Fine Structure Constant derived from Principle Theory.

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Article

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Abstract

Derivation of mass (m), charge (e) and fine structure constant (FSC) from theory are unsolved problems in physics up to now. Neither the Standard Model (SM) nor the General theory of Relativity (GR) has provided a complete explanation for mass, charge and FSC. The question “of what is rest mass” is therefore still essentially unanswered. We will show that the combination of two Principle Theories, General Relativity and Thermodynamics (TD), is able to derive the restmass of an electron (m) which surprisingly depends on the (Sommerfeld) FSC (same for the charge (e)).

1. Introduction

Since the introduction of the Higgs mechanism, the Standard Model (SM) presents an explanation of mass in the following way: „mass is built up by exchange-particles“, so-called Higgs particles ^[1].

The mass of the Higgs mechanism is greater, the stronger the field is coupled to the elementary particle. The Higgs Model (at the moment) is not able to derive the restmass of the electron completely but the Higgs Field has become a physical reality because there must be a Higgs boson from theory with a certain mass - theoretically predicted by Peter Higgs - and confirmed experimentally in the meantime ^[1].

If the moving electron interacts with the Higgs field, then the mass of the electron must be greater. The mass should be smaller if the electron is at rest. So the main part of restmass cannot be explained by the Higgs-mechanism alone is a possible conclusion.

So there must be an alternative to the SM deriving the (main part of) rest energy of the electron independently of the Higgs-Hypothesis. This alternative is well known as Einstein's theory of relativity ^[2]. Mass and its gravity are the basis of theoretical considerations there.

"... insofern, dass man von Punktteilchen (mit Masse) ausgehen darf, ist die Thermodynamik eine vollständige Theorie." (A.E)

Thus, we have to give up first the point particle hypothesis and second we need to combine GR and Thermo-Dynamics (TD) because each alone cannot predict the existence of the electron mass. This is exactly the path that Einstein practically set in his wordings, but did not cover it himself

"A theory that sets mass and charge a priori is incomplete." (A.E)

Newton	Momentum P	G-Field Acceleration g	Velocity
	dP/dt	$g=G \cdot M_E / R_E^2$	$v=g \cdot t$
		M_E : Mass Earth, R_E : Radius Earth, set a priori	g : "used", v ="derivation in m/s"
Author	Momentum P	G-Field Differential Equation	Velocity
	dP/dt	$-Ro^2 \frac{\partial}{\partial t} \frac{\partial}{\partial t} r(t) = Go \cdot m(t)$	$\frac{\partial r(t)}{\partial t} = \frac{c}{2\pi} \Psi(t) < c$
		Go : Gravitational Constant, Ro : Planck-Length, $c=wo \cdot Ro$, set a priori	Ψ : "used", $m(t)$ = "derivation in kg"
Klein Gordon	Momentum P	Differential Equation	Velocity
	p^2	$\hbar \cdot \left(\nabla^2 - \frac{1}{c^2} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \right) \psi = (m \cdot c)^2 \psi$	$c=P/m$
		h : Planck-Constant, c : velocity of light, set a priori	m : "used", Ψ ="derivation"

Table 1: Overview. Derivation of $m(t)$ from a Differential Equation is new in physics

In the booklet "Fundamentals of the Theory of Relativity" Einstein writes on Page 22 (Chapter: Space and Time in Pre-Relativistic Physics) ^[2]:

Durch Vertauschung der Indizes und nachfolgender Subtraktion erhält man den Momentensatz:

$$1.0 \quad \frac{d}{dt} \left[m \cdot \left(\xi_\mu \frac{d\xi_\nu}{dt} - \xi_\nu \frac{d\xi_\mu}{dt} \right) \right] = \xi_\mu X_\nu - \xi_\nu X_\mu$$

Hinweis: ξ_ν ist die Differenz der Koordinaten des materiellen Punktes (m) und derjenigen eines raumfesten Punktes. So haben die ξ_ν Vektorcharakter. Für den Fall, daß die Kräfte X_μ konservativ sind, ist der Vektorcharakter von X_ν leicht zu erkennen. Denn dann existiert eine potentielle Energie (Φ) mit der Kraft $X_\nu = -d\Phi/dx_\nu$ eine Folge unserer allgemeinen Gesetze (Erweiterung eines Tensors vom Rang 0).

