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Dynamical Modelling and Simulation of Spur Gears with Flank Pitch Error

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Abstract: Gearbox is commonly regarded as the most important power section of wind turbines which has been widely valued for its high malfunction rate. Gear fault researches mainly include wearing, pitting, spalling, breakage, falling off, etc, while little attention was paid to tooth Flank Pitch Error(FPE). Taking a single-stage parallel shaft spur gear as the research object, an 8-DOF gear transmission model and the FPE model were established in this paper and the gear's time-varying meshing stiffness (TVMS) models with & without tooth FPE were obtained respectively, which the dynamic models with various tooth FPE values under different rotating speeds were simulated after. The simulation results showed that the TVMS mathematical model proposed in the paper under tooth FPE is practical at both low and high rotating speeds. Under the FPE model, side-bands are formed around each multiple of meshing frequency whose peaks are distributed by a fixed fault characteristic frequency f_{fp} interval. The gearbox vibrates severely as the tooth FPE values and rotational speed grow. The peak value of the vibration signal is about 3 times that in case of fault-free state when the FPE value reaches 0.001rad, thus the impact of FPE on gearboxes cannot be neglected.

Keywords: spur gear, fault, dynamics simulation, time/frequency domain analysis, vibration

1. Introduction

Gearbox has great research value as the most critical link in the transmission chain of speed-increasing wind turbines which refers to a multi-stage complex transmission mechanism. A single-stage gear transmission model is generally selected for mechanism research in order to study the dynamic characteristics of gears for its precision and reliability. Commonly, gear failure includes wearing, fracture, pitting, spalling, etc. An evaluation of influences of tooth surface wear (TSW) on stiffness of the gear mesh considering wear evolution process was presented in[1]. A vibration-based scheme for updating a wear prediction model is proposed in[2] and a dynamic model of a spur gear system is firstly developed to generate realistic vibrations, which allows a quantitative study of the effects of gear tooth surface wear on gearbox vibration responses. Gear tooth crack detection by investigating the effects of tooth crack on the vibration behaviors of a nonlinear spur gear system was deeply investigated in [3]. The dynamic response of a single stage spur gear transmission is computed by using analytical gear-mesh issued from analytical modelling[4]. When tooth fault occurs, the vibration response characteristics caused by the change of time-varying mesh stiffness play an important role in crack fault diagnosis[5]. An analytical model is proposed[6] to investigate the effect of gear tooth crack on the gear mesh stiffness. Both the tooth crack propagations along tooth width and crack depth are incorporated in this model to simulate gear tooth root crack, especially when it is at very early stage. A correlation coefficient based parameter for detecting the natural progression of pitting fault in spur gear was presented in[7]. A novel spur gear dynamical model, validated by various experimental tests, to analytically investigate the effects of tooth pitting and spalling on the vibration responses of a gear transmission was built in[8], while study focuses on the nonlinear dynamic and vibration characteristics of spur gear pair with local spalling defect to explore the spalling mechanism[9].

The periodic change of the time-varying meshing stiffness which results in torsional vibration should be paid more attention to when studying the dynamics of gearbox. A new gear mesh kinematic model that can evaluate the actual contact positions of tooth engagement with time varying gear mesh center distance was proposed[10]. A model for determining mesh stiffness of cylindrical gears is proposed using a combination of finite element method (FEM) and local contact analysis of elastic bodies[11]. An improved analytical method (IAM) suitable for gear pairs with

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tip relief is established to determine time-varying mesh stiffness (TVMS), where the effects of ETC, nonlinear contact stiffness, revised fillet-foundation stiffness, and tooth profile modification are considered[12].

The study of gear dynamics is of great significance for gearbox vibration reduction. Generally, the vibration signal of the faulty gear model is obtained through simulation & experiment, of which the extraction and mechanism analysis involves multiple disciplines. Based on the dynamic equations of a single-stage gear pair and some reasonable simplifications, frequency responses of the gear pair in healthy state and those suffering from different faults are analyzed, respectively[13]. The correlation analysis between gear modification and vibration characteristics of transmission system was difficult to quantify, thus a novel small sample vibration of gearbox prediction method based on grey system theory and bootstrap theory was presented[14]. Vibration responses of faulty gear contain modulation components that are commonly used as the indicators to detect the fault. Aiming at the vibration modulation mechanism of fault gear, the gear meshing stiffness with frequency modulation components is deduced considering the actual speed fluctuation, as well as its influence on vibration responses[15]. The application of the improved empirical mode decomposition (EMD) theory in gearbox fault diagnosis has been studied in [16], and the transient features of gearbox vibration signals are shown. The application of vibration signals to detect and diagnose gear fault is an important part of preventive gearbox maintenance for its great significance in reducing maintenance costs and improving gearbox reliability. The focused benefit of condition monitoring is to increase the reliability, functionality, machining efficiency and surface property of the component and to reduce the downtime and power consumption[17]. Fault diagnosis of a wind turbine gearbox can schedule maintenance strategy on a wind turbine and save operational cost for wind farms[18].

