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General Planar Motion Modeling and Control of a Smart Rigid-Flexible Satellite Considering Large Deformations

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Abstract

In this paper, the motion of a smart rigid-flexible satellite by considering large deformations for its flexible appendages in general planar motion is modeled. Therefore, the satellite can experience translational and rotational motions also its flexible appendages can vibrate arbitrarily in the motion plane. Two control forces perpendicular to each other and one control torque are responsible for controlling the motion of the satellite on the desired trajectories. Also, piezoelectric actuators and sensors suppress vibrations and estimate the transverse displacement of the satellite's flexible appendages, respectively. The coupled ordinary-partial differential equations of motion, equations of the sensors, and boundary conditions of the system are obtained using extended Hamilton's principle. Then, these equations are discretized using the Galerkin method. The discretized equations of motion are a set of coupled nonlinear ordinary differential equations due to the consideration of the large rotation angle of the satellite and large deformations for its flexible appendages. Adaptive super-twisting global nonlinear sliding mode controller is designed to satisfy the control objectives including position and attitude control, as well as suppressing vibrations of the flexible appendages in the presence of uncertainties and external disturbances. Eventually, numerical simulations are presented to illustrate the effectiveness of the proposed controller.

Keywords

Smart rigid-flexible satellite, Partial differential equations, Galerkin method, Piezoelectric, Adaptive super-twisting global nonlinear sliding mode control

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1. Introduction

In recent decades, the need for the existence of satellites has increased day by day. By closely looking at the devices around us, it can be realized that without satellites, we can no longer use features such as navigation, satellite channels, etc. In addition, wide climate information, Earth changes, and researches about the other planets all refer to the need for satellites. Satellites with flexible appendages are a large group of satellites. Due to the use of solar panels (flexible appendages), they can receive their required energy from the Sun and even store this energy in their batteries. Therefore, due to their automatic energy supply from the Sun, these satellites can perform missions with longer duration and distance from the Earth. With this explanation, it is clear that the modeling, control and construction of these satellites have a special place for researchers.

In general, flexible satellites modeling can be categorized as follows:

- Investigation of the vibrations of flexible appendages [1, 2].
- Investigation of the attitude maneuver of the satellite along with vibrations of its flexible appendages [3, 4].
- Investigation of the orbital motion by considering the attitude maneuver of the satellite and vibrations of its flexible appendages [5, 6].
- Investigation of the general motion of the satellite, which includes the translation and rotation of the satellite along with the vibrations of its flexible appendages [7].

Modeling of a rotating smart Euler-Bernoulli beam [8], rotating conical Timoshenko beam [9], general planar motion of an Euler-Bernoulli beam [10] and planar motion of a constrained rigid-flexible robotic manipulator [11], all are researches about the systems which have the same motion equations with the flexible satellites. In addition, [12-15] are other studies conducted in this field.

In order to design a controller for flexible satellites, two approaches can be taken: (i) controller design based on the coupled ordinary-partial differential equations (ODE_PDEs) of motion [1-4, 7] or (ii) controller design based on the coupled ordinary differential equations (ODEs) of motion [5, 6]. Of course, ODEs of motion can be obtained directly (e.g., the Rayleigh-Ritz method) or ODE-PDEs can be obtained first and then they can be discretized and converted to a set of ODEs (e.g., the Galerkin method) [16]. Other controllers designed based on ODEs of motion of the flexible satellites and similar systems in previous researches include robust adaptive control for three axes attitude maneuver of a flexible satellite in orbital motion [5], hybrid adaptive sliding mode/Lyapunov control for one axis maneuver of a satellite with nonlinear flexible appendages in orbital motion using the singular perturbation theory [6], fractional-order sliding mode control for a rotating Euler-Bernoulli beam [8], multivariable predictive control for active vibration control of a flexible one-link manipulator [17], variable structure control for a flexible spacecraft in attitude maneuver [18], nonlinear PD control for three axes attitude stabilization of a flexible satellite [19] and active vibration suppression and high accuracy attitude control for a flexible satellite with piezoelectric actuators via fixed-time prescribed performance control [20].

Many researches have been performed in the field of sliding mode controllers. The reason why researchers are interested in this type of controllers is their significant advantages and solvable disadvantages. Some of these researches are reviewed as follows:

Slotin [21] introduced a simple sliding mode control scheme for MIMO systems, which has the ability to deal with parametric uncertainties. Huang et al. [22] developed an adaptive sliding mode controller for nonlinear systems with uncertainties and external disturbances. Liu et al. [23] proposed a global sliding mode controller with a new reaching law to reduce undesired effects of chattering phenomenon and used it to control a chaotic systems. Mobayen and Tchier [24] investigated a robust adaptive second-order sliding mode control

scheme for uncertain linear systems with external disturbances. In this method, the derivatives of the control signals contain the sign function. By integrating them, continuous control signals without the chattering problem are generated. Also, they designed an adaptive super-twisting global nonlinear sliding mode (ASTGNSM) controller for robotic manipulators in the presence of uncertainties and external disturbances [25]. The super-twisting algorithm in this control scheme is responsible for eliminating the undesirable chattering phenomenon. Soltanpour et al. [26] designed a robust fuzzy sliding mode controller by considering uncertainties for robotic manipulators in joints space. Adaptive sliding mode controller based on a neural network for robotic manipulators was designed by Sun et al. [27]. It should be noted that this control scheme also has the ability to deal with uncertainties and external disturbances. Chen et al. [28] designed an adaptive sliding mode backstepping controller for moving robotic manipulators with nonholonomic constraints in the presence of uncertainties and disturbances. Chen et al. [29] designed and applied a dynamic adaptive sliding mode controller to a wheeled mobile robot. Zeinali and Notash [30] developed an adaptive sliding mode controller with an uncertainty estimator for robotic manipulators. The uncertainty estimator in this control scheme is responsible for estimating unmodeled dynamics, external disturbances and time-varying parameters. Li et al. [31] designed an adaptive sliding mode controller for uncertain MIMO nonlinear systems, which do not require to be aware of the upper bound of uncertainties due to the adaptive control design. Chen and Yu [32] designed an adaptive sliding mode control based on a new disturbance observer. In this control scheme, uncertainties and disturbances are estimated by the observer.

In this research, in the first step, the motion of a smart flexible satellite by considering large deformations for its flexible appendages in general planar motion is modeled. In the next step, to achieve control objectives, including position and attitude controlling and

suppressing vibrations of the flexible appendages in the presence of uncertainties and external disturbances, the adaptive global nonlinear sliding mode (AGNSM) and ASTGNSM controllers are designed. The main innovations of this paper are summarized as follows:

- Modeling the motion of a smart rigid-flexible satellite by considering large deformations for its flexible appendages in general planar motion.
- Considering two control thrust forces for a smart satellite with nonlinear flexible appendages in general planar motion.
- Deriving ODE-PDEs of motion and ODEs of sensors using the extended Hamilton's principle. Then discretizing these equations by the Galerkin method.
- Designing the AGNSM and ASTGNSM controllers to satisfy the mentioned control aims in the presence of uncertainties and external disturbances.

The structure of this paper is as follows. In section 2, governing equations of system and corresponding boundary conditions are derived by using the extended Hamilton's principle. Then, these governing equations are converted to a set of ODEs by applying the Galerkin method. The AGNSM and ASTGNSM controllers are designed to satisfy the control objectives in the presence of uncertainties and external disturbances in section 3. In section 4, numerical simulations are provided to illustrate the dynamic behavior of the system with and without controllers. Eventually, the major conclusions of this study are presented in section 5.

2. Dynamic modeling

In this section, model definition and dynamic model formulation for a smart satellite with nonlinear flexible appendages in general planar motion are presented.

Important parts of the smart flexible satellite include: (i) central rigid body, (ii) flexible appendages, (iii) piezoelectric actuators and sensors, (iv) thrust forces and (v) control torque (see Fig. 1).

As can be seen in Fig. 1, the satellite consists of the connection of two Euler-Bernoulli beams to a square rigid body located between them. There are N piezoelectric patches on each of the beams. N_a and N_s represent the number of piezoelectric actuators and sensors, respectively. The j th piezoelectric actuator and sensor for the left flexible appendages are denoted by LA_j and LS_j , respectively. Similarly, RA_j and RS_j represent the j th piezoelectric actuator and sensor for the right panel.

Fig. 1 Smart satellite with nonlinear flexible appendages along with coordinate systems, piezoelectric actuators and sensors, control forces and control torque.

The control forces generated by thrusters, control torque generated by Reaction Wheels (RW) or Control Moment Gyroscopes (CMG) and piezoelectric actuators voltages are control inputs which are responsible for controlling the smart flexible satellite on the desired trajectory and suppressing vibrations of its flexible appendages. Also, piezoelectric sensors are responsible for estimating the lateral displacements of the flexible appendages.

According to Fig. 1, XY and $x_b y_b$ are orbital and body coordinate systems, respectively. The XY coordinate system is the inertial reference frame and the $x_b y_b$ coordinate system is introduced as the fixed body coordinate system on the CRB. In addition, θ is the rotation angle of the satellite which expresses the rotation of the x_b -axis relative to X -axis.

Other considered assumptions are listed as follows:

- i. Ignoring the fluid viscosity damping around the satellite because of the low density of the atmosphere and structural damping of its flexible appendages due to its insignificant effect on the translation and rotation of satellite.
- ii. Modeling the flexible appendages as Euler-Bernoulli beams due to their significant length-to-thickness ratio.
- iii. Considering large deformations for the flexible appendages based on the Von-Karman strain.

