Classicalization of Quantum Fluctuations at the Planck Scale in the $R_h=ct$ Universe

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Classicalization of Quantum Fluctuations at the Planck Scale in the $R_h = ct$ Universe

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Abstract

The quantum to classical transition of fluctuations in the early universe is still not completely understood. Some headway has been made incorporating the effects of decoherence and the squeezing of states, though the methods and procedures continue to be challenged. But new developments in the analysis of the most recent *Planck* data suggest that the primordial power spectrum has a cutoff associated with the very first quantum fluctuation to have emerged into the semi-classical universe from the Planck domain at about the Planck time. In this paper, we examine the implications of this result on the question of classicalization, and demonstrate that the birth of quantum fluctuations at the Planck scale would have been a ‘process’ supplanting the need for a ‘measurement’ in quantum mechanics. Emerging with a single wavenumber, these fluctuations would have avoided the interference between different degrees of freedom in a superposed state. Moreover, the implied scalar-field potential had an equation-of-state consistent with the zero active mass condition in general relativity, allowing the quantum fluctuations to emerge in their ground state with a time-independent frequency. They were therefore effectively quantum harmonic oscillators with classical correlations in phase space from the very beginning.

Key words: cosmology, quantum fluctuations, inflation

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1. Introduction

In standard inflationary Λ CDM cosmology, the early universe underwent a phase of quasi-exponential expansion due to the action of a scalar (inflaton) field with a near-flat potential, a process typically referred to as ‘slow-roll’ inflation [1–5].

The concept of inflation was introduced to solve several global inconsistencies with standard Big Bang cosmology, notably its horizon problem, but an even more important consequence of inflation was realized with the introduction of quantum effects [6–10]. Small inhomogeneous fluctuations on top of an otherwise isotropic and homogeneous background are now broadly believed to be the explanation for the anisotropies in the cosmic microwave background (CMB) and the formation of large-scale structure [11]. Beginning as quantum seeds in the distant conformal past, well before the Planck time t_{Pl} , these inhomogeneities grew with the expansion of the universe and produced a near scale-free power spectrum as they crossed the Hubble

horizon [12]. Indeed, the small departure from a perfectly scale-free distribution is viewed (in this model) as a consequence of ‘Hubble friction,’ whose strength and duration are directly attributable to the detailed shape of the inflaton potential. Considerable work has been carried out over the past four decades to use this framework in order to constrain the physical conditions in the early universe [13–17], at energies up to the grand unified scale at $\sim 10^{15}$ GeV.

But the CMB anisotropies and galaxy clusters are classical, so we are faced with the problem of understanding how quantum fluctuations in the inflaton field transitioned into purely classical objects—an issue closely related to the long-standing ‘measurement’ problem in quantum mechanics, i.e., how does a deterministic outcome appear in a measurement process performed on a quantum system prepared in a superposed state? Different authors use a range of descriptions to characterize the distinctions between quantum and classical states (see, e.g., refs. [18–20]) but, at a fundamental level, a successful transition from the former to the latter requires the substantial elimination of interference between the various degrees of freedom in a superposed quantum state and the subsequent appear-

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ance of macroscopic variables. The second condition is the emergence of classical correlations in the phase space of such canonical variables. In other words, classical trajectories (e.g., orbits) with well-defined values of these variables need to be established.

The age-old (mostly heuristic) Copenhagen interpretation suggests that a quantum system remains in its superposed state until an observer performs a measurement on it, the interaction of which causes it to ‘collapse’ into one of the eigenstates of the observable being measured [21]. The original version of this concept has since been refined with the phenomenon of decoherence, in which the system becomes entangled with its environment [22], consisting of a very large number of degrees of freedom, bringing the original quantum state into an ensemble of classical looking ones. The manifestation of this problem specifically in the cosmological context has sometimes been framed in the sense that the CMB anisotropies constitute a measurement of the field variable [23–25].

The formalism based on decoherence, however, is subject to considerable debate. It has been argued by some [26–28] that decoherence by itself does not solve the problem of a single outcome. As we shall discuss later in this paper, this mechanism is arguably even less likely to be relevant to the early universe—a ‘closed’ system where the distinction between the quantum state and an ‘environment’ is essentially nonexistent. Other types of approaches, such as Bohmian mechanics [29,30], do not by themselves predict testable features that may be verified or refuted.

Our entry point into this discussion is motivated by several significant new developments in the updated analysis of the latest *Planck* data release [31], which build on many attempts made over the past two decades to find specific features associated with the primordial power spectrum, $\mathcal{P}(k)$. For example, several large-angle anomalies have been present in the CMB maps since the 1990’s. The most recent studies [32,33] of the *Planck* data have extended our ability to examine whether these issues are more likely due to instrumental or other systematic effects, or whether they truly represent real characteristics in $\mathcal{P}(k)$. And in § 3.1, we shall briefly discuss why these anomalies may now be viewed as possibly being due to a cutoff $k_{\min}$ in the power spectrum. Its value, however, calls into question whether slow-roll inflation remains viable [64], at least based on the inflaton potentials proposed thus far. As we shall explore in § 3.2, a more interesting interpretation for the appearance of $k_{\min}$ is that this represents the first mode to exit from the Planck scale into the semi-classical universe at about the Planck time. If correct, this interpretation would clearly have a considerable impact on the quantum-to-classical transition.

In this paper, we focus on how this interpretation could alter our view of the long-standing classicalization problem in the early universe. The conventional view has been that quantum fluctuations were seeded in the Bunch-Davies vacuum, though without much guidance other than they emerged in the ground state. But if they appeared at the

Planck scale at about the Planck time, the manner in which the quantum states were ‘prepared’ may have some bearing on the classicalization problem. In § 2, we shall summarize the current status with the quantum-to-classical transition of fluctuations in the inflaton field, and then transition into a discussion of the analogous situation for a non-inflationary scalar field with the zero active mass equation-of-state (relevant to the above interpretation of $k_{\min}$) in § 3. We shall describe the impact of these new developments on the classicalization question in § 4 and end with our conclusions in § 5.

2. Classicalization of Quantum Fluctuations in the Inflaton Field

2.1. Quantum Fluctuations in the Inflaton Field

The essential steps for deriving the perturbation growth equation are by now well known, and one may find many accounts of this procedure in both the primary and secondary literature. For the sake of brevity, we here show only some key results—chiefly those that will also be relevant to our discussion of quantum fluctuations at the Planck scale in §§ 3 and 4—and refer the reader to several other influential publications for all the details.

The perturbed Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime for linearized scalar fluctuations is given by the line element [34–37]

$$ds^2 = (1 + 2A) dt^2 - 2a(t)(\partial_i B) dt dx^i - a^2(t) [(1 - 2\psi)\delta_{ij} + 2(\partial_i \partial_j E) + h_{ij}] dx^i dx^j, \quad (1)$$

where indices i and j denote spatial coordinates, $a(t)$ is the expansion factor, and A , B , ψ and E describe the scalar metric perturbations, while h_{ij} are the tensor perturbations.

For small perturbations about the homogeneous scalar field $\phi_0(t)$,

$$\phi(t, \mathbf{x}) = \phi_0(t) + \delta\phi(t, \mathbf{x}), \quad (2)$$

one can identify the curvature perturbation $\mathcal{R}$ on hypersurfaces orthogonal to worldlines in the comoving frame written as a gauge invariant combination of the scalar field perturbation $\delta\phi$ and the metric perturbation ψ [34]:

$$\mathcal{R} \equiv \psi + \left(\frac{H}{\dot{\phi}} \right) \delta\phi. \quad (3)$$

A solution for $\mathcal{R}$ may then be obtained by (i) using an expansion in Fourier modes,

$$\mathcal{R}(\tau, \mathbf{x}) \equiv \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \mathcal{R}_{\mathbf{k}}(\tau) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (4)$$

and (ii) inserting the linearized metric (Eq. 1) into Einstein’s equations, which together yield the following perturbed equation of motion for each mode $\mathbf{k}$:

$$\mathcal{R}_{\mathbf{k}}'' + 2 \left(\frac{z'}{z} \right) \mathcal{R}_{\mathbf{k}}' + k^2 \mathcal{R}_{\mathbf{k}} = 0. \quad (5)$$

Overprime denotes a derivative with respect to conformal time $d\tau = dt/a(t)$, and we have defined the variable

$$z \equiv \frac{a(t)(\rho_\phi + p_\phi)^{1/2}}{H}. \quad (6)$$

Using the canonically normalized Mukhanov-Sasaki variable [35,36]

$$u_{\mathbf{k}} \equiv z\mathcal{R}_{\mathbf{k}}, \quad (7)$$

Equation (5) may be recast into the more familiar form of a parametric oscillator with a time-dependent frequency, known (in this context) as the Mukhanov-Sasaki equation,

$$u_{\mathbf{k}}'' + \omega_{\mathbf{k}}^2(\tau) u_{\mathbf{k}} = 0, \quad (8)$$

where the time-dependent frequency is

$$\omega_{\mathbf{k}}(\tau)^2 \equiv k^2 - \frac{z''}{z}. \quad (9)$$

In Minkowski spacetime, the scale factor $a(\tau)$ is constant and $\omega_{\mathbf{k}} = k$, which reduces Equation (8) to that of the more basic harmonic oscillator with a constant frequency.

