

Critical Chains and Its Randomness Study

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Research Article

Keywords: critical chain, critical path method, comparative study, randomness, duration optimization, resource constraints

Posted Date: May 20th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-522605/v1>

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13 **Conflicts of interest/Competing interests**

14 The authors declare that they have no conflict of interest.

15 **Acknowledgements**

16 This work was supported in part by the National Natural Science Foundation of China under Grant

17 51808424 and under Grant 51478384, and in part by the Industrial Building Environment and Energy

18 Conservation Innovation Team of China under Grant 2017KCT-14.

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Abstract: It has been widely accepted in the academic community that the Critical Chain Method (CCM) has significant advantages over the Critical Path Method (CPM) in solving the problem with resource constraints. However, this paper conducted a study on comparing the two methods of Critical Chain Method and Critical Path Method, and found that the only difference between those two methods lies in how to determine the priority of resources allocating, and on the assumption of not setting buffer zone, those two methods have no essential distinctions at all. By establishing the relationship between CCM and CPM, this paper also enriched and improved CCM to some extent, and pointed out that the buffer zone setting in CCM is merely subjective and short of scientificity. In the meantime, for the problem of unclear representation of critical chains, it proposed two ways of representing critical chains and related rules to follow. To verify the conclusion of this paper, further detailed case study of comparing CCM and CPM was performed. Affected by various uncertain factors, the project construction sequence is random, the total construction duration is random, and the critical chain is also random, so it is unable to determine how to direct construction. Aiming at the randomness of the critical chain, this article analyzed various uncertain factors of the critical chain, and on the basis of solving the critical chain sequence time, it proposed the approach to determine the completion probability of the total construction duration and control the construction of the critical chain to direct the construction, in the meantime, the inverse algorithm was adopted to determine of the construction duration under the condition of required completion probability.

Keywords: critical chain; critical path method; comparative study; randomness; duration optimization; resource constraints

1. Introduction

In order to solve the problem of limit resources in the construction project, Dr. Goldratt proposed the Critical Chain Method (CCM)[1,2]. Once proposed, this method had aroused great interests and extensive researches in academia. CCM has been widely accepted to be superior to traditional Critical Path Method (CPM). However, through a comparative study by the authors, it is concluded that there is no essential distinctions between the two, and the only difference between those two is about the method of determining the priority of resource allocation and the buffer zone[3].

The critical chain proposed based on the constraint theory has significant advantages in solving resource constraints and construction time waste caused by inefficiency of workers. The critical chain has two technical points when solving the problem, the identification of the critical chain and the setting buffer zone[4].

Adopting a heuristic algorithm

$$ACTIM = T_{total} - LS_{i-j} \quad (1)$$

determine the start sequence, adjust the logical relationship between the work predecessors and successors, and obtain the longest chain in time that meets the resource limit requirements which is the critical chain identified. However, the heuristic algorithm can only solve the problem of determining the operation time of the sequence.

In the middle of construction process, affected by various uncertain factors, the time factors such as the earliest start time and the latest finish time of the activity are uncertain. It also leads to uncertainties in

construction sequence variance and activity durations. The randomness of sequence duration leads to the uncertainty of the total construction duration after the project logical sequence change. The ACTIM algorithm cannot identify critical chains with uncertain sequence duration and total duration. When there is more than one critical chain, the variance of different critical chain sequences' duration is different, and there are differences between the critical chains. The uncertainty of the sequence duration of the critical chains, the uncertainty of the total construction duration, and the differences between the critical chains lead to the randomness of the critical chains.

The purpose of the buffer zone is to absorb the risks caused by reducing the duration of critical chain processes. However, the setting of buffer zone is not scientific and reasonable, and there exists a problem that the buffer zone may be set too large or too small.

The traditional method is not applicable to the identification of critical chains with random characteristics and the problem of unreasonable buffer settings. Based on this, the article studied the randomness of critical chains and the problem of buffer setting.

2. Review on Critical Chains

On solving the problem of limited resources, CCM has two technical key points of critical chain identification and buffer zone setting.

2.1 Identification of critical chain

The identification of critical chain is an algorithm that allocates resources reasonably under the resource constraints. There are quite a few methods for identifying, the most commonly used is a heuristic algorithm based on priority rules proposed by Goldratt, also known as ACTIM value method. Here is formula (2)

$$ACTIM = T - LS_{i-j} \quad (2)$$

In this formula, T -Total Duration, LS_{i-j} -Latest Start Time of an activity.

When resource conflicts occur between activities, the priority of resource allocation can be determined by the ACTIM value: the activity with larger ACTIM value has higher priority over the activity with smaller ACTIM value on resource allocation. This may cause changes in the logical relationships between related activities in the original network diagram and delays in the total construction duration, and there will be multiple critical chains. However, there has not been sufficiently studied regarding this issue up to present, and neither on how to represent the critical chains under the resource constraints.

2.2 Setting buffer zone and its problems

In the execution of a project, at the beginning people are more tend not to hurry to work at full strength until it's getting closer to the project finish phase. It's like students tend to waste most of their time outside of studying, whereas study extremely hard right before the exam coming. This phenomenon is called "Student Syndrome", which is expressed in Parkinson's Law- How much time left will automatically become the time needed to complete that work, that is, the work will always be dragged down to be completed to the last minute, and never be completed ahead of time[6].

To mitigate the impact of "Student Syndrome" or Parkinson's Law on the project duration, the critical chain technology proposed a method of setting a buffer zone. The essence of it is to reduce the working time of each process and gather the reduced time together for setting up a buffer area for use. Common buffer setting methods are 50% clipping method and root variance method.