In order to obtain the equations of motion, the equations of sensors and the boundary conditions, the extended Hamilton's principle is employed [16].

$$\int_{t_1}^{t_2} [\delta T - \delta U + \delta W_{nc}] dt = 0 \quad (1)$$

As can be seen, the kinetic energy of system, the potential energy of system and the work done by the control forces and torque on the system which are denoted by T , U and W_{nc} , respectively, are required to use this principle.

In relations (2), the velocity vector of the CRB and elements of the left and right flexible appendages are represented by $\mathbf{V}_{CRB}$, $\mathbf{V}_{LAE}$ and $\mathbf{V}_{RAE}$, respectively.

$$\mathbf{V}_{CRB} = \dot{x}_{CRB}(t)\mathbf{i} + \dot{y}_{CRB}(t)\mathbf{j} = \dot{\mathbf{X}}(t)\mathbf{I} + \dot{\mathbf{Y}}(t)\mathbf{J} \quad (2)$$

$$\begin{aligned}\mathbf{V}_{LAE} &= (\dot{x}_{CRB}(t) - \dot{\theta}w_L)\mathbf{i} + (\dot{y}_{CRB}(t) - (x+r)\dot{\theta}(t) + \dot{w}_L(x,t))\mathbf{j} \\ &= (\dot{X}(t)\cos\theta + \dot{Y}(t)\sin\theta - \dot{\theta}w_L)\mathbf{i} + (\dot{Y}(t)\cos\theta - \dot{X}(t)\sin\theta - (x+r)\dot{\theta}(t) + \dot{w}_L(x,t))\mathbf{j}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_{RAE} &= (\dot{x}_{CRB}(t) - \dot{\theta}w_R)\mathbf{i} + (\dot{y}_{CRB}(t) + (x+r)\dot{\theta}(t) + \dot{w}_R(x,t))\mathbf{j} \\ &= (\dot{X}(t)\cos\theta + \dot{Y}(t)\sin\theta - \dot{\theta}w_R)\mathbf{i} + (\dot{Y}(t)\cos\theta - \dot{X}(t)\sin\theta + (x+r)\dot{\theta}(t) + \dot{w}_R(x,t))\mathbf{j}\end{aligned}$$

where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors along the $x_b y_b$ coordinate system axes. Also, $\mathbf{I}$ and $\mathbf{J}$ are the unit vectors along the XY coordinate system axes. $w_L(x,t)$ and $w_R(x,t)$ are the lateral displacement of the left and right flexible appendages, respectively. $\theta(t)$ is the rotation angle of the satellite, which indicates the rotation of the x_b -axis relative to the X -axis.

By using relations (2), the kinetic energy of the smart flexible satellite is written as the summation of the kinetic energy of the CRB, the kinetic energy of the flexible appendages and the kinetic energy of the piezoelectric patches as:

$$\begin{aligned}T &= T_{CRB} + T_{appendage} + T_{PZT} = \frac{1}{2}M\dot{X}^2 + \frac{1}{2}M\dot{Y}^2 + \frac{1}{2}J\dot{\theta}^2 + \frac{1}{2}(\rho A)_{La} \int_0^{l_a} |\mathbf{V}_{LAE}|^2 dx \\ &+ \frac{1}{2}(\rho A)_{Ra} \int_0^{l_a} |\mathbf{V}_{RAE}|^2 dx + \sum_{i=1}^N \frac{1}{2}(\rho A)_{LP_i} \int |\mathbf{V}_{LAE}|^2 dx + \sum_{i=1}^N \frac{1}{2}(\rho A)_{RP_i} \int |\mathbf{V}_{RAE}|^2 dx \quad (3) \\ |\mathbf{V}_{LAE}|^2 &= (\dot{X}(t)\cos\theta + \dot{Y}(t)\sin\theta - \dot{\theta}w_L)^2 + (\dot{Y}(t)\cos\theta - \dot{X}(t)\sin\theta - (x+r)\dot{\theta}(t) + \dot{w}_L(x,t))^2 \\ |\mathbf{V}_{RAE}|^2 &= (\dot{X}(t)\cos\theta + \dot{Y}(t)\sin\theta - \dot{\theta}w_R)^2 + (\dot{Y}(t)\cos\theta - \dot{X}(t)\sin\theta + (x+r)\dot{\theta}(t) + \dot{w}_R(x,t))^2\end{aligned}$$

where LAE and RAE represent the elements of the left and right flexible appendages of the satellite, respectively.

The displacement field for Euler Bernoulli beams is considered as follows:

$$u_i(x,t) = -y \frac{\partial w_i(x,t)}{\partial x}, i = La, Ra \quad (4)$$

where the left or right flexible appendage's displacements in x and y directions are illustrated by $u_i(x,t)$ and $w_i(x,t)$, respectively.

The Von-Karman strain ε_x in the x -axis direction corresponding to each of the satellite's nonlinear flexible appendages can be expressed as follows [33].

$$\varepsilon_{x_i} = \frac{\partial u_i}{\partial x} + \frac{1}{2} \left[\left(\frac{\partial u_i}{\partial x} \right)^2 + \left(\frac{\partial w_i}{\partial x} \right)^2 \right], i = La, Ra \quad (5)$$

Since the expression $\frac{\partial u_i}{\partial x}$ is small in the satellites' translational and rotational motion, the expression $\frac{1}{2} \left(\frac{\partial u_i}{\partial x} \right)^2$ against $\frac{\partial u_i}{\partial x}$ will be ignored. Thus, by substituting the Euler-Bernoulli beam displacement field (4) in relation (5), the longitudinal normal strain ε_{x_i} and normal stress σ_{x_i} of the Euler-Bernoulli beam by considering large deformations can be written as follows:

$$\varepsilon_{x_i} = -y \frac{\partial^2 w_i(x,t)}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w_i(x,t)}{\partial x} \right)^2, \sigma_{x_i} = E \varepsilon_{x_i} = E \left[-y \frac{\partial^2 w_i(x,t)}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w_i(x,t)}{\partial x} \right)^2 \right], i = La, Ra \quad (6)$$

where E is the Young's modulus of the appendages.

The potential energy of the piezoelectric patches U_{PZT} includes the mechanical, electromechanical and electrical parts which are denoted by U_{PZT}^{mech} , $U_{PZT}^{elec-mech}$ and U_{PZT}^{elec} , respectively [6, 34, 35].

$$U_{PZT} = U_{PZT}^{mech} + U_{PZT}^{elec-mech} + U_{PZT}^{elec} = \frac{1}{2} \int_{PZT} E_p \varepsilon_x^2 dV - \int_{PZT} e_{31} E_z \varepsilon_x dV + \frac{1}{2} \int_{PZT} \varepsilon_p E_z^2 dV \quad (7)$$

where E_p is the Young's modulus of the piezoelectric patch, e_{31} is the equivalent piezoelectric coefficient, ε_p is the dielectric constant of the piezoelectric material and $E_z = V_p/t_p$ is the electric field of each piezoelectric patch. Also, V_p and t_p are the voltage and thickness of them, respectively.

Therefore, by using Eqs. (6) and (7), the potential energy of the flexible appendages

$U_{appendage}$ and the potential energy of the piezoelectric patches U_{PZT} can be written as follows:

$$\begin{aligned}
 U_{appendage} &= \sum_i \frac{1}{2} \int \sigma_{x_i} \varepsilon_{x_i} dV, i = La, Ra \\
 &= \frac{1}{2} \int_{La} E_{La} \left[y^2 \left(\frac{\partial^2 w_L}{\partial x^2} \right)^2 - y \left(\frac{\partial^2 w_L}{\partial x^2} \right) \left(\frac{\partial w_L}{\partial x} \right)^2 + \frac{1}{4} \left(\frac{\partial w_L}{\partial x} \right)^4 \right]_{La} dV \\
 &+ \frac{1}{2} \int_{Ra} E_{Ra} \left[y^2 \left(\frac{\partial^2 w_R}{\partial x^2} \right)^2 - y \left(\frac{\partial^2 w_R}{\partial x^2} \right) \left(\frac{\partial w_R}{\partial x} \right)^2 + \frac{1}{4} \left(\frac{\partial w_R}{\partial x} \right)^4 \right]_{Ra} dV
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 U_{PZT} &= \sum_{i=1}^N \left(\frac{1}{2} \int_{LP_i} E_{LP_i} \left[y^2 \left(\frac{\partial^2 w_L}{\partial x^2} \right)^2 - y \left(\frac{\partial^2 w_L}{\partial x^2} \right) \left(\frac{\partial w_L}{\partial x} \right)^2 + \frac{1}{4} \left(\frac{\partial w_L}{\partial x} \right)^4 \right]_{LP_i} dV \right. \\
 &\quad \left. - \int_{LP_i} (e_{31} E_z)_{LP_i} \left(-y \frac{\partial^2 w_L}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w_L}{\partial x} \right)^2 \right)_{LP_i} dV + \frac{1}{2} \int_{LP_i} (\varepsilon_p E_z^2)_{LP_i} dV \right) \\
 &+ \sum_{i=1}^N \left(\frac{1}{2} \int_{RP_i} E_{RP_i} \left[y^2 \left(\frac{\partial^2 w_R}{\partial x^2} \right)^2 - y \left(\frac{\partial^2 w_R}{\partial x^2} \right) \left(\frac{\partial w_R}{\partial x} \right)^2 + \frac{1}{4} \left(\frac{\partial w_R}{\partial x} \right)^4 \right]_{RP_i} dV \right. \\
 &\quad \left. - \int_{RP_i} (e_{31} E_z)_{RP_i} \left(-y \frac{\partial^2 w_R}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w_R}{\partial x} \right)^2 \right)_{RP_i} dV + \frac{1}{2} \int_{RP_i} (\varepsilon_p E_z^2)_{RP_i} dV \right)
 \end{aligned} \tag{9}$$

