The Space-Evolution Frame as an Alternative to Space-Time

Xiaonan Du (x.du@gsi.de)
GSI Helmholtzzentrum für Schwerionenforschung https://orcid.org/0000-0002-8088-1252

Research Article

Keywords: space-evolution, four dimensional Euclidean space, space-time, special relativity, Lorentz transform

Posted Date: June 9th, 2021

DOI: https://doi.org/10.21203/rs.3.rs-528264/v4

The Space-Evolution Frame as an Alternative to Space-Time

Xiaonan Du
GSI Helmholtzzentrum für Schwerionenforschung

***Abstract*—As an alternative to the Minkowski space-time frame, this paper proposes a four-dimensional Euclidean space that combines three spatial dimensions with proper time instead of time. We call this space evolution, in which proper time is interpreted as an evolutionary position and time is considered world line length and absolute. The space-evolution frame provides a new perspective for our understanding of time, space and special relativity. The new frame is self-consistent and compatible to spacial relativity, the Lorentz transform and its predictions could be derived geometrically by simple coordinate rotation.**

I. Introduction

In 1905, Einstein introduced special relativity in its modern understanding as a theory of space and time[1]. Around 1907, Minkowski recognized that the work of Hendrik Antoon Lorentz (1904) and Einstein on the theory of relativity can be understood in a non-Euclidean space. In 1908, Hermann Minkowski presented a geometric interpretation of special relativity that fused time and the three spatial dimensions of space into a single four-dimensional continuum now known as Minkowski space. In the publication [2], Hermann Minkowski introduced the concepts of space-time interval, proper time and world line.

Subsequent work of Hermann Minkowski, in which he introduced a 4-dimensional geometric "space-time" model for Einstein's version of special relativity, paved the way for Einstein's later development of his general theory of relativity and laid the foundations of relativistic field theories.

Although Minkowski achieved an important step for physics, space time is, in particular, not a metric space and not a Riemannian manifold with a Riemannian metric. In Minkowski's space-time model, the position of an event is given by x, y, z and time t. Unfortunately, space and time are separately not invariant, which is to say that, under the proper conditions, different observers will disagree regarding the length of time between two events. However, special relativity provides a new invariant, called the space-time interval, which combines distances in space and in time. All observers who measure time and distance carefully will find the same space-time interval between any two events. Then, the space-time interval $(\Delta s)^2$ between the two events that are separated by a distance Δx in space and by Δct in the time coordinate is:

$$(\Delta s)^2 = (\Delta ct)^2 - (\Delta x)^2 \tag{1}$$

It seems mathematically feasible to write the equation as

$$(\Delta ct)^2 = (\Delta s)^2 + (\Delta x)^2 \tag{2}$$

to make the equation more elegant, but such a rewrite lacks motivation from the perspective of physics.

This paper proposes a brand new reinterpretation of proper time, coordinate time and event interval so that they could be described with a standard Euclidean space. We call this "space-evolution". In section 2, we clarify the concept of object evolution. Furthermore, evolutionary position and evolutionary speed are introduced as physical quantities that are similar to spatial position and spatial speed. In section 3, we integrate the evolutionary position with three spatial positions to establish the four-dimensional Euclidean space-evolution frame. Section 4 discusses the coordinate transformation between different observers, with which the relativistic effects are explained. Section 5 derives the Lorentz transformation from the rotational transformation of the space-evolution frame. Section 6 demonstrates the coordinate invariance of causality with a thought experiment.

II. Evolutionary Position and Evolutionary Speed.

Evolution in this paper refers to the progress of the observed rest subject, e.g., aging of a person, timing of a clock, stellar evolution, and decay of an element. We exclusively discuss isolated subjects whose internal processes are not affected by any field. A clock is a perfect model to explain some important concepts in this paper; its reading at any time is uniquely determined by inner structure and the laws of physics, and ticking is independent of spatial position. The evolutionary position, correspondingly, is a physical quantity that determines the status of the subject's evolution process. For a rest clock, the most convenient way to coordinate/calibrate the evolutionary position is by its reading; for any other subject, the evolutionary position can be calibrated according to the reading of a mass-less clock carried by it. In some sense, evolutionary position is a reinterpretation of proper time, and we represent it with τ. Though such reinterpretation is quite essential, thus the term "evolution" is preserved; it also helps to avoid confusion with coordinate time. The term "time" specifically refers to coordinate time (denoted by t) in rest of this paper.