Since $\omega_{\mathbf{k}}(\tau)$ depends solely on τ and k (not the direction of $\mathbf{k}$), the most general solution to the Mukhanov-Sasaki equation may be written

$$u_{\mathbf{k}} = a_{\mathbf{k}}^- u_{\mathbf{k}}(\tau) + a_{\mathbf{k}}^+ u_{\mathbf{k}}^*(\tau), \quad (10)$$

where $u_{\mathbf{k}}(\tau)$ and its complex conjugate are linearly independent solutions to Equation (8), and are the same for all Fourier modes with $k = |\mathbf{k}|$. In general, however, the integration constants $a_{\mathbf{k}}^-$ and $a_{\mathbf{k}}^+$ may depend on the direction of $\mathbf{k}$. From Equations (4) and (7), one may thus write the complete solution to the Mukhanov-Sasaki equation as

$$u(\tau, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} [a_{\mathbf{k}}^- u_{\mathbf{k}}(\tau) e^{i\mathbf{k}\cdot\mathbf{x}} + a_{\mathbf{k}}^+ u_{\mathbf{k}}^*(\tau) e^{-i\mathbf{k}\cdot\mathbf{x}}], \quad (11)$$

which is manifestly real since $a_{\mathbf{k}}^+ = (a_{\mathbf{k}}^-)^*$.

Our derivation of Equation (11) has made no reference to the definite size of the inhomogeneities. This result therefore merely shows us the dynamics of such inhomogeneities if they came into existence. To address their presence and magnitude, one must consider states at the quantum level, where the fluctuation field becomes an operator. The next step is therefore the quantization of the field, noting that the Hilbert space for the quantum field will be a direct product of individual Hilbert spaces for the Fourier modes.

The field $u(\tau, \mathbf{x})$ is quantized like the harmonic oscillator, except that the frequency (Eq. 9) is now time dependent, which is itself a consequence of the curved spacetime within which these modes are evolving. To properly normalize the quantum fluctuation, one must therefore choose the vacuum carefully, since the spacetime is generally not Minkowskian.

The field $u(\tau, \mathbf{x})$ and its canonical conjugate ‘momentum’ $\pi(\tau, \mathbf{x}) \equiv u'(\tau, \mathbf{x})$ are promoted to quantum operators $\hat{u}$ and $\hat{\pi}$, satisfying the standard equal-time canonical quantization relations

$$\begin{aligned} [\hat{u}(\tau, \mathbf{x}_1), \hat{\pi}(\tau, \mathbf{x}_2)] &= i\delta^3(\mathbf{x}_1 - \mathbf{x}_2) \\ [\hat{u}(\tau, \mathbf{x}_1), \hat{u}(\tau, \mathbf{x}_2)] &= 0 \\ [\hat{\pi}(\tau, \mathbf{x}_1), \hat{\pi}(\tau, \mathbf{x}_2)] &= 0. \end{aligned} \quad (12)$$

The constants of integration $a_{\mathbf{k}}^-$ and $a_{\mathbf{k}}^+$ in the mode expansion of $u(\tau, \mathbf{x})$ become operators $\hat{a}_{\mathbf{k}}^-$ and $\hat{a}_{\mathbf{k}}^+$, so that

$$\hat{u}(\tau, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} [\hat{a}_{\mathbf{k}}^- u_{\mathbf{k}}(\tau) e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}_{\mathbf{k}}^+ u_{\mathbf{k}}^*(\tau) e^{-i\mathbf{k}\cdot\mathbf{x}}]. \quad (13)$$

We thus see from Equations (12) and (13) that

$$\begin{aligned} [\hat{a}_{\mathbf{k}}^-, \hat{a}_{\mathbf{k}'}^+] &= \delta^3(\mathbf{k} - \mathbf{k}') \\ [\hat{a}_{\mathbf{k}}^-, \hat{a}_{\mathbf{k}'}^-] &= 0 \\ [\hat{a}_{\mathbf{k}}^+, \hat{a}_{\mathbf{k}'}^+] &= 0, \end{aligned} \quad (14)$$

which allows us to interpret $\hat{a}_{\mathbf{k}}^-$ and $\hat{a}_{\mathbf{k}}^+$ as annihilation and creation operators, respectively, creating and annihilating excitations, or particles, of the field $u(\tau, \mathbf{x})$. And in the usual fashion, the quantum states in the Hilbert space are constructed by a repeated application of the creation operator,

$$|n_{\mathbf{k}_1}, n_{\mathbf{k}_2}, \dots\rangle = \frac{1}{\sqrt{n_{\mathbf{k}_1}! n_{\mathbf{k}_2}! \dots}} \{(\hat{a}_{\mathbf{k}_1}^+)^{n_{\mathbf{k}_1}} (\hat{a}_{\mathbf{k}_2}^+)^{n_{\mathbf{k}_2}} \dots\} |0\rangle, \quad (15)$$

starting with the vacuum state $|0\rangle$, which is defined by

$$\hat{a}_{\mathbf{k}}^- |0\rangle = 0. \quad (16)$$

In de Sitter space, where $a(\tau) = -(H\tau)^{-1}$ (in terms of the Hubble constant H during inflation), the exact solution to Equation (8) is

$$u_{\mathbf{k}}(\tau) = A_k \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) + B_k \frac{e^{ik\tau}}{\sqrt{2k}} \left(1 + \frac{i}{k\tau}\right), \quad (17)$$

where A_k and B_k are integration constants to be fixed by the choice of vacuum. In a time-independent spacetime, the normalization of the wavefunction is obtained by imposing canonical quantization or, equivalently, by minimizing the expectation value of the Hamiltonian in the vacuum state. For example, in Minkowski space, one would find from Equation (8) with $\omega_k = k$, and the Hamiltonian written in terms of u_k and π_k , that

$$u_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \quad (\text{Minkowski}). \quad (18)$$

In de Sitter spacetime, such a solution would therefore correspond to $|\tau| \rightarrow \infty$, and $A_k = 1$, $B_k = 0$. This is interpreted to mean that in the remote conformal past, all modes of current interest were much smaller than the Hubble radius, allowing us to ignore curvature effects on the mode normalization, thereby defining a unique physical vacuum known as the Bunch-Davies vacuum [38].

With the time evolution

$$u_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) \quad (\text{de Sitter}), \quad (19)$$

Equation (13) gives us the full form of the field operator $\hat{u}(\tau, \mathbf{x})$, which may be used to find the quantum states of

the inflaton scalar field. As we shall see in § 3, much of this derivation remains intact for the (non-inflationary) numen field as well, though with several crucial differences we shall discuss shortly. (As we shall see in § 3.2, it will be necessary to clearly distinguish between inflaton scalar fields and those that do not produce an inflated expansion. We shall therefore informally refer to the latter as ‘numen’ fields.)

We now turn our attention to describing the vacuum state with a wavefunctional that gives the probability amplitudes for different field configurations in Fourier space. Though we have developed the expression for the field operator, $\hat{u}(\tau, \mathbf{x})$, in the Heisenberg picture, finding the various occupation numbers in a quantum state such as Equation (15) is more easily accomplished using a functional approach in the Schrödinger picture (see, e.g., refs. [39,40]).