The classification method of malfunction varies with the categories of equipment, but can basically be divided into latent faults and functional faults. The former one that refers to a tiny local fault which does not directly affect the established function of the overall equipment may evolve into a functional fault in a short time if neglected, such as FPE. At present, this kind of fault has not received enough attention, thus the author studied the basic characteristics and action laws of FPE fault and explored the influence of the fault on gear's dynamic performances. The research background and necessity of tooth FPE fault was introduced in Chapter1. A single-stage cylindrical spur gear was taken as the research object and the dynamic differential equation of the spur gear transmission was proposed in Chapter2. The characteristics of the cylindrical spur gear transmission was analyzed and the mathematical expression of the gear's TVMS was established under error-free condition in Chapter3. The FPE geometric model was illustrated, based on which the mathematical expression of the gear's TVMS was deduced, verified and improved in Chapter4. Time & frequency domain analysis to study the influence of FPE on gearbox's vibration characteristics under different rotational speeds was performed in Chapter5. The results of FPE research was discussed and reference guidance for gear's dynamic load optimization was provided in Chapter6.

2. Cylindrical spur gear transmission modelling

2.1 DOF of parallel gear transmission

The periodic TVMS variation in gear is the most important factor triggering gearbox vibration. A set of cylindrical spur gears was chosen as the research object which can better reflect the vibration characteristics of gears and the impact of failure on gearbox. The spur gear transmission model is shown in Fig.1 which mainly includes input Shaft1, big gear(Gear), output Shaft2, small gear(Pinion), and gearbox housing(HSG).

Fig.1 Spur Gear Transmission System

X axis was set as the rotation axis and the positive direction of the X axis was defined as the driving gear rotating direction. The Gear is fixed on the input Shaft1 and the two sections constitute the first module Block1 which shares the same freedom of rotation around X axis and translation in Y&Z direction. The Pinion is fixed on the output Shaft2 and they form the second module Block2 which share the same freedom of rotation around X axis and translation in Y&Z direction. The gearbox housing (HSG) owns the translation freedom in Y&Z direction. Thus, 8 degrees of freedom shall be taken into account in this transmission system.

2.2 Dynamic modelling of spur gear transmission

The 8 degrees of freedom are respectively marked in Fig.2, according to which 8 sets of equations are required to ascertain the dynamic characteristics of the gear meshing process.

Fig.2 Degrees of Freedom of the Spur Gear Transmission System

According to the serial number of the DOF in Fig.2, the corresponding dynamic equations are as following: module1's dynamic rotation equation around X, module2's dynamic rotation equation around X, module1's dynamic translation equation in Z, module1's dynamic translation equation in Y, module2's dynamic translation equation in Z, module2's dynamic translation equation in Y, HSG's dynamic translation equation in Z, HSG's dynamic translation equation in Y.

$$I_{block1}\ddot{\theta}_g + R_{bg}F_{pg} = T_{in} \quad (1)$$

$$I_{block2}\ddot{\theta}_p - R_{bp}F_{pg} = -T_{out} \quad (2)$$

$$m_{block1}\ddot{z}_g + C_{b1}(\dot{z}_g - \dot{z}_H) + K_{b1}(z_g - z_H) - F_{pg} \sin \alpha = 0 \quad (3)$$

$$m_{block1}\ddot{y}_g + C_{b1}(\dot{y}_g - \dot{y}_H) + K_{b1}(y_g - y_H) + F_{pg} \cos \alpha = 0 \quad (4)$$

$$m_{block2}\ddot{z}_p + C_{b2}(\dot{z}_p - \dot{z}_H) + K_{b2}(z_p - z_H) + F_{pg} \sin \alpha = 0 \quad (5)$$

$$m_{block2}\ddot{y}_p + C_{b2}(\dot{y}_p - \dot{y}_H) + K_{b2}(y_p - y_H) - F_{pg} \cos \alpha = 0 \quad (6)$$

$$m_H\ddot{z}_H + C_{zh}\dot{z}_H + C_{b1}(\dot{z}_H - \dot{z}_g) + C_{b2}(\dot{z}_H - \dot{z}_p) + K_{zh}z_H + K_{b1}(z_H - z_g) + K_{b2}(z_H - z_p) = 0 \quad (7)$$

$$m_H\ddot{y}_H + C_{yh}\dot{y}_H + C_{b1}(\dot{y}_H - \dot{y}_g) + C_{b2}(\dot{y}_H - \dot{y}_p) + K_{yh}y_H + K_{b1}(y_H - y_g) + K_{b2}(y_H - y_p) = 0 \quad (8)$$

θ_g, θ_p represent the rotation angles of the Gear and the Pinion. $T_{in}, T_{out}, C_{Damp}$ represent the input torque, load torque and load damping separately. $I_{block1}, I_{block2}, m_{block1}, m_{block2}$ represent the moment of inertia and mass of the module1 and module2 respectively. $z_g, y_g, z_p, y_p, z_H, y_H$ represent displacement of the Gear, the Pinion and the HSG in Z&Y direction respectively. $C_{b1}, K_{b1}, C_{b2}, K_{b2}$ indicate the bearing damping and bearing stiffness of the input shaft and output shaft respectively. C_{zh}, K_{zh} represent translation damping and stiffness of HSG in Z. C_{yh}, K_{yh} indicate translation damping and stiffness of HSG in Y. F_{pg} represents the total gear meshing force and its expression is as following.

