High obliquity, high angular momentum Earth as Moon's origin revisited by Advanced Kinematic Model of Earth-Moon System

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Research Letter

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High obliquity, high angular momentum Earth as Moon's origin revisited by Advanced Kinematic Model of Earth-Moon System

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Abstract

Matija Cuk et.al (2016) have proposed a new model for the birth and tidal evolution of our natural satellite Moon, born from lunar accretion of impact generated terrestrial debris in the equatorial plane of high obliquity, high angular momentum Earth. This paper examines their findings critically in the light of Advanced Kinematic Model (AKM) which includes Earth's obliquity(ϕ), Moon's orbital plane inclination (α) , Moon's obliquity (β) and lunar's orbit eccentricity (e). It is shown that AKM's valid range of application is from $45R_E$ to $60.33R_E$. The evolution of α, β, e is in correspondence with the simulation results of Matija Cuk et.al (2016) but evolution of Earth's obliquity has a break at $45R_E$. According to AKM, earlier than $45R_E$ Earth should achieve 0° obliquity in order to achieve the modern value of eco-friendly 23.44° obliquity. Cuk et al (2016) silent on this point. AKM stands vindicated because using protocol exchange algorithm http://doi.org/10.1038/protex.2019.017, AKM has successfully given precise theoretical formalism of Observed LOD curve for the last 1.2Gy time span opening the way for early warning and forecasting methods for Earth-quake and sudden Volcanic eruptions. This paper gives us an algorithm to determine the short term and long term changes in Earth's

obliquity which is related to Weather and Climate Extremes. Hence this paper gives us the mathematical tool for predicting the Earth's climate extreme.

Keywords:

Geosynchronous orbits; Earth's Obliquity Angle; Moon's Obliquity Angle; Lunar Plane Inclination; Lunar Orbit's Eccentricity; tidally interlocked;

1 Introduction-Keplerian Era.

The Kepler's Third Law for a given Planet-Sun configuration is:

$$a^3\Omega^2 = G(M+m) \quad 1$$

Eq. (1) does not specify if the given orbital configuration is stable. Newton derived this law assuming that centripetal force (GMm/a^2) = centrifugal force(mv_{tang}^2/a) where a = semi-major axis of Earth-Moon orbital configuration, M = mass of the Earth and m = mass of our Moon. By implication it was assumed that all planetary configurations predicted by (1) are stable. By the end of 19th century George Howard Darwin put a question mark on this stability by publishing two papers on E-M system (Darwin, 1879, 1880).

In 18th Century, German Philosopher Kant had suggested the theory of retardation of Earth's spin based on the ancient records of Solar Eclipses (Stephenson &Houldon, 1986; Stephenson 2003). Similar kind of studies had been carried out by Kevin Pang at Jet propulsion Laboratory at Pasadena (Morrison1978; Jong & Soldt 1989). He happened to step upon certain ancient records regarding Solar Eclipses. A total Solar Eclipse had been observed in the town of Anyang, in Eastern China, on June 5, 1302 B.C. during the reign of Wu Ding. Had Earth maintained the present rate of spin, the Eclipse should have been observed in middle of Europe. This implies that in 1302 B.C. i.e. 3,291 years ago Earth's spin period was shorter by 0.047 seconds. This leads to a slowdown rate of 1.428 seconds per 100,000 years.

In 1879 George Howard Darwin carried out a complete theoretical analysis of Earth-Moon System and put forward a sound hypothesis for explaining the slowdown of Earth's spin on its axis. This marked the end of Keplerian Era. Gravitationally bound bodies were necessarily tidally interacting and tidal interaction led to tidal dissipation with inherent instability and hence a post-Keplerian physics was required to deal with gravitationally bound binary pairs. Tidally dissipative system because of loss of energy cannot be stable. The system will evolve to a minimum energy state which is a stable configuration by necessity.