Bei dieser Darstellung wird es offenbar, daß die Momente von Vektoren nicht wieder Vektoren, sondern Tensoren sind wegen des antisymmetrischen Charakters gibt es aber nicht neun, sondern nur drei selbständige Gleichungen dieses Systems: Die Möglichkeit, antisymmetrische Tensoren zweiten Ranges im Raume von 3 Dimensionen durch Vektoren zu ersetzen, beruht auf .... Die Auffassung der antisymmetrischen Tensoren zweiten Ranges als Vektoren im Raum von 3 Dimensionen hat den Vorteil einer gewissen Anschaulichkeit, aber sie wird der eigentlichen Natur der betreffenden Größen nicht so unmittelbar gerecht wie die Tensorauffassung.

Notice: Generally we have to assume $m(t)$ within Einstein's equation of motion.

2. Derivation of mass

The general theory of relativity (GR) in connection with the principles of thermodynamics (TD) together makes it possible to answer the question of the rest mass of elementary particles if the following two hypotheses are taken into account.

1. Hypothesis:

The restmass ($m(t)$) of the elementary particle cannot be an invariant so $E = m \cdot c^2 + dQ$ (dQ not zero) must be taken into account.

2. Hypothesis:

The main laws of thermodynamics and principles of GR are applied.

Remark:

This way of thinking based on the two hypothesis leads to the fact that restmass depends on the Sommerfeld FSC as well as the elementary charge

does.

Let us first look at Newton's equation of motion ($\mathbf{F} = d\mathbf{P}(t) / dt$) or Einstein's equation of moments of pre-relativistic time in the vector representation (1.0) as a basis for further discussion. In a special case, Newton and Einstein discuss the interaction of two masses with respect to their gravitational force (space time curvature instead of force within Einstein's approach). In purely formal terms, the G-field is a "free invention" like the Higgs field is. The physical reality of a G-Field arises from the fact that an apple is attracted and accelerated by the earth's mass. This is not the focus on this paper.

The following common equation of momentum assumes that all quantities are time-dependent, i.e. mass $m(t)$, distance $r(t)$, and unit vector $\mathbf{u}(t)$.

$$2.0 \quad \frac{d}{dt}[m(t) \cdot \frac{d}{dt}\{r(t) \cdot \vec{u}(t)\}] = \vec{f}_\mu$$

In 2.0 $\vec{f}_\mu$ describes the existence of internal forces. So " $\vec{f}_\mu$ due to an internal G-Field" is now again a free invention. The physical reality of internal force of an "internal G-Field" arises from the fact that we have experimental reality of an electrons restmass. So restmass is made generally from this internal G-Field-like action (due to $\vec{f}_\mu$) and of course accompanied by the external interaction with the Higgs-Field which we want to neglect in a first approximation.

Then we carry out the common mathematical steps (left side 2.0). Formally by the product rule we get five "internal" force components f_1, f_2, f_3, f_4, f_5 of a particle $m(t)$: ("point" means partial derivation)

$$2.1 \quad \dot{m} \cdot \dot{r} \cdot \vec{u} + m \cdot \ddot{r} \cdot \vec{u} + 2m \cdot \dot{r} \cdot \dot{\vec{u}} + \dot{m} \cdot r \cdot \ddot{\vec{u}} + m \cdot r \cdot \ddot{\vec{u}} = \vec{f}_\mu$$

$m(t)$ is the mass in question in kilograms. $r(t)$ is a distance function in meters due to internal action assumed to come up with restmass $m(t)$ and $\mathbf{u}(t)$ is a unit vector allowed to rotate. Here (t) is the time of observation.

We have to define a "math-structure" of each $\vec{f}_\mu$ if we want to complete physically the momentum equation due to internal action of one particle with center at rest. With f_2 (the second force of five from 2.1) we define an internal force combined with inertial acceleration ($-r''$). Thus presenting the following differential equation 2.1.1 in a general time dependent form:

$$2.1.1 \quad m(t) = -\frac{4\pi \cdot r^2(t)}{4\pi \cdot G(t)} \cdot \frac{\partial^2 r(t)}{\partial t^2}$$

(This Differential Equation (DE) looks like but is not similar to any other DE from textbooks.) A solution $r(t)$, (let us say a mass generating function) gives first $m(t)$ and second allows to calculate the “effective mass value” (m), i.e. rest mass from a time average. The time average is from a mean square giving the effective value of mass (2.1.7 and 2.1.8) to be compared with the experimental value.