$$F_{pg} = C_m[r_{bg}\dot{\theta}_g - r_{bp}\dot{\theta}_p - (z_g - z_p)\sin\alpha - (y_g - y_p)\cos\alpha] + K_m[r_{bg}\theta_g - r_{bp}\theta_p - (z_g - z_p)\sin\alpha - (y_g - y_p)\cos\alpha] \quad (9)$$

C_m, K_m represent meshing damping and meshing stiffness respectively.

3. Time-varying meshing stiffness of cylindrical spur gears

An obvious phase of alternate meshing between single & double-pair-teeth will occur during the meshing process of cylindrical spur gears, which is the essential reason for periodic changes of TVMS. It is necessary to obtain the varying stiffness K and the coincidence degree c of a single gear during its entire meshing cycle so as to obtain the gear's TVMS.

3.1 Coincidence degree calculation

The simplified model of the gear transmission geometry is shown in Fig.3. The standardized pressure angle of the gear is α . P_2 is the starting point of the gear meshing and the intersection point of Pinion addendum circle and B_1B_2 . P_1 is the intersection point of Gear addendum circle and B_1B_2 . B_1B_2 is the two gears' involute on which all the meshing points are distributed and the meshing force holds the same direction as it, thus P_1P_2 is the actual meshing line. The friction between gears is ignored.

Fig.3 Mesh area of Gear and Pinion

$$P_1P_2 = \frac{m}{2} [Z_p(\tan\alpha_p \cos\alpha - \sin\alpha) + Z_g(\tan\alpha_g \cos\alpha - \sin\alpha)] \quad (10)$$

α_p is the Pinion's addendum circle meshing angle and α_g is the Gear's addendum circle meshing angle. Coincidence degree c is the ratio of the actual meshing line to the normal tooth pitch of the base circle which ranges from 1 to 2 in order to ensure a smooth transmission process.

$$\alpha_p = \cos^{-1}(R_{bp}/R_{ap}) \quad (11)$$

$$\alpha_g = \cos^{-1}(R_{bg}/R_{ag}) \quad (12)$$

$$c = \frac{P_1P_2}{\pi m \cos\alpha} = \frac{1}{2\pi} [Z_p(\tan\alpha_p - \tan\alpha) + Z_g(\tan\alpha_g - \tan\alpha)] \quad (13)$$

3.2 Meshing stiffness with a single pair of teeth

There is a pair of interaction forces acting on the actual meshing line without friction which is represented by

F in Fig.4. It can be seen from Fig.3 that the actual meshing line track corresponds to the meshing force direction. The meshing force is divided into F_t and F_n respectively, of which the former refers to normal portion while the latter refers to radial portion. The expression is shown in equation (14).

$$\begin{cases} F_t = F \cos \alpha \\ F_n = F \sin \alpha \end{cases} \quad (14)$$

Fig.4 Mesh Force of the Single Tooth in Gear

Taking Gear as the research object, F can be decomposed into the circumferential force that makes the gear rotate and the radial force that presses the gear, which respectively cause the gear's bending and radial deformation. Fillet-foundation deflections should also be considered. The energy that a pair of gears possess in meshing can be described as:

$$U_{total} = \frac{F^2}{2K} = \frac{F^2}{2} \left(\frac{1}{K_{tg}} + \frac{1}{K_{tp}} + \frac{1}{K_{fp}} + \frac{1}{K_{fg}} + \frac{1}{K_n} \right) \quad (15)$$

The influence that the normal portion of meshing force has on the gear's deformation is basically constant with the position of meshing point changing. Therefore, the radial meshing stiffness is considered to be a certain value which is only related to the tooth width and gear material and it's defined as:

$$K_n = \frac{\pi E W}{4(1 - \nu^2)} \quad (16)$$

E is the Young's modulus of the material. W is the tooth width. ν is the Poisson's ratio. As the position of the meshing point changes the corresponding torque of the circumferential force changes, which has a periodic effect on the bending deformation of the teeth. A single gear tooth is viewed as a cantilever beam and the contact area is divided into n units in order to study the bending deformation of the gear, which is expressed by $S_i (i = 1, 2, \dots, n)$. The beam is regarded as a balanced state, so the bending stiffness can be obtained by equation (17),(18):

$$\frac{1}{K_t} = \cos^2 \alpha \sum_{i=1}^n L_i \left[\frac{\left(d_i - L_i d_i + \frac{1}{3} L_i^2 \right)}{E' \bar{I}_i} + \frac{1}{s_h G \bar{A}_i} + \frac{\tan^2 \alpha}{\bar{A}_i E'} \right] \quad (17)$$

$$\begin{cases} E' = \frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)} \\ \frac{1}{\bar{I}_i} = \left(\frac{1}{I_i} + \frac{1}{I_{i+1}} \right) / 2 \\ \frac{1}{\bar{A}_i} = \left(\frac{1}{A_i} + \frac{1}{A_{i+1}} \right) / 2 \end{cases} \quad (18)$$