As long as the various k -modes are all independent, the Hilbert space for the field operator $\hat{u}(\tau, \mathbf{x})$, containing eigenstates $|u\rangle$, is simply a direct product of Hilbert spaces for the different Fourier components. If a state is initially decomposed as $|\Psi\rangle$, the probability amplitude of measuring the field configuration $u(\tau, \mathbf{x})$ is then the wave functional $\Psi[u(\tau, \mathbf{x})] \equiv \langle u|\Psi\rangle$ where, in the Schrödinger approach, one may factor it into independent mode components according to:

$$\Psi[u(\tau, \mathbf{x})] = \prod_k \Psi_{\mathbf{k}}[u_{\mathbf{k}}(\tau)] , \quad (20)$$

each of which is a solution to Schrödinger’s equation:

$$i \frac{\partial \Psi_{\mathbf{k}}}{\partial \tau} = \hat{H}_{\mathbf{k}} \Psi_{\mathbf{k}} . \quad (21)$$

The Hamilton operator in Fourier space may be written

$$\hat{H} = \frac{1}{2} \int d^3 \mathbf{k} \hat{H}_{\mathbf{k}} = \frac{1}{2} \int d^3 \mathbf{k} \left[\hat{\pi}_{\mathbf{k}} \hat{\pi}_{\mathbf{k}}^\dagger + \omega_{\mathbf{k}}^2 \hat{u}_{\mathbf{k}} \hat{u}_{\mathbf{k}}^\dagger \right] . \quad (22)$$

The solution to Equations (21) and (22) for a harmonic oscillator is well known, and has the form of a Gaussian function:

$$\Psi_{\mathbf{k}}[u_{\mathbf{k}}(\tau)] = N_{\mathbf{k}}(\tau) e^{-\Omega_{\mathbf{k}}(\tau)(u_{\mathbf{k}})^2} , \quad (23)$$

where

$$|N_{\mathbf{k}}| \equiv \left(\frac{2 \text{Re}\{\Omega_{\mathbf{k}}\}}{\pi} \right)^{1/4} \quad (24)$$

is the normalization factor, and

$$\Omega_{\mathbf{k}} \equiv -\frac{i}{2} \frac{f'_k}{f_k} \quad (25)$$

is written in terms of the function f_k , the solution to Equation (8), i.e.,

$$f_k = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\tau} \right) e^{-ik\tau} . \quad (26)$$

The functional $\Psi[u(\tau, \mathbf{x})]$, representing a quantum fluctuation in the inflaton field, is a superposition of many separate Fourier states $\Psi_{\mathbf{k}}$ (Eqn. 20). Somehow, this quantum description, which includes interference between the various modes and no *a priori* classical correlations in

phase space between the field amplitudes and their canonical partners, must have transitioned into classical fluctuations, characterized in part by a diagonalized density matrix and well defined dynamical trajectories. How this might have happened is the issue we shall address next.

2.2. The Quantum to Classical Transition

The question concerning whether or not the state $\Psi[u(\tau, \mathbf{x})]$ can complete the quantum to classical transition based on the factors we have summarized above has received considerable attention over the past four decades, and is still an open one [41–44,39,45–51]. As noted in the introduction, the cosmological scalar perturbations must satisfy at least two conditions to classicalize: (i) the system must undergo decoherence, so that quantum interference becomes negligible and macroscopic variables appear. We see a universe with well-defined, classical perturbation field values, not superpositions of multiple components represented by Equation (20). (ii) One must see the establishment of classical correlations in phase space, meaning that the allowed dispersion in the canonical variables (u, π) becomes insignificantly smaller than their classical orbital values.

Different authors have proposed somewhat different schemes for this transition, indicating that no consensus has yet been reached. In part, this is due to the fact that the conventional rules one may rely on in ordinary quantum mechanical applications are not necessarily all available in cosmology. To begin with, the system being studied quantum mechanically is the entire Universe. There is no possible separation into a subsystem of interest, its environment and the observer. Moreover, there is just one universe, obviating any possibility of adopting the statistical ensemble interpretation of quantum mechanical measurements. Even more importantly, the observer is here a consequence of the quantum to classical transition, and could not have played any causal part in it. For these (and other) reasons, the system being studied is thus unusual by typical quantum mechanical standards.

A common step taken by the various approaches to solving this problem is to relegate most of the very large number of degrees of freedom in $\Psi[u(\tau, \mathbf{x})]$ to an ‘environment’ and then to ignore them, allowing one to evolve the reduced density matrix $\hat{\rho}$ for the subset of remaining (presumably more interesting) observables. Under some circumstances, also involving suitable time averaging, this procedure eventually diagonalizes $\hat{\rho}$, representing a complete mitigation of any interference effects, which is interpreted as the emergence of classical behavior.

This is not entirely satisfactory, however, because decoherence has not yet successfully solved the long-standing ‘measurement’ problem in quantum mechanics. Certainly, the diagonalization of $\hat{\rho}$ removes certain quantum traits from the system, but it is not clear that a non-interfering set of simultaneous co-existing possibilities is necessarily

classical (see, e.g., ref. [52]). Note, for example, that changing the Hilbert space of the quantum system to a different basis would destroy the diagonal nature of the density matrix. The interpretation of $\hat{\rho}$ is therefore subject to various observer-dependent choices. What is lacking is a clear understanding of how to choose the basis and an interaction specific eigenstate for the preferred observable. In our everyday experience, some progress may be made by allowing the measurement device to ‘select’ the basis, and adopting the ensemble interpretation for the density matrix. But these features are obviously missing in the cosmological context [46,53].

To solve the classicalization problem using decoherence in a cosmological setting, one must therefore identify a physical mechanism and a preferential basis it selects. It is also necessary to find a criterion for separating the large number of degrees of freedom into the ‘interesting set’ and ‘the environment’ dictated by the physical problem at hand. At least some of the proposed treatments then appeal to ‘a specific realization’ of the stochastic variables [39,49], sounding very much like a conventional ‘collapse’ from the statistical description of the universe to one of the members in the statistical ensemble. But no insight is provided into how and when such a transition occurred in the real universe.

To address the second requirement, we begin by noting that in the remote conformal past, where $\Omega_{\mathbf{k}} \rightarrow k/2$, the wavefunctional $\Psi_{\mathbf{k}}$ represented the ground state of a harmonic oscillator, consistent with the previously described Bunch-Davies vacuum. For the largest modes (those of relevance to the structure we observe today), the function $\Omega_{\mathbf{k}}$ acquired a non-trivial time dependence and Equation (23) evolved into a squeezed state—meaning that, for these modes, there exists a direction in the $(u_{\mathbf{k}}, \pi_{\mathbf{k}})$ plane where the dispersion is exponentially small, while the dispersion in a perpendicular direction is very large. The resultant linear combination of $u_{\mathbf{k}}$ and $\pi_{\mathbf{k}}$ for which the dispersion is minimized is often viewed as the emerging classical phase-space trajectory.

A convenient tool to study this process, and its possible relevance to the classicalization question, is the Wigner function (see, e.g., the reviews in refs. [54,55]; for a more pedagogical account, see also ref. [56]), defined by

$$\begin{aligned} \mathcal{W}(u_{\mathbf{k}}^R, u_{\mathbf{k}}^I, \pi_{\mathbf{k}}^R, \pi_{\mathbf{k}}^I) &= \frac{1}{(2\pi)^2} \int dx dy \\ &\Psi_{\mathbf{k}} \left(u_{\mathbf{k}}^R - \frac{x}{2}, u_{\mathbf{k}}^I - \frac{y}{2} \right) e^{-i\pi_{\mathbf{k}}^R x - i\pi_{\mathbf{k}}^I y} \times \\ &\Psi_{\mathbf{k}} \left(u_{\mathbf{k}}^R + \frac{x}{2}, u_{\mathbf{k}}^I + \frac{y}{2} \right), \end{aligned} \quad (27)$$

in terms of the real and imaginary parts of $u_{\mathbf{k}}$ and $\pi_{\mathbf{k}}$. Quantum mechanics inherently deals with probabilities, while classical physics deals with well-defined trajectories in phase space. The Wigner function provides a means of representing the density distributions in $u_{\mathbf{k}}$ and $\pi_{\mathbf{k}}$ for comparison with the ensemble of trajectories one would get using classical means.

The literature on Wigner functions is extensive (including the aforementioned reviews in refs. [54–56]). Its properties suggest that it behaves like a probability distribution in (u, π) phase space, except that it can sometimes take on negative values, so it is generally not a true probability distribution. Moreover, points in the (u, π) space do not represent actual states of the system because the values of $u_{\mathbf{k}}$ and $\pi_{\mathbf{k}}$ cannot be determined precisely at the same time. Nevertheless, $\mathcal{W}$ can be used to visualize correlations between $u_{\mathbf{k}}$ and $\pi_{\mathbf{k}}$, particularly for Gaussian states, such as we have in Equation (23), for which the Wigner function is indeed always positive definite.

It is trivial to show in the case of Equation (23) that the explicit form of $\mathcal{W}$ is

$$\begin{aligned} \mathcal{W}(u_{\mathbf{k}}^R, u_{\mathbf{k}}^I, \pi_{\mathbf{k}}^R, \pi_{\mathbf{k}}^I) &= \frac{1}{\pi^2} e^{-\text{Re}\{\Omega_{\mathbf{k}}\}[(u_{\mathbf{k}}^R)^2 + (u_{\mathbf{k}}^I)^2]} \times \\ &e^{-(\pi_{\mathbf{k}}^R + \text{Im}\{\Omega_{\mathbf{k}}\}u_{\mathbf{k}}^R)^2 / \text{Re}\{\Omega_{\mathbf{k}}\}} e^{-(\pi_{\mathbf{k}}^I + \text{Im}\{\Omega_{\mathbf{k}}\}u_{\mathbf{k}}^I)^2 / \text{Re}\{\Omega_{\mathbf{k}}\}}, \end{aligned} \quad (28)$$

with which one may clearly see the effect of strong squeezing as $\Omega_{\mathbf{k}}$ changes dramatically with time (Eqns. 25 and 26) during the cosmic expansion.