No matter which method is used to set the buffer zone, it is a subjective method and lacks scientific basis. The purpose of setting up the buffer zone is to solve the problem of “Student Syndrome”, but this is not the only solution. It is entirely possible to make a scientific and reasonable estimate of the activity duration by studying various uncertain factors affecting the activity duration, including “Student Syndrome” that may occur. Therefore, it is not necessary to set the buffer zone.

3. Determination and representation of critical chains

3.1 Rules for determining critical chains

According to the ACTIM value method proposed by Goldratt, it mainly solves the issue of resource allocating priority when the resource is limited, and meets the constraint condition of resource priority allocation order, which may change the logical relationships between some activities s in the original network and form new paths. It increases the difficulty of identifying critical chains, for which the following basic rules must be followed:

- (1) Meet the constraints of resource limits.
- (2) New paths formed after changes in the logical relationships between certain activities must be considered.
- (3) The sum of the durations of all activities on the same path is the longest.

This often leads to multiple critical chains. How to present the critical chain clearly and intuitively is a problem to be solved.

3.2 Representation of critical chains

There are many researches on the critical chain, but there is still no consensus has been reached on the representation of the critical chain, and the problem that the critical chain representation is unclear still exists. The key to solving this problem is to clearly show the change in the logical relationships between some activities after meeting the resource constraints. This paper proposed two representations of critical chains.

Taking Fig.1 as an example, assuming that all activities use the same resource and the buffer zone setting is not considered, the numbers in parentheses on the arrow line indicate the resource requirements (cubic meters / week), and the numbers below the arrow line indicate the duration of activity (week). The maximum resource limit is 20 cubic meters per week.

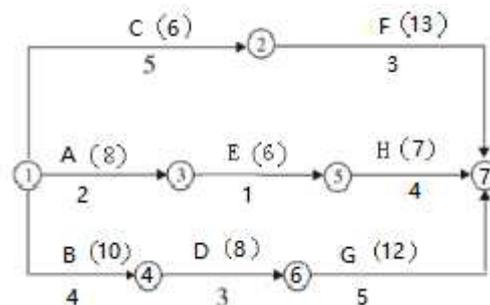


Figure 1 Directly mark out the changes in critical chains and related activity logic relationships on the network diagram

According to the resource constraints and critical chain determination rules, determine the new logical relationships and critical chains between the activities, and use the double solid line or thick solid line to

directly mark out on the network diagram. The double solid lines in Fig.2 represent two critical chains: A-C-F-G and B-D-F-G. If the work in the network diagram has no logical relationships, after adjusting the resources, it becomes a Finish-Start relationship, and it is located on the critical chain, and the operations are still marked with double solid lines. For example, activity A and activity C originally had a parallel relationship, and after adjustment of resources, there was a Finish-Start relationship, and a double solid line was artificially added to form a complete critical chain. In this way, the critical chain and the change in the logical relationships between the activities that meet the resource limit can be seen intuitively and clearly from the figure. The wavy line in the figure indicates the float time.

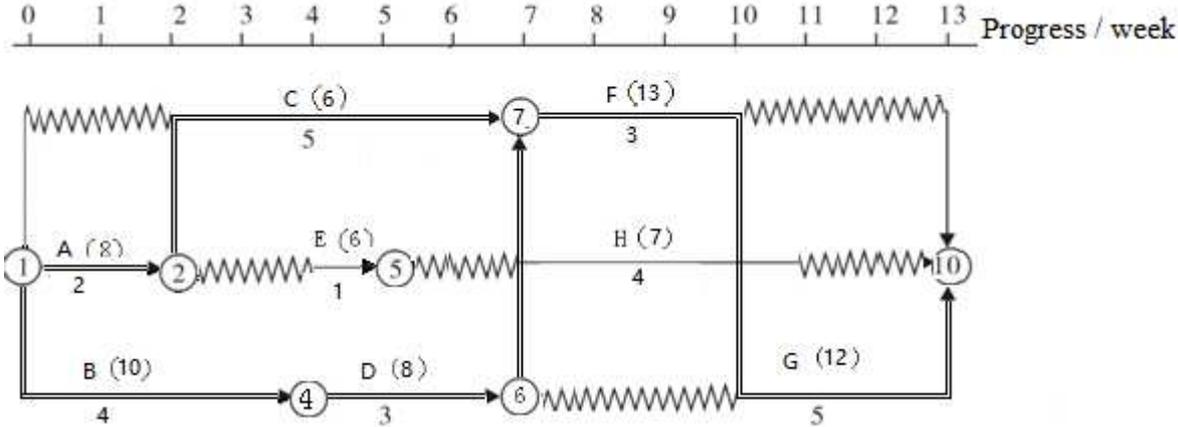


Figure 2 Critical chain representation

(2) Adding nodes and dummy activities to annotate the changes of critical chains and relevant activity logic relationships.

The first representation method is relatively simple and easy to operate, but there will arrow line crosses that do not meet the drawing rules of the network diagram. If the work in the network graph originally had no logical relationships, after the resource adjustment, the logical relationship of Finish-Start can be represented by adding nodes and dummy activities. The critical chain is still represented by double solid line or thick solid line. As shown in Fig.3, node 3 and dummy activity 2-3 are added between activities A and C. Similarly, nodes 8 and 9 are new nodes. The critical chains are still A-C-F-G and B-D-F-G. The expression is clearer, especially when there are multiple key chains, which is more advantageous.