1.1. The beginning of Evolutionist view of Universe – Post-Keplerian Era.

By mid 20th century it was increasingly felt that celestial bodies pair behave as electrons orbiting the nucleus in individual atoms. Within an atom electrons had radiation-less stable permissible orbits propounded by Niels Bohr in 1913:

$$Angular\ Momentum\ of\ electron = I \times \omega = m \times r^2 \times \frac{v_{Tang}}{r} = m \times v_{Tang} \times r$$

$$= n \times \frac{h}{2\pi} \qquad 2$$

Eq. (2) Simplifies to de Broglie standing wave condition:

$$2\pi r = n \times \frac{h}{m \times v_{Tang}} = n \times \frac{h}{p} = n \times \lambda_{de-Broglie} \qquad 3$$

Eq.(3) Simply states that electrons are permitted to stay in radiation-less stable orbits where electrons behaves as matter wave which forms a Standing Wave and is inhibited from making synchrotron radiation and is ensured stable orbits. Any other orbit would collapse and electron would be launched on a death spiral towards its respective nucleus.

It is postulated by the Author that in exactly the same manner celestial body binaries are born at a_{G1} (inner Clarke's Orbit) which is a Keplerian Orbit , an equilibrium orbit where centripetal force is exactly balanced by centrifugal force but it is an energy maxima (Sharma 2011) hence the secondary tumbles short or tumbles long of a_{G1} . In 2002 at World Space Cogress, Houston, Author proposed Kinematic Model(KM) (Sharma & Ishwar 2002) . According to KM celestial body pairs have two triple synchrony orbits (a_{G1} and a_{G2}) where they are conservative systems and no dissipation of energy is involved (Sharma, Ishwar & Rangesh,2009; Sharma, 2011; Krasinsky, 2002). Here triple synchrony orbits imply:

$$\omega(spin\ angular\ velocity\ of\ the\ primary) = \Omega(orbital\ angular\ velocity)$$
$$= \Omega'(spin\ angular\ velocity\ of\ the\ secondary \qquad 4$$

The orbits of triple synchrony are defined as geo-synchronous orbits in E-M system and Clarke's orbits in context of planet-satellite pairs, star-planet pairs, star-star pairs, neutron star-neutron star(NS) pairs and NS and BH (black hole) pairs.

i. Planet-satellite pairs (with high mass ratio fractions above 0.2), star-planet pairs (with high mass ratio fraction above 0.2) and star pairs are non-relativistic systems. They, within months/years, lock-in at outer Clarke’s orbit. Non-relativistic systems are stable at outer triple synchrony orbits.

ii. NS pairs, NS and BH pairs or BH pairs are relativistic systems. Relativistic systems are radiating gravitational waves and they are being driven towards coalescence hence they are always experiencing in-spiral orbital motion until the final ring-down and merger and always negatively off-setted with respect to outer Clarke’s Orbit and never locked-in at outer Clarke’s orbit. The magnitude of off-set is decided by the relativistic strength of NS pair/NS-BH pair/BH pair which in turn is decided by the rate of apsidal precession (long axis of the elliptical orbit of the secondary turning in the same direction as the secondary’s orbital motion).

iii. Tidally interacting binary pair such as planet-moon may have mass ratios from 10^{-4} to 0.2. These experience tidal evolution from inner Clarke's Orbit to Outer Clarke's orbit with time constant of evolution τ which is inverse power law of mass ratio (q = secondary mass/ primary mass). As q decreases from 0.2 to 10^{-4} correspondingly time constant of evolution τ increases from year to Ky to My to Gy respectively (Sharma 2011).

iv. If mass ratio is infinitismal as in case Communication Satellite and Earth then time constant of evolution τ becomes infinite and Communication Satellite remains stay put in its orbit if it is shielded from Poynting Robertson Drag and radiation pressure.

1.2. Two competing schools of thought on tidally interacting binaries.

The whole scientific community had been using the elasto-viscous model for analyzing the tidally interacting binaries but this was based on the knowledge of Love number and Q factor of the celestial bodies which in turn depended on the knowledge of density, rigidity, viscosity and rate of periodic forcing. These parameters are known with large uncertainties for different Planets and their Satellites and hence their Tidal Evolutionary History will be arrived at with equal uncertainty in Seismic Model based analysis (Shi et.al. 2013). The Author developed a Kinematic Model (KM) of E-M system to study its tidal evolutionary history from its birth at inner geo-synchronous orbit (a_{G1}) to the final lock-in orbit at outer geo-synchronous orbit (a_{G2}) or to its final doom in glancing angle collision with Earth (Sharma 2011). The KM required only the globe-orbit parameters and the age of the system. Since the system parameters were known with high degree of confidence level hence the results arrived at were reliable and accurate. In spite of this improvement KM failed to resolve the conundrum in E-M puzzle. This had baffled the whole scientific community.