The original idea was inspired by time dilation in special relativity, which states that a moving clock with spatial speed u ticks slower from the perspective of a rest observer. The time between two ticks for moving clock ($\Delta t'$) and rest clock

(Δt) exhibits the following relation

$$\Delta t' = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\Delta t \tag{3}$$

We define evolutionary speed $v_\tau = \Delta t/\Delta t'$ to quantify the relative ticking rate of the moving clock or the progression rate of any other process. Denoting spatial position with x, normalized spatial speed is defined as

$$v_x = u/c = \frac{dx}{cdt} \tag{4}$$

that satisfy $(-1 \leq v_x \leq 1)$, Eqn. (3) could be rewritten as

$$v_\tau = \sqrt{1 - v_x^2} \tag{5}$$

or

$$v_\tau^2 + v_x^2 = 1 \tag{6}$$

Eqn. (5) imposes a constraint on evolutionary speed: $(0 \leq v_\tau \leq 1)$; and according to general relativity, the ticking rate of a clock can be slowed down by a gravitational field, although there is no way to speed up the ticking rate of a clock. The above facts indicate that evolutionary speed is capped at 1, similar to spatial speed. The upper limit for spatial speed has been noticed and widely discussed. However, one fact that has long been neglected is that the aging rate of any evolvable subject also has an upper limit.

According to Eqn. (5), any subject with spatial speed v_x is evolving with speed v_τ; obviously, v_τ will drop to zero if v_x approaches 1c. Eqn. (6) strongly suggests that v_τ and special speed v_x are components of a unit velocity vector, and thus we consider the evolutionary position as a component of the coordinate system in the next section.

In addition, the interpretation of evolutionary position suggests that it is more qualified than coordinate time to unite with three spatial positions to form a four-dimensional coordinate system. The evolutionary position describes the status of the subject itself, similar to the spatial position; τ is also independent/orthogonal to the spatial position: an astronaut would stay at the same position if he was aging at full speed and would not age if he was traveling at the speed of light. Though time is reading of a rest clock that is irrelevant to the subject, it is quite distinguishable from spatial position components.

III. The Space-Evolution Frame

Minkowski space differs from four-dimensional Euclidean space because time is, unlike the 3 spatial coordinates, reading of the lab clock rather than description of the subject itself. In this section, we fuse the evolutionary position τ space and the three spatial positions x, y, and z into a single four-dimensional manifold. We call such a new coordinate system "space evolution", which is distinguishable from "space time", which can potentially be a promising alternative to the well-known Minkowski space time.

The status of a subject can be coordinated by a raw vector, which we refer to as four-position

$$\vec{P} = \begin{bmatrix}\tau & x & y & z\end{bmatrix} \tag{7}$$

and space evolution is the collection of such points. First, we assume that such space evolution is Euclidean. Similar to space time, we define the world line of a subject as its path traces in 4-dimensional space evolution. Accordingly, we define an infinitesimal interval between two statuses as line element dl, which is coordinate-independent, written as

$$dl^2 = d\tau^2 + dx^2 + dy^2 + dz^2 \tag{8}$$

Eqn. (8) can be rewritten as a differential equation

$$1 = (\frac{d\tau}{dl})^2 + (\frac{dx}{dl})^2 + (\frac{dy}{dl})^2 + (\frac{dz}{dl})^2 \tag{9}$$

By comparing Eqn. (9) with Eqn. (6), we speculate that the world line element and time may be related as

$$dl = c\,dt \tag{10}$$

In the space-evolution diagram, the stretch of the world line drives variation in the 4-position of the endpoint; in physics, time drives succession of the subject's evolutionary position and spatial position. Therefore, this paper accepts world line length as a geometric representation of time; in other words, the stretch of a worldline is driven only by the growth of time. Furthermore, we rewrite Eqn. (9) as

$$\begin{aligned} 1 &= v_\tau^2 + v_x^2 + v_y^2 + v_z^2 \\ &= ||\vec{v}|| \end{aligned} \tag{11}$$

where

$$\vec{v} = \begin{bmatrix}v_\tau & v_x & v_y & v_z\end{bmatrix} \tag{12}$$

which is a four-dimensional unit vector called four-velocity, the tangent vector of the world line at a point in space evolution. This is quite different from that of special relativity. An infinitesimal stretch of the world line (or time interval) can be expressed as

$$d\vec{P} = \vec{v}dl = \begin{bmatrix}d\tau & dx & dy & dz\end{bmatrix} \tag{13}$$

