

The Space-evolution Frame as Alternative to Space-time

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Article

Keywords: space-evolution, four dimensional Euclidean space, space-time, space and special relativity

Posted Date: May 20th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-528264/v1>

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The Space-evolution Frame as Alternative to Space-time

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Abstract—As a alternative to Minkowski spacetime frame, this paper propose a four dimensional Euclidean space that combine three spacial dimension with evolution instead of time. It is called space-evolution, in which time is considered as world line length and is absolute. The space-evolution frame provide a new perspective for understanding of time, space and special relativity. It is self-consistent without losing compatibility to special relativity, the Lorentz transform and its predictions could be derived geometrically by simple coordinate rotation.

I. INTRODUCTION

In 1905, Einstein introduced special relativity in its modern understanding as a theory of space and time[1]. Around 1907, Minkowski recognized that the work of Hendrik Antoon Lorentz (1904) and Einstein on the theory of relativity can be understood in a non-Euclidean space. In 1908, Hermann Minkowski presented a geometric interpretation of special relativity that fused time and the three spatial dimensions of space into a single four-dimensional continuum now known as Minkowski space. In the publication [2] Hermann Minkowski introduced the the concepts of spacetime interval, proper time and worldline.

Subsequent work of Hermann Minkowski, in which he introduced a 4-dimensional geometric "spacetime" model for Einstein's version of special relativity, paved the way for Einstein's later development of his general theory of relativity and laid the foundations of relativistic field theories.

Though Minkowski took an important step for physics, spacetime is, in particular, not a metric space and not a Riemannian manifold with a Riemannian metric. In Minkowski spacetime, the position of an event is given by x, y, z and time t . Unfortunately, space and time are separately not invariant, which is to say that, under the proper conditions, different observers will disagree on the length of time between two events. But special relativity provides a new invariant, called the spacetime interval, which combines distances in space and in time. All observers who measure time and distance carefully will find the same spacetime interval between any two events. Then the spacetime interval $(\Delta s)^2$ between the two events that are separated by a distance Δx in space and by Δct in the time coordinate is:

$$(\Delta s)^2 = (\Delta ct)^2 - (\Delta x)^2 \quad (1)$$

It seems mathematically feasible to write the equation as

$$(\Delta ct)^2 = (\Delta s)^2 + (\Delta x)^2 \quad (2)$$

to make the equation more elegant, but such rewrite lack of motivation from physics perspective.

This paper propose a brand new reinterpretation of time, space and event interval, so that they could be described with a standard Euclidean space. We call it 'space-evolution'. In section 2, we clarify the concept of evolution, and then evolutionary position and evolutionary speed are introduced as Physical quantities that similar to spacial position and spacial speed. In section 3 we integral evolutionary position with spacial positions to establish the Euclidean space-evolution frame. Section 4 discuss the coordinate transformation between different observers, with which the Relativistic effects are explained. Section 5 in further derives the Lorentz transformation from the rotational transformation in space-evolution.

II. THE INTRODUCTION OF EVOLUTION POSITION AND EVOLUTION SPEED.

Evolution in this paper refers to progress of observed resting subject, e.g ageing of person, timing of a clock, stellar evolution, decay of elements. The path and outcome of evolution is unique, determined by law of physics, invariant to observers. Evolutionary position, correspondingly, is a physical quantity showing how much is the subject progressed towards its fate, just like age to a person. In this paper, evolutionary position is represented by τ .

The original idea was inspired by the time dilation of special relativity, which saying that a moving clock with spacial speed u will tick slower seems to observer. And according to General Relativity, the clock ticking speed only could be slowed down rather than speedup, indicating that evolution speed also have a upper limit. The time between two ticks for moving clock($\Delta t'$) and rest clock(Δt)has relation

$$\Delta t' = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \Delta t \quad (3)$$