As long as $\Omega_{\mathbf{k}} \rightarrow k/2$, $\mathcal{W}$ is peaked over a small region of phase space, representing the Wigner function of a coherent state, i.e., the ground state of a harmonic oscillator. We shall return to this in § 4 below, where we consider the corresponding Wigner function for the numen quantum fluctuations. We shall see that this limiting situation takes on added significance in that case, given that $\Omega_{\mathbf{k}}$ actually remains constant for a numen field. But $\Omega_{\mathbf{k}}$ is definitely not constant here. As τ advances, $\mathcal{W}$ spreads and acquires a cigar shape typical of squeezed states [57], with a drastically reduced dispersion around $\pi_{\mathbf{k}}$ and a correspondingly enormous dispersion around $u_{\mathbf{k}}$. But the overall dispersion may be minimized, as we alluded to earlier, by choosing an appropriate linear combination of $u_{\mathbf{k}}$ and $\pi_{\mathbf{k}}$, an outcome that some argue represents the emerging correlation in classical phase space.

One may therefore think of a squeezed state as a state with the minimal uncertainty, though not in terms of the original (u, π) variables [39,47]. In the cosmological context, the inflationary expansion created an uncertainty on the value of the field and its conjugate momentum that was much larger than the minimum uncertainty implied by the Heisenberg uncertainty principle. The minimum uncertainty was instead associated with a new pair of ‘rotated’ canonical variables. But this outcome has also been challenged as not necessarily representing a classical system [44,51].

Take the following situation as an example. Consider an electron in a minimal wavepacket localized at the origin, with an uncertainty Δx in position and $\Delta p = \hbar/(2\Delta x)$ in momentum. Then form a superposition of this state with an identical one after a translation by a large distance D . The overall uncertainty is now $\sim D\hbar/(2\Delta x)$, which can be increased arbitrarily by simply choosing a sufficiently large distance D . This situation is clearly analogous to one of our

squeezed states, but the superposition we have created is nonetheless not classical.

And then there is the issue of definite outcomes, also known as the ‘measurement problem’ [58], as we alluded to in the introduction. This aspect of quantum mechanics has been with us for over a century. Even if decoherence were successful in diagonalizing the density matrix, it cannot solve the definite outcome problem, which is far worse in cosmology than it is in typical laboratory situations. The usual approach of adopting the Copenhagen interpretation of quantum mechanics, in which a measurement ‘collapses’ the state vector of the system into an eigenstate corresponding to the measurement result, does not work in the early Universe, where there were no measurement devices or observers present.

While decoherence might have diagonalized the reduced density matrix of the system to an ensemble of classically observable universes, it does not explain how a certain universe was singled out to be observed [28]. Perhaps the Copenhagen interpretation is just not well suited to cosmology. In addition, if decoherence resulted from an environment comprised of the ‘non-interesting’ degrees of freedom in $\Psi[u(\tau, \mathbf{x})]$, how could this happen when inflation would have driven all such fields towards their vacuum states? Moreover, why do we have the privilege of deciding which degrees of freedom to relegate to the background based solely on whether our current technology allows us to observe them today?

We shall now divert our attention away from the traditional inflationary scenario we have been describing, and seriously consider the implications of a novel feature emerging from the latest release of the *Planck* data: the observational measurement of a cutoff in the primordial spectrum bears directly on the question of how and when quantum fluctuations were generated in the early Universe. We shall study what new ideas and constraints this brings to the classicalization process.

3. Quantum Fluctuations at the Planck Scale

3.1. Emergence of a Cutoff in the primordial power spectrum $\mathcal{P}(k)$

All three of the major satellite missions designed to study the CMB—COBE [59]; WMAP [60]; and *Planck* [31]—have uncovered several anomalies in its fluctuation spectrum. The two most prominent among them are: (1) an unexpectedly low level—perhaps even a complete absence—of correlation at large angles (i.e., $\theta \gtrsim 60^\circ$), manifested via the angular correlation function, $C(\theta)$; and (2) relatively weak power in the lowest multipole moments of the angular power spectrum, C_ℓ . Their origin, however, is still subject to considerable debate, many arguing in favor of a misinterpretation or the result of unknown systematics, such as an incorrect foreground subtraction (see refs. [61,62] for reviews). This uncertainty is also fueled in large part by a

persistent lack of clarity concerning how quantum fluctuations were seeded in the early universe [63,64].

The large-scale anomalies stand in sharp contrast to our overall success interpreting the CMB anisotropies at angles smaller than 1° . Over the past several decades, a concerted effort has therefore been made in attempting to identify features in the primordial power spectrum, $\mathcal{P}(k)$, responsible for their origin. For example, in their study of the CMB angular power spectrum, Shafieloo & Souradeep [65] assumed an exponential cutoff at low k -modes, and identified a turnover generally consistent with the most recent measurement we shall discuss below (see Eq. 29). This early treatment was based on WMAP observations [60], however, not the higher precision *Planck* measurements [31] we have today, so the reality of a non-zero $k_{\min}$ remained somewhat controversial.

In followup work, Nicholson et al. [66] and Hazra et al. [67] inferred a ‘dip’ in $\mathcal{P}(k)$ on a scale $k \sim 0.002 \text{ Mpc}^{-1}$ for the WMAP data, confirming the outcome of an alternative approach by Ichiki et al. [68] that identified an oscillatory modulation around $k \sim 0.009 \text{ Mpc}^{-1}$. In closely aligned work, Tocchini et al. [69] modeled both a dip at $k \sim 0.035 \text{ Mpc}^{-1}$ and a ‘bump’ at $k \sim 0.05 \text{ Mpc}^{-1}$. Like the others, though, these features were based solely on WMAP observations and therefore appeared to be merely suggestive rather than compelling. Tocchini et al. [70] improved on this analysis considerably, and found evidence for three features in $\mathcal{P}(k)$, one of which was a cutoff at $\sim 0.0001 - 0.001 \text{ Mpc}^{-1}$ at a confidence level of $\sim 2\sigma$. Complementary work by Hunt & Subir [71,72] confirmed the likely existence of a cutoff $k < 5 \times 10^{-4} \text{ Mpc}^{-1}$ but, as before, also concluded that more accurate *Planck* data would eventually be needed to confirm these results more robustly.

The subsequent *Planck* observations have not only largely confirmed these earlier results, but have provided us with a greatly improved precision in the ‘measurement’ of $k_{\min}$. For example, the Planck Collaboration [73] fit a cutoff to the CMB angular power spectrum and found a value $\sim 3 - 4 \times 10^{-4} \text{ Mpc}^{-1}$. Still, though these studies all pointed to the likely existence of a cutoff in $\mathcal{P}(k)$, a non-zero $k_{\min}$ could not be claimed with a confidence level exceeding $1 - 2\sigma$.

This situation improved considerably when, instead of looking solely at the power spectrum, the impact of a cutoff was also considered on the angular correlation function, independently of the angular power spectrum. Two separate (though complementary) studies of the latest *Planck* data release have provided more compelling evidence that the two large-angle features in the CMB anisotropies may be real. The first of these [32] demonstrates that the most likely explanation for the missing large-angle correlations is a cutoff,

$$k_{\min} = \frac{4.34 \pm 0.5}{r_{\text{cmb}}}, \quad (29)$$

in the primordial power spectrum, $\mathcal{P}(k)$, where r_{cmb} is the comoving distance to the surface of last scattering. For the *Planck*- Λ CDM parameters, $r_{\text{cmb}} \approx 13,804 \text{ Mpc}$ [31], and

we therefore have $k_{\min} = (3.14 \pm 0.36) \times 10^{-4} \text{ Mpc}^{-1}$. A zero cutoff (i.e., $k_{\min} = 0$) is ruled out by the $C(\theta)$ data at a confidence level exceeding $\sim 8\sigma$.