Notice: Furthermore the invention of a similar f_1 (Coulomb-Contribution) leads to the reality of the elementary charge of the electron. f_4 leads to the magnetic moment of the electron. f_3 is due to a Coriolis contribution. (Not discussed here.)

Since the speed of light is an invariant within the theory of relativity, we can get the following equation if we multiply by c^2 (invariant GR-value) and introduce the Einstein kappa instead of Newton's G value.

$$2.1.2 \quad m(t) \cdot c^2 = -\frac{2 \cdot 4\pi \cdot r^2(t)}{\kappa(t)} \cdot \frac{\partial^2 r(t)}{\partial t^2}$$

We see immediately that now $m(t)$, the mass of the particle, can no longer be an invariant without introducing a non-adiabatic action. In order to be able to reconcile the equation with the conservation of energy, we allow a “non-adiabatic change of energy-state” of mass included into the energy conservation concept, while applying the Second Law of Thermodynamics for that with respect to the mass generating function $r(t)$.

$$2.1.2.1 \quad E = m(t) \cdot c^2 + Q(t) = \text{const}$$

Notice: The basic discussion is adopted from and related to “Einstein’s Matter-Wave-Equation” due to rest mass m_e not zero. $E = m_e \cdot c^2$ and $E = h \cdot c / \lambda_e$ and $P = h / \lambda_e$. So m_e is the effective value of $m(t)$, see (2.1.6 and 2.1.7). In so far as the electron is at rest λ_e is the Compton wavelength and not de Broglie’s Matter Wave wavelength (which would be infinite in this case)!

2a Mass $m(t)$ generating function is $r(t)$

$$2.1.3 \quad r(t) = R_0 \cdot \left(\frac{\omega_0}{2\pi} \int \psi(t) \cdot dt \right)$$

$$2.1.4 \quad \psi(t) = \cos \{ e^{-(t-t_0)/\tau} \cdot \omega \cdot (t-t_0) \}$$

$\Psi(t)$ has to be imagined as a periodic wave-function. (t) is the time of observation (here and now). However, the “loss energy $dQ < 0$ ” (frequency-loss), executed by exponential decay while including the electron life-span τ (about the age of the universe), must be taken into account for each periodic process (II- Law applied). Here we only deal with (today's) rest mass non zero, based on action-velocity with less than speed of light, and $dQ < 0$ for that. The electron therefore necessarily loses energy (starting at t_0). The loss for today is unimaginably little because of the very huge lifespan τ .

2b Important Prediction

This concept, “lifespan” τ (due to non-adiabatic “internal” action) leads to the derivation of the FSC (revealed from a principle theory, as Pauli required, see 2.1.11, 2.1.12, 2.1.13).

2c Newtonian Approximation Calculation

$$a(t) := (t - t_0) / \tau$$

$$2.1.5 \quad \dot{r}(t) = \frac{c}{\sqrt{2\pi}} \cos(e^{-(t-t_0)/\tau} \cdot \omega \cdot (t - t_0))$$

$$2.1.6 \quad m(t) = - \left(\frac{c}{\sqrt{2\pi}} [\omega \cdot (1-a) \cdot e^{-a}] \cdot \sin(e^{-a} \cdot \omega \cdot t) \frac{R_0^2}{G_0} \right)$$

$$2.1.7 \quad \overline{m^2} = \frac{1}{2\pi} \int_0^{2\pi} (c \cdot [\omega \cdot (1-a) \cdot e^{-a}] \cdot \sin(u) \frac{R_0^2}{G_0})^2 \partial u$$

$$2.1.8 \quad m = \frac{c}{\sqrt{4\pi}} [\omega \cdot (a-1) \cdot e^{-a}] \cdot \frac{R_0^2}{G_0} = \frac{c}{\sqrt{4\pi}} \left[\frac{\omega_0}{N/f} \cdot (a-1) \cdot e^{-a} \right] \cdot \frac{R_0^2}{G_0}$$

$$2.1.9 \quad m_e = m_0 \cdot 6 \cdot (1-a) / N_e = 9.107 e^{-31} \text{ kg and } a = \ln 3 \text{ und } N = 10^{22} \text{ und } M_0 = (h \cdot c / G_0)^{1/2}$$