In other words, moving clock ticks slower(ticking speed is lower) than rest clock. Generalize to all other subjects with observable evolutionary process, if the observed subject is moving with speed u relative to observer, its evolutionary speed (v_τ) measured by observer will be slowed down with relation

$$v_\tau = \sqrt{1 - v_x^2} \quad (4)$$

Where $v_x = u/c$ is the spacial speed in unit of light-speed, satisfy $-1 \leq v_x \leq 1$, v_τ is the evolutionary speed satisfy $0 \leq v_\tau \leq 1$. It says that any subject with spacial speed v_x is

evolving with speed $\sqrt{1 - v_x^2}$. Especially, any subject at rest is evolving with speed of light. Rewrite Eqn. 4,

$$v_\tau^2 + v_x^2 = 1 \quad (5)$$

The relation of Eqn.5 strongly suggests that subject's evolutionary speed, v_τ and special speed v_x are components of a unit velocity vector. The evolution axis should be considered as part of our coordinate system. Obviously the evolutionary speed will down to zero if the subject has spacial speed of c .

The evolutionary position of the subject could be calibrated with proper time, which is analogous to arc length of world line in Minkowski spacetime. Say reading of a clock that stick with the subject is t_1 and the reference time is t_0 , the evolutionary position τ will be

$$\tau = c(t_1 - t_0)$$

Even if the evolution axis is mathematically identical to proper time in spacetime, in order to preserve the physical interpretation and to avoid confusion with coordinate time, the name "evolution" is preferred. The phrase "time" is specifically refer to coordinate time in this paper.

Since the evolutionary position records the subject itself like spacial position did, rather than records a rest clock that irrelevant to the subject, possibly, evolution is more qualified than coordinate time to unite with spacial position and form a coordinate system.

III. THE SPACE-EVOLUTION COORDINATE

The heuristic thought is that we treat evolutionary interval as geometrical length that identical to special distance. Minkowski space differs from four-dimensional Euclidean space, because time is, unlike 3 spacial coordinate, reading of lab clock rather than description of the object itself. In this section we merge space and evolution to form a new four dimensional Euclidean space. Such coordinate system may potentially replace the well know Minkowski space without violate spacial relativity. We call such new coordinate system as "space-evolution", which is distinguishable to "spacetime".

According to definition, τ is a scalar recording subject's evolutionary position relative to the origin. Together with three spacial positions, the subject is coordinated by a raw vector we refer to as 4-position

$$\vec{P} = [\tau \quad x \quad y \quad z] \quad (6)$$

In space-evolution, evolutionary axis is independent to three spacial axis, the space is Euclidean. an infinitesimal length of the world line The history of an subject traces a curve in space-evolution, called its world line. The length of the world line is represented by l . Consider an infinitesimal world line dl , the geometrical interpretation could be written as

$$dl^2 = d\tau^2 + dx^2 + dy^2 + dz^2 \quad (7)$$

By comparing Eqn.7 with Eqn.2, we could speculate that the worldline represent time

$$dl = cdt \quad (8)$$

Rewrite Eqn. 7 as a differential equation

$$\begin{aligned} 1 &= \left(\frac{d\tau}{dl}\right)^2 + \left(\frac{dx}{dl}\right)^2 + \left(\frac{dy}{dl}\right)^2 + \left(\frac{dz}{dl}\right)^2 \\ &= v_\tau^2 + v_x^2 + v_y^2 + v_z^2 \\ &= \|\vec{v}\| \end{aligned} \quad (9)$$

where

$$\vec{v} = [v_\tau \quad v_x \quad v_y \quad v_z] \quad (10)$$

is a four dimensional unit vector represents 4-velocity of the subject. By comparing Eqn.10 with Eqn.5, we confirm that the speculation addressed by Equation 8 is correct.