G is the shear modulus. s_h is the shear coefficient. L_i is the width of the i-th unit. d_i is the distance from point S_i to P. A_i and I_i are the area and moment of inertia of S_i respectively. The stiffness of the fillet foundation can be obtained from formula (19) [8,19]

$$\frac{1}{K_f} = \frac{\cos^2 \alpha}{WE} \left[L^* \left(\frac{uf}{S_f} \right)^2 + M^* \left(\frac{uf}{S_f} \right) + P^* (1 + Q^* \tan^2 \alpha) \right] \quad (19)$$

The stiffness change curve of a single pair of teeth $K_{STj}(t)$ ($j = 1, 2, 3$) can be obtained according to the total meshing stiffness expression (20),

$$K = \frac{1}{\left(\frac{1}{K_{tg}} + \frac{1}{K_{tp}} + \frac{1}{K_{fp}} + \frac{1}{K_{fg}} + \frac{1}{K_n} \right)} \quad (20)$$

3.3 Mesh stiffness modelling of spur gear

The time-varying meshing stiffness of a single pair of teeth is represented by a black dashed line in Fig.5 whose meshing period is cT_{GP} calculated by equation (20). The time-varying meshing stiffness $K_n(t)$ corresponds to the periodic superposition of the meshing stiffness of a single pair of teeth that is represented by a thick blue solid line whose meshing period is T_{GP} . The peak value section indicates double-pair-teeth meshing and the valley value section corresponds to single-pair-teeth meshing as shown in Fig.5.

Fig.5 Time Varying Mesh Stiffness of Tooth and Gear

The transfer function $f(t)$ is used to express the meshing stiffness equation of a single pair of teeth, where only a single pair of teeth is considered in its meshing period as shown in equation (21):

$$f(t) = \begin{cases} K_{ST}(t), & t \in [0, cT_{GP}) \\ 0, & Else \end{cases} \quad (21)$$

The TVMS of an error-free gear-meshing can be described as the consequence of countless single-pair-teeth meshing stiffness superimposed at a certain periodic interval, as shown in equation (22):

$$K_n(t) = \sum_{n=0}^{\infty} f(t - nT_{GP}), \quad t \in [0, +\infty) \quad (22)$$

4. Tooth flank pitch error failure

The biggest source of failure from gear transmission is gear vibration caused by changes in gear meshing stiffness while the factors that trigger gear meshing stiffness changing vary. Tooth width, tooth side clearance, manufacturing error, gear center distance and external excitation will cause variations in meshing stiffness. Short-term high-intensity vibration will cause gear damage and fracture and long-term fatigue vibration will aggravate the wear of the gear and cause the gear surface to stick. The FPE that a phenomenon of serious wear of individual teeth is mainly caused by manufacturing errors, uneven materials etc, which will seriously reduce the service life of the gearbox and endanger the stable power generation characteristics of the whole machine, thus it should be paid special attention to.

4.1 FPE Modelling

Tooth FPE is a kind of manufacturing error or wear failure which refers to the offset of a side of the gear tooth relative to the normal standard gear on the pitch circle. The offset is towards the interior of the tooth with positive values while the negative towards the exterior. The gear transmission effect caused by FPE can be ignored if the tooth flank pitch error is on the non-meshing surface in the gear meshing process. Therefore, the direction of the

tooth side shall be illustrated when the error value is defined. The thumb is pointing to the direction of gear rotation and the red tooth surface is the right side in the light of the right-hand screw rule, shown in Fig.6. Angle definition method was adopted to define the tooth flank pitch error in this paper, by which the offset angle value is utilized to characterize the error.

Fig.6 Direction Definition of Flank Pitch Error

The positive flank pitch error on the left side of Pinion was taken as an example where the red dotted line corresponds to the practical profile of the faulty gear and the left-side tooth flank pitch error is θ in rad[10], as seen in Fig.7.

Fig.7 Geometric Model of Flank Pitch Error

4.2 Theoretical time-varying meshing stiffness modelling

The time-varying meshing stiffness of the gear will change significantly in a fault state. It can be seen from Fig.6 that the second pair of meshing teeth shall start to contact at the point P_2 under normal circumstance. The actual meshing point shifts to point R due to the tooth FPE, which causes the original double-pair-teeth meshing area to mesh in a single pair of teeth and its corresponding time period is t_1 (corresponding to section AB in the figure). The error tooth that was not in contact suddenly comes in when the first pair of teeth is out of contact and this process is completed in an instant (corresponding to section BB1 in the figure). Then the error tooth meshes with a single pair of teeth (corresponding to fig.8 B1C section). A pair of gears located behind the error tooth enters the meshing area (corresponding to section CC1 in the figure) after that. The original error tooth can't mesh in time with a certain gap when the new meshing tooth enters, so the new tooth is forced to be in the state of tooth-disengagement (corresponding to section C1D in Figure). Only when the second normal tooth enters the meshing area can the gear return to its original stable state of non-fault cycle operation. The time-varying meshing stiffness under the establishment of tooth flank pitch failure is shown in Fig.8 based on the analysis above, where the relationship between t_1, t_2, t_3 is shown in equation (23):

$$\begin{cases} t_2 = t_1 + (c-1)T_{GP} \\ t_3 = t_1 + T_{GP} \end{cases} \quad (23)$$