Substitution of the relations $\int_A dA = A$, $\int_A y dA = Q$ and $\int_A y^2 dA = I$, which are the cross-

sectional area and the first and second cross-sectional area moment of inertia [36, 37], respectively, in the potential energy relations (8) and (9) leads to the potential energy of the system U as:

$$\begin{aligned}
 U &= U_{appendage} + U_{PZT} = \frac{1}{2} \int_{La} E_{La} \left[I_a (w_L'')^2 - Q_a w_L'' (w_L')^2 + \frac{1}{4} A_a (w_L')^4 \right]_{La} dx \\
 &+ \frac{1}{2} \int_{Ra} E_{Ra} \left[I_a (w_R'')^2 - Q_a w_R'' (w_R')^2 + \frac{1}{4} A_a (w_R')^4 \right]_{Ra} dx \\
 &+ \sum_{i=1}^N \left(\frac{1}{2} \int_{LP_i} E_{LP_i} \left[I_p (w_L'')^2 - Q_p w_L'' (w_L')^2 + \frac{1}{4} A_p (w_L')^4 \right]_{LP_i} dx \right. \\
 &\quad \left. - \int_{LP_i} \left(e_{31} \frac{V_p}{t_p} \right)_{LP_i} \left(-Q_p w_L'' + \frac{1}{2} A_p (w_L')^2 \right)_{LP_i} dx + \frac{1}{2} \left(\frac{\varepsilon_p A_p l_p}{t_p^2} \right)_{LP_i} V_{LP_i}^2 \right) \\
 &+ \sum_{i=1}^N \left(\frac{1}{2} \int_{RP_i} E_{RP_i} \left[I_p (w_R'')^2 - Q_p w_R'' (w_R')^2 + \frac{1}{4} A_p (w_R')^4 \right]_{RP_i} dx \right. \\
 &\quad \left. - \int_{RP_i} \left(e_{31} \frac{V_p}{t_p} \right)_{RP_i} \left(-Q_p w_R'' + \frac{1}{2} A_p (w_R')^2 \right)_{RP_i} dx + \frac{1}{2} \left(\frac{\varepsilon_p A_p l_p}{t_p^2} \right)_{RP_i} V_{RP_i}^2 \right)
 \end{aligned} \tag{10}$$

Virtual work done by the control forces and torque on the satellite can be expressed as:

$$\delta W = \tau \delta \theta + (f_{x_b} \cos \theta - f_{y_b} \sin \theta) \delta X + (f_{x_b} \sin \theta + f_{y_b} \cos \theta) \delta Y \quad (11)$$

where f_{x_b} , f_{y_b} and τ are thruster forces in x_b and y_b axes directions and control torque, respectively.

Substituting Eqs. (3), (10), and (11) in the extended Hamilton's principle and applying the variation operator to them (see Appendix A) along with using the integration by part (see Appendix B) with some straightforward mathematical operations lead to the equations of motion (12)-(16), boundary conditions (17) and sensors Eqs. (18) and (19) of motion of the smart satellite with nonlinear flexible appendages in general planar motion.

$$\begin{aligned} & M\ddot{X} + (\rho A)_{La} \int_0^{l_a} [\ddot{X} + (x+r)\ddot{\theta} \sin \theta + (x+r)\dot{\theta}^2 \cos \theta - \ddot{w}_L \sin \theta - 2\dot{w}_L \dot{\theta} \cos \theta - w_L(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta)] dx \\ & + \sum_{i=1}^N (\rho A)_{LP_i} \int_{LP_i} [\ddot{X} + (x+r)\ddot{\theta} \sin \theta + (x+r)\dot{\theta}^2 \cos \theta - \ddot{w}_L \sin \theta - 2\dot{w}_L \dot{\theta} \cos \theta - w_L(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta)] dx \\ & + (\rho A)_{Ra} \int_0^{l_a} [\ddot{X} - (x+r)\ddot{\theta} \sin \theta - (x+r)\dot{\theta}^2 \cos \theta - \ddot{w}_R \sin \theta - 2\dot{w}_R \dot{\theta} \cos \theta - w_R(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta)] dx \\ & + \sum_{i=1}^N (\rho A)_{RP_i} \int_{RP_i} [\ddot{X} - (x+r)\ddot{\theta} \sin \theta - (x+r)\dot{\theta}^2 \cos \theta - \ddot{w}_R \sin \theta - 2\dot{w}_R \dot{\theta} \cos \theta - w_R(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta)] dx \\ & = f_{x_b} \cos \theta - f_{y_b} \sin \theta \end{aligned} \quad (12)$$

$$\begin{aligned} & M\ddot{Y} + (\rho A)_{La} \int_0^{l_a} [\ddot{Y} - (x+r)\ddot{\theta} \cos \theta + (x+r)\dot{\theta}^2 \sin \theta + \ddot{w}_L \cos \theta - 2\dot{w}_L \dot{\theta} \sin \theta - w_L(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta)] dx \\ & + \sum_{i=1}^N (\rho A)_{LP_i} \int_{LP_i} [\ddot{Y} - (x+r)\ddot{\theta} \cos \theta + (x+r)\dot{\theta}^2 \sin \theta + \ddot{w}_L \cos \theta - 2\dot{w}_L \dot{\theta} \sin \theta - w_L(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta)] dx \\ & + (\rho A)_{Ra} \int_0^{l_a} [\ddot{Y} + (x+r)\ddot{\theta} \cos \theta - (x+r)\dot{\theta}^2 \sin \theta + \ddot{w}_R \cos \theta - 2\dot{w}_R \dot{\theta} \sin \theta - w_R(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta)] dx \\ & + \sum_{i=1}^N (\rho A)_{RP_i} \int_{RP_i} [\ddot{Y} + (x+r)\ddot{\theta} \cos \theta - (x+r)\dot{\theta}^2 \sin \theta + \ddot{w}_R \cos \theta - 2\dot{w}_R \dot{\theta} \sin \theta - w_R(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta)] dx \\ & = f_{x_b} \sin \theta + f_{y_b} \cos \theta \end{aligned} \quad (13)$$

$$\begin{aligned} & J\ddot{\theta} + (\rho A)_{La} \int_0^{l_a} [(\ddot{Y} \cos \theta - \ddot{X} \sin \theta - (x+r)\ddot{\theta} + \ddot{w}_L)(-(x+r)) + (\ddot{X} \cos \theta + \ddot{Y} \sin \theta - \ddot{\theta} w_L - 2\dot{\theta} \dot{w}_L)(-w_L)] dx \\ & + \sum_{i=1}^N (\rho A)_{LP_i} \int_{LP_i} [(\ddot{Y} \cos \theta - \ddot{X} \sin \theta - (x+r)\ddot{\theta} + \ddot{w}_L)(-(x+r)) + (\ddot{X} \cos \theta + \ddot{Y} \sin \theta - \ddot{\theta} w_L - 2\dot{\theta} \dot{w}_L)(-w_L)] dx \\ & + (\rho A)_{Ra} \int_0^{l_a} [(\ddot{Y} \cos \theta - \ddot{X} \sin \theta + (x+r)\ddot{\theta} + \ddot{w}_R)(x+r) + (\ddot{X} \cos \theta + \ddot{Y} \sin \theta - \ddot{\theta} w_R - 2\dot{\theta} \dot{w}_R)(-w_R)] dx \\ & + \sum_{i=1}^N (\rho A)_{RP_i} \int_{RP_i} [(\ddot{Y} \cos \theta - \ddot{X} \sin \theta + (x+r)\ddot{\theta} + \ddot{w}_R)(x+r) + (\ddot{X} \cos \theta + \ddot{Y} \sin \theta - \ddot{\theta} w_R - 2\dot{\theta} \dot{w}_R)(-w_R)] dx = \tau \end{aligned} \quad (14)$$

As can be seen, the five equations of motion are in the form of nonlinear coupled ODE-PDEs. The first three Eqs. (12)-(14) are in the form of ODEs which express the translational and rotational motion of the satellite and the last two Eqs. (15) and (16) are in the form of PDEs which express the vibrations of the flexible appendages.

$$\begin{aligned}
& (\rho A)_{La} \int_0^{l_a} (\ddot{Y} \cos \theta - \ddot{X} \sin \theta - (x+r)\ddot{\theta} - \dot{\theta}^2 w_L + \ddot{w}_L) dx \\
& + \sum_{i=1}^N (\rho A)_{LP_i} \int_{LP_i} (\ddot{Y} \cos \theta - \ddot{X} \sin \theta - (x+r)\ddot{\theta} - \dot{\theta}^2 w_L + \ddot{w}_L) dx \\
& + \int_0^{l_a} [(EI)_{La} w_L'''' - \frac{3}{2}(EA)_{La} w_L'' (w_L')^2] dx + \sum_{i=1}^N \int_{LP_i} [(EI)_{LP_i} w_L'''' - \frac{3}{2}(EA)_{LP_i} w_L'' (w_L')^2] dx \\
& = \sum_{i=1}^{N_s} \left(\frac{e_{31} Q_p}{t_p} \right)_{LA_i} V_{LA_i} \int_{LA_i} [(\delta'(x - x_{2LA_i}) - \delta'(x - x_{1LA_i}))] dx
\end{aligned} \tag{15}$$

$$\begin{aligned}
& (\rho A)_{Ra} \int_0^{l_a} (\ddot{Y} \cos \theta - \ddot{X} \sin \theta + (x+r)\ddot{\theta} - \dot{\theta}^2 w_R + \ddot{w}_R) dx \\
& + \sum_{i=1}^N (\rho A)_{RP_i} \int_{RP_i} (\ddot{Y} \cos \theta - \ddot{X} \sin \theta + (x+r)\ddot{\theta} - \dot{\theta}^2 w_R + \ddot{w}_R) dx \\
& + \int_0^{l_a} [(EI)_{Ra} w_R'''' - \frac{3}{2}(EA)_{Ra} w_R'' (w_R')^2] dx + \sum_{i=1}^N \int_{RP_i} [(EI)_{RP_i} w_R'''' - \frac{3}{2}(EA)_{RP_i} w_R'' (w_R')^2] dx \\
& = \sum_{i=1}^{N_s} \left(\frac{e_{31} Q_p}{t_p} \right)_{RA_i} V_{RA_i} \int_{RA_i} [(\delta'(x - x_{2RA_i}) - \delta'(x - x_{1RA_i}))] dx
\end{aligned} \tag{16}$$