The world line will be curved if the subject experiences acceleration, and an extended discussion about this will be carried out in other papers. Assuming that the coordinate of a subject when time $l = a$ is $\vec{P}(a) = \begin{bmatrix}\tau_a & x_a & y_a & z_a\end{bmatrix}$, then its coordinate when time $l = b$ can be calculated by

$$\vec{P}(b) = \int_a^b \vec{v}(l)dl + \vec{P}(a) \tag{14}$$

accordingly, the time spent on the process is equal to the stretched arc length

$$c\,\Delta t = \int_a^b dl = b - a \tag{15}$$

Or, if the entire world line is known to us, the coordinate of the subject could be written as a set of univariate time series

$$\vec{P}(l) = \begin{bmatrix}\tau(l) & x(l) & y(l) & z(l)\end{bmatrix}$$

For the sake of simplicity and a two-dimensional display, from now on, we assume $y = z = 0$ and $v_y = v_z = 0$. Fig. 1 displays a space-evolution diagram with world line to illustrate the geometric relationship between time, evolutionary coordinates, and spatial coordinates. The world line is the

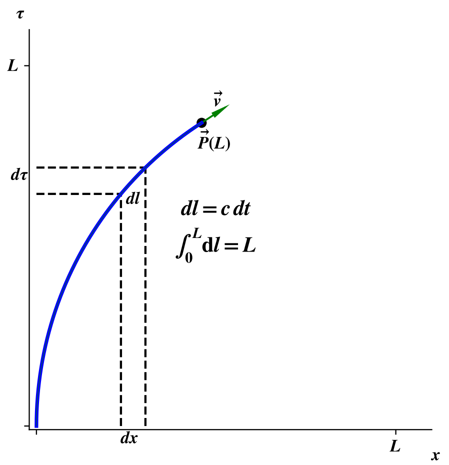

Fig. 1. The space-evolution diagram with a world line that stretches from 0 to L. The blue curve represents the world line of an accelerating subject, the black dot at the end point of the world line represents the current status, and $\vec{v}$ is the current four-velocity.

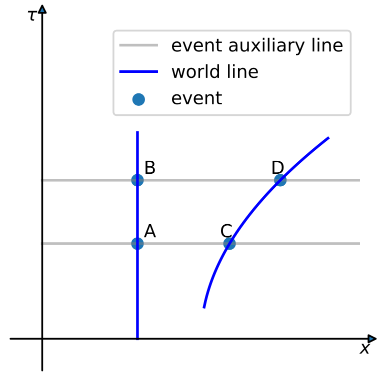

Fig. 2. The blue line on the left side is the world line for stationary subjects, and the blue line on the right side is for accelerating subjects. They start at different locations, but the total lengths are both equal to universal time interval between two observation. Events A and B occur on stationary subjects, and events C and D occur on accelerating subjects. According to the Synchronous World Line Postulate, the sequence of those events is C, A, D, B. The sequence may vary from observers with different spatial velocity.

trace of a subject in the time period $0 \leq l \leq L$. The subject was accelerated; thus, the world line was curved. It should be pointed out that the limitation of spatial speed is implemented by a geometric fact that arc length dl is never shorter than its spatial projection dx; thus, $v_x = dx/dl \leq 1$.

One may argue that this paper simply switches the roles of coordinate time and proper time in space-time diagrams, but in space evolution, the concepts of a point and a world line are quite different. The space-time diagram describes events, though the space-evolution diagram describes evolvable subjects. An event occurs instantaneously at a single point in space time, represented by a set of coordinates x, y, z and t. The space-time observer waits until the subject reaches a specific evolutionary stage and then records the time of the rest clock (length of stretched world line) and the spatial position of the subject. However, from the perspective of space evolution, an event is considered to be a subject that evolves to the evolutionary position specified by the event. Taking a time bomb as an example, space time describes an explosion and tracks the location and time when it happens. However, space evolution is a description of the bomb, tracking its timer reading and spatial coordinates, and the explosion occurrence is a specified evolutionary position (stage) marked on the τ axis.