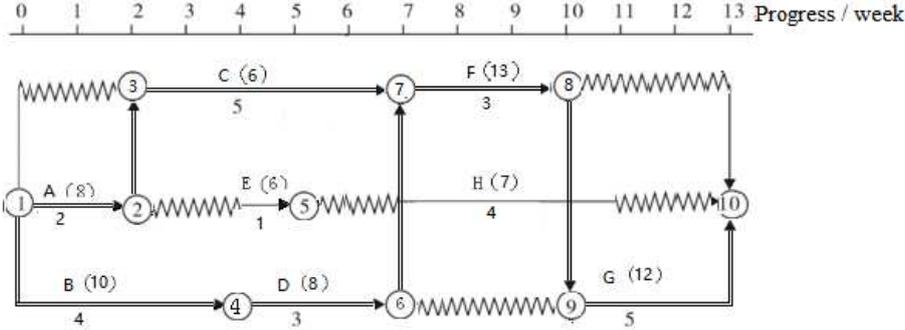


Figure 3 Critical chain representation

4. Comparison of CCM and CPM

A comparative study of the Critical Chain Method and Critical Path Method using the network plan shown in Fig.4 is based on the following assumptions:

- (1) The duration of every activity is a fixed value and does not change;
- (2) Do not consider the buffer zone setting for the critical chain;
- (3) Once a activity is started, it must not be interrupted to ensure the integrity of the construction process;
- (4) The resource allocation intensity of each activity is balanced and reasonable, and will not be changed during the optimization process.
- (5) All activities use the same resource, and the resource limit is $R_{\max} = 12$ (cubic meters / week)

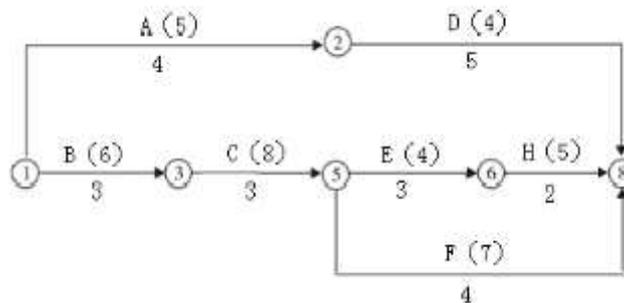


Figure 4 A comparative study of the Critical Chain Method and Critical Path Method

4.1 CPM solution process

(1) CPM optimization fundamentals

Using the peak-cutting method in CPM, assuming that the total resource demand exceeds the resource limit at a certain time period, there are several activities being performed at the same time, and any two activities A and B are changed from the original parallel operation to A is performed before B[7], as shown in Fig.5.

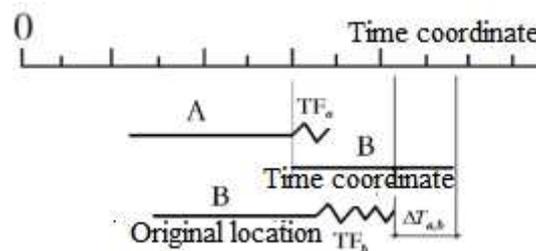


Figure 5 Optimization principle diagram

Then proceed with the new sequence, the extended construction duration is:

$$\Delta T_{a,b} = EF_a + D_b - LF_b = EF_a - LS_b \quad (3)$$

where, $\Delta T_{a,b}$ —when activity A is arranged before activity B, the corresponding duration is extended.
 $\Delta T_{a,b}$ —The smallest one will get top priority on resource allocation.

EF_a —Earliest Finish time of activity A

LS_b —Latest Start Time for activity B.

Similarly, we can get $\Delta T_{b,a}$, compare $\Delta T_{a,b}$ and $\Delta T_{b,a}$, and choose the smallest one as the optimization solution.

(2)Main optimization steps

During the period (3,4), A and C running simultaneously.

$$R_{(3,4)}=R_a+R_c=5+8=13, R_{(3,4)} > R_{\max}$$

Priority of resource allocating of resources is required. There are two possible solutions: activity A is performed before activity C and the duration is not extended; Activity C is performed before activity A and the duration is extended by 2 days. So, choose activity A before C. As shown in Table1.

Table 1 Working time parameters for periods exceeding the resource limit

Activity	EF	LS	sequence	plan 1	plan 2
A	4	4	Predecessors	A	C
C	6	7	Successors	C	A
			$\Delta T_{a,b}$	-3	2

During the period of (7,9), there are D, E, and F working simultaneously.

$$R_{(7,9)}=R_d+R_e+R_f=4+4+7=15$$

$$R_{(7,9)} > R_{\max}$$

Priority of resource allocation is required. There are 6 possible solutions. Activity D is performed before activity E and the duration is extended by 1 day. Activity D is performed before activity F and the duration is not extended. Activity E is performed before activity F and the duration is extended by 1 day; Activity E is performed before activity D and the duration is extended by 2 days; Activity F is performed before activity D and the duration is extended by 5 days; Activity F is performed before activity E and the duration is extended by 1 day. So select activity D before activity F, as shown in Table 2.

Table 2 Activity time parameters for periods exceeding the resource limit

Activity	EF	LS	sequence	plan 1	plan 2	plan 3	plan 4	plan 5	plan 6
D	9	8	Predecessors	D	D	E	E	F	F
E	10	8	Successors	E	F	F	D	D	E

F	13	9	$\Delta T_{a,b}$	1	0	1	2	5	5
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(3) CPM-based determination and representation of critical chain

To meet the resource constraints, the critical chain is A-D-F, as shown in Fig.6. Among them, activities A and C, and activities D and F are new logical relationships.