The essential boundary conditions that are known from the geometry of the system and the natural boundary conditions that are derived from the extended Hamilton's principle can be presented as follows:

$$\begin{cases} w_L(0, t) = w_L'(0, t) = w_R(0, t) = w_R'(0, t) = 0 \\ w_L''(l_a, t) = w_R''(l_a, t) = 0 \\ E(Aw_L'^3(l_a, t) - 2Iw_L'''(l_a, t)) = 0 \\ E(Aw_R'^3(l_a, t) - 2Iw_R'''(l_a, t)) = 0 \end{cases} \tag{17}$$

According to the research [6], since the higher-order term in Eq. (6) has an insignificant effect on the electro-elastic properties of piezoelectric materials, the expressions obtained from this higher-order term in the equations related to actuators and sensors are omitted. Therefore, the equations of piezoelectric sensors are also the results of using the extended Hamilton's principle, which are obtained as:

$$\left(\frac{e_{31}Q_p}{t_p}\right)_{LS_i} \int_{LS_i} w_L'' dx + \left(\frac{\varepsilon_p A_p l_p}{t_p^2}\right)_{LS_i} V_{LS_i} = 0$$

$$(i = 1, 2, \dots, N_s)$$
(18)

$$\left(\frac{e_{31}Q_p}{t_p}\right)_{RS_i} \int_{RS_i} w_R'' dx + \left(\frac{\varepsilon_p A_p l_p}{t_p^2}\right)_{RS_i} V_{RS_i} = 0$$

$$(i = 1, 2, \dots, N_s)$$
(19)

The Galerkin method is used to discretize the equations of motion (12)-(16) and the equations of piezoelectric sensors (18) and (19). Therefore, the transverse displacements of the flexible appendages are approximated as follows:

$$w_L(x, t) \approx \sum_{j=1}^m q_{L_j}(t) \phi_j(x)$$

$$w_R(x, t) \approx \sum_{j=1}^m q_{R_j}(t) \phi_j(x)$$
(20)

where $q_{L_j}(t)$ and $q_{R_j}(t)$ are j th generalized coordinates of the left and right flexible appendages, respectively. Also, $\phi_j(x)$ is j th known independent comparison function.

Eqs. (21) - (23) are discretized form of Eqs. (12) - (14), respectively, which are obtained by direct substitution of relations (20).

$$\begin{aligned} & M\ddot{X} + (\rho A)_{La} \int_0^{l_a} [\ddot{X} + (x+r)\ddot{\theta} \sin \theta + (x+r)\dot{\theta}^2 \cos \theta - (\sum_{j=1}^m \ddot{q}_{L_j}(t) \phi_j(x)) \sin \theta - 2(\sum_{j=1}^m \dot{q}_{L_j}(t) \phi_j(x)) \dot{\theta} \cos \theta \\ & - (\sum_{j=1}^m q_{L_j}(t) \phi_j(x)) (\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta)] dx \\ & + \sum_{i=1}^N (\rho A)_{LP_i} \int_{LP_i} [\ddot{X} + (x+r)\ddot{\theta} \sin \theta + (x+r)\dot{\theta}^2 \cos \theta - (\sum_{j=1}^m \ddot{q}_{L_j}(t) \phi_j(x)) \sin \theta - 2(\sum_{j=1}^m \dot{q}_{L_j}(t) \phi_j(x)) \dot{\theta} \cos \theta \\ & - (\sum_{j=1}^m q_{L_j}(t) \phi_j(x)) (\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta)] dx \\ & + (\rho A)_{Ra} \int_0^{l_a} [\ddot{X} - (x+r)\ddot{\theta} \sin \theta - (x+r)\dot{\theta}^2 \cos \theta - (\sum_{j=1}^m \ddot{q}_{R_j}(t) \phi_j(x)) \sin \theta - 2(\sum_{j=1}^m \dot{q}_{R_j}(t) \phi_j(x)) \dot{\theta} \cos \theta \\ & - (\sum_{j=1}^m q_{R_j}(t) \phi_j(x)) (\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta)] dx \\ & + \sum_{i=1}^N (\rho A)_{RP_i} \int_{RP_i} [\ddot{X} - (x+r)\ddot{\theta} \sin \theta - (x+r)\dot{\theta}^2 \cos \theta - (\sum_{j=1}^m \ddot{q}_{R_j}(t) \phi_j(x)) \sin \theta - 2(\sum_{j=1}^m \dot{q}_{R_j}(t) \phi_j(x)) \dot{\theta} \cos \theta \\ & - (\sum_{j=1}^m q_{R_j}(t) \phi_j(x)) (\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta)] dx = f_{x_b} \cos \theta - f_{y_b} \sin \theta \end{aligned}$$
(21)

$$\begin{aligned}
& M\ddot{Y} + (\rho A)_{La} \int_0^{l_a} [\ddot{Y} - (x+r)\ddot{\theta} \cos \theta + (x+r)\dot{\theta}^2 \sin \theta + (\sum_{j=1}^m \ddot{q}_{L_j}(t)\phi_j(x)) \cos \theta - 2(\sum_{j=1}^m \dot{q}_{L_j}(t)\phi_j(x))\dot{\theta} \sin \theta \\
& - (\sum_{j=1}^m q_{L_j}(t)\phi_j(x))(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta)] dx \\
& + \sum_{i=1}^N (\rho A)_{LP_i} \int_{LP_i} [\ddot{Y} - (x+r)\ddot{\theta} \cos \theta + (x+r)\dot{\theta}^2 \sin \theta + (\sum_{j=1}^m \ddot{q}_{L_j}(t)\phi_j(x)) \cos \theta - 2(\sum_{j=1}^m \dot{q}_{L_j}(t)\phi_j(x))\dot{\theta} \sin \theta \\
& - (\sum_{j=1}^m q_{L_j}(t)\phi_j(x))(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta)] dx \\
& + (\rho A)_{Ra} \int_0^{l_a} [\ddot{Y} + (x+r)\ddot{\theta} \cos \theta - (x+r)\dot{\theta}^2 \sin \theta + (\sum_{j=1}^m \ddot{q}_{R_j}(t)\phi_j(x)) \cos \theta - 2(\sum_{j=1}^m \dot{q}_{R_j}(t)\phi_j(x))\dot{\theta} \sin \theta \\
& - (\sum_{j=1}^m q_{R_j}(t)\phi_j(x))(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta)] dx \\
& + \sum_{i=1}^N (\rho A)_{RP_i} \int_{RP_i} [\ddot{Y} + (x+r)\ddot{\theta} \cos \theta - (x+r)\dot{\theta}^2 \sin \theta + (\sum_{j=1}^m \ddot{q}_{R_j}(t)\phi_j(x)) \cos \theta - 2(\sum_{j=1}^m \dot{q}_{R_j}(t)\phi_j(x))\dot{\theta} \sin \theta \\
& - (\sum_{j=1}^m q_{R_j}(t)\phi_j(x))(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta)] dx = f_{x_b} \sin \theta + f_{y_b} \cos \theta
\end{aligned} \tag{22}$$