$f=3!$ Is a degeneration number due to a fermion N discussion and M_0 is the Planck mass or $m_0 = M_0 / \sqrt{4\pi}$ and R_0 is the Planck length or $r_0 = R_0 / \sqrt{4\pi}$ and ω_0 is the Planck frequency (see 2.1.10 and for $\ln 3$ see 2.1.15 and 2.1.16)

$$2.1.10 \quad M_0 = \sqrt{\frac{h \cdot c}{G_0}} = \frac{c \cdot \omega_0 \cdot R_0^2}{G_0} = \frac{c^2 \cdot R_0^1}{G_0}$$

Excuse 3-Fermions Mass

So 3 quantized mass values from experiment require 3 quantum numbers N from theory.

Hypothesis N(N1,N2,N3): Electron $N_e = \mathbf{N}_1 \mathbf{e}^a \cdot \mathbf{N}_2 \cdot 3\mathbf{N}_3$, Myon $N_y = \mathbf{N}_1 \mathbf{e}^a \cdot (1) \cdot 4\mathbf{N}_3$, Touon $N_t = \mathbf{N}_1 \mathbf{e}^a \cdot \mathbf{N}_2 \cdot (1)$ with $\mathbf{N}_1 \mathbf{e}^a$ to be that new fermion-invariant instead of the mass itself. f (N1,N2,N3-permutations)=3! And $a = \ln 3$ (see 2b Entropy discussion). So $\mathbf{e}^a = 3$ and Ne, Ny, Nt are now a fact from theory. (N1, N2, N3 are new open questions now.)

Thus we can write fermion mass (m) with open question due to N in the follwing way:

$$2.1.11 \quad m = m_0 \cdot \left[\frac{3!}{N} \cdot (a-1) \right] = m_0 \cdot \sqrt{\left[\frac{36}{N^2} \cdot (a-1)^2 \right]} = m_0 \cdot \sqrt{\frac{24}{N} \cdot 2 \cdot [3/4 \cdot (a-1)^2]}$$

Notice: $[3/4 \cdot (a-1)^2]$ is simply a number derived by assuming a frequency decay process.

Extended GR-invariance condition (Planck Mass $m_{pl} = M_0$ and also $m_0 = m_{pl} / \sqrt{4\pi}$)

$$E1 \quad (\mathbf{m}_{pl} \cdot c^2)^2 = E_e^2 - (c \cdot P)^2 + \sum \left[\left(\frac{n}{n_i} c \cdot P_e \right)^2 - \left(\frac{n}{n_i} E_e \right)^2 \right]$$

$$E2 \quad E_N = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \cdot \left[\frac{1}{\sqrt{1 - \left(\frac{n}{n_i}\right)^2}} \cdot \dots \right] \cdot \frac{\sqrt{24 \cdot 2\alpha}}{N_1 \cdot N_2 \cdot N_3} \cdot \frac{\mathbf{m}_{pl}}{\sqrt{4\pi}} \cdot c^2$$

$$E3 \quad E_{me} = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \cdot \mathbf{m}_{Ne} \cdot c^2$$

Excample $n/n_i = 3/5 < 1$. So $\gamma(3/5) = 1/\sqrt{1 - (3/5)^2} = 1/\sqrt{25 - 9/25} = 1/\sqrt{16/25} = 5/4$

	$\mathbf{e}^a \cdot \mathbf{N}_1$ (n/ni)-invariant	N2 (n/ni)	N3 (n/ni)
Ne	$3 \cdot (3 \cdot \mathbf{19} \cdot \mathbf{61}) \cdot (5 \cdot 2)^{12}$	$(3/5 \cdot 4/5 \cdot 11/61 \cdot 35/37) \cdot (7 \cdot \mathbf{13} \cdot 37)$	$3/5 \cdot (5 \cdot \mathbf{19} \cdot \mathbf{61})$
Ny	$3 \cdot (3 \cdot \mathbf{19} \cdot \mathbf{61}) \cdot (5 \cdot 2)^{12}$	(1)	$4/5 \cdot (5 \cdot \mathbf{19} \cdot \mathbf{61})$
Nt	$3 \cdot (3 \cdot \mathbf{19} \cdot \mathbf{61}) \cdot (5 \cdot 2)^{12}$	$(3/5 \cdot 4/5 \cdot 11/61 \cdot 35/37) \cdot (7 \cdot \mathbf{13} \cdot 37)$	(1)