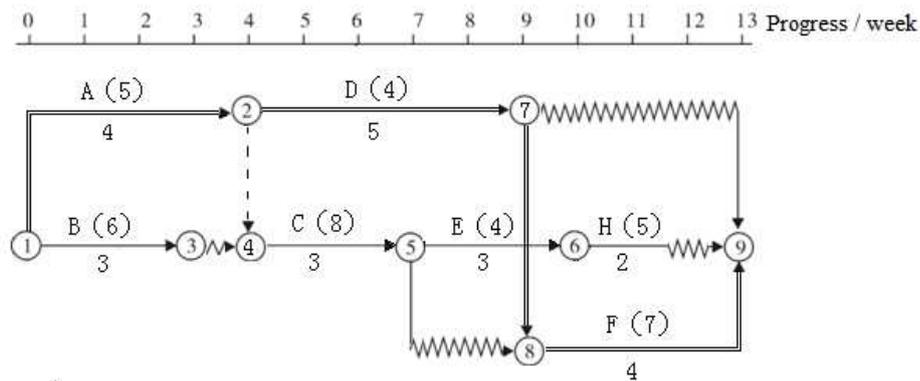


Figure 6 Schematic of the key chain based on CPM and CCM

4.2CCM solution process

To solve the network plan resource constraint issue shown in Fig.6 by applying key chain method, the main steps are indicated as the following:

- (1) Calculate the latest start time (LS) of each activity and the total duration of the project.

The total duration of the network plan was determined to be 13 weeks by the CPM method. The latest start time of each activity is shown in the second row of Table 1.

- (2) Calculate the priority of resource allocation for each activity

Adopt the heuristic algorithm and obtain the ACTIM value of each activity according to formula (1), see the third row of Table 1.

- (3) Determine the start time and end time of the activity

For allocating resources based on the ACTIM value and constraints, if the amount of simultaneous resources demand in a certain period exceeds the maximum resource limit. The resource with the highest ACTIM value is allocated first, and the smaller one is pushed backward. According to this method, until all the activities meet the resource limit requirements, the start time and finish time of each activity are re-determined. The specific results are shown in rows 6 and 7 of Table 1.

- (4) Identify critical chains

In order to meet the resource limit requirements, activities A and C, and activities D and F are transformed from the original parallel relationship to the immediately Finish-Start relationship. At this time, the length of the line formed by steps A, D, and F is the longest, so the critical chain is A-D-F, as shown in Table 3.

Table 3 The critical chain identification and duration calculation influenced by resources

1	Activity name	A	B	C	D	E	F	H
2	LS	2	0	3	6	6	7	9
3	ACTIM	9	11	8	5	5	4	2
4	Activity duration	4	3	3	5	3	4	2
5	Resource demand	5	6	8	4	4	7	5
6	Adjusted start time	0	0	4	4	7	9	10
7	Adjusted finish time	4	3	7	9	10	13	12

4.3 Comparison of CCM and COM results

From the above results, there is no difference between CCM and CPM when the buffer zone setting is not considered.

The article compared the Critical Chain Method and Critical Path Method through case studies, and drew the following conclusions:

(1)The buffer zone setting in the Critical Chain Method is a subjective method and lacks scientificity. To solve the problem of “Student Syndrome”, other methods can be adopted;

(2)Two representation methods of the critical chain were proposed to make the expression of the critical chain clearer and more intuitive;

(3) If the buffer zone setting is not considered, there is essentially no difference between Critical Chain Method and Critical Path Method; the only difference is about the method of determining the priority of resource allocation.

Therefore, the comparative study of this article enriches and improves the Critical Chain Method and opens up the connections between CCM and CPM.

5. Uncertainty Factor Analysis

The randomness of the critical chain is mainly caused by the uncertainty of the total duration of the construction sequence[11]. The critical chain projects with large time span and complicated procedures are affected by various uncertain factors such as the local economy, politics, and technics , resulting in turbulence of the durations of activities, which ultimately affect the total duration of the project. The critical chain sequence duration is determined by four factors: construction quantity, planned productivity rate, manpower and equipment manhour invested, and the number of shifts in work[5].

(1) Construction quantity

Uncertain factors affect the construction sequence duration by affecting the construction quantity. Once the construction drawings are determined, the project's quantity will usually not change, but design drawings' defects, construction problems and other factors can cause the project's quantity to change, and activity durations and the total construction duration would change consequently.

(2) Productivity rate

Uncertainty affects the duration of a sequence by affecting the planned productivity rate. Although the planned productivity rate is reasonable and scientific, deviation always exists between the plan and the actual productivity. In fact, unexpected situations happen in real-world construction jobs. For example, the impact of bad weather on workers' efficiency makes the critical chain impossible to be implemented with

the planned productivity rate, which affects the construction sequence duration.

(3) Manpower and manhours

During project implementation, in the manpower and equipment manhours are maintained in a constant state. Therefore, once personnel and work shift change would affect the construction sequence duration and project duration[11].

(4) Working shifts

To complete the work as soon as possible, the construction shift change causes the actual progress to be inconsistent with the planned progress, which affected the activity duration.

Therefore, the activity duration is affected by construction quantity, planned productivity rate, manpower and equipment manhours, and work shifts. Any changes in any single factor would affect the construction sequence duration of the critical chain, and the duration of the construction sequence is random.

6. Estimation of a activity duration

Due to the existence of many uncertain factors mentioned above, the duration of a activity is uncertain, which makes the start and finish time of each activity and the total construction duration uncertain. For this reason, it is necessary to make a scientific and reasonable estimation of the activity duration. Commonly used approaches are theoretical calculation, three-point estimation and two-point probability estimation, etc[8].

Theoretical calculation refers to the estimation about activity duration based on construction quantity, productivity rate, manpower and equipment manhours, and work shifts, but the theoretical calculation does not take into account the impact of uncertain factors on the project activity duration, and the estimation results are not accurate. In order to make the activity duration estimation as scientific and accurate as possible, the most optimistic time, common time and most pessimistic time of the activity duration are estimated under the influence of uncertain factors, and the three estimated times are used as the process under uncertainty basic data for working hours[9].