Fig.8 Theoretical TVMS under Flank Pitch Error

The time-varying-mesh-stiffness is established based on Fig.8 and the TVMS mathematical expression for the fault zone is concluded as:

$$K_{te}(t) = \begin{cases} K_{ST3}, t \in [t_1, t_2) \\ K_{ST2} = K_n(t), t \in [t_2, t_3) \\ K_{ST1}(t), t \in [t_3, t_1 + cT_{GP}) \end{cases} \quad (24)$$

The fault characteristic frequency of partial FPE in Gear&Pinion is shown in (25) (the FPE fault studied in this paper is a single-tooth error, corresponding to a local fault):

$$\begin{cases} f_m = \frac{n_i Z_i}{60} \\ f_{fi} = \frac{f_m}{Z_i} \end{cases} \quad (i = g, p) \quad (25)$$

f_m represents the meshing frequency. n_i represents the corresponding shaft speed of i (i indicates Pinion or Gear). f_{fi} represents the fault characteristic frequency of FPE.

4.3 Improved model of TVMS

The variation of meshing stiffness is extremely complicated in the realistic meshing process of spur gears. The sensitivity of meshing stiffness is especially strong under the FPE state in the process of meshing teeth alternation, where even a small error value may bring great changes in meshing stiffness. In addition, there is also a highly nonlinear relationship between time-varying meshing stiffness, the rotating speed and the error value under FPE fault. A reasonable mathematical model was established to express the time-varying meshing stiffness under FPE by simplifying the relationship where the time of the tooth-disengagement state is processed by linearization. A certain low rotating speed was taken as an example, under which the TVMS was shown in Fig.9. A short interval of teeth separation before and after the meshing process of the faulty tooth can be seen and the phenomenon of intermittent tooth-separation is more obvious especially after the single faulty tooth meshing period. The gear's meshing stiffness in the tooth-disengagement region is considered to be represented by rectangular waves, as section CC1.

Fig.9 A Special TVMS Case under Flank Pitch Error

The duration of the tooth-disengagement zone corresponding to FPE state is linearly related to the magnitude of the error value. The duration of the tooth-disengagement zone will increase accordingly as the error value increases. A mathematical relationship between the time-varying meshing stiffness and the error value during the fault period was established in order to study the influence of FPE value on the vibration characteristics of the gear. The offset spacing corresponding to FPE in Figure 7 is K_3K_4 which is approximately converted to the normal tooth pitch $L=M_1K_4-M_2K_3$, so L can be expressed as:

$$L = R_p [\sin(\alpha_p + \theta) - \sin \alpha_p] \quad (26)$$

CC1's corresponding duration is linear with L and inversely proportional to the gear rotating speed which is recorded as μL , where μ represents duration coefficient, positively related to rotating speed; the tooth-disengagement zone BB1 is an instantaneous zone and its duration is recorded as $\lambda \mu L$, where λ indicates scale factor. The value ranges of λ & μ shall be based on enough samples of actual operating test data.

$$K_{pe}(t) = \begin{cases} \sum_{n=0}^{\infty} f(t - nT_{GP}) - K_{ST1}(t), t \in [t_1, t_2) \\ \min\{K_{ST2}\} * \text{rectpuls}(t - t_2 - \lambda \kappa L/2, \lambda \kappa L/2), t \in [t_2, t_2 + \lambda \kappa L) \\ K_n(t) = \sum_{n=0}^{\infty} f(t - nT_{GP}), t \in [t_2 + \lambda \kappa L, t_3) \\ \min\{K_{ST2}\} * \sum_{i=1}^n \text{rectpuls}(t - t_3 - i \kappa L/n, \kappa L/2n), t \in [t_3, t_3 + \kappa L) \\ \sum_{n=0}^{\infty} f(t - nT_{GP}) - K_{ST3}(t), t \in [t_3 + \kappa L, t_3 + (c-1)T_{GP}) \end{cases} \quad (27)$$

The mathematical model of meshing stiffness under FPE is shown in formula (27). The phenomenon of tooth-disengagement during the meshing process will be different with the change of error values and rotating speeds. The mathematical model above will lose its practical significance if the FPE value is too negligible to cause tooth-disengagement in the meshing area of the faulty tooth, thus this part of error is not studied in this paper. The meshing stiffness may also fluctuate significantly as the speed changes. A certain degree of FPE value is enough to cause the abnormal fault meshing stiffness shown in Fig.9 at a low rotating speed, and it is likely to trigger the meshing of non-faulty teeth to malfunction if the speed is adequately high, so simulation is needed to study the dynamic characteristics of gear at different speeds and different error values.