$$\begin{aligned}
& J\ddot{\theta} + (\rho A)_{La} \int_0^{l_a} [(\ddot{Y} \cos \theta - \ddot{X} \sin \theta - (x+r)\ddot{\theta}) + (\sum_{j=1}^m \ddot{q}_{L_j}(t)\phi_j(x))(-(x+r)) + (\ddot{X} \cos \theta + \ddot{Y} \sin \theta - \ddot{\theta}(\sum_{j=1}^m q_{L_j}(t)\phi_j(x))) \\
& - 2\dot{\theta}(\sum_{j=1}^m \dot{q}_{L_j}(t)\phi_j(x))(-(\sum_{j=1}^m q_{L_j}(t)\phi_j(x)))] dx \\
& + \sum_{i=1}^N (\rho A)_{LP_i} \int_{LP_i} [(\ddot{Y} \cos \theta - \ddot{X} \sin \theta - (x+r)\ddot{\theta}) + (\sum_{j=1}^m \ddot{q}_{L_j}(t)\phi_j(x))(-(x+r)) + (\ddot{X} \cos \theta + \ddot{Y} \sin \theta - \ddot{\theta}(\sum_{j=1}^m q_{L_j}(t)\phi_j(x))) \\
& - 2\dot{\theta}(\sum_{j=1}^m \dot{q}_{L_j}(t)\phi_j(x))(-(\sum_{j=1}^m q_{L_j}(t)\phi_j(x)))] dx \\
& + (\rho A)_{Ra} \int_0^{l_a} [(\ddot{Y} \cos \theta - \ddot{X} \sin \theta + (x+r)\ddot{\theta}) + (\sum_{j=1}^m \ddot{q}_{R_j}(t)\phi_j(x))(x+r) + (\ddot{X} \cos \theta + \ddot{Y} \sin \theta - \ddot{\theta}(\sum_{j=1}^m q_{R_j}(t)\phi_j(x))) \\
& - 2\dot{\theta}(\sum_{j=1}^m \dot{q}_{R_j}(t)\phi_j(x))(-(\sum_{j=1}^m q_{R_j}(t)\phi_j(x)))] dx \\
& + \sum_{i=1}^N (\rho A)_{RP_i} \int_{RP_i} [(\ddot{Y} \cos \theta - \ddot{X} \sin \theta + (x+r)\ddot{\theta}) + (\sum_{j=1}^m \ddot{q}_{R_j}(t)\phi_j(x))(x+r) + (\ddot{X} \cos \theta + \ddot{Y} \sin \theta - \ddot{\theta}(\sum_{j=1}^m q_{R_j}(t)\phi_j(x))) \\
& - 2\dot{\theta}(\sum_{j=1}^m \dot{q}_{R_j}(t)\phi_j(x))(-(\sum_{j=1}^m q_{R_j}(t)\phi_j(x)))] dx = \tau
\end{aligned} \tag{23}$$

Eqs. (24) and (25) are the discretized and reduced-order form of Eqs. (15) and (16), respectively, which are formed by applying the Galerkin method and then using integration by part (see Appendix C). $\phi_s(x)$ is the s th approximating function which plays the role of the s th weight function of the Galerkin method. However, due to the order reduction process, it is sufficient for these functions in Eqs. (21) - (25) to satisfy only the essential boundary conditions of the system.

$$\begin{aligned}
& (\rho A)_{La} \int_0^{l_a} \phi_s(x) (\ddot{Y} \cos \theta - \ddot{X} \sin \theta - (x+r)\ddot{\theta} - \dot{\theta}^2 (\sum_{j=1}^m q_{L_j}(t)\phi_j(x)) + (\sum_{j=1}^m \ddot{q}_{L_j}(t)\phi_j(x))) dx \\
& + \sum_{i=1}^N (\rho A)_{LP_i} \int_{LP_i} \phi_s(x) (\ddot{Y} \cos \theta - \ddot{X} \sin \theta - (x+r)\ddot{\theta} - \dot{\theta}^2 (\sum_{j=1}^m q_{L_j}(t)\phi_j(x)) + (\sum_{j=1}^m \ddot{q}_{L_j}(t)\phi_j(x))) dx \\
& + (EI)_{La} \int_0^{l_a} \phi_s''(x) (\sum_{j=1}^m q_{L_j}(t)\phi_j'(x)) dx + \frac{1}{2} (EA)_{La} \int_0^{l_a} \phi_s'(x) (\sum_{j=1}^m q_{L_j}(t)\phi_j'(x))^3 dx \\
& + \sum_{i=1}^N (EI)_{LP_i} \int_{LP_i} \phi_s''(x) (\sum_{j=1}^m q_{L_j}(t)\phi_j'(x)) dx + \frac{1}{2} \sum_{i=1}^N (EA)_{LP_i} \int_{LP_i} \phi_s'(x) (\sum_{j=1}^m q_{L_j}(t)\phi_j'(x))^3 dx \\
& = - \sum_{i=1}^{N_s} \left(\frac{e_{31} Q_p}{t_p} \right)_{LA_i} V_{LA_i} \int_{LA_i} \phi_s''(x) dx \\
& (s = 1, 2, \dots, m)
\end{aligned} \tag{24}$$

$$\begin{aligned}
& (\rho A)_{Ra} \int_0^{l_a} \phi_s(x) (\ddot{Y} \cos \theta - \ddot{X} \sin \theta + (x+r)\ddot{\theta} - \dot{\theta}^2 (\sum_{j=1}^m q_{R_j}(t)\phi_j(x)) + (\sum_{j=1}^m \ddot{q}_{R_j}(t)\phi_j(x))) dx \\
& + \sum_{i=1}^N (\rho A)_{RP_i} \int_{RP_i} \phi_s(x) (\ddot{Y} \cos \theta - \ddot{X} \sin \theta + (x+r)\ddot{\theta} - \dot{\theta}^2 (\sum_{j=1}^m q_{R_j}(t)\phi_j(x)) + (\sum_{j=1}^m \ddot{q}_{R_j}(t)\phi_j(x))) dx \\
& + (EI)_{Ra} \int_0^{l_a} \phi_s''(x) (\sum_{j=1}^m q_{R_j}(t)\phi_j'(x)) dx + \frac{1}{2} (EA)_{Ra} \int_0^{l_a} \phi_s'(x) (\sum_{j=1}^m q_{R_j}(t)\phi_j'(x))^3 dx \\
& + \sum_{i=1}^N (EI)_{RP_i} \int_{RP_i} \phi_s''(x) (\sum_{j=1}^m q_{R_j}(t)\phi_j'(x)) dx + \frac{1}{2} \sum_{i=1}^N (EA)_{RP_i} \int_{RP_i} \phi_s'(x) (\sum_{j=1}^m q_{R_j}(t)\phi_j'(x))^3 dx \\
& = - \sum_{i=1}^{N_s} \left(\frac{e_{31} Q_p}{t_p} \right)_{RA_i} V_{RA_i} \int_{RA_i} \phi_s''(x) dx \\
& (s = 1, 2, \dots, m)
\end{aligned} \tag{25}$$

The discretized equations of the sensors can also be expressed by directly substituting the relations (20) in Eqs. (18) and (19) as follows:

$$\begin{aligned}
& \left(\frac{e_{31} Q_p}{t_p} \right)_{LS_i} \int_{LS_i} \left(\sum_{j=1}^m q_{L_j}(t)\phi_j''(x) \right) dx + \left(\frac{\varepsilon_p A_p l_p}{t_p^2} \right)_{LS_i} V_{LS_i} = 0 \\
& (i = 1, 2, \dots, N_s)
\end{aligned} \tag{26}$$

$$\begin{aligned}
& \left(\frac{e_{31} Q_p}{t_p} \right)_{RS_i} \int_{RS_i} \left(\sum_{j=1}^m q_{R_j}(t)\phi_j''(x) \right) dx + \left(\frac{\varepsilon_p A_p l_p}{t_p^2} \right)_{RS_i} V_{RS_i} = 0 \\
& (i = 1, 2, \dots, N_s)
\end{aligned} \tag{27}$$

3. Controller design

In this section, adaptive super-twisting global nonlinear sliding mode control laws are designed to control position, attitude and suppress vibrations of the flexible appendages of the smart flexible satellite in general planar motion along with uncertainties and external disturbances.

Discretized and reduced-order dynamic equations of the smart flexible satellite in general planar motion by considering uncertainties and external disturbances can be written as follows:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \boldsymbol{\tau}_d = \boldsymbol{\tau} \quad (28)$$

where $\mathbf{q} \in \mathbb{R}^{2m+3}$ is a vector which includes position, angular position of the satellite and generalized coordinates of its flexible appendages. Also, $\dot{\mathbf{q}} \in \mathbb{R}^{2m+3}$ and $\ddot{\mathbf{q}} \in \mathbb{R}^{2m+3}$ represent its first and second time-derivatives, respectively. The inertia matrix of the system and the nonlinear vector including the effects of centrifugal and Coriolis accelerations and flexible appendages equations are represented by $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{(2m+3) \times (2m+3)}$ and $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{2m+3}$, respectively. Finally, $\boldsymbol{\tau} \in \mathbb{R}^{2N_a+3}$ ($N_a = m$) is the control inputs vector and $\boldsymbol{\tau}_d \in \mathbb{R}^{2N_a+3}$ ($N_a = m$) is the disturbances vector.

In order to become closer to the practical works, it is necessary to consider $\mathbf{M}(\mathbf{q})$ and $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ as follows:

$$\begin{aligned} \mathbf{M}(\mathbf{q}) &= \mathbf{M}_0(\mathbf{q}) + \Delta\mathbf{M}(\mathbf{q}) \\ \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) &= \mathbf{C}_0(\mathbf{q}, \dot{\mathbf{q}}) + \Delta\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \end{aligned} \quad (29)$$

where $\mathbf{M}_0(\mathbf{q})$ and $\Delta\mathbf{M}(\mathbf{q})$ are the known and uncertain parts of $\mathbf{M}(\mathbf{q})$, respectively. Similarly, $\mathbf{C}_0(\mathbf{q}, \dot{\mathbf{q}})$ and $\Delta\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ are the known and uncertain parts for $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, respectively.

The dynamic equations of the system (28) can be rewritten as follows:

$$\begin{aligned} \ddot{\mathbf{q}} &= -\mathbf{M}_0^{-1}(\mathbf{q})[\mathbf{C}_0(\mathbf{q}, \dot{\mathbf{q}}) + \boldsymbol{\tau}_d + \Delta\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \Delta\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) - \boldsymbol{\tau}] \\ &= \mathbf{f}_0(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{M}_0^{-1}(\mathbf{q})\boldsymbol{\tau} + \mathbf{D} \end{aligned} \quad (30)$$

where $\mathbf{f}_0(\mathbf{q}, \dot{\mathbf{q}}) = -\mathbf{M}_0^{-1}(\mathbf{q})\mathbf{C}_0(\mathbf{q}, \dot{\mathbf{q}})$ is a known and bounded nonlinear function and $\mathbf{D} = -\mathbf{M}_0^{-1}(\mathbf{q})[\boldsymbol{\tau}_d + \Delta\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \Delta\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})]$ represents the lumped uncertainties and disturbances of the system.