Assume that the activity duration follows the β distribution, and estimate the expectation of each activity duration. The calculation formula is as follows:

$$\mu_i = \frac{a + 4b + c}{6} \quad (4)$$

Among them, a is the most optimistic operation time of process i ;

b is the most probable operating time of process i ;

c is the most pessimistic operation time of process i .

The corresponding variance of operation time is:

$$\sigma_i = \left(\frac{c - a}{6} \right)^2 \quad (5)$$

7. Probability estimation and determination of critical chains

7.1 Estimate of total construction period and completion probability

In order to solve the problem of the randomness and the irrational buffer setting of the critical chain, it is proposed to use the completion probability of the critical chain to control the time limit of the critical chain to ensure that the efficiency of the project can be effectively solved. It also avoids the problem that the buffer is too large or too small due to the irrational buffer settings. Fundamentally, the points that are prone to problems in the critical chain are deeply controlled to achieve the purpose of scientific and reasonable management and control of the project.

The expected value of the operation time of each process on the key chain is μ_i , and the variance is σ_i^2 . According to the nature of the expectation and variance, the total duration expectation and variance can be obtained:

$$\mu = \sum \mu_i \quad (6)$$

$$\sigma^2 = \sum \sigma_i^2 \quad (7)$$

Assume that the operating time of each process of the engineering project follows the same distribution. According to the limit theorem of the same distribution center, it can be known that when there are enough processes on the critical chain, the total construction period follows the normal distribution. Its completion probability function at a specific time is:

$$P_i = P(T \leq T_r) = \int_0^{T_r} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(T-\mu)^2}{2\sigma^2}} dT \quad (8)$$

Under certain conditions, the completion probability of a project's total duration is:

$$P = P_1 \times P_2 \times \dots \times P_n \quad (9)$$

Among them: T_r is the designated duration or contract duration;

μ is the expected duration of the critical chain, that is, the expected value of the total duration;

σ^2 is the variance of the total duration on the critical chain;

n is the nth critical chain.

7.2 Identification and representation of critical chains

When the activity duration is uncertain, the critical chain is random. At this time, the determination of the critical chain must also meet the following conditions[1]:

(1) Meet the constraints of resource limits;

(2) The sum of the duration of various activities on the critical chain is the longest;

(3) The probability of completion is the lowest. When there are multiple critical chains, the total completion probability of the network plan is the product of the completion probability of each critical chain.

When the operation time of the process is uncertain, the specific steps of the critical chain representation based on the double-coded time-scale network diagram are as follows:

(1) Based on the determined duration of each activity, it is expected to obtain the start time and finish

time of each activity that meets the resource limit conditions based on the ACTIM value;

(2) Draw a time-scaled network diagram based on the start time and finish time, and draw double solid lines for the sequence in which the time on the critical chain is sequentially connected, and at the same time, add nodes manually at the node-free time sequence transfer. As shown in Fig.2, when there are multiple key chains, all can be represented by one icon.

8. Examples

(1) Known conditions: The relevant information of a project is shown in Table 4. The resource limit is $R_a = 12$ units, and the specified construction period is $Tr = 14d$.

Table 4 Operation time related parameters

Activity name	Successor	Expected duration	Duration variance
B	C	3	0.3
A	D	4	0.5
C	E, F	3	0.6
E	G	3	0.7
D	/	5	0.2
G	/	3	0.8
F	/	4	0.2

(2) Draw network diagram. Based on the known conditions shown in Table 1, the network is drawn as shown in Fig.7. The numbers above the arrows indicate the resource intensity of the activity, and the numbers below the arrow indicate the expected activity duration.

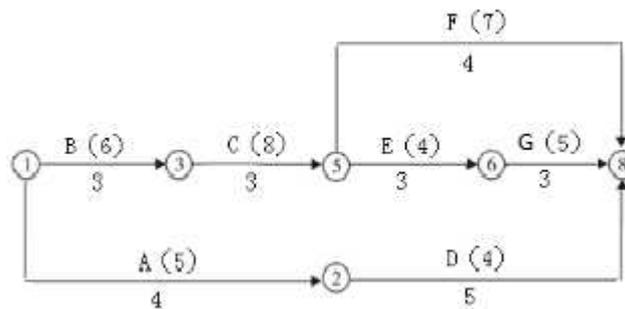


Figure 7 Network for a case project

(3) Identify critical chains. ACTIM heuristic algorithm is adopted to identify the critical chain, determine the ACTIM value, and the expected start time and finish time of each activity.

Table 5 The critical chain identification and duration calculation influenced by resources

Activity	B	A	C	E	D	F	G
LS_{i-j}	0	3	3	6	7	9	8
<i>ACTIM</i>	12	9	9	6	5	3	4
Activity duration	3	4	3	3	5	3	4
Resource demand	6	5	8	4	4	5	7

Start time	0	0	4	7	4	9	10
Finish time	3	4	7	10	9	13	13

According to the above method, two critical chains A-C-E-G and A-D-F can be obtained, and the expected construction duration is 13d.

(4) According to the data in Table 5, draw the diagram of the critical chain representation in the network diagram, as shown in Fig.8 below.