5. Dynamic simulation

The system turns from an unsteady state to a steady one during the gear meshing process at an early stage. The simulation results are all based on the data derived from a stable operating speed. The FPE fault is set on the tenth pinion tooth. Gear dynamic characteristics simulation mainly refers to the time domain and frequency domain analysis of the relevant dynamic parameters during the gear meshing process. The main time domain operation

parameters are set as follows: The integration method SODASRT 2, General absolute tolerance 10^{-5} , Relative tolerance 10^{-7} , MBS Formalism Explicit, Gravity Field, Duration 5s, Sampling frequency 20000hz. Spur gears related design parameters and bearing parameters are available in Appendix1. The tooth flank pitch fault model may show characteristic differences with the shaft speed, so it shall be simulated and analyzed at different rotating speeds .

5.1 Simulation analysis at a low speed

The low-speed shaft angular velocity was set as 1.36rad/s, thus meshing frequency f_m was 11hz and fault characteristic frequency f_{fp} equalled 0.65hz. Multiple sets of error models were arranged, with which the gear meshing stiffness was obtained as shown in Fig.10. The intersecting single-pair-teeth meshing appears in the early stage of the double-pair-teeth meshing which should have been operating when the error value =0.0002rad, which has little effect on the gear vibration characteristics, thus the gear meshing process is quite similar to that of the error-free model and its fault characteristics are not obvious. The gear meshing experiences obvious overtime single-pair-teeth meshing in the period when the single-pair-teeth and double-pair-teeth should have been meshing alternately when the error value reaches 0.0004rad. More obvious complete tooth-disengagement will occur in the early stage of single-pair-teeth meshing, which will cause considerable impact on the gear and this part of the error model is also the focus of simulation research. More apparent tooth-disengagement will occur at the single & double-pair-teeth alternate meshing section as the error value continues to increase to 0.001rad, but the basic dynamic law remains the same. The meshing force of the faulty meshing zone is shown in Fig.11. It can be seen from the fig.that the maximum meshing force is approximately linear with its FPE value. The FPE value holds a positive correlation with its maximum meshing force accordingly, which might bring greater impact load on the gear. The maximum meshing force under each error value is marked in Fig.11.

Fig.10 TVMS under Different Flank Pitch Error Values **Fig.11** Mesh Force under Different Flank Pitch Error Values

It can be known that the TVMS changes periodically regardless whether the FPE exists from the time-domain simulation results, but the period might vary under different situations. The author took the tooth FPE value $\theta = 0$ & $\theta = 0.001$ rad as samples. Fast Fourier transform was performed on the time-domain simulation results of TVMS and the amplitude in the frequency domain was obtained, shown in Fig.12.

Fig.12 Frequency Domain Analysis of TVMS

(a) represents the fault-free model, (b) represents the tooth flank pitch error value $\theta = 0.001\text{rad}$, (c) is partial enlarged view of (b). It can be seen from Fig.12 that the maximum amplitude values all appear at the point where the frequency is 0, which corresponds to a stable meshing stiffness value. The frequency corresponding to the second largest amplitude value is meshing frequency f_m , which is also fixed as the interval frequency value between peaks of side-bands. It can be seen from (a) that the meshing stiffness amplitude is only related to the multiplier of the meshing frequency which mainly corresponds to frequency like $1f_m, 2f_m, 4f_m, 7f_m$, etc. There are narrow frequency side-bands surrounding the peak at the multiplier of meshing frequency, whose peaks keep a steady interval—fault frequency f_{fp} in (b) and (c). It can be seen from Fig.11 that the most crucial factor causing the periodic change of TVMS is meshing frequency. The meshing stiffness amplitude under meshing frequency is $1.73 \times 10^9 \text{N/m}$ within error-free model while the corresponding meshing stiffness amplitude descends to $1.5 \times 10^9 \text{N/m}$ under error condition whose overall reduction is 13.3%. The meshing stiffness amplitude at fault frequency is $6.1 \times 10^8 \text{N/m}$ which is much greater than that at 4 times the meshing frequency whose amplitude meshing stiffness is $4.7 \times 10^8 \text{N/m}$. It can be inferred that FPE has great influence on the fluctuation of TVMS, so the effects on the gear's vibration characteristics cannot be neglected.

The gearbox vibration signals in Y & Z directions and its frequency domain analysis results were shown in Fig.13 and Fig.14 when the FPE value θ was 0 & 0.001rad separately.

Fig.13 Time/Frequency Domain Analysis of HSG Acceleration in Z Direction **Fig.14** Time/Frequency Domain Analysis of HSG Acceleration in Y Direction

It can be seen from Fig.13 & Fig.14 that the HSG's vibration acceleration amplitude in Z direction is within 5m/s^2 and within 1.6m/s^2 in Y direction. Besides, the vibration signal maintains a periodic cycle. The gearbox vibration acceleration in Z & Y directions increases to 24m/s^2 and 7m/s^2 respectively at the tooth FPE fault meshing area whose impact will seriously shorten the life of the gearbox. The gearbox's vibration excitation frequencies in Z direction without FPE fault are mainly concentrated on $f=829.2\text{Hz}$ and $f=1138.8\text{Hz}$ while relatively scattered under FPE model where fault frequency bands will be formed in addition to the obvious reduction in the amplitude of the two relatively concentrated frequencies, which precisely causes the violent vibration of the fault mesh zone. The frequency domain analysis result of the vibration signal in Y direction is basically similar to the situation above.