A subsequent study [33] focused on the CMB angular power spectrum itself (i.e., C_ℓ versus ℓ), and its results (i) confirmed that the introduction of this cutoff in $\mathcal{P}(k)$ does not at all affect the remarkable consistency between the standard inflationary model prediction and the *Planck* measurements at $\ell > 30$ [31], where the underlying theory is widely believed to be correct; and (ii) showed that such a cutoff ($k_{\min}$) also self-consistently explains the missing power at large angles, i.e., the low multipole moments ($\ell = 2 - 5$). The cutoff optimized by fitting the angular power spectrum over the whole range of ℓ 's is $k_{\min} = (2.04^{+1.4}_{-0.79}) \times 10^{-4} \text{ Mpc}^{-1}$, while a fit to the restricted range $\ell \leq 30$, where the Sachs-Wolfe effect is dominant [74], gives $k_{\min} = (3.3^{+1.7}_{-1.3}) \times 10^{-4} \text{ Mpc}^{-1}$. The outcome based on the CMB angular power spectrum therefore rules out a zero cutoff at a confidence level $\gtrsim 2.6\sigma$. In either case, the inferred value of $k_{\min}$ is fully consistent with the cutoff implied by missing correlations in $C(\theta)$, and one concludes that both of these large-angle anomalies are probably due to the same truncation, i.e., $k_{\min} \sim 3 \times 10^{-4} \text{ Mpc}^{-1}$, in $\mathcal{P}(k)$. The confidence with which one may make such a claim depends on whether the cutoff is used to address the missing power at low ℓ 's, or the missing correlations at large angles. Nevertheless, the fact that the same $k_{\min}$ apparently solves both anomalies makes the assumption of a cutoff quite reasonable.

These results reinforce the perception that the small-angle anisotropies (for $\ell \gtrsim 30$), which are mostly due to acoustic oscillations, are well understood, while the fluctuations associated with angular correlations at $\theta \gtrsim 60^\circ$, due to the Sachs-Wolfe effect, continue to be problematic for the standard inflationary picture [59,60,31]. The evidence for a non-zero $k_{\min}$ speaks directly to the cosmological expansion itself. At $\ell \lesssim 30$, we are probing ever closer to the beginning of inflation, culminating with the cutoff $k_{\min}$, signaling the very first mode that would have crossed the horizon when the quasi-de Sitter phase started [36,64,17].

But if one insists on inflation simultaneously fixing the horizon problem and accounting for the observed primordial power spectrum, $\mathcal{P}(k)$, the implied time at which the accelerated expansion began would have suppressed the comoving size of the universe to a tenth of the required value [64,17]. Moreover, neither a radiation-dominated, nor a kinetic-dominated, phase preceding inflation could have alleviated this disparity [75,76].

3.2. A Reinterpretation of $k_{\min}$

To be clear, the measurement of $k_{\min}$ does not completely rule out inflation, nor even the idea that some slow-roll variant may eventually be constructed to address the inconsistency described above. At a minimum, however, the currently proposed inflaton potentials $V(\phi)$ require at least

some modification. Moreover, a cutoff in $\mathcal{P}(k)$ does not argue *against* the influence of a scalar field, ϕ , nor anisotropies arising from its quantum fluctuations, but there is now some motivation to question whether $V(\phi)$ was truly inflationary.

Over the past decade, some evidence has been accumulating that the cosmic fluid may possess a zero active mass equation-of-state, $\rho + 3p = 0$ (in terms of its total energy density ρ and pressure p), supported by over 27 different kinds of observation at low and high redshifts (see Table 2 in ref. [77] for a recent summary of these results). Such a universe lacks a horizon problem [78,79], so the lack of a fully self-consistent inflationary paradigm may be telling us that the universe does not need it. This is the key assumption we shall make to reinterpret $k_{\min}$ in this paper. In addition to the growing body of empirical evidence favoring this approach, there is also theoretical support for the zero active mass equation-of-state from the ‘Local Flatness Theorem’ in general relativity [80].

As noted earlier, we shall clearly distinguish between the roles played by a non-inflaton ϕ and a conventional inflaton field by informally referring to the former as a ‘numen’ field, based on our inference that it may represent the earliest form of substance in the universe. Its equation-of-state is assumed to be $\rho_\phi + 3p_\phi = 0$, and we shall see shortly why this property appears to provide a more satisfactory interpretation of $k_{\min}$ than an inflaton field.

The background numen field is homogeneous, so its energy density ρ_ϕ and pressure p_ϕ are simply given as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (30)$$

and

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \quad (31)$$

The zero active mass equation-of-state therefore uniquely constrains the potential to be

$$V(\phi) = \dot{\phi}^2, \quad (32)$$

with the explicit solution

$$V(\phi) = V_0 \exp \left\{ -\frac{2\sqrt{4\pi}}{m_{\text{Pl}}} \phi \right\}, \quad (33)$$

in terms of the Planck mass

$$m_{\text{Pl}} \equiv \frac{1}{\sqrt{G}}. \quad (34)$$

Some may recognize this as a special member of the category of minimally coupled fields explored in the 1980's [81–84], intended to produce so-called power-law inflation. But unlike the other fields in this cohort, the numen field's zero active mass equation-of-state makes it the only member of this group that does *not* inflate, since the Friedmann equations with this density and pressure lead to an expansion factor $a(t) = t/t_0$, written in terms of the age of the universe, t_0 . This normalization of $a(t)$ is appropriate for a spatially flat FLRW metric, which the observations appear to be telling us.

With this expansion factor, the conformal time may be written

$$\tau(t) = t_0 \ln a(t) , \quad (35)$$

such that the zero of τ coincides with $t = t_0$. The parameter z in Equation (6) thus becomes

$$z = \frac{m_{\text{Pl}}}{\sqrt{4\pi}} a(t) , \quad (36)$$

so that $z'/z = 1/t_0$ and $z''/z = 1/t_0^2$. The resulting curvature perturbation equation analogous to Equation (8) may thus be written

$$u_k'' + \alpha_k^2 u_k = 0 , \quad (37)$$

where the frequency (analogous to Eqn. 9) is now given by the expression

$$\alpha_k \equiv \frac{1}{t_0} \sqrt{\left(\frac{2\pi R_h}{\lambda_k}\right)^2 - 1} , \quad (38)$$

in terms of the proper wavelength of mode k ,

$$\lambda_k \equiv \frac{2\pi}{k} a(t) . \quad (39)$$

In this expression, the quantity $R_h \equiv c/H = ct$ is the apparent (or gravitational) radius [85], which defines the Hubble horizon in a spatially flat universe. The most critical difference between Equations (8) and (9), and Equations (37) and (38), is that here both R_h and $a(t)$ scale linearly with t , and therefore the frequency α_k of the numen quantum fluctuations is *always time-independent*. This is because the ratio $R_h/\lambda_k \propto kR_h/a(t)$ is constant for each k , and therefore numen fluctuations do not criss-cross the horizon; once λ_k is established upon the mode's exit into the semi-classical universe, it remains a fixed fraction of R_h as they expand with time. This feature is crucial to understanding how and why numen quantum fluctuations provide a more satisfactory explanation than inflation for the origin of k_{min} .

The solution to Equation (37) is that of the standard harmonic oscillator:

$$u_k(\tau) = \begin{cases} B(k) e^{\pm i\alpha_k \tau} & (2\pi R_h > \lambda_k) \\ B(k) e^{\pm |\alpha_k| \tau} & (2\pi R_h < \lambda_k) \end{cases} . \quad (40)$$

That is, all modes with $\lambda_k < 2\pi R_h$ oscillate, while the super-horizon ones do not, mirroring the behavior of the more conventional inflaton field. Here, however, the mode with the longest wavelength relevant to the formation of structure is the one for which $\lambda_k(\tau) = 2\pi R_h(\tau)$, i.e.,

$$k^{\text{numen}}|_{\text{min}} = 1/t_0 . \quad (41)$$

One's intuition would immediately suggest that $k^{\text{numen}}|_{\text{min}}$ ought to be identified with the cutoff k_{min} measured in the CMB, and it is not difficult to demonstrate why that has to be the case for a numen scalar field, as we shall see shortly.

A slightly different (and simpler) definition of the Planck scale than that appearing in Equation (34) is based on the length λ_{Pl} at which the Compton wavelength $\lambda_C \equiv 2\pi/m$

for mass m equals its Schwarzschild radius $R_h \equiv 2Gm$. That is,

$$\lambda_{\text{Pl}} \equiv \sqrt{4\pi G} . \quad (42)$$

The Compton wavelength λ_C grows as the gravitational radius R_h shrinks, so the standard inflationary picture conflicts with quantum mechanics in its interpretation of wavelengths shorter than $2\pi\lambda_{\text{Pl}}$ (the factor 2π arising from the definition of λ_{Pl} in terms of R_h). This is a serious problem for the standard model because the fluctuation amplitude A_s measured in the CMB anisotropies requires quantum fluctuations in the inflaton field to have been born in the Bunch-Davies vacuum, long before the Planck time $t_{\text{Pl}} \equiv \lambda_{\text{Pl}}$, a conundrum commonly referred to as the ‘‘Trans-Planckian Problem’’ [86].