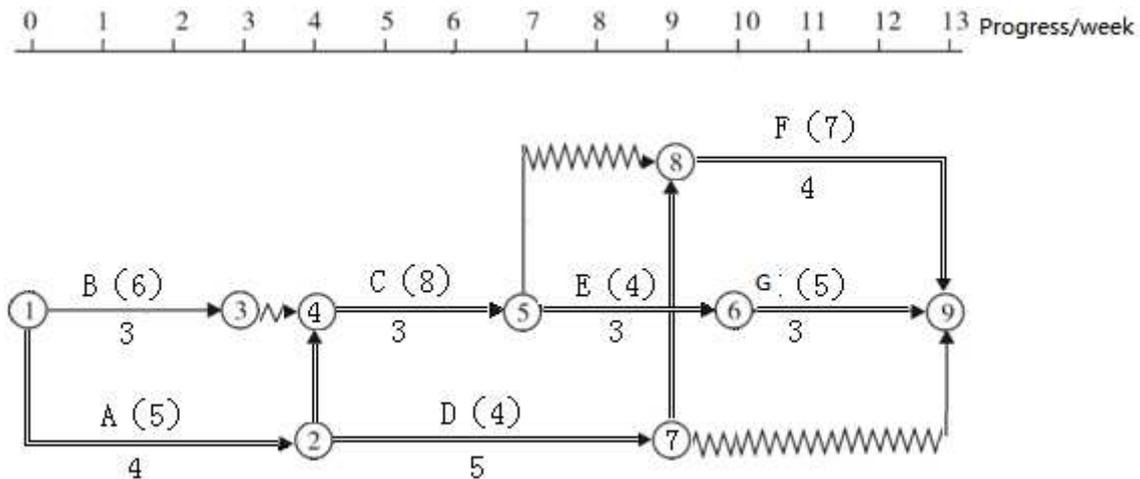


Figure 8 Critical chain network for a project

(5) There are two critical chains. The completion probability of different critical chains in the specified construction duration are:

① For the critical chain A-D-F,

Variance: $\sigma_1^2 = \sigma_A^2 + \sigma_D^2 + \sigma_F^2 = 1.2$, When the specific duration is 14d, according to formula (6), $P_1 = 0.8$ can be obtained.

② For the critical chain A-C-E-G,

Variance: $\sigma_2^2 = \sigma_A^2 + \sigma_C^2 + \sigma_E^2 + \sigma_G^2 = 0.5 + 0.6 + 0.7 + 0.8 = 2.6$, when the specific duration is 14d, $P_2 = 0.7$ can be obtained.

At 14d, the total completion probability of the critical chain is $P = P_1 \times P_2 = 0.56$, and critical chain A-C-E-G need to be controlled well to ensure completion on time.

9. Conclusion

By analyzing the uncertain factors of the critical chain, the article had made the following conclusions:

(1) The uncertainty of the activity duration of the critical chain sequence can be confirmed by the expectation of the activity duration;

(2) Using fixed probability to determine the completion probability at a specific time can achieve the purpose of determining which critical chain to direct the construction;

(3) The critical chain representation method based on the time-scaled network graph is used to make the critical chain representation more intuitive and clearer.

Declarations

Not applicable.

Funding

This work was supported by the National Natural Science Foundation of China (51808424 and 51478384) and the Industrial Building Environment and Energy Conservation Innovation Team of China (2017KCT-14).

Conflicts of interest/Competing interests

The authors declare that they have no conflict of interest.

Ethics approval

Any submission that has data collected from human subjects requires ethics approval.

Consent to participate

Written informed consent was obtained from individual or guardian participants.

Consent for publication

Not applicable.

Availability of data and material

All data generated or analysed during this study are included in this published article.

Code availability

Not applicable.

Authors' contributions

X.H. conceptualized the main idea of this research project; X.H. designed and conducted the experiments; W.Y. and T.L. checked the results; X.H. wrote the whole paper. All authors have read and agreed to the published version of the manuscript.

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Figures

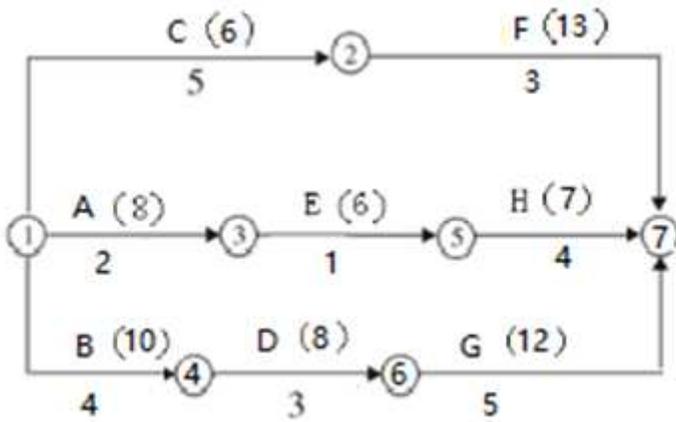


Figure 1

Directly mark out the changes in critical chains and related activity logic relationships on the network diagram