The global nonlinear sliding surface for the expressed system by Eq. (30) can be defined as follows:

$$s = \mathbf{G}(\mathbf{r} - \mathbf{r}_0 \exp(-\beta t)), (\mathbf{r} = \dot{\mathbf{e}} + \lambda \mathbf{e}) \quad (31)$$

where $\mathbf{e} = \mathbf{q} - \mathbf{q}_d$ and $\dot{\mathbf{e}} = \dot{\mathbf{q}} - \dot{\mathbf{q}}_d$ are position and velocity tracking errors, respectively. $\mathbf{q}_d$, $\dot{\mathbf{q}}_d$ and $\mathbf{G}$ are the desired position and velocity trajectories and a constant row vector, respectively. In addition, λ and β are constant positive coefficients.

Remark 1. The global nonlinear sliding surface (31) compared to the sliding surface $\hat{s} = \mathbf{G}(\dot{\mathbf{e}} + \lambda \mathbf{e})$, forces the error state variables from the initial time to reach the surface. Therefore, robustness of the control system in the presence of uncertainties and disturbances is confirmed.

Taking the derivative of the global nonlinear sliding surface (31) with respect to time leads to the following equation

$$\begin{aligned} \dot{s} &= \mathbf{G}(\dot{\mathbf{r}} + \beta \mathbf{r}_0 \exp(-\beta t)) \\ &= \mathbf{G}(\ddot{\mathbf{e}} + \lambda \dot{\mathbf{e}} + \beta \mathbf{r}_0 \exp(-\beta t)) \end{aligned} \quad (32)$$

where the acceleration tracking error is defined as $\ddot{\mathbf{e}} = \ddot{\mathbf{q}} - \ddot{\mathbf{q}}_d$. By substituting Eq. (30) in Eq. (32), the following equation is obtained.

$$\begin{aligned} \dot{s} &= \mathbf{G}(\ddot{\mathbf{q}} - \ddot{\mathbf{q}}_d + \lambda \dot{\mathbf{e}} + \beta \mathbf{r}_0 \exp(-\beta t)) \\ &= \mathbf{G}(\mathbf{f}_0(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{M}_0^{-1}(\mathbf{q})\boldsymbol{\tau} + \mathbf{D} - \ddot{\mathbf{q}}_d + \lambda \dot{\mathbf{e}} + \beta \mathbf{r}_0 \exp(-\beta t)) \end{aligned} \quad (33)$$

In order to find the equivalent control law $\boldsymbol{\tau}_0$, it is sufficient to set Eq. (33) equal to zero ($\dot{s} = 0$) by ignoring the existence of uncertainties and disturbances ($\mathbf{D} = \mathbf{0}$). Therefore, the equivalent control law will be obtained as follows:

$$\boldsymbol{\tau}_0 = -\mathbf{M}_0(\mathbf{q})(\mathbf{f}_0(\mathbf{q}, \dot{\mathbf{q}}) - \ddot{\mathbf{q}}_d + \lambda \dot{\mathbf{e}} + \beta \mathbf{r}_0 \exp(-\beta t)) \quad (34)$$

When uncertainties and disturbances are taken into account ($\mathbf{D} \neq \mathbf{0}$), the equivalent control law alone can no longer guarantee the efficiency of the control system and bring the system's errors on the sliding surface to the origin from any initial condition. Therefore, the need for an auxiliary control law to deal with uncertainties and disturbances is quite clear. In practical works, the upper bound of uncertainties and disturbances of the system is often unknown but bounded. Therefore, assuming the upper bound of lumped uncertainties and disturbances of the system is bounded, an unknown positive constant such as Γ can be considered which confirms the inequality $\|\mathbf{D}\| < \Gamma$.

The auxiliary control law can be presented as follows:

$$\boldsymbol{\tau}_1 = -\mathbf{M}_0(\mathbf{q})(k s \mathbf{G}^T + \hat{\Gamma} \text{sign}(s \mathbf{G}^T)) \quad (35)$$

where $\hat{\Gamma}$ is the estimated value of Γ and k is a positive constant. If the upper bound of the lumped uncertainties and disturbances of the system Γ is known, it is sufficient to insert the same specified value in the auxiliary control law instead of $\hat{\Gamma}$. Otherwise, the value of $\hat{\Gamma}$ should be updated by the following adaptation law.

$$\dot{\hat{\Gamma}} = \kappa \|s \mathbf{G}^T\| \quad (36)$$

In the above relation κ is a positive constant.

Finally, the main control law $\boldsymbol{\tau}$ can be obtained as the summation of the equivalent control law (34) and the auxiliary control law (35) as follows:

$$\begin{aligned} \boldsymbol{\tau} &= \boldsymbol{\tau}_0 + \boldsymbol{\tau}_1 \\ &= -\mathbf{M}_0(\mathbf{q})(\mathbf{f}_0(\mathbf{q}, \dot{\mathbf{q}}) - \ddot{\mathbf{q}}_d + \lambda \dot{\mathbf{e}} + \beta \mathbf{r}_0 \exp(-\beta t) + k s \mathbf{G}^T + \hat{\Gamma} \text{sign}(s \mathbf{G}^T)) \end{aligned} \quad (37)$$

Theorem 1. Suppose that the global nonlinear sliding surface (31) is selected as the sliding surface and the lumped uncertainties and disturbances of the system is unknown but bounded, That is, $\|\mathbf{D}\| < \Gamma$ such that Γ is an unknown positive constant. Assume that $\hat{\Gamma}$ is an

estimation of Γ which is updated by the adaptation Law (36). If the adaptive control law (37) is used, then the error trajectories of the system are converged to the global nonlinear sliding surface ($s = 0$) in a finite time and remained on it [25].

Proof 1. If the control law (37) is placed in the time derivative equation of the global nonlinear sliding surface (33), the following equation will be obtained.

$$\dot{s}(t) = \mathbf{G}(-ks(t)\mathbf{G}^T - \hat{\Gamma} \text{sign}(s(t)\mathbf{G}^T) + \mathbf{D}) \quad (38)$$

In this stage, the stability of the control system will be proved by the Lyapunov stability theorem. For this purpose, the following positive-definite Lyapunov function candidate is considered

$$V = \frac{1}{2}s^2 + \frac{1}{1}\tilde{\mathcal{H}}^2 \quad (39)$$

where $\tilde{\Gamma} = \hat{\Gamma} - \Gamma$ is defined as the difference between the estimated value and the actual value of the upper bound of lumped uncertainties and disturbances. The following relation is obtained by differentiating the Lyapunov function candidate with respect to time and substituting equations (36) and (38) in it.

$$\begin{aligned} \dot{V} &= s\dot{s} + \gamma\tilde{\mathcal{H}}\dot{\tilde{\Gamma}} \\ &= s\mathbf{G}(\mathbf{D} - ks\mathbf{G}^T - \hat{\Gamma} \text{sign}(s\mathbf{G}^T)) + \gamma(\hat{\Gamma} - \Gamma)\dot{\hat{\Gamma}} \\ &= s\mathbf{G}(\mathbf{D} - ks\mathbf{G}^T - \hat{\Gamma} \text{sign}(s\mathbf{G}^T)) + \gamma\kappa(\hat{\Gamma} - \Gamma)\|s\mathbf{G}^T\| \\ &\leq \|s\mathbf{G}\|\|\mathbf{D}\| - \hat{\Gamma}\|s\mathbf{G}\| + \gamma\kappa\|s\mathbf{G}^T\|(\hat{\Gamma} - \Gamma) + \|s\mathbf{G}^T\|\Gamma - \|s\mathbf{G}^T\|\Gamma \\ &= -\|s\mathbf{G}\|(\Gamma - \|\mathbf{D}\|) - (1 - \gamma\kappa)\|s\mathbf{G}\|(\hat{\Gamma} - \Gamma) \end{aligned} \quad (40)$$

Since $\|\mathbf{D}\| < \Gamma$ and $\gamma\kappa < 1$, inequality (40) will be converted to the following inequality.

$$\begin{aligned} \dot{V} &\leq -\sqrt{2}\|\mathbf{G}\|(\Gamma - \|\mathbf{D}\|)\left(|s|/\sqrt{2}\right) - \sqrt{2/\gamma}(1 - \gamma\kappa)\left(\tilde{\Gamma}/\sqrt{2/\gamma}\right)\|s\mathbf{G}\| \\ &\leq -\min\left\{\sqrt{2}\|\mathbf{G}\|(\Gamma - \|\mathbf{D}\|), \sqrt{2/\gamma}(1 - \gamma\kappa)\|s\mathbf{G}\|\right\}\left(|s|/\sqrt{2} + \tilde{\Gamma}/\sqrt{2/\gamma}\right) = -\Theta V^{\frac{1}{2}} \end{aligned} \quad (41)$$

In the above relation, the inequality $\Theta = \min \left\{ \sqrt{2} \|\mathbf{G}\| (\Gamma - \|\mathbf{D}\|), \sqrt{2/\gamma} (1 - \gamma\kappa) \|s^T \mathbf{G}\| \right\} > 0$ is true. Eventually, the Lyapunov stability theorem confirms that the use of an adaptive global nonlinear sliding mode control algorithm, the control system stability and its finite-time convergence to the global nonlinear sliding surface in the presence of uncertainties and disturbances without knowing their lumped upper bound is guaranteed.