Before we proceed, a fundamental postulate must be posited:

Synchronous World Line Postulate: In a flat space-evolution frame, all subjects' world lines stretch the same length between two observations.

This postulate is based on some common sense reasoning: in different locations of flat space, rest clocks of identical structure tick with the same rate, and the upper limits of spatial speed are the same for all subjects. The postulate states that the universe has a unified time, and the world lines of all matters in the universe stretch synchronously over this time, which we call 'universal time'. In the rest of this paper, time, l and ct all represent the universal time. We speculate that the postulate also holds in the gravitational field, as long as the coordinate system is established in a proper way. The postulate allows us to measure the world line length of all other subjects by checking the reading of a rest clock. A time interval ($c\triangle t$) recorded by a rest lab clock is actually the mutual geometrical length ($\triangle l$) stretched by the world lines of all subjects.

Fig. 2 demonstrates events in the space-evolution diagram. We define the event-auxiliary line as an auxiliary line perpendicular to the evolution axis and through the evolutionary position that determines the event occurrence. An event is the intersection point of a world line and an event auxiliary line. An example for understanding is that the expression “the observer sees the event happen on the subject“ is equal to “the observer sees the subject reach the evolutionary position defined by the event“.

Two events may or may not occur on the same subject: if they do, they may or may not occur on a stationary subject. A spacetime diagram does not specify those conditions. However, there is a hidden premise that the space-time observer always presumes that the event happens on the rest object before coordinate transformation, so in a space-evolution diagram preparing for Lorentz transformation, the world line of this subject should be parallel to the τ axis.

In the frame of space evolution, a rest clock records an evolution interval $\triangle\tau = c\triangle t = \triangle l$ but a zero spatial interval, although a light speed clock shows zero evolution interval to the observer, but its spatial interval $\triangle x = \triangle l$. The speed of light c is treated as a conversion constant used to normalize the spatial distance and evolutionary distance. Although we have not found a way to properly describe photons and EM fields with space evolution, as they have no evolution component, treating photons as light speed subjects will cause trouble in coordinate transformation.

IV. Rotation of Space-Evolution Frame

Consider space-evolution frames S and define a subject B to have coordinate $[\tau\ x]$ and velocity $[v_\tau\ v_x]$. Define another subject C to have velocity $[\mu_\tau\ \mu_x]$ in S, where S' is the frame of reference for C. In frame S', what is B's coordinate $[\tau'\ x']$ and velocity $[v'_\tau\ v'_x]$?

Given that $\vec{v}$ is a coordinate-invariant vector in a Euclidean space, we use rotation instead of addition and subtraction to describe changes in $\vec{v}$. First, we aim to find S', in which C is stationary. In contrast to spacetime evolution, in space-evolution, it is not necessary to assign the origin of S' to the moving subject so that the subject is relatively stationary. Instead, we need only to rotate S with angle θ, which satisfies $\sin(\theta) = \mu_x$ and $\cos(\theta) = \mu_\tau$.

$$\begin{bmatrix}1 & 0\end{bmatrix} = \begin{bmatrix}\mu_\tau & \mu_x\end{bmatrix} \begin{bmatrix}\mu_\tau & -\mu_x \\ \mu_x & \mu_\tau\end{bmatrix} \tag{16}$$

where $[1, 0]$ is the velocity of subject C in frame S', indicating that the subject C has zero spatial velocity but evolves with the speed of light. We identify S' as the frame of reference for the subject C. Accordingly, B's coordinate and velocity in the new frame S' could be calculated with the same transformation

$$\begin{bmatrix}\tau' & x'\end{bmatrix} = \begin{bmatrix}\tau & x\end{bmatrix} \begin{bmatrix}\mu_\tau & -\mu_x \\ \mu_x & \mu_\tau\end{bmatrix} \tag{17}$$

$$\begin{bmatrix}v'_\tau & v'_x\end{bmatrix} = \begin{bmatrix}v_\tau & v_x\end{bmatrix} \begin{bmatrix}\mu_\tau & -\mu_x \\ \mu_x & \mu_\tau\end{bmatrix} \tag{18}$$

Naturally, in such rotational transformation:

- The distance between any two subject points is preserved, including the origin. Therefore, the infinitesimal world line length dl is preserved, and even the shape of all world lines is preserved.

$$dl^2 = d\tau^2 + dx^2 = d\tau'^2 + dx'^2$$

- The angle between any pair of velocity vectors is preserved.
- The origin is preserved and fixed, rather than being a regular evolvable subject.