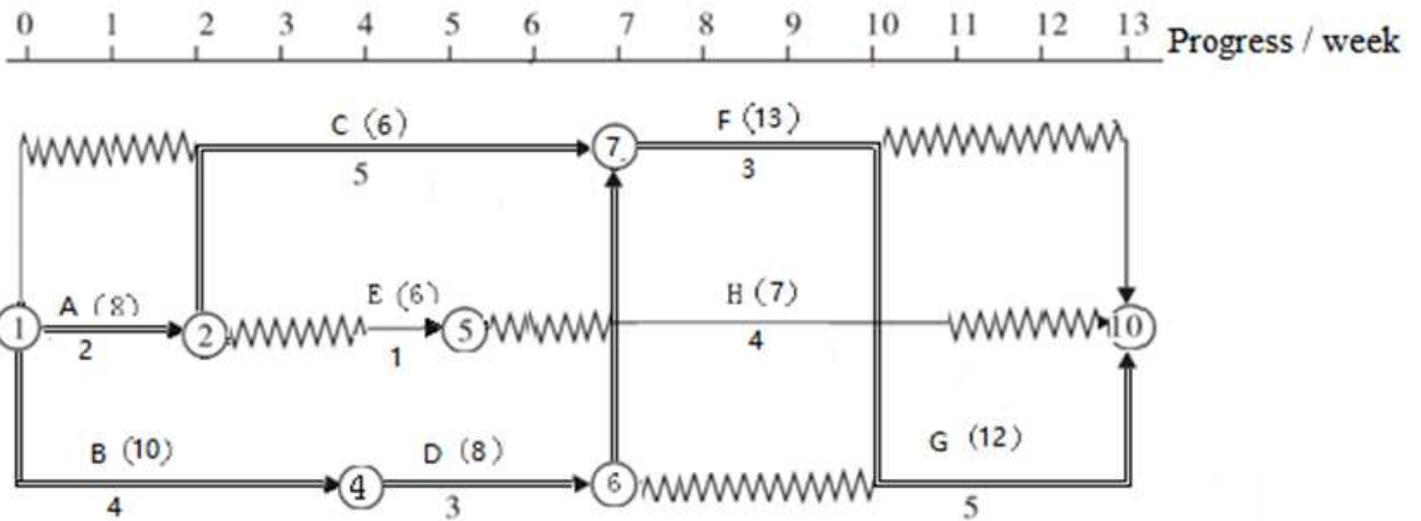


Figure 2

Critical chain representation

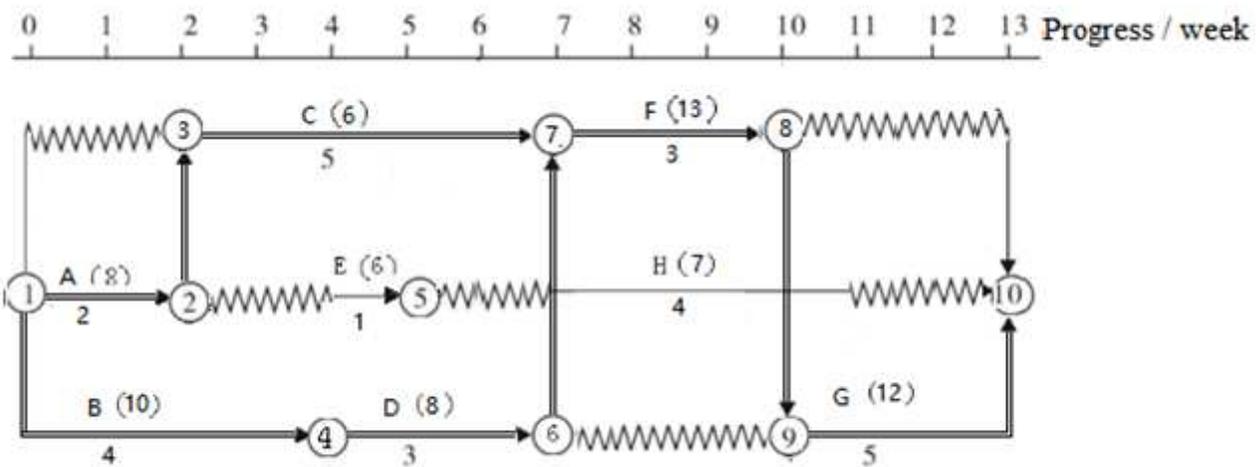


Figure 3

Critical chain representation

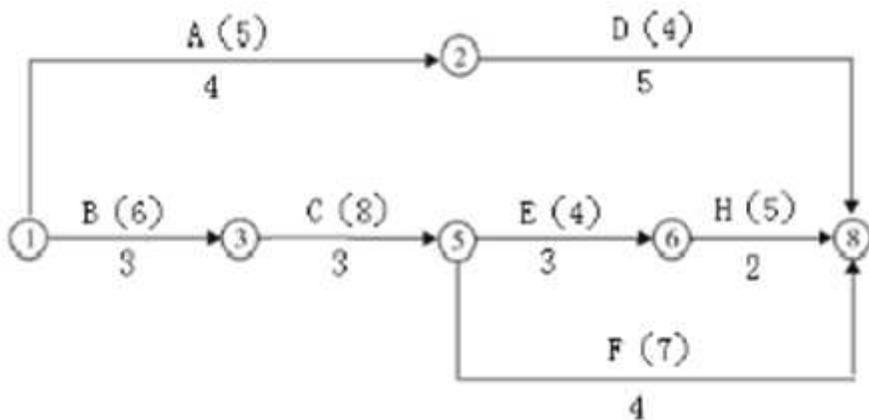


Figure 4

A comparative study of the Critical Chain Method and Critical Path Method

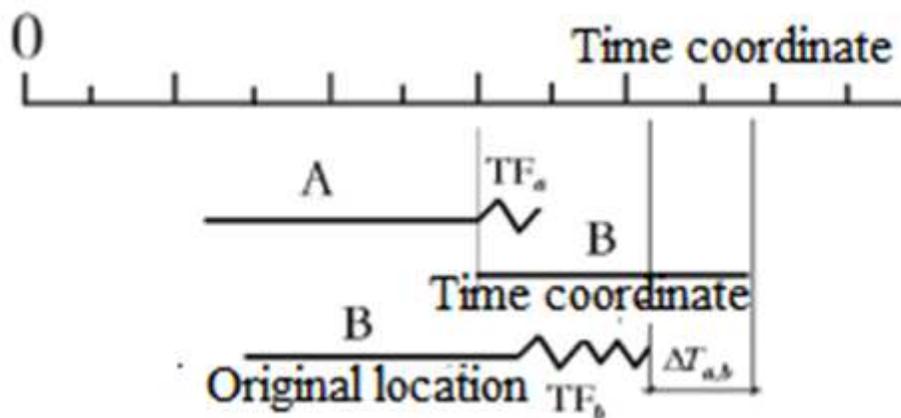


