personal interest of any nature or kind in any product, service and/or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled, “The optimization solution of the vertical rolling force by using Unified Yield criterion”.

Availability of data and materials: The datasets used or analyzed during the current study are available from the corresponding author on reasonable request.

References

- [1] Ginzburg VB (1989) Steel-rolling technology: theory and practice. Marcel Dekker, New York
- [2] Okado M, Ariizumi T, Noma Y, Yabuuchi K, Yamazaki Y (1981) Width behaviour of the head and tail of slabs in edge rolling in hot strip mills. *Tetsu To Hagane* 67:2516-2525. doi: 10.2355/tetsutohagane1955.67.15_2516
- [3] Tazoe N, Honjyo H, Takeuchi M (1984) New forms of hot strip mill width rolling installations. In: AISE Spring Conference, Dearborn, USA, pp 85-88
- [4] Shibahara T, Misaka Y, Kono T, Koriki M, Takemoto H (1981) Edger set-up model at roughing train in a hot strip mill. *Tetsu To Hagane* 67:2509-2515. doi: 10.2355/tetsutohagane1955.67.15_2509
- [5] Ginzburg VB, Kaplan N, Bakhtar F, Tabone CJ (1991) Width control in hot strip mills. *Iron Steel Engineer* 68:25-39
- [6] Xiong SW, Zhu XL, Liu XH, Wang G, Zhang Q, Li H, Meng X, Han L (1997) Mathematical model of width reduction process of roughing trains of hot strip mills. *Shanghai Metal* 19(1):39-43
- [7] Xiong SW, Rodrigues JMC, Martins PAF (2003) Three-dimensional modelling of the vertical-horizontal rolling process. *Finite Elem Anal Des* 39:1023-1037. doi: 10.1016/s0168-874x(02)00154-3
- [8] Huisman HJ, Huetink J (1985) A combined Eulerian-Lagrangian three-dimensional finite-element analysis of edge-rolling. *J Mech Work Technol* 11:333-353. doi: 10.1016/0378-3804(85)90005-1
- [9] Chung WK, Choi SK, Thomson PF (1993) Three-dimensional simulation of the edge rolling process by the explicit finite-element method. *J Mater Process Technol* 38(1): 85-101.

doi: 10.1016/0924-0136(93)90188-C

- [10] Forouzan MR, Salehi I, Adibi-sedeh AH (2009) A comparative study of slab deformation under heavy width reduction by sizing press and vertical rolling using FE analysis J Mater Process Technol 209(2): 728-736. doi: 10.1016/j.jmatprotec.2008.02.063
- [11] Ruan JH, Zhang LW, Gu SD (2015) Finite element simulation based plate edging model for plan view pattern control during wide and heavy plate rolling. Ironmak Steelmak 41(3): 199-205.
- [12] Yun D, Lee D, Kim J, Hwang S (2012) A new model for the prediction of the dog-bone shape in steel mills. ISIJ Int 52:1109-1117. doi: 10.2355/isijinternational.52.1109
- [13] Zhang YF, Di HS, Li X, Peng W, Zhao DW, Zhang DH (2020) A novel approach for the edge rolling force and dog-bone shape by combination of slip-line and exponent velocity field. SN Applied Sciences 2:2055. doi: 10.1007/s42452-020-03770-3
- [14] Yu MH, He LN, Liu CY (1992) Generalized twin-shear stress yield criterion and its generalization. Chinese Sci Bull 37(24): 2085-2089.
- [15] Zhao DW, Li J, Liu XH, Wang GD (2009) Deduction of plastic work rate per unit volume for Unified yield criterion and its application. Trans Nonferrous Met Soc China 19:657-660. doi: 10.1016/S1003-6326(08)60329-5.
- [16] Zhang YF, Zhang HY, Chen LJ, Zhao DW, Zhang DH (2018) Calculation of vertical rolling force through slip-line field. J Plast Eng 25:288-291.
- [17] Liu YM, Ma GS, Zhang DH, Zhao DW (2015) Upper bound analysis of rolling force and dog-bone shape via sine function model in vertical rolling. J Mater Process Technol 223:91-97. doi: 10.1016/j.jmatprotec.2015.03.051
- [18] Zhang SH, Wen HT, Lei D (2021) A novel yield criterion and its application to calculate the rolling force of a thick plate during hot rolling. J Braz Soc Mech Sci 43:12. doi:10.1007/s40430-020-02761-0
- [19] Zhang DH, Liu YM, Sun J, Zhao DW (2016) A novel analytical approach to predict rolling force in hot strip finish rolling based on cosine velocity field and equal area criterion. Int J Adv Manuf Tech 84:843-850. doi: 10.1007/s00170-015-7692-z

Figures

Figure 1

Slip-line field and hodograph for edge rolling.

Figure 3

Yield loci in π -plane.

Figure 4

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Figure 5

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Figure 6

Comparison between calculated rolling force under different b and actual measured data.

Figure 7

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Figure 8

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