Remark 2. In order to prevent the chattering phenomenon, the boundary layer method can be employed. For this purpose, in the control law (37), instead of the sign function, saturation or hyperbolic tangent functions are used. Therefore, the modified control laws are given as [25, 36, 38]:

$$\boldsymbol{\tau} = -\mathbf{M}_0(\mathbf{q})(\mathbf{f}_0(\mathbf{q}, \dot{\mathbf{q}}) - \ddot{\mathbf{q}}_d + \lambda \dot{\mathbf{e}} + \beta \mathbf{r}_0 \exp(-\beta t) + ks\mathbf{G}^T + \hat{\Gamma} \text{sat}(\frac{s\mathbf{G}^T}{\delta})) \quad (42)$$

$$\boldsymbol{\tau} = -\mathbf{M}_0(\mathbf{q})(\mathbf{f}_0(\mathbf{q}, \dot{\mathbf{q}}) - \ddot{\mathbf{q}}_d + \lambda \dot{\mathbf{e}} + \beta \mathbf{r}_0 \exp(-\beta t) + ks\mathbf{G}^T + \hat{\Gamma} \tanh(\frac{s\mathbf{G}^T}{\delta})) \quad (43)$$

where δ is the thickness of the boundary layer. Furthermore, the saturation and hyperbolic tangent functions are denoted by $\text{sat}(\cdot)$ and $\tanh(\cdot)$, respectively.

The use of the boundary layer method in sliding mode controllers is not without its drawbacks. Two main drawbacks of this method are:

- A thick boundary layer may cause the steady-state error in the control system.
- A thin boundary layer may not be able to eliminate the chattering problem.

Remark 3. The use of the super-twisting algorithm is an appropriate alternative to eliminate the chattering phenomenon instead of the boundary layer method. In this method, the control system is adaptively adjusted so that the chattering phenomenon does not occur. The adaptive super-twisting global nonlinear sliding mode control law can be presented as follows [25, 39, 40]:

$$\begin{aligned}
\boldsymbol{\tau} &= \boldsymbol{\tau}_0 + \boldsymbol{\tau}_1 = -\mathbf{M}_0(\mathbf{q})(\mathbf{f}_0(\mathbf{q}, \dot{\mathbf{q}}) - \ddot{\mathbf{q}}_d + \lambda \dot{\mathbf{e}} + \beta \mathbf{r}_0 \exp(-\beta t) \\
&+ k s \mathbf{G}^T + \mu_0 \|s \mathbf{G}^T\|^{\frac{1}{2}} \text{sign}(s \mathbf{G}^T) + \mathbf{v}) \\
\dot{\mathbf{v}} &= \hat{\Gamma} \text{sign}(s \mathbf{G}^T) \\
\dot{\hat{\Gamma}} &= \kappa \|s \mathbf{G}^T\|
\end{aligned} \tag{44}$$

where μ_0 is a positive constant.

4. Simulation results

In this section, numerical simulations are provided to demonstrate the efficiency of the proposed AGNSM and ASTGNSM controllers. Properties and parameters of the flexible appendages, piezoelectric patches and CRB are given in Tables 1 and 2, respectively.

Table 1 Properties and parameters of the flexible appendages and piezoelectric patches [6].

	E (N/m ²)	ρ (kg/m ³)	Length(m)	Width (m)	Thickness (m)	PZT constant (mV ⁻¹)
Flexible appendages	76×10^9	2840	5	0.5	0.08	-
Piezoelectric patches	2×10^9	1800	-	0.5	0.01	22×10^{-12}

$q_{L_1}(0) = 0.03m$, $q_{L_2}(0) = 0.02m$, $q_{R_1}(0) = 0.04m$ and $q_{R_2}(0) = 0.01m$ are the non-zero initial conditions of the system. Also, the desired values for position, velocity and acceleration are $q_{d_i} = 0$ $q_{d_i} = 0.175 \text{ rad}$ $\dot{q}_{d_i} = 0$ and $\ddot{q}_{d_i} = 0$ $i = 1, \dots, 9 \text{ \& } i \neq 3$, $i = 3$, $i = 1, \dots, 9$ and $i = 1, \dots, 9$, respectively. All the simulations for under control systems are performed in the presence of 20% uncertainty for the EI , $(EI)_p$, ρA , $(\rho A)_p$ terms and $0.2 \cos(8\pi t) \boldsymbol{\tau} \in \mathbb{R}^{9 \times 1}$ as the vector of external disturbances.

Table 2 Parameters of the CRB [7].

Parameter	value
CRB mass (M)	800 kg
CRB moment inertia (J)	400 kg.m ²
Distance from the beginning of the appendages to the center of the CRB (r)	0.5 m

The normal bending modes of clamped-free Euler-Bernoulli beams $W_n(x)$ have been selected as approximating functions of the Galerkin method as follows:

$$\begin{aligned}
 W_n(x) &= C_n [\sin \beta_n x - \sinh \beta_n x - \alpha_n (\cos \beta_n x - \cosh \beta_n x)] \\
 \left\{ \begin{aligned} \alpha_n &= \left(\frac{\sin \beta_n l + \sinh \beta_n l}{\cos \beta_n l + \cosh \beta_n l} \right) \\ \beta_1 l &= 1.875104, \beta_2 l = 4.694091, \beta_3 l = 7.854757, \beta_4 l = 10.995541, \dots \end{aligned} \right. \quad (45)
 \end{aligned}$$

where $\beta_n l$ is obtained from $\cos \beta_n l \cdot \cosh \beta_n l = -1$ and C_n is an arbitrary coefficient [41, 42].

After investigating the solutions, the result was that considering three generalized coordinates, three piezoelectric actuators and three piezoelectric sensors for each of the flexible appendages are appropriate selections to study the system's behavior. The locations of the first, second and third piezoelectric actuators and piezoelectric sensors on both of the flexible appendages are the same and are given as $(x=[0,0.2], x=[2,2.2], x=[3.8,4])$ and $(x=[1,1.2], x=[3,3.2], x=[4.8,5])$, respectively.

Parameters and gains of the AGNSM and ASTGNSM controllers are stated as $\mathbf{G} = [0.1 \ 0.15 \ 1 \ 0.2 \ 0.2 \ 0.2 \ 0.2 \ 0.2 \ 0.2]$, $\lambda = 4$, $\beta = 1$, $k = 0.2$, $\kappa = 0.2$ and $\delta = 0.01$.

Elastic deformations of the flexible appendages with and without the proposed controllers are presented in Fig. 2. The conversion of Fig. 2(a) to zero-size surfaces of Figs. 2(b) and 2(c)

indicates that the AGNSM and ASTGNSM controllers are able to completely suppress the vibrations of the flexible appendages in less than 4 seconds, respectively.

(a)

(b)

(c)

Fig. 2 Vibrations of the flexible appendages: (a) system without controller, (b) system under AGNSM controller and (c) system under ASTGNSM controller.

It can be seen from the slopes formed in Fig. 2(a) that the assumption $\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 \ll \frac{\partial u}{\partial x}$ in dynamic modeling of the smart satellite with nonlinear flexible appendages is valid.

Figs. 3 and 4 show the tip vibrations of the flexible appendages, position and angular position of the CRB without control, respectively.

Fig. 3 Tip vibrations of the flexible appendages without control.

Fig. 4 Position and angular position of the satellite without control.

According to Fig. 5, tip vibrations of the flexible appendages with proposed controllers are suppressed just in 3 seconds.

Fig. 5 Tip vibrations of the flexible appendages under control.

Fig. 6 indicates the position and velocity of the CRB under proposed controllers. Also, the angular position and velocity of the satellite with the mentioned controllers are presented in Fig. 7.

Fig. 6 Position and velocity of the CRB under control

According to Figs. 6 and 7, all state variables related to translation and rotation of the satellite have reached their desired values in the presence of uncertainties and external disturbances. The insignificant difference between some states and their desired values is due to the consideration of the boundary layer for the AGNSM controller and the super-twisting algorithm in the ASTGNSM controller, which can be reduced to less and less by performing the parameter optimization process. However, the orientation of the satellite, which is the most important control target, has reached exactly the desired value and remained there.

Fig. 7 Angular position and angular velocity of the satellite under control

Control signals generated by the proposed controllers are presented in Figs. 8-10. Fig. 8 includes control forces and control torque applied to the CRB, Figs. 9 and 10 illustrate input voltages to the attached piezoelectric actuators on the left and right flexible appendages, respectively.

Fig. 8 Control forces and control torque.

By investigating Figs. 8-10, it can be seen that the generated control signals related to the input voltages of the piezoelectric actuators by both proposed controllers are very similar and have a higher quality than the generated signals for the control forces and control torque. Adaptive controllers have adjusted their adaptive parameters faster and with higher weight for vibrations of the flexible appendages due to the higher speed of vibration of the flexible appendages compared to the satellite's translation and rotation maneuver.

Fig. 9 Voltages of the attached piezoelectric actuators on the left flexible appendage.

In both of the proposed controllers, thanks to their adaptability, the upper bound of the lumped uncertainties and external disturbances is estimated adaptively and there is no need to preknowledge of them. In addition, in the ASTGNSM controller, unlike the AGNSM controller with the boundary layer, there is no need to determine the size of the boundary layer and the control system is automatically and adaptively adjusted such that it does not have the chattering problem.

Fig. 10 Voltages of the attached piezoelectric actuators on the right flexible appendage.

Temporal changes of the adaptive parameters and gains of the proposed controllers are given in Fig. 11. According to Eq. (44), gamma reaches a constant value due to the existence of the global nonlinear sliding surface in its updating law.