Table 2: Fermion Quantum Numbers restriction from Einstein new Gamma-factor (n/ni) condition and possible degeneration $(x \cdot n)/(x \cdot n_i)$

So that with $N_3(n/ni)$ and then $N_e/N_t = 3 \cdot \mathbf{19} \cdot \mathbf{61} = 3177$ is close to the experimental ratio $m_t/m_e = 3177.23$ (we are missing the Higgs-Contribution)

N_e/N_μ	=	$\frac{3}{4}$	N2	$206.7639 <$	206.7682
N_e/N_t	=	3	N3	$3477.0000 <$	3477.23

Table 3 The Higgs-Field mass contribution might explain the difference between GR+TD results and experimental ratio.

End of excuse

In so far as the Fermion-Quantum-Number N cannot be explained completely by physical arguments at this moment, we accept that N remains an unsolved problem in equation E3.1.

$$E3.1 \quad m = \sqrt{h \cdot c / G_0 \cdot 2\alpha / 4\pi \cdot 24 / N^2}$$

But the main result is not the existence of a Quantum-Number N but the derivation of the FSC is (α , not depending on N). So we focus on that result next.

Notice:

The restmass $m(\alpha)$ depends on the FSC!

2c Fine structure constant from the point of view of GR + TD

The Newtonian approximation provides for the fine structure constant, if SR and GR influences are not taken into account:

$$2.1.12 \quad \alpha_0 = 3/4 \cdot (-\ln 3 + 1)^2 = 1/137.112$$

If SR and GR influences are taken into account this leads to the following general formula for the FSC:

$$2.1.13 \quad \alpha = \frac{1}{g_{44}} \frac{1}{(\beta_{\text{int}})^2} (3/4) \cdot (1/\beta_{\text{int}} \cdot \ln(1/3) + 1)^2$$

The corresponding DG of the principle theory (GR + TD) which gives 2.1.13 is 2.1.14 assuming the particle at rest ($c \cdot dt = d\lambda$):

$$2.1.14 \quad m(\lambda) = -\frac{8\pi \cdot r^2(\lambda)}{\kappa(\lambda)} \cdot \frac{\partial^2 r(\lambda)}{g_{44} \cdot \partial \lambda^2}$$

Corresponds to 2.1.2

Remark: Within GR we have to define applied and restricted to internal action:

$$2.1.14a \quad G(\lambda) = G_0 \cdot \left(\frac{\omega_0}{\sqrt{2\pi}} \int \cos(e^{-(\lambda-\lambda_0)/\tau} \cdot \omega \cdot (\lambda - \lambda_0)) d\lambda \right)^2$$

$$r^2/G = \text{const} = R_0^2/G_0$$

$$2.1.14b \quad r(\lambda) = R_0 \cdot \frac{\omega_0}{\sqrt{2\pi}} \int \cos(e^{-(\lambda-\lambda_0)/\tau} \cdot \omega \cdot (\lambda - \lambda_0)) \partial \lambda$$

2d Entropy discussion

$$(-\Delta\lambda / \tau) = -\sqrt{1-v_{\text{int}}^2 / c^2} \cdot \Delta t / \tau_0 = -\beta_{\text{int}} \cdot a(t) = -\ln 3$$

We define the GR invariant $(-\ln 3)$ being a consequence of a 3D space (or vice versa).

$$2.1.15 \quad -(\Delta\lambda \cdot h) / (\tau \cdot h) = -\Delta Q / (k \cdot T) = -\ln 3 = +\ln(1/3)$$

$$2.1.16 \quad S = (\Delta\lambda \cdot h \cdot k) / (\tau \cdot h) = \Delta Q / T = k \cdot \ln W = k \cdot \ln(1/3)$$

Notice:

Probability $W=1/3$ is physically equal to the possibility of movement in one of the 3 spatial directions.