The 4-velocity in space-evolution is quite differ from that in special relativity. It is unit tangent vector of the subject's world line. Given its property, we use rotation to describe changes of \vec{v} instead of adding and subtracting. A infinitesimal stretch of world line (or time interval) could be expressed as

$$d\vec{l} = \vec{v}dl = [d\tau \quad dx \quad dy \quad dz] \quad (11)$$

The finite of spacial speed is implemented in the geometric fact that arc length dl is always longer than its spacial projection dx , dy or dz . The world line will be curved if the subject experienced acceleration. Assume the coordinate of a subject at time $l = a$ is $[\tau_a \quad x_a \quad y_a \quad z_a]$, then its coordinate at time $l = b$ is

$$[\tau_b \quad x_b \quad y_b \quad z_b] = \int_a^b \vec{v}(l)dl + [\tau_a \quad x_a \quad y_a \quad z_a] \quad (12)$$

accordingly, the stretched arc length(time spent)

$$b - a = \int_a^b dl \quad (13)$$

Or if the entire world line is known to us, the coordinate of the subject could be written as a set of univariate time series

$$\vec{P}(l) = [\tau(l) \quad x(l) \quad y(l) \quad z(l)]$$

In other words, the stretch of world line is driving only by growth of time l .

One may argue that this paper just simply switches the role of coordinate time and proper time in spacetime diagram, but in space-evolution, the concept of a point and a world line is quite different. The spacetime diagram describe events, though the space-evolution diagram describe evolve-able subjects. An event is something that happens instantaneously at single point in spacetime, represented by a set of coordinates x , y , z and t . The spacetime observer wait until subject reaches specific evolutionary stage, then record the time of rest clock(length of stretched world line) and the spacial position of subject. But in perspective of space-evolution, a event is considered to be subject that evolve to the evolutionary position that specified by the event. Take a time bomb as example, spacetime describe explosion, track its location and time when it happen. But space-evolution describe the bomb, track its timer reading and spacial coordinate, explosion is a specified evolutionary position(stage).

For the sake of simplicity and two dimensional display, from now on we assume $y = z = 0$ and $v_y = v_z = 0$. Figure 1 drawing a space-evolution diagram to illustrate the geometric

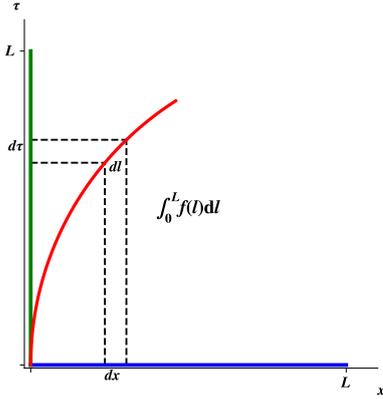


Fig. 1. World line of three subjects, green path for a rest clock, blue path for a light speed clock, red path for a clock under acceleration. Three curve lengths grow from 0 to L simultaneously over time.

relationship between time, evolution, space and subject's world line.

From metric written by Equation 7 and demonstration of Figure 1, we are safe to say that the evolutionary axis is orthonormal to three spacial axis, the space-evolution that we established is a perfect Euclidean space.

Before we proceed, a fundamental postulate must be posted:

Synchronous World Line Postulate: In flat space-evolution frame, all subject's world line stretch the same length between two observations.

This postulate is based on some common sense, that in different places of flat space, rest clocks tick with same rate, and upper limit of spacial speed is the same. The postulate saying that the universe has a unified time, and the world lines of all matters in the universe stretch synchronously over this time. We would guess that in gravitational field, world line may stretch slower because of improper coordination instead of violation of the Synchronous postulate.

The postulate allows us to measure the worldline length of all other subject by checking the reading of a rest clock. The time interval ($c\Delta t$) that recorded by resting clock is, actually, mutual geometrical length(Δl) that all subject's world line stretched.

Fig. 2 demonstrate events in space-evolution diagram. We define the event-auxiliary-line as a auxiliary line that perpendicular to evolution axis and through the evolutionary position that determines the event occur. An event is the cross point of a world line and event-auxiliary-line. An example for understanding, the expression "the observer sees the event happen on the subject" is equal to "the observer sees the subject reach the evolutionary position that defined by the event".