Fig. 11 Adaptive gains and parameters of the AGNSM and ASTGNSM controllers.

Figs. 12 and 13 illustrate output voltages of the attached piezoelectric sensors on the left and right flexible appendages, respectively. The strain has zero value at the end of the cantilever beam and has the highest value in the middle region. Also, Figs. 12 and 13 confirm that the output voltage of the third sensor has the lowest values and the output voltage of the second sensor has the highest values corresponding to lowest and highest value of strain.

Fig. 12 Voltages of the attached piezoelectric sensors on the left flexible appendage.

Fig. 13 Voltages of the attached piezoelectric sensors on the right flexible appendage.

5. Conclusion

In this paper, the motion of a smart rigid-flexible satellite by considering the large deformations for its flexible appendages in general planar motion has been investigated. Therefore, the translation and rotation of the smart satellite along with the arbitrary vibrations of its nonlinear flexible appendages are considered. The thrust forces, control torque and piezoelectric actuators are responsible for controlling the position and attitude of the satellite and suppressing the vibrations of the flexible appendages, respectively. In addition, piezoelectric sensors estimate the transverse displacement of flexible appendages. The ODE-PDEs of Motion, sensors equations, and boundary conditions are derived using extended Hamilton's principle. Then, by applying the Galerkin method to the equations of motion and sensors equations, two sets of ODEs were obtained as discretized equations of motion and sensors, respectively. The AGNSM and ASTGNSM controllers were designed to satisfy control objectives such as position and attitude control and vibration suppression of the flexible appendages in the presence of 20% uncertainty for the EI , $(EI)_p$, ρA , $(\rho A)_p$ terms and $0.2\cos(8\pi t)\boldsymbol{\tau} \in \mathbb{R}^{9 \times 1}$ as the vector of external disturbances. The results show that the ASTGNSM controller, due to the super-twisting algorithm, is able to eliminate the chatting problem and unlike the AGNSM controller with the boundary layer, there is no need to determine the width of the boundary layer. Besides, both proposed controllers are able to meet all the mentioned control objectives with appropriate speed and quality. Also, thanks to the use of adaptive laws to estimate the upper bound of lumped uncertainties and external disturbances, there is no need to preknowledge of this upper bound. In addition, adaptive controllers adjust their adaptive parameters faster and with higher weight for vibrations of the flexible appendages due to the higher speed of vibration of the flexible appendages compared to the satellite's translation and rotation maneuvers.

Appendix A

Applying the variation operator to Eqs. (3) and (10) leads to Eqs. (A.1) and (A.2), respectively.

[illegible]

$$\begin{aligned}
\delta U = & \int_{La} \left[(EI)_{La} w_L'' - \frac{1}{2} (EQ)_{La} (w_L')^2 \right] \delta w_L'' dx + \int_{La} \left[- (EQ)_{La} w_L'' w_L' + \frac{1}{2} (EA)_{La} (w_L')^3 \right] \delta w_L' dx \\
& + \int_{Ra} \left[(EI)_{Ra} w_R'' - \frac{1}{2} (EQ)_{Ra} (w_R')^2 \right] \delta w_R'' dx + \int_{Ra} \left[- (EQ)_{Ra} w_R'' w_R' + \frac{1}{2} (EA)_{Ra} (w_R')^3 \right] \delta w_R' dx \\
& + \sum_{i=1}^N \int_{LP_i} \left[(E_p I_p)_{LP_i} w_L'' - \frac{1}{2} (E_p Q_p)_{LP_i} (w_L')^2 \right] \delta w_L'' dx + \int_{LP_i} \left[- (E_p Q_p)_{LP_i} w_L'' w_L' + \frac{1}{2} (E_p A_p)_{LP_i} (w_L')^3 \right] \delta w_L' dx \\
& + \sum_{i=1}^N \int_{LA_i} \left(\frac{e_{31} (-Q_p) V_p}{t_p} \right) \delta w_L'' dx - \int_{LA_i} \left(\frac{e_{31} A_p V_p}{t_p} \right) w_L' \delta w_L' dx \\
& + \sum_{i=1}^N \int_{RP_i} \left[(E_p I_p)_{RP_i} w_R'' - \frac{1}{2} (E_p Q_p)_{RP_i} (w_R')^2 \right] \delta w_R'' dx + \int_{RP_i} \left[- (E_p Q_p)_{RP_i} w_R'' w_R' + \frac{1}{2} (E_p A_p)_{RP_i} (w_R')^3 \right] \delta w_R' dx \\
& + \sum_{i=1}^N \int_{RA_i} \left(\frac{e_{31} (-Q_p) V_p}{t_p} \right) \delta w_R'' dx - \int_{RA_i} \left(\frac{e_{31} A_p V_p}{t_p} \right) w_R' \delta w_R' dx \\
& + \sum_{i=1}^N \left\{ \left(\frac{e_{31} Q_p}{t_p} \right)_{LS_i} \int_{LS_i} w_L'' dx - \frac{1}{2} \left(\frac{e_{31} A_p}{t_p} \right)_{LS_i} \int_{LS_i} (w_L')^2 dx + \left(\frac{\varepsilon_p A_p l_p}{t_p^2} \right)_{LS_i} V_{LS_i} \right\} \delta V_{LS_i} \\
& + \sum_{i=1}^N \left\{ \left(\frac{e_{31} Q_p}{t_p} \right)_{RS_i} \int_{RS_i} w_R'' dx - \frac{1}{2} \left(\frac{e_{31} A_p}{t_p} \right)_{RS_i} \int_{RS_i} (w_R')^2 dx + \left(\frac{\varepsilon_p A_p l_p}{t_p^2} \right)_{RS_i} V_{RS_i} \right\} \delta V_{RS_i}
\end{aligned} \tag{A.2}$$

Appendix B

The relations obtained from integration by part (B.1) are used in the simplification of Eqs. (A.1) and (A.2) to obtain the equations of motion, the equations of sensors and the boundary conditions of the system.

$$\begin{aligned}
& \int_{t_1}^{t_2} \int_a^b \dot{w} \delta \dot{w} dx dt = \int_a^b \left(\dot{w} \delta w \Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} \ddot{w} \delta w dt \right) dx \\
& \int_a^b w' \delta w' dx = w' \delta w \Big|_a^b - \int_a^b w'' \delta w dx \\
& \int_a^b w'' \delta w'' dx = w'' \delta w' \Big|_a^b - \int_a^b w''' \delta w' dx = \int_a^b w''' \delta w dx + w'' \delta w' \Big|_a^b - w''' \delta w \Big|_a^b \\
& \int_a^b \delta w'' dx = \int_a^b [\delta(x-b) - \delta(x-a)] \delta w' dx = [\delta(x-b) - \delta(x-a)] \delta w \Big|_a^b - \int_a^b [\delta'(x-b) - \delta'(x-a)] \delta w dx
\end{aligned} \tag{B.1}$$

Appendix C

The relations obtained from integration by part (C.1) are used in the process of reducing the order of Eqs. (15) and (16) after the discretization process.

$$\begin{cases}
\int_a^b v w''' dx = \int_a^b v'' w'' dx + v w'' \Big|_a^b - v' w'' \Big|_a^b \\
\int_a^b v w'' (w')^2 dx = \frac{1}{3} v (w')^3 \Big|_a^b - \frac{1}{3} \int_a^b v' (w')^3 dx \\
\int_a^b v [\delta'(x-b) - \delta'(x-a)] dx = v [\delta(x-b) - \delta(x-a)] \Big|_a^b - \int_a^b v' [\delta(x-b) - \delta(x-a)] dx \\
= v [\delta(x-b) - \delta(x-a)] \Big|_a^b - [v'(b) - v'(a)] = v [\delta(x-b) - \delta(x-a)] \Big|_a^b - \int_a^b v''(x) dx
\end{cases} \quad (C.1)$$

Data availability Statement

The datasets generated and analyzed during the current study, details of the process of obtaining the equations and data supporting Figs. 2-13 are available from the corresponding author Dr. Ramin Vatankhah on request. The mechanical properties and parameter values of the central rigid body and the flexible appendages of the satellite given in Tables 1 and 2 are taken from sources <https://doi.org/10.1016/j.jsv.2018.06.052> [7] and <https://doi.org/10.1177/1077546312439913> [6], respectively. Also, the electromechanical properties and Parameter values of the piezoelectric actuators and sensors are available in research <https://doi.org/10.1177/1077546312439913> [6].

Conflict of Interest

The authors declare that they have no conflict of interest.

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Figures

Figure 1

Smart satellite with nonlinear flexible appendages along with coordinate systems, piezoelectric actuators and sensors, control forces and control torque.

(a)

(b)

(c)

Figure 2

Vibrations of the flexible appendages: (a) system without controller, (b) system under AGNSM controller and (c) system under ASTGNSM controller.

Figure 3

Tip vibrations of the flexible appendages without control.

Figure 4

Position and angular position of the satellite without control.

Figure 5

Tip vibrations of the flexible appendages under control.

Figure 6

Position and velocity of the CRB under control

Figure 7

Angular position and angular velocity of the satellite under control

Figure 8

Control forces and control torque.

Figure 9

Voltages of the attached piezoelectric actuators on the left flexible appendage.

Figure 10

Voltages of the attached piezoelectric actuators on the right flexible appendage.

Figure 11

Adaptive gains and parameters of the AGNSM and ASTGNSM controllers.

Figure 12

Voltages of the attached piezoelectric sensors on the left flexible appendage.

Figure 13

Voltages of the attached piezoelectric sensors on the right flexible appendage.