Such transformation is self-consistent from a geometric point of view, but quite different from the Lorentzian transformation of velocities. The velocities in the Lorentz transform refer to the "speed of event" rather than the "speed of subject". The question is, does Eqn.17,18 explain physical phenomena properly, especially those in special relativity?

Fig. 3 demonstrates that the frame rotation expressed by Eqn. 17 is able to explain the typical consequences derived from the Lorentz transformation. Frame S is represented by the solid axis, and frame S' is represented by the dotted axis. S' is the frame of reference for spatial speed $v_x = \sin(\pi/6)$ in the figure. The Lorentz factor γ is introduced so that the result is comparable to the space-time diagram

$$\gamma = \frac{1}{v_\tau} = \frac{1}{\sqrt{1 - v_x^2}}$$

Fig. 3(a) shows a subject in frame S with velocity vector $\vec{v} = [v_\tau\ \ v_x]$. However, in the transformed frame S', the velocity vector is $[1\ \ 0]$, parallel to the evolution axis. Thus, the subject is identified as a spatial stationary subject with coordinates $[\tau'\ x']$, and frame S' is considered the frame of reference for the subject.

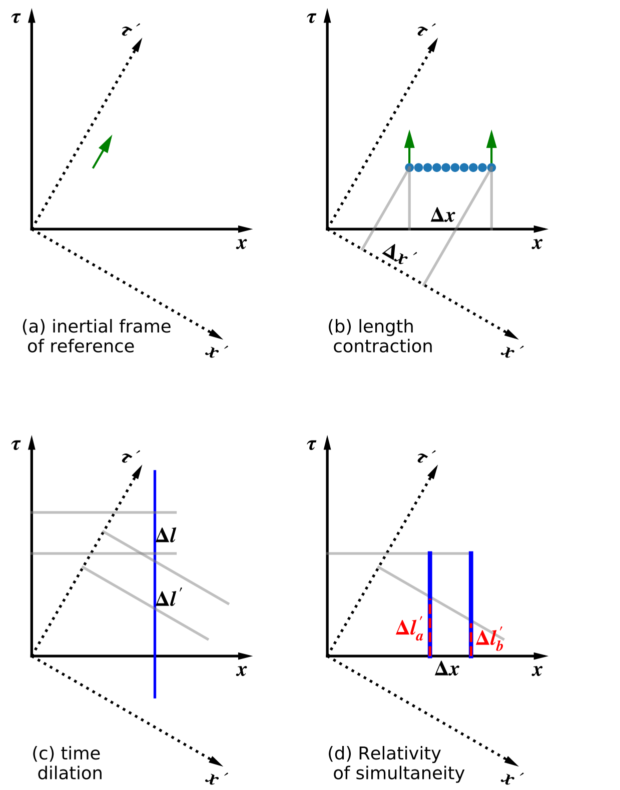

Fig. 3. The gray lines are event auxiliary lines for projection, world lines are represented with blue lines, and dotted lines with arrows are axes for S'. A green arrow represents a subject and its 4-velocity vector.

Fig. 3(b) demonstrates the length contraction of a measuring rod. The rod is drawn with dots so that it is distinguishable from the world line. The rod is at rest and aligned along the x-axis in frame S. In this frame, the length of this rod is written as Δx, but in frame S', the rod is moving towards the origin with spatial velocity $-v_x$, and the spatial length projection

$$\Delta x' = \Delta x\sqrt{1 - v_x^2} = \Delta x/\gamma \tag{19}$$

One should also notice that two synchronized clocks in frame S, placed at the two ends of the rod, are not synchronized in frame S' since they have different evolutionary positions. However, $\Delta x'^2 + \Delta\tau'^2 = \Delta x^2$ is invariant under coordinate transformation, indicating that the 4-position distance between two subjects is also preserved.