Figure 5

Optimization principle diagram

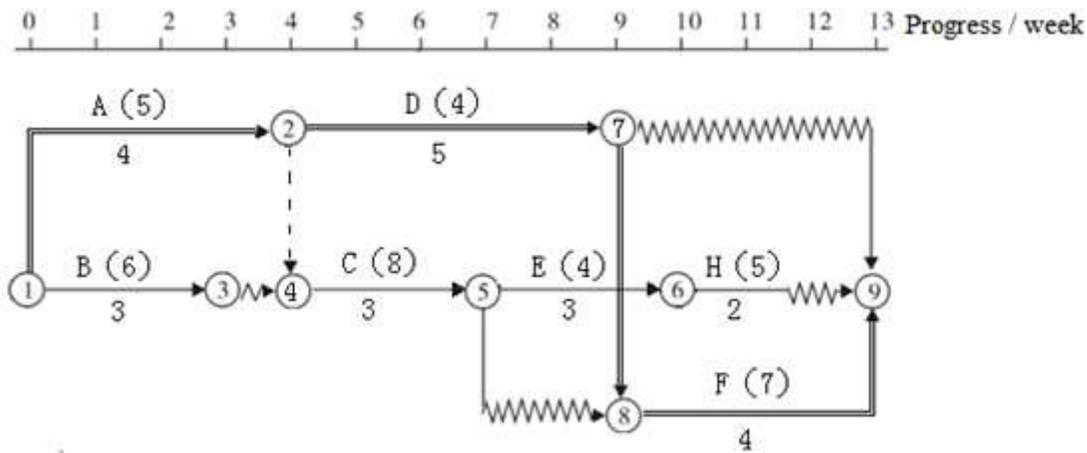


Figure 6

Schematic of the key chain based on CPM and CCM

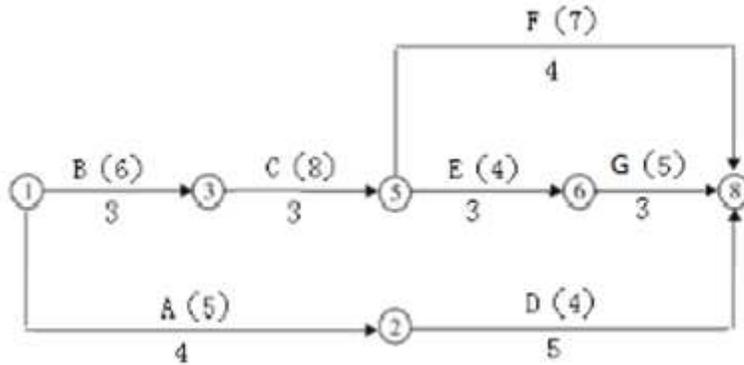


Figure 7

Network for a case project

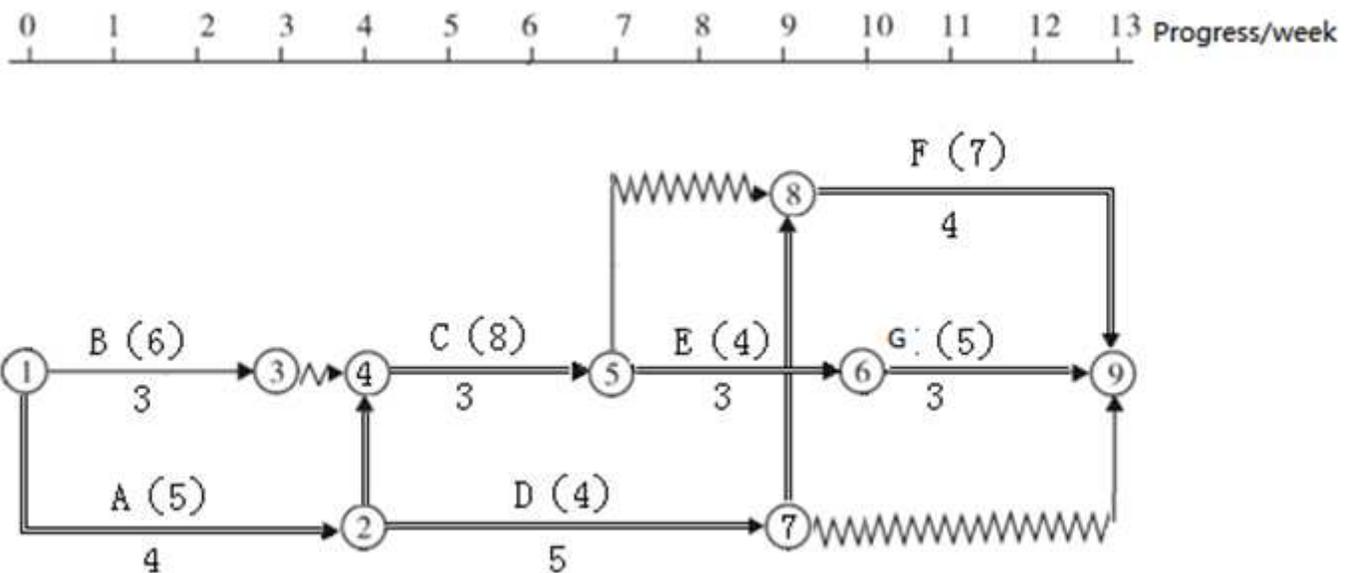


Figure 8

Critical chain network for a project