5.2 Simulation analysis at high rotating speed

Set the angular velocity of the input shaft as 40.86 rad/s , thus the meshing frequency f_m equals 331.6Hz and the fault characteristic frequency f_{fp} equals 19.6Hz . The gear's TVMS and maximum meshing force were obtained after simulation analysis under different FPE values had been performed, shown in Fig.15 & Fig.16.

Fig.15 TVMS under Different Flank Pitch Error Values

Fig.16 Mesh Force under Different Flank Pitch Error Values

It can be seen from Fig.15 that the gear's meshing stiffness maintains a stable periodicity without tooth FPE. There is no tooth-disengagement in the initial stage of faulty tooth meshing when the tooth FPE value reaches 0.0002rad, but the unstable fluctuation gradually causes tooth-disengagement in the rear meshing area where the meshing stiffness drops to zero. The error value results in single-pair-teeth meshing in the fault section and instantaneous tooth disengagement when tooth FPE value θ increases to 0.0004rad, but has little effect on the gear meshing after the error tooth. The gear meshing state after the faulty tooth meshing is more unstable when the tooth FPE value θ reaches 0.001rad and the tooth disengagement phenomenon becomes obvious. It can be seen from Fig.16 that the tooth FPE failure would lead the maximum meshing force to increasing significantly. The increments and their ratios were marked in the figure.

Fig.17 TVMS Analysis within Time/Frequency Domain

It can be seen from Fig.17 that the excitation frequencies of the gear's TVMS are mainly $1f_m$, $2f_m$, $4f_m$, $7f_m$. The meshing stiffness amplitude under the fault characteristic frequency f_{fp} cannot be ignored when the tooth FPE value θ reaches 0.0004rad. The TVMS model is basically consistent with that proposed in 4.3. The fault characteristic frequency at this time changes into the main excitation of TVMS when tooth FPE value θ grows to 0.001rad, which results in great changes in meshing-stiffness's periodic characteristics.

The analysis of vibration signals is a very important lesson that reveals the operating status of gearboxes through time domain/frequency domain analysis which is an effective fault diagnosis method in researching gear faults. The

vibration signals in Y & Z direction derived from fault meshing section 1.02~1.05s are selected as the research and analysis objects and the time&frequency domain analysis results are shown in Fig.18.

Fig.18 Vibration Signal Analysis within Time/Frequency Domain

It can be seen from Fig.18.a & Fig.18.b that the gearbox vibration signals are basically the same outside the faulty meshing zone regardless of the error value, which also indirectly illustrates the good match between the simulation method and the actual situation. The gearbox vibration is more obvious, and the acceleration in Z direction is totally beyond safety range with the increasing of tooth FPE value. It can be seen from Fig.18c & Fig.18d that the excitation source of the vibration signal is basically a frequency multiplier of the meshing frequency in the non-fault mode and its frequency were $3f_m$, $2f_m$, $4f_m$, $1f_m$ according to the amplitude descending order. The excitation amplitude of the triple meshing frequency is obviously reduced when the FPE value θ reaches 0.0004rad and the excitation amplitude of the other times frequencies decreases. Narrow frequency bands with fault frequency interval appears around the meshing frequency and they are no longer negligible as the faulty excitation. The amplitude under the fault frequency increases significantly when the tooth flank pitch error value continues to increase to $\theta = 0.001\text{rad}$. The vibration signal in Y direction is mainly excited by $2f_m$ and $3f_m$ and it is mainly excited by $2f_m$ and double fault frequency in Z direction. Thus, a conclusion can be drawn that the gearbox's vibration excitation frequency may change when FPE value increases to a certain extent.

6. Summary and discussion

The author took parallel-axis spur gears as the research object, established the mathematical expression of the time-varying meshing stiffness of the gear without failure, studied the tooth flank pitch error fault and discussed its failure mechanism, and accomplished the gear's time-varying mesh stiffness mathematical modelling, performed time&frequency domain analysis of different error models at low and high rotating speeds. The main conclusions are as follows:

(1) The main excitation frequencies of TVMS is the meshing frequency multiplication which mainly concentrate on $1f_m, 2f_m, 4f_m, 7f_m$ in the frequency domain analysis. Each meshing frequency multiplication forms a side-band under the fault model whose peaks are equally spaced by the fault frequency f_{fp} . The periodicity of the time-varying meshing stiffness may change when the fault value increases to a certain extent.