Two events may or may not occur on same subject, if dose, may or may not on a stationary subject. Space time diagram doesn't specify those conditions. But there is a hidden premises, spacetime observer always presume that the event happens on rest object before coordinate transformation, so in space-evolution diagram that preparing for Lorentz transformation, the world line of this subject should parallel to

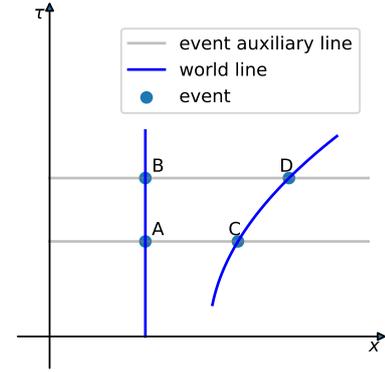


Fig. 2. The left side blue line is world line for stationary subject, the right side blue line is for a accelerating subject, they start at different location but total length are equal to coordinate time. Event A and B occur on stationary subject, event C and D occur on accelerating subject. According to the Synchronous World Line Postulate, the sequence of those events is C,A,D,B.

evolution axis.

We have to discuss the role of photons in space-evolution and its speed c . for most case, the photon we observed is created by distant event, rather than subject, and such event like decay or explosion is not likely to create sequence of photon so that we could track the evolutionary process of the subject. The constant c is considered as a factor that used to geometrically normalize evolutionary axis τ and spacial axis x . It could be measured by dividing proton's interval with rest clock's interval, both should have the same geometric length according to the postulate. In frame of space-evolution, a rest clock record a evolution interval $\Delta\tau = c\Delta t = \Delta l$ but zero spacial interval, thou a light speed clock shows zero evolution interval to observer but its spacial interval $\Delta x = \Delta l$. The speed of light c is treated as conversion constant for spacial distance and evolutionary distance.

IV. ROTATION OF SPACE-EVOLUTION FRAME

Consider a space-evolution frames S , define a subject B to have coordinate $[\tau, x]$ and velocity $[v_\tau, v_x]$. define another subject C to have velocity $[\mu_\tau, \mu_x]$ in S , S' is the frame of reference for C . In frame S' , what is B 's coordinate $[\tau', x']$ and velocity $[v'_\tau, v'_x]$?

First we aim to find S' , in which C is stationary. It is not necessary to stick the origin of S' to the moving subject so that the subject is relatively stationary, there has to be a fundamental change /shift in thinking for space-evolution diagram. Instead, we just need to rotate S with angle θ , which satisfy $\sin(\theta) = \mu_x$ and $\cos(\theta) = \mu_\tau$.

$$[1, 0] = [\mu_\tau, \mu_x] \begin{bmatrix} \mu_\tau & -\mu_x \\ \mu_x & \mu_\tau \end{bmatrix} \quad (14)$$

Where $(1, 0)$ is the velocity of subject C in frame S' , indicating the subject C have zero spatial velocity but evolve with speed of light, we identify S' as frame of reference for the subject C . Accordingly, B 's coordinate and velocity in new

frame S' could be calculated with the same transformation

$$[\tau', x'] = [\tau, x] \begin{bmatrix} \mu_\tau & -\mu_x \\ \mu_x & \mu_\tau \end{bmatrix} \quad (15)$$

$$[v'_\tau, v'_x] = [v_\tau, v_x] \begin{bmatrix} \mu_\tau & -\mu_x \\ \mu_x & \mu_\tau \end{bmatrix} \quad (16)$$

Naturally, in such rotational transformation:

- distance between any two subject points is preserved, including the origin. Therefore infinitesimal world line length dl is preserved, even the shape of all world lines are preserved.

$$dl^2 = d\tau^2 + dx^2 = d\tau'^2 + dx'^2$$

- angle between any pair of velocity vector is preserved
- The origin is preserved and fixed, rather than be a regular evolvable subject.

Such transformation is self-consistent from a geometric point of view, but quite differ from Lorentz transformation of velocities. Though the velocities in Lorentz transform refer to “speed of event” rather than “speed of subject”. The question is, dose Eqn.15,16 explain physical phenomena properly, especially those in special relativity?