The numen field can completely avoid this inconsistency if we argue that each mode k emerged into the semi-classical universe when

$$\lambda_k = 2\pi\lambda_{\text{Pl}} , \quad (43)$$

and then evolved subject to the oscillatory solution in Equation (40). As we shall see shortly, the fact that each succeeding k -mode emerges at later times in this picture produces a near scale-free power spectrum $\mathcal{P}(k)$ with $n_s \lesssim 1$ [63]. Such an idea—that modes could have been born at a particular spatial scale—has already received some attention in the past, notably by Hollands & Wald [87]. In their case, however, the fundamental scale was not related to λ_{Pl} . Others supporting this proposal include Brandenberger et al. [88] and Hassan et al. [89].

It is not difficult to understand why the fundamental scale for the numen field must be λ_{Pl} . If we interpret $k^{\text{numen}}|_{\text{min}}$ to be k_{min} , the latter defines the time t_{min} at which the first quantum fluctuation emerged out of the Planck domain into the semi-classical universe. From Equation (29) and the expression for r_{cmb} in a universe with zero active mass,

$$r^{\text{numen}}|_{\text{cmb}} = \frac{c}{H_0} \ln(1 + z_{\text{cmb}}) , \quad (44)$$

one therefore finds that

$$t_{\text{min}} = \frac{4.34 t_{\text{Pl}}}{\ln(1 + z_{\text{cmb}})} . \quad (45)$$

Its dependence on z_{cmb} is so weak that t_{min} is approximately equal to t_{Pl} regardless of where the last scattering surface was located. For example, in *Planck*- Λ CDM, $z_{\text{cmb}} \sim 1080$, for which $t_{\text{min}} \sim 0.63 t_{\text{Pl}}$. But even if we were to adopt a very different value $z_{\text{cmb}} = 50$, the first quantum fluctuation would have emerged at $t_{\text{min}} \sim 1.1 t_{\text{Pl}}$.

If the spatially largest fluctuation we see in the CMB was due to a numen fluctuation, one concludes from this analysis that it must have emerged out of the Planck regime at roughly the Planck time. In other words, this fluctuation would have physically exited into the semi-classical universe shortly after the Big Bang—indeed, it would have appeared as soon as it could, given what we currently understand about the Planck time t_{Pl} . No other scalar field introduced thus far, inflaton or otherwise, has this very interesting property.

But if a Bunch-Davies vacuum in the remote conformal past is not used for these fluctuations, how does one then determine the normalization constant $B(k)$ of the modes in Equation (40)? As we saw in § 2.1, a principal complication with the inflaton field is the significant spacetime curvature encountered by its quantum fluctuations as they cross the Hubble radius. This led to the introduction of a Bunch-Davies vacuum in the distant conformal past, where the modes could have been seeded in Minkowski space. We made reference to the fact that this situation actually creates an inconsistency with quantum mechanics, often referred to as the trans-Planckian problem. This issue is largely beyond the scope of the present paper, however, because the numen field completely avoids this inconsistency.

Even though the numen quantum fluctuations emerged at the Planck scale—with a wavelength comparable to the gravitational radius R_h —the zero active mass equation-of-state in the cosmic fluid (leading to Eqn. 32) ensures that the frame into which they emerged from the Planck regime was geodesic. That is, in spite of the Hubble expansion, the universe was always in free fall, with *zero internal acceleration*. One can easily confirm this from the fact that the frequency α_k in Equation (38) is always time-independent. One therefore does not need an ad hoc construction of a Bunch-Davies vacuum, and we may simply set

$$B(k) = \frac{1}{\sqrt{2\alpha_k}} \quad (46)$$

for the numen quantum fluctuations (in Eqn. 40), following the same minimization of the Hamiltonian argument that led to Equation (18).

We complete this brief survey by considering the spectrum $\mathcal{P}(k)$ one should expect from the birth of quantum fluctuations at the Planck scale. Right away, we can see from Equation (38) that the difference between α_k and k is what distorts the primordial power spectrum away from a pure, scale-free distribution, as we shall confirm below. This is most easily recognized if we rewrite the mode frequency in the form $\alpha_k = \sqrt{k^2 - 1/t_0^2}$ (which is $< k$).

From the definition of u_k and the curvature perturbation, we see that

$$|\mathcal{R}_k|^2 = \frac{2\pi}{m_{\text{Pl}}^2} \frac{1}{\alpha_k a^2}. \quad (47)$$

The power spectrum $\mathcal{P}_{\mathcal{R}}(k)$ is defined as

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2} |\mathcal{R}_k|^2, \quad (48)$$

and therefore

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{1}{(2\pi)^2} \left[\frac{a(t_k)}{a(t)} \right]^2 \left[1 - \left(\frac{k_{\min}}{k} \right)^2 \right]^{-1/2}. \quad (49)$$

What is not known yet is how these quantum fluctuations devolved into grand unified theories particles, after which the perturbation amplitude remained frozen. Nevertheless, it is likely that the dynamics of this decay/evolution is associated with a particular length (or energy) scale, L_* [90,91], not unlike the Planck scale λ_{Pl} .

Mode k reached this scale at $a(t_k^*) = L_* k / 2\pi$ or, equivalently, at time $t_k^* = t_0 L_* k$. And so Equation (49) may be re-written

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{1}{(2\pi)^2} \left[\frac{\lambda_{\text{Pl}}}{L_*} \right]^2 \left[1 - \left(\frac{k_{\min}}{k} \right)^2 \right]^{-1/2}, \quad (50)$$

which one then needs to compare with the observed CMB power spectrum $\mathcal{P}(k) = A_s (k/k_0)^{n_s-1}$. In the context of the standard model, the *Planck* optimizations [31] give $A_s = (2.1 \pm 0.04) \times 10^{-9}$ and $n_s = 0.9649 \pm 0.0042$. And it is not difficult to see that $L_* \sim 3.5 \times 10^3 \lambda_{\text{Pl}}$. Therefore, with the Planck scale set at $m_{\text{Pl}} \approx 1.22 \times 10^{19}$ GeV, one finds that L_* corresponds to an energy of roughly 3.5×10^{15} GeV, remarkably consistent with the energy scale expected in grand unified theories. Of course, much of this is mere speculation at the present time, given that the physics of this process lies beyond the standard model. Nevertheless, the quantum fluctuations in this picture would have oscillated until $t \sim 3.5 \times 10^3 t_{\text{Pl}}$, after which the numen field would have devolved into grand unified theory particles, with a freezing of the perturbation amplitude thereafter.

The primordial power spectrum (Eq. 50) is almost scale-free, but not exactly. Using the conventional definition of the scalar index, we find that

$$n_s = 1 + \frac{d \ln \mathcal{P}_{\mathcal{R}}(k)}{d \ln k} = 1 - \frac{1}{(k/k_{\min})^2 - 1}. \quad (51)$$

The index n_s is therefore slightly less than 1, and we confirm that the deviation from a pure scale-free distribution is due to the aforementioned difference between α_k and k , which in the end arises from the Hubble expansion (or ‘frictional’) term $\mathcal{R}'_k$ in the growth Equation (5).

At least qualitatively, this result agrees with the value of n_s measured by *Planck*. Of course, the numen optimization may produce a different outcome than that seen with Λ CDM, but it would be difficult to see why the ‘red’ tilt ($n_s < 1$) should be converted into ‘blue’ ($n_s > 1$) with a change in background cosmology. An average of n_s over k in Equation (51) gives $\langle n_s \rangle \sim 0.96$ over the range $k_{\min} \lesssim k \lesssim 50 k_{\min}$. The spectral index would approach closer to one at larger values of k .

The caveat here, of course, is that there are still several unknowns with this process that make it impossible for us to know all the factors influencing the value of n_s . For example, we have assumed that the length scale at which the modes emerge from the Planck regime is always fixed at λ_{Pl} . But this constraint depends critically on the nature of quantum gravity and its transition into general relativity. It may turn out that the actual length scale varies as the Universe expands. If so, this evolution would produce an additional deviation of n_s from one. As a quick illustration, suppose we were to represent such a variation with the scaling $\lambda_{\text{PL}} \rightarrow \lambda_{\text{Pl}} k^{-\beta}$. In that case, Equation (51) would become

$$n_s = 1 - 2\beta - \frac{1}{(k/k_{\min})^2 - 1}, \quad (52)$$

and the *Planck* data would then imply that $\beta \lesssim 0.02$.