(2) The gearbox vibration amplitude and meshing force are positively related to the rotating speed and FPE value. The gearbox vibrates more severely with higher rotating speed and greater FPE value. The main excitation frequency of the vibration signal will change when the error value is large to some extent. The maximum meshing force is about 3 times that in the case of error-free when FPE value reaches 0.001rad, thus the impact on gear can't be ignored.

(3) The gear's dynamic characteristics varies even if with a constant FPE value when they are operating at different rotating speeds. The gear transmission instability decreases with the addition of error value when the error value is not enough to cause single tooth meshing in the fault-tooth zone at high rotating speeds. However, the gear transmission instability will increase with the addition of error value when the error value is large enough to cause single tooth meshing in the fault-tooth zone and so as the vibration signal.

(4) The frequency domain analysis result of the tooth FPE fault has practical reference value for fault diagnosis. The time domain load analysis result of the tooth FPE fault showed that a reasonable setting of the tooth side clearance can effectively reduce the impact load, which has instructive significance to the research of gear structure optimization and gear box load reduction.

Even if the tooth flank pitch error fault and its failure mechanism were in-depth studied and the vibrational characteristics were obtained, some insufficiency remains in the literature, like neglecting the friction, shortage of experimental validation and they will be studied in the future work.

7. Declarations

Availability of data and materials

The datasets used and analyzed during the current study are available from the corresponding author on reasonable request.

Competing interests

The authors declare that they have no competing interests.

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Authors' contributions

Lizhuang Tao: proposed the idea of tooth flank pitch error research; established the fault model; analyzed the simulation results; drafted the manuscript.

De Tian: offered some suggestions and revised the literature.

Shize Tang: offered some relevant data and revised the literature.

Xiaoxuan Wu: offered some relevant data and revised the literature.

Bei Li: offered some relevant data and revised the literature.

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Appendix 1: Parallel gear parameters

Module	$m=25$ mm	Mass of block2	$m_{\text{block2}}=1097$ kg
Number of Gear teeth	$Z_g=51$	Mass of gearbox housing	$m_H=6000$ kg
Number of Pinion teeth	$Z_p=17$	Moment inertia of block1	$I_{\text{block1}}=255\text{kg}\cdot\text{m}^2$
Pressure angle	$\alpha=20$ deg	Moment inertia of block2	$I_{\text{block2}}=17$ kg $\cdot\text{m}^2$
Tooth width of Gear	$W_G=0.34$ m	Torque-in	$T_{\text{in}}=1.352\times 10^5$ N $\cdot\text{m}$
Tooth width of Pinion	$W_P=0.35$ m	Torque of load	$T_{\text{out}}=1.352\times 10^5/3$ N $\cdot\text{m}$
Addendum circle radius of Gear	$R_{ag}=0.6625$ m	Stiffness of shaft1 bearing	$K_{b1}=1\times 10^{11}$ N/m
Base circle radius of Gear	$R_{bg}=0.599$ m	Damping of shaft1 bearing	$C_{b1}=1\times 10^8$ N $\cdot\text{s}/\text{m}$
Addendum circle radius of Pinion	$R_{ap}=0.2375$ m	Stiffness of shaft2 bearing	$K_{b2}=4\times 10^{10}$ N/m
Base circle radius of Pinion	$R_{bp}=0.1997$ m	Damping of shaft2 bearing	$C_{b2}=4\times 10^7$ N $\cdot\text{s}/\text{m}$
Contact ratio	$\epsilon=1.634$	Trans stiffness of HSG in Z	$K_{zh}=3\times 10^8$ N/m
Young's Modulus	$E=2.09\times 10^{11}$ Pa	Trans damping of HSG in Z	$C_{zh}=3\times 10^5$ N $\cdot\text{s}/\text{m}$
Poisson's Ratio	$\nu=0.3$	Trans stiffness of HSG in Y	$K_{yh}=2\times 10^8$ N/m
Mass of block1	$m_{\text{block1}}=3258$ kg	Trans damping of HSG in Y	$C_{yh}=2\times 10^5$ N $\cdot\text{s}/\text{m}$

Figures

Figure 1
Spur Gear Transmission System

Figure 2

Degrees of Freedom of the Spur Gear Transmission System

Figure 3

Mesh area of Gear and Pinion

Figure 4

Mesh Force of the Single Tooth in Gear

Figure 5

Figure 6

Direction Definition of Flank Pitch Error

Figure 7

Geometric Model of Flank Pitch Error

Figure 8

Theoretical TVMS under Flank Pitch Error

Figure 9

A Special TVMS Case under Flank Pitch Error

Figure 10

TVMS under Different Flank Pitch Error Values

Figure 11

Mesh Force under Different Flank Pitch Error Values

Frequency Domain Analysis of TVMS

Figure 13

Time/Frequency Domain Analysis of HSG Acceleration in Z Direction

Figure 14

Time/Frequency Domain Analysis of HSG Acceleration in Y Direction

Figure 15

TVMS under Different Flank Pitch Error Values

Figure 16

Mesh Force under Different Flank Pitch Error Values

Figure 17

TVMS Analysis within Time/Frequency Domain

Figure 18

Vibration Signal Analysis within Time/Frequency Domain