Fig. 3(c) supposes a clock is at rest in frame S; its world line is the blue straight line. The clock ticks when the subject evolves to a specific evolutionary position. The two ticks are intercepted by two event-auxiliary lines. The world line length intercepted by two event-auxiliary lines is Δl, which is the time interval between two ticks that seems to be the observer. Though in frame S', the intercepted world line length is

$$\Delta l' = \frac{\Delta l}{\sqrt{1 - v_x^2}} = \gamma\Delta l \tag{20}$$

It should be pointed out that at a specific moment of universal time, an event that already happened to one observer may not have happened yet for another observer, depending on how the stretching world line intercepts the event auxiliary line.

The two blue lines in Fig. 3(d) are the world lines of two rest subjects with different spatial locations. From the perspective of space time, the two events occur simultaneously when two stretching world lines cross the same evolution plane, which denotes the occurrence of the event. The event plane (same-evolution plane) of frame S' is tilted, and the world lines that stretched before the event occurred in S' are represented by red dashed lines. The length differences between them could be calculated geometrically as

$$\Delta l'_a - \Delta l'_b = \frac{v_x \Delta x}{\sqrt{1 - v_x^2}} = \gamma v_x \Delta x \tag{21}$$

which is the time interval of two events that occur in frame S'. When we consider an event located in the space-evolution coordinate, the rotation of coordinates works very well in explaining the time dilation, length contraction and relativity of simultaneity.

As mentioned above, photons, which are unevolvable, cannot be described by the space-evolution frame and its transformation. However, particles with mass can be treated as evolvable subjects and their worldlines may be drawn, though the meaning of the evolutionary position for stable particles is unknown.

V. Lorentz transformation

Lorentz covariance is considered to be the fundamental postulate of special relativity. In this section, we attempt to derive the Lorentz transform from the rotation of space-evolution coordinates. Considering two space-evolution frames S and S', S' is the frame of reference for a moving space-time observer, whose spatial velocity is μ_x in S. As an example, in frame S, we define a world line for a rest subject at spatial position d as follows:

$$\begin{bmatrix}\tau(l) & x(l)\end{bmatrix} = \begin{bmatrix}l & d\end{bmatrix} \tag{22}$$

According to Eqn. 17, the function of the same world line in S' is

$$\begin{bmatrix}\tau'(l) & x'(l)\end{bmatrix} = \begin{bmatrix}l & d\end{bmatrix} \begin{bmatrix}\mu_\tau & -\mu_x \\ \mu_x & \mu_\tau\end{bmatrix} \tag{23}$$

Fig. 4 demonstrates how the space-time coordinate $[ct' \;\; x']$ in S' can be calculated geometrically. The world line and its start/end point are invariant with respect to coordinate transformation, but the event-auxiliary line is rotated with the frame thanks to its definition. Thus, the cross point with the world line differs from that in frame S.

Based on Fig. 4, by inspecting the geometric relation between τ, ct, x, the space-time coordinate of the event in S' is calculated as

$$\begin{aligned} t' &= AK'/c = (CK' - CA)/c = \gamma(t - u_x x) \\ x' &= OB = OC - BC = \gamma(x - u_x ct) \end{aligned} \tag{24}$$

The Lorentz transform is obtained.

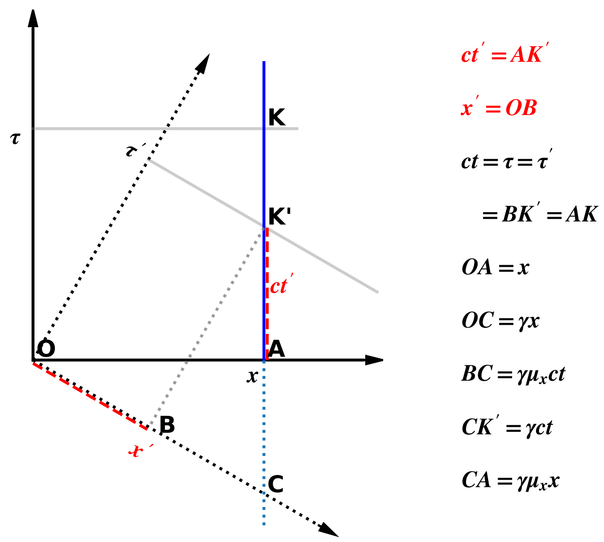

Fig. 4. Geometrical interpretation of Lorentz transformation in space-evolution configuration. The moving space-time observer-measured time ct' and position x' are parameters to be solved, highlighted in red. The blue line is the world line with start point A, K is when the stationary observer confirms the event occurrence, and K' is when the moving observer confirms the event occurrence.