4. Classicalization of Quantum Fluctuations at the Planck Scale

4.1. The Birth of Quantum Fluctuations at the Planck Scale

The acute classicalization problem plaguing the inflaton field stems directly from the nature of the wave functional $\Psi[u(\tau, \mathbf{x})]$ describing its fluctuations (Eqn. 20). As discussed in § 2.2, there is no consensus yet on how the interference between its mode components could have been completely removed. This problem does not exist for numen quantum fluctuations due to the way they were seeded—unlike their inflaton counterparts, each of the numen fluctuations emerged with a single k -mode. Pure, single k -mode quantum states were established from the very beginning as a result of the distinct *time* at which they entered the semi-classical universe out of the Planck domain.

A numen fluctuation born at time t_b , had a unique comoving wavenumber defined by the relation

$$k_b \equiv \frac{2\pi a(\tau_b)}{\lambda_{P1}} \quad (53)$$

or, more explicitly,

$$k_b = \frac{2\pi}{\lambda_{P1} t_0} t_b. \quad (54)$$

Thus, the oscillating modes in Equation (40) should more accurately be written as follows:

$$u_k(\tau, \tau_b) = \frac{1}{\sqrt{2\alpha_k}} e^{\pm i\alpha_k \tau} \delta(k - k_b), \quad (55)$$

where clearly k_b is uniquely related to τ_b via Equations (35) and (54). In other words, it is not sufficient to merely track the temporal evolution of a numen quantum fluctuation. One must also specify its time of birth.

For the numen quantum fluctuation operator, one thus has

$$\hat{u}(\tau, \tau_b, \mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3 \mathbf{k}}{\sqrt{2\alpha_k}} \delta(k - k_b) \times \left[\hat{a}_{\mathbf{k}}^- e^{i(\mathbf{k} \cdot \mathbf{x} - \alpha_k \tau)} + \hat{a}_{\mathbf{k}}^+ e^{-i(\mathbf{k} \cdot \mathbf{x} - \alpha_k \tau)} \right]. \quad (56)$$

The integral is straightforward to evaluate, and one finds that the numen field operator, analogous to Equation (13) for the inflaton field, is

$$\hat{u}(\tau, \tau_b, \mathbf{x}) = \frac{k_b^2}{\sqrt{\pi \alpha_{k_b}}} j_0(k_b |\mathbf{x}|) \left[\hat{a}_{\mathbf{k}_b}^- e^{-i\alpha_{k_b} \tau} + \hat{a}_{\mathbf{k}_b}^+ e^{i\alpha_{k_b} \tau} \right], \quad (57)$$

where $j_0(k_b |\mathbf{x}|)$ is a spherical Bessel function of the first kind. We emphasize again that, in this picture, the entire numen quantum fluctuation born at τ_b is characterized by a single wavenumber k_b . A schematic diagram showing the amplitude of this operator at any given time, $\tau \geq \tau_b$, is shown in figure 1.

Several of the characteristics we see in the numen field were anticipated by the model proposed by Hollands and

Fig. 1. Schematic diagram showing the amplitude (plotted along $\hat{z}$) of a numen field operator projected onto the xy -plane. The spatial variation (in comoving coordinates $\mathbf{x}$) is proportional to the spherical Bessel function of the first kind, $j_0(k_b |\mathbf{x}|)$, where k_b is the unique wavenumber corresponding to the time τ_b at which the fluctuation emerged into the semi-classical universe across the Planck scale λ_{P1} (see Eqns. 54 and 57).

Wald [87], though their basic concept and scales were quite different. The essence underlying the numen field's behavior is its inferred potential given in Equation (33). As in our case, Hollands and Wald also started by considering the dynamics of a scalar field in a background cosmological setting, though the equation-of-state in their cosmic fluid was inconsistent with the zero active mass condition. Nevertheless, in both their and our cases we require that the fluctuations were born in their ground state at a fixed length scale. The work described in this paper is strongly informed by the recent *Planck* data release and its subsequent analysis, which were not available at the time Hollands and Wald proposed their model. As we have seen, the latest measurements strongly suggest that the cutoff in the observed primordial spectrum is associated with the very first quantum fluctuation that exited the Planck domain at about the Planck time. There is no room, in this proposal, for the basic length scale at which the numen fluctuations were born to be anything but the Planck wavelength.

But though the proposal by Hollands and Wald lacked the rigor imposed by the latest *Planck* observations, already Perez et al. [51] noted a very important trait of quantum fluctuations born in this fashion. They pointed out that such a model clearly demonstrates the need for some process to be responsible for their birth, playing a role analogous to that of quantum mechanical *measurement*. The birth of the mode, they argued, is effectively the step in which quantum mechanical uncertainty is removed. This is precisely the situation we describe in Equation (55), whereby each quantum fluctuation emerging into the semi-classical universe is characterized by a unique wavenumber, constrained by the time at which the fluctuation was created.

Perez et al. [51] also wondered what process could be associated with the particular time of such an occurrence. With the insights we have gained from the *Planck* data, we can now suggest that the length scale at which the numen modes were born is not at all random. It carries sig-

nificant physical meaning. As we have seen, the Planck length is the scale at which the Compton wavelength equals the Schwarzschild radius, meaning that below this scale is the realm of quantum gravity. If the picture we are describing in this paper is correct, the birth of numen quantum fluctuations was associated with the release of these modes into the semi-classical universe, where gravity is adequately described by (the classical theory of) general relativity. Moreover, as far as we know, the Planck scale never changes. Thus, as quantum fluctuations stochastically exited in time, their wavenumber had to reflect their time of ‘birth’ and it is this correlation that built the near scale-free power spectrum seen in the cosmic microwave background [31,63].

The manner in which numen fluctuations were born thus already removes a major hurdle in the classicalization process. Rather than having to deal with quantum mechanical interference between many degrees of freedom, here we have distinct quantum fluctuations possessing unique wavenumbers. Two issues remain, however, one having to do with how these modes acquired classical correlations in phase space and the mechanism that converted a homogeneous, isotropic universe into an inhomogeneous one. We shall consider the latter next, and then revisit the Wigner function (Eqn. 27) to resolve the former.

4.2. Anisotropies

A missing ingredient from much of the past discussion concerning classicalization in the early universe has been the process by which a perfectly homogeneous and isotropic state transformed into an inhomogeneous and anisotropic state described by the density fluctuations. A chief reason for this handicap has been the absence of an obvious ‘external’ source of asymmetry. Broadly speaking, such a transition can be effected as part of an ‘R’ process, e.g., measurement or collapse, but not a U process, i.e., a unitary evolution via a Schrödinger type of equation. It has been recognized that without a measurement-like process, the required transition could not have happened. In hindsight, the problem has actually been a lack of appreciation for the importance of the gravitational (or ‘apparent’) horizon $R_h(\tau) = c/H(\tau)$ [85].

Yes, the universe is homogeneous and isotropic, but only when described using the ‘community’ coordinates of myriads of observers dispersed throughout the cosmos [92]. But from the perspective of a single observer at a fixed spacetime point, the universe does not appear to be homogeneous. His/her description of the physical state of the system, using their coordinates centered at their location, must take into account the effects of spacetime curvature. Take R_h , as a prime example. The gravitational radius is an apparent horizon that separates null geodesics approaching the observer from those that are receding. One could not argue that such a divided congruence of null geodesics is consistent with homogeneity. But let us affirm that there

is no conflict between these two descriptions, because the universe is indeed homogeneous when viewed relative to the comoving frame.

Given the thesis developed in this paper, it should be clear why the role of R_h is central to the manner in which numen quantum fluctuations were born and how they classicalized. It is this gravitational horizon that delimited the size of the fluctuations, which were isotropic, but nevertheless initially restricted in size to the Planck scale. Remember that λ_{P1} actually equals R_h at the time, t_b , when mode k emerged into the semi-classical universe.

Crucially, an apparent horizon in the cosmological context is not an event horizon [85]. It may turn into one in our asymptotic future, but has not been static up until today. It has been growing, and causally-connected regions of spacetime continue to change as R_h expands. The ‘measurement-like’ process that created the numen fluctuations at the Planck scale therefore also introduced an inhomogeneity relative to the observer who appeared later, at time $t > t_b$, outside the region bounded by $R_h(t_b)$. In the numen context, the largest CMB fluctuation we see today corresponds to R_h at decoupling. Since $\lambda_{\max}$ (corresponding to $k_{\min}$) grew at the same rate as R_h , this happens to be the largest-size mode created right after the Big Bang, at the Planck time t_{P1} . This region has been exposed to us by the expanding universe, so that we can now see the impact of that first quantum fluctuation on the CMB. The seeding of numen fluctuations at the Planck scale not only created pure, single k -mode fluctuations, but also explains why these quantum fluctuations have always been finite in size, and why the universe we see is therefore a patchwork of inhomogeneities. At least in this context, the breakdown of spherical symmetry was not due to some unknown process in quantum mechanics itself. It was the result of an inherent property of the FLRW spacetime when we take into account the physical characteristics of the gravitational (or, apparent) horizon.