Fig. 3 demonstrate that the frame rotation expressed by Eqn. 15 is able to explain the typical consequences derived from the Lorentz transformation. Frame S is represented by solid axis and frame S' by dotted axis. S' is the frame of reference for spacial speed $v_x = \sin(\pi/6)$ in the figure. The Lorentz factor γ is introduced so that the result is comparable to spacetime diagram

$$\gamma = \frac{1}{v_\tau} = \frac{1}{\sqrt{1 - v_x^2}}$$

Fig. 3(a) show a subject in frame S with velocity vector $\vec{v} = [v_\tau, v_x]$. But in transformed frame S' , the velocity vector is $(1,0)$, parallel to evolution axis. Thus the subject is identified as a spacial stationary subject with coordinates (τ', x') , the frame S' is considered as the frame of reference for the subject.

Fig. 3(b) demonstrate length contraction of a measuring rod. The rod is draw with dots so that it distinguishable to world line. The rod is at rest and aligned along the x -axis in the frame S . In this frame, the length of this rod is written as Δx , but in frame S' the rod is moving towards the origin with spacial velocity $-v_x$, the spacial length projection

$$\Delta x' = \Delta x \sqrt{1 - v_x^2} = \Delta x / \gamma$$

One should also notice that two synchronized clocks in frame S , placed at the two ends of the rod, is not synchronized in frame S' since they have different evolutionary position. Though $\Delta x'^2 + \Delta \tau'^2 = \Delta x^2$ is invariant under coordinate transformation, indicating that the 4-distance between two subject is also preserved.

Fig. 3(c) suppose a clock is at rest in the unprimed system S , its world line is the blue strate line. The clock ticks when the subject evolve to specific evolutionary position. The two ticks are intercepted by two event-auxiliary-lines. The world line length that intercepted by two event-auxiliary-lines is Δl ,

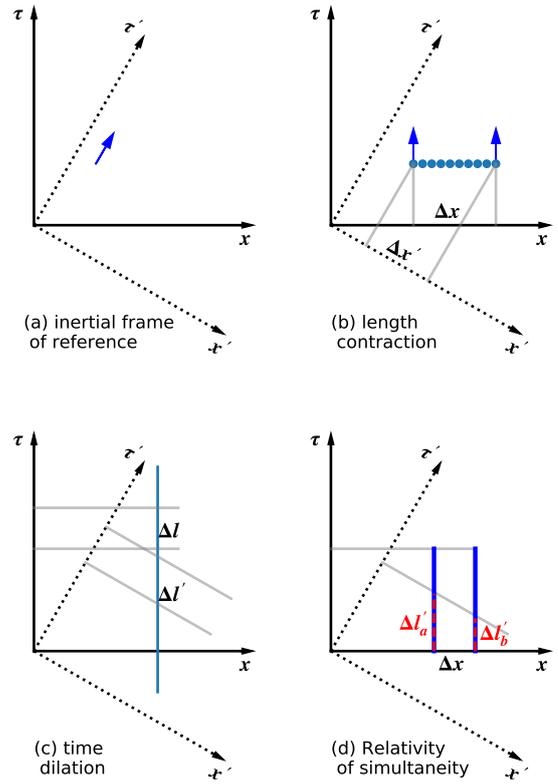


Fig. 3. The gray lines are auxiliary line for projection, blue lines are world line, dotted lines with arrow are axes for S' . blue arrow is subject and its velocity vector.

which is time interval between two ticks seems to observer. Though in frame S' , the intercepted world line length is

$$\Delta l' = \frac{\Delta l}{\sqrt{1 - v_x^2}} = \gamma \Delta l$$

The two blue lines in Fig. 3(d) is world line of two subjects with different spacial locations. In perspective of spacetime, the two event occur simultaneously when two world lines cross the same-evolution-plane that denote the occurring of event. The event plane (same-evolution-plane) of frame S' is tilt, the world lines that stretched before the event occur in S' is represented by red dashed lines. The length differences between them could be calculated geometrically as

$$\Delta l'_a - \Delta l'_b = \frac{v_x \Delta x}{\sqrt{1 - v_x^2}} = \gamma v_x \Delta x$$

,which is time interval of two events occur in frame S'

when we looking at a event located in the space-evolution coordinate, rotation of coordinate works very well on explaining the Time dilation, Length contraction and Relativity of simultaneity.