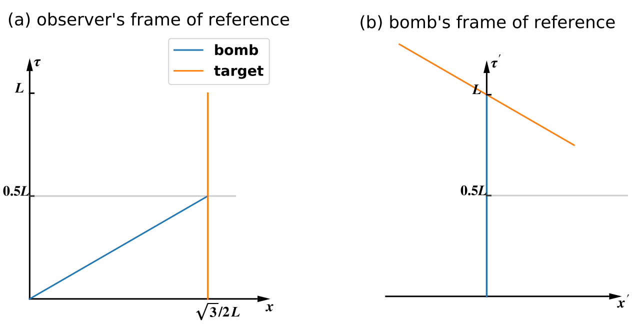

Fig. 5. The same progress presented from the perspective of the target and the time bomb. (a) shows the world lines of the target and bomb in the observer's frame of reference. (b) shows them in the moving bomb's frame of reference. The observation lasts from $l = 0$ to $l = L$. The event "Bomb explosion" is coordinated at $\tau = 0.5L$ and the event "target exposure" is coordinated at $\tau = L$.

VI. Discussion

The coordinate independence of world lines and causality becomes clear by carrying out a thought experiment. As shown in Fig. 5, consider at moment $l = 0$ that a rest target is hidden in a bunker located at $x = \sqrt{3}/2\,L$. The target pops out of the bunker for a very short time when $l = L$. A time bomb is located at $x = 0$, moving towards the target with spatial speed $v_x = \sqrt{3}/2$, and its explosion radius is much smaller than L. The bomb's drifting will not be blocked by the bunker. The bomb timer's initial countdown is set to $0.5L$. The question is: will the target be destroyed? According to special relativity, the answer is clear: the target can be destroyed only when the timer is initially set to $0.5L$. Now, we attempt to inspect the process in the space-evolution frame for both the target and bomb.

Fig. 5 (a) shows the world lines of the target and the bomb;

both stretch from 0 to L synchronously. When $l = L$, the bomb and target encounter each other at $x = \sqrt{3}/2$, and the evolutionary position of the bomb is $\tau = 0.5L$; thus, it explodes. The evolutionary position of the target is L and thus is positioned outside of the bunker. The outcome is destruction of the target.

Fig. 5 (b) shows the same process, but from the perspective of the bomb. The space-evolution frame is established as the bomb's frame of reference. The target moves towards the bomb with spatial speed $v_x = -\sqrt{3}/2$, and its location coincides with the rest of the bomb at $x = 0$. When $l = 0.5L$, the worldline of the bomb stretches to $[0, 0.5L]$, so it explodes; the target's world line stretches to $[0, L]$, so it pops out of the bunker. The target is destroyed at $l = 0.5L$.

What the bomb and the target can reach a consensus regarding is the physical fact that the "bomb destroys the target", which is supposed to be coordinate-independent, although they may interpret the process in different ways. We conclude that the coordinate transformation of space evolution does not violate causality. The Synchronous-World-Line-Postulate is critical for causality; otherwise, explosion of the bomb and exposure of the target may occur at different places, and the target escapes from explosion and the causality breaks.

VII. Conclusion

By introducing the evolution axis, we successfully established space evolution as a Euclidean orthonorm coordinate system without losing compatibility with special relativity. The coordinate transformation for different observers is achieved by rotation, which is a typical property of Euclidean space. In this frame, the speed of light c is nothing but a constant used to normalize the space-evolution coordinate system. The evolutionary and spatial speed naturally have an upper limit c as a geometric fact. Lorentz transformation is not the founding principle, but rather a simple consequence of the geometrical nature of the theory, and its consequences, such as length contraction and time dilation, can be obtained without much mathematical effort. The excellent geometric property of space evolution suggests that the evolution axis is more qualified than time to combine with the three spatial axis. The space-evolution frame is likely to further improve our understanding of time and space, although difficulties are encountered in explaining photons. Further attention is expected to reveal the potential value of the space-evolution frame in physics.

References

[1] Einstein, Albert. Zur Elektrodynamik bewegter Körper", Annalen der Physik 17: 891 (1905).

[2] Minkowski, Hermann. Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern . Nachrichten der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse: 53–111 (1908).