4.3. Classical Phase Space Correlations

A distinguishing feature of quantum fluctuations in the inflaton field is that their frequency changed with time in response to the spacetime curvature they encountered as they grew into the expanding universe. This led to the very strong time dependence in $\Omega_{\mathbf{k}}$ (Eqn. 25) which, according to the Wigner function (Eqn. 28), then produced a highly squeezed state. For the various reasons outlined in § 2.2, however, it is not clear that this restructuring of the dispersions about $u_{\mathbf{k}}$ and $\pi_{\mathbf{k}}$ is consistent with emerging classical correlations in phase space.

The situation with numen quantum fluctuations was completely different, principally because their frequency $\alpha_{\mathbf{k}}$ (Eqn. 38) was constant in time. As we alluded to earlier, this property results from the zero active mass equation-of-state ($\rho_\phi + 3p_\phi = 0$), which produced a constant expansion rate with $a(t) \propto t$. The numen modes therefore mimicked

Fig. 2. Schematic diagram showing the Wigner function of the coherent numen state (see Eqn. 63). The state begins at the right, where $\alpha_{k_b} \tau = \pi/4$, and moves clockwise about $(0, 0)$. The second peak is plotted at $\alpha_{k_b} \tau = 3\pi/4$, and the third at $\alpha_{k_b} \tau = 5\pi/4$. In generating these plots, the following values were chosen for the constants in Equation (63): $\alpha_{k_b} = 3$, and $\mu = 3$.

a classical harmonic oscillator.

Such a coherent state is considered to be the most classical of all states because their Wigner function peaks over tiny regions in phase space, and retains this configuration for all times. Given a value of u_k for such a state, one may then obtain the corresponding canonical momentum π_k very close to the value calculated using classical dynamics. The Wigner function thus follows a classical trajectory, with minimal outward spread in all phase-space directions.

To see this quantitatively, let us examine the Wigner function for a numen fluctuation,

$$\mathcal{W}_{\Psi_{k_b}}(u_{k_b}, \pi_{k_b}) \equiv \int \frac{dy}{\pi} \Psi_{k_b}^*(u_{k_b} + y) e^{2i\pi_{k_b}y} \Psi_{k_b}(u_{k_b} - y), \quad (58)$$

which is reminiscent of the quantum harmonic oscillator. Its derivation is very well known, so we won't dwell on the details (see, e.g., ref. [56]). With the ground-state solution to the Schrödinger Equation (21),

$$\Psi_{k_b}[u_{k_b}(\tau)] = \left(\frac{\alpha_{k_b}}{\pi}\right)^{1/4} e^{-\alpha_{k_b} u_{k_b}^2/2}, \quad (59)$$

we may easily evaluate the integral and find that

$$\mathcal{W}_{\Psi_{k_b}}(u_{k_b}, \pi_{k_b}) = \frac{1}{\pi} e^{-\alpha_{k_b} u_{k_b}^2 - \pi_{k_b}^2 / \alpha_{k_b}}. \quad (60)$$

To see how $\mathcal{W}_{\Psi_{k_b}}$ varies in time, let us briefly return to Equations (40) and (46), and solve for the time evolution of $u_{k_b}(\tau)$ and $\pi_{k_b}(\tau)$ in terms of their values at $\tau = 0$: $u_{k_b}(0)$ and $\pi_{k_b}(0)$. We find that

$$\begin{aligned} u_{k_b}(0) &= u_{k_b}(\tau) \cos(\alpha_{k_b} \tau) - \frac{\pi_{k_b}}{\alpha_{k_b}} \sin(\alpha_{k_b} \tau) \\ \pi_{k_b}(0) &= \pi_{k_b}(\tau) \cos(\alpha_{k_b} \tau) + \alpha_{k_b} u_{k_b} \sin(\alpha_{k_b} \tau). \end{aligned} \quad (61)$$

Now take the Wigner function at time $\tau = 0$ to represent the lowest energy state for the field amplitude $u_{k_b}(0)$ shifted by a constant μ . That is,

$$\mathcal{W}_{\Psi_{k_b}}(u_{k_b}, \pi_{k_b}, 0) = \frac{1}{\pi} e^{-\alpha_{k_b} (u_{k_b} - \mu)^2 - \pi_{k_b}^2 / \alpha_{k_b}}. \quad (62)$$

With the use of Equation (61), this becomes

$$\mathcal{W}_{\Psi_{k_b}}(u_{k_b}, \pi_{k_b}, \tau) = \frac{1}{\pi} e^{-\alpha_{k_b} [u_{k_b}(\tau) - \mu \cos(\alpha_{k_b} \tau)]^2} \times e^{-[\pi_{k_b}(\tau) + \alpha_{k_b} \mu \sin(\alpha_{k_b} \tau)]^2 / \alpha_{k_b}}. \quad (63)$$

The Wigner function thus oscillates back and forth with an amplitude μ and angular frequency α_{k_b} —a coherent state with a very small dispersion relative to the classicalized trajectory in the (u_{k_b}, π_{k_b}) plane (see fig. 2).

This result has a rather simple interpretation. The fact that numen fluctuations were born into the semi-classical universe in their ground state, with a time-independent frequency α_{k_b} , means that they functioned exactly like the coherent states of a conventional quantum harmonic oscillator. They therefore acquired classical correlations in the (u_{k_b}, π_{k_b}) phase space from the very beginning.

5. Conclusions

The *Planck* data suggest that the primordial power spectrum has a cutoff, $k_{\min}$, probably associated with the first quantum fluctuation to emerge out of the Planck domain into the semi-classical universe. Moreover, this birth must have occurred at about the Planck time—the earliest moment permitted by our current theories following the Big Bang. It is easy to appreciate the physical significance of such an interpretation, because our current (classical) theory of gravity is valid only down to the Planck scale, not beyond. It is therefore tempting to view the manner in which these (numen) quantum fluctuations were born as a physical consequence of the transition from quantum gravity to general relativity.

Insofar as the question of classicalization is concerned, this picture effectively removes the hurdles faced by inflation. Leaving aside the related issue that the existence of $k_{\min}$ challenges the viability of slow-roll inflation to simultaneously explain the fluctuation spectrum and mitigate the horizon problem, which has been dealt with elsewhere [64], one cannot ignore the challenge of explaining how fluctuations in the inflaton field successfully transitioned from the quantum to classical domains.

With the interpretation we have examined in this paper, two factors stand out clearly as the most influential. These are (i) the birth of quantum fluctuations with a fixed length scale at the Planck time constitutes a ‘process’ that effectively replaces the need for a ‘measurement’ in quantum mechanics; and (ii) the implied scalar field potential has an equation-of-state consistent with the zero active mass condition in general relativity. The spacetime associated with the expansion profile in this cosmology would have allowed the fluctuations to emerge in their ground state, with a time-independent frequency, α_k . This makes all the difference because the numen quantum fluctuations were therefore essentially quantum harmonic oscillators, with classi-

cal correlations of their canonical variables from the very beginning.

Should the proposal we have made in this paper turn out to be correct, an obvious question concerns the need for inflation in cosmology. For a paradigm that has been with us for over four decades, it is somewhat troubling that no complete theory yet exists. Some of the complications appear to be that the data tend to shy away from its predictions as the measurement precision continues to improve, rather than confirm the basic inflationary premise with increasing confidence. The large-scale anomalies in the CMB anisotropies are a good illustration of this point. Today, after three major satellite missions have completed their work studying the CMB, one would reasonably have expected all of the nuances associated with the primordial power spectrum $\mathcal{P}(k)$ to have fallen into place. But they don't, and it is difficult to discount the observational constraint that a zero $k_{\min}$ is now ruled out by the data at a relatively high level of confidence [32,33].

On the flip side, inflation may not even be necessary to solve the horizon problem, which actually does not exist in cosmologies that avoided an early phase of deceleration [78,93]. In that case, the fact that this paradigm struggles explaining the angular correlation function in the CMB, and that its quantum fluctuations have no obvious path to classicalization, may be indicators that it simply never happened. The many observational tests already completed to examine this question would not reject such a view (see, e.g., Table 2 in ref. [77]).

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Figures

Figure 1

Schematic diagram showing the amplitude (plotted along z) of a numen field operator projected onto the xy -plane. Please see the Manuscript PDF file for the complete figure caption.

Figure 2

Schematic diagram showing the Wigner function of the coherent numen state. Please see the Manuscript PDF file for the complete figure caption.