One should notice that photon, which is unevolvable, can not be described by the space-evolution frame and its transformation. But particle with mass is possible to treat as a evolvable subject and draw its world line, though the meaning of evolutionary position for particles is unknown.

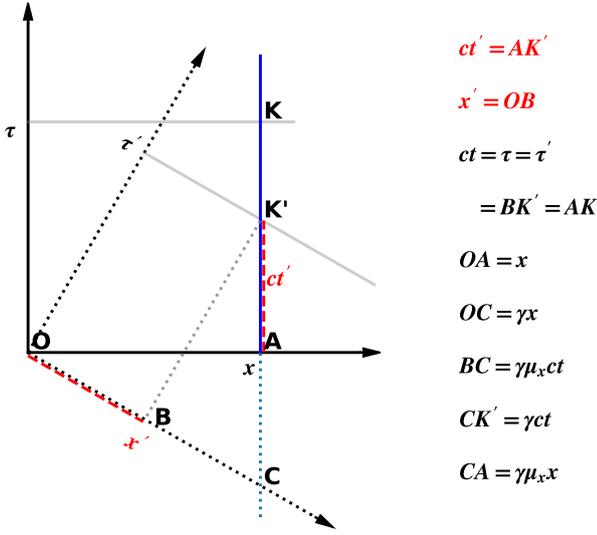


Fig. 4. Geometrical interpretation of Lorentz transformation in space-evolution configuration. The moving spacetime observer measured time ct' and position x' are parameters to be solved, highlighted with red color. The blue line is world line with start point A , K is when stationary observer confirms the event occurring, K' is when moving observer confirms the event occurring.

V. LORENTZ TRANSFORMATION

Lorentz covariance is considered to be the fundamental postulate of special relativity. In this section, we try to drive the Lorentz transform from the rotation of space-evolution coordinate. Consider two space-evolution frame S and S' , S' is frame of reference for a moving spacetime observer, whose spacial velocity is μ_x in S . As a example, in frame S we define a world line for a rest subject at spacial position d as follows:

$$[\tau(l) \quad x(l)] = [l \quad d] \quad (17)$$

According to Eqn. 15, the expression in S' for the same world line is

$$[\tau'(l), x'(l)] = [l, d] \begin{bmatrix} \mu_\tau & -\mu_x \\ \mu_x & \mu_\tau \end{bmatrix}$$

Fig. 4 demonstrate how is the spacetime coordinate (ct', x') in S' is calculate geometrically. The world line and its start point is invariant of coordinate transformation, but the event-auxiliary-line is rotated with the frame thanks to its definition. Thus the cross point with world line is differ from which in frame S .

Based on Fig. 4, by inspecting the geometric relation between τ, ct, x , the spacetime coordinate of the event in S' is calculated as

$$\begin{aligned} t' &= AK'/c = (CK' - CA)/c = \gamma(t - u_x x) \\ x' &= OB = OC - BC = \gamma(x - u_x ct) \end{aligned} \quad (18)$$

The Lorentz transform is obtained.

VI. CONCLUSION

By introduce the evolution axis, we successfully established the space-evolution as an Euclidean othournorm coordinate

system, without losing compatibility to special relativity. It is confirmed that the evolution axis is more qualified than time to integral with spacial axis. The coordinate transformation for different observers is achieved by rotation, which is typical property of Euclidean space. In such frame, the speed of light c is nothing but a constant used to normalize space-evolution coordinate system. The evolutionary and spacial speed naturally have a upper limit c as a geometric fact. Lorentz transformation is not founding principle but rather a simple consequence of the geometrical nature of the theory, and it's consequences such as length contraction and time dilation could be obtained without much mathematical effort. The space-evolution frame is likely to further improve our understanding of time and space. Though difficulties encountered in explaining photons. Further attention is expected to discover its potential value in physics.

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