The increase in λ is compensated by the heat-loss dQ (dark matter production), so that the spatial dimension $(-\ln 3 = +\ln(1/3))$ remains constant.

Conclusion:

So the FSC is depending on $GR(g_{44})$ and $SR(\beta)$ parameters. **We are dealing with an electron at rest, so β is defined by internal action only. Due to the metric influence the FSC must be different on the moon compared to the earth. So mass (and charge) must be slightly different as well. This is new in physics out of GR+TD and must be proved by experiment**

3. Application and a new Experiment

3.1 The FSC on white dwarfs are different due to the metric influence g_{44} ^[3].

3.2 The same to the moon (private investigation, suggested experiment 2020).

(earth: $1/137.035999046(27)$ ^[4] moon: $1/137.035999239(27)$)

Appendix

Literature:

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- (2) Albert Einstein: Grundzüge der Relativitätstheorie Vieweg & Sohn, Braunschweig, 1969
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- (4) R.H. Parker (2018): Measurement of the fine-structure constant as a test of the Standard Model, Atomic Physics, Science 13 Apr 2018: Vol. 360, Issue 6385, pp. 191-195

Appendix I

Einstein Energy Equation (dQ=0)

Particle not at rest (m not zero, v not c)

$$E=m \cdot c^2$$

$$P=m \cdot v$$

$$E^2=(P/2m)^2+(mc^2)^2$$

Photon not at rest (with v=c)

$$P=h/\lambda$$

$$E^2=(cP)^2+(m_{ph}c^2)^2$$

Einstein Matter-Wave-Equation Electron at rest (v=0)

$$E=m_e \cdot c^2$$

$$E=h \cdot c/\lambda_e$$

$$P=h/\lambda_e$$

De Broglie (Phase) Wave Equation of moving Electron (v not c)

$$\lambda_B=h/m_e \cdot v$$

Compton Wavelength Electron at rest (v=0)

$$\lambda_C=h/m_e \cdot c$$

Effective Mass Value and its zero contribution

Einstein's non-point like Electron at rest

$$h/T=m_e \cdot c^2$$

(not De Broglie's moving matter wave)

$$T=10^{-18}s$$

And tau about more than age of the universe

$$\tau \frac{\partial}{\partial t} \int_0^T [(t/\tau) \cdot e^{-t/\tau}] \cdot \partial t = t \cdot e^{-t/\tau} \Big|_0^T = (0s - 10^{-18} \cdot e^{-0}s)$$

Integral-Contribution which can be neglected due to T (Periodic Time) and tau (Life-Span).

$$\frac{\partial}{\partial t} \int_0^T [(t/\tau) \cdot e^{-t/\tau}] \cdot \partial t = \int_0^T \frac{\partial}{\partial t} [(t/\tau) \cdot e^{-t/\tau}] \cdot \partial t = \int_0^T [(1/\tau) \cdot e^{-t/\tau} + (t/\tau) \cdot (-1/\tau) \cdot e^{-t/\tau}] \cdot \partial t$$

$$\frac{\partial}{\partial t} \int_0^T [(t/\tau) \cdot e^{-t/\tau}] \cdot \partial t = 1/\tau \int_0^T [(1 - (t/\tau)) \cdot e^{-t/\tau}] \cdot \partial t$$

Multiply by Tau (last integral) and you will have that part of the integral which can be neglected when calculating the effective value of mass from 2.1.7

The function $\Psi(t)$ formally looks like:

Figure 1 simulation of the frequency decay (life-span a few seconds)

Appendix II

$$\gamma=(n_i/n_j)$$

n_i	n	n_j	$\gamma(n/n_i)$
5	4	3	5/3
5	3	4	5/4
13	12	5	13/5
10	8	6	10/6
25	24	7	25/7
10	6	8	10/8
17	15	8	17/8
15	12	9	15/9
41	40	9	41/9
26	24	10	26/10
61	60	11	61/11

$$\gamma=(n_i/n_j)$$

2*(3)*(5)*7*11*(13)*17*((19))*23*29*31*(37)*41*43*47*53*59*((61))

$$\gamma=(n_i/n_j)$$