1.3. The conundrum of Earth-Moon puzzle.

The Apollo mission had confirmed the age of E-M system with high degree of certainty as 4.467Gy (Stevenson 2008, Toubol et.al.2007). With this age of E-M system the present rate of recession of Moon should have been 2.4cm/y but Lunar Laser Experiment operational since 20th July 1969 was giving a recession rate of 3.82±0.07cm/y (Dickey et.al.1994) which indicated anomalously high dissipation rate in Earth's oceans and continents .If the present rate of recession was assumed and extrapolated into the past it led to the birth of Moon at 2.8Gy. In addition the KM with age as 4.467Gy was not giving a matching theoretical formalism of observed LOD curve based on Coral fossils (Well 1963, 1966) and ancient tidalites (.Sonnett & Chan 1997). The reconstruction of the history of Lunar recession from existing data of tidal rhythmites (Coughenour et.al. 2009) and that of length of Earth day (LOD) record in coral fossils (Wells 1963,1966) indicate that Earth-Moon is not just a Clockwork or orbitally fixed pair of monoliths as viewed by Newton in Mathematica Principia but instead Earth-Moon system is a tidally evolving system where Moon is tidally receding from Earth since its birth. The findings of Stephenson (1997) and Lambeck (1980) firmly established that Earth-Moon system is a non-linearly tidally evolving system with a complex history of interrupted tidal evolution due to intermittent gravitational resonances with lunar M_2 tides and solar S_2 tides . Kant (1754) hypothesis included the lunar tidal perturbation as well as solar tidal perturbation on tidal evolution of E-M system. G.H. Darwin (1889,1890) clearly established that Moon's tidal brake and Solar tidal brake have slowed down the Earth's spin from 5 hrs and orbital radius of 18,000 Km (just beyond Roche's Limit) (Ida, Canup & Stewart 1997) to the present Earth's spin rate of 24 h and orbital radius of 384,400 Km and in the process the Earth's angular excess spin energy released has led to tidal heating of Earth and spin angular momentum of Earth spin has been transferred to E-M orbital angular momentum and Lunar tidal recession. The results seem to indicate a non-linear variable rate of tidal dissipation throughout E-M system's history. Oceans may enter and exit M_2 and S_2 tide resonance in geologically short time intervals (Kagan 1997). It was this tension between observation and theory which compelled the Author to remove the first degree approximations from E-M model and propose a more comprehensive and detailed model with an elaborate dynamics of E-M system which the Author is referring to as Advanced Kinematic Mode;(AKM) and as will be shown in this paper AKM has dramatically improved the model to real world correlation.

1.4. The origin of Moon from high obliquity, high angular momentum (AM) Earth and impact generated circum-terrestrial debris disk.

Matija Cuk et.al (2016) have proposed a new model for the birth and tidal evolution of our natural satellite Moon in which lunar tidal dissipation due to lunar obliquity tides during Cassini State transition plays an important role in stabilizing and allowing E-M system to arrive at climatically favorable E-M configuration with a low Earth's obliquity (ϕ =23.44°). High angular momentum and high obliquity Earth provides a more robust mechanism to remove excess AM and provides Earth's mantle like isotopic composition properties of Moon.

Here there is a conundrum. As Moon's orbital plane inclination drops from 15° to 5°, Earth's obliquity must rise from 0° to 23.5°. This requires that at Cassini State Transition Earth's obliquity Φ must be 0.

Rubicam (1993) has discussed this problem. At present, the Earth's mean obliquity is slowly increasing as a result of tidal interactions with the Moon. Simultaneously the lunar inclination is decreasing, so that the angular momentum of the Earth–Moon system is conserved.

The conservation of total angular momentum is given as follows (Rubicam 1993):

$$Sin\left[\frac{\Delta\varphi}{2}\right] = -\frac{J_{orbit}}{J_{spin\_Earth}} \times Sin\left[\frac{\Delta\alpha}{2}\right] \qquad 5$$

This implies that if lunar orbital plane inclination angle decreases by 5° and J_{orbit}/ J_{spin_Earth} =10 then Earth's obliquity must increase by 60.6°. This precisely is predicted by Advanced Kinematic Model as seen by close examination of Figure 1.

2 Materials and Methods

2.1. Calculation of Total Angular Momentum(AM) of Earth-Moon System as the vector sum of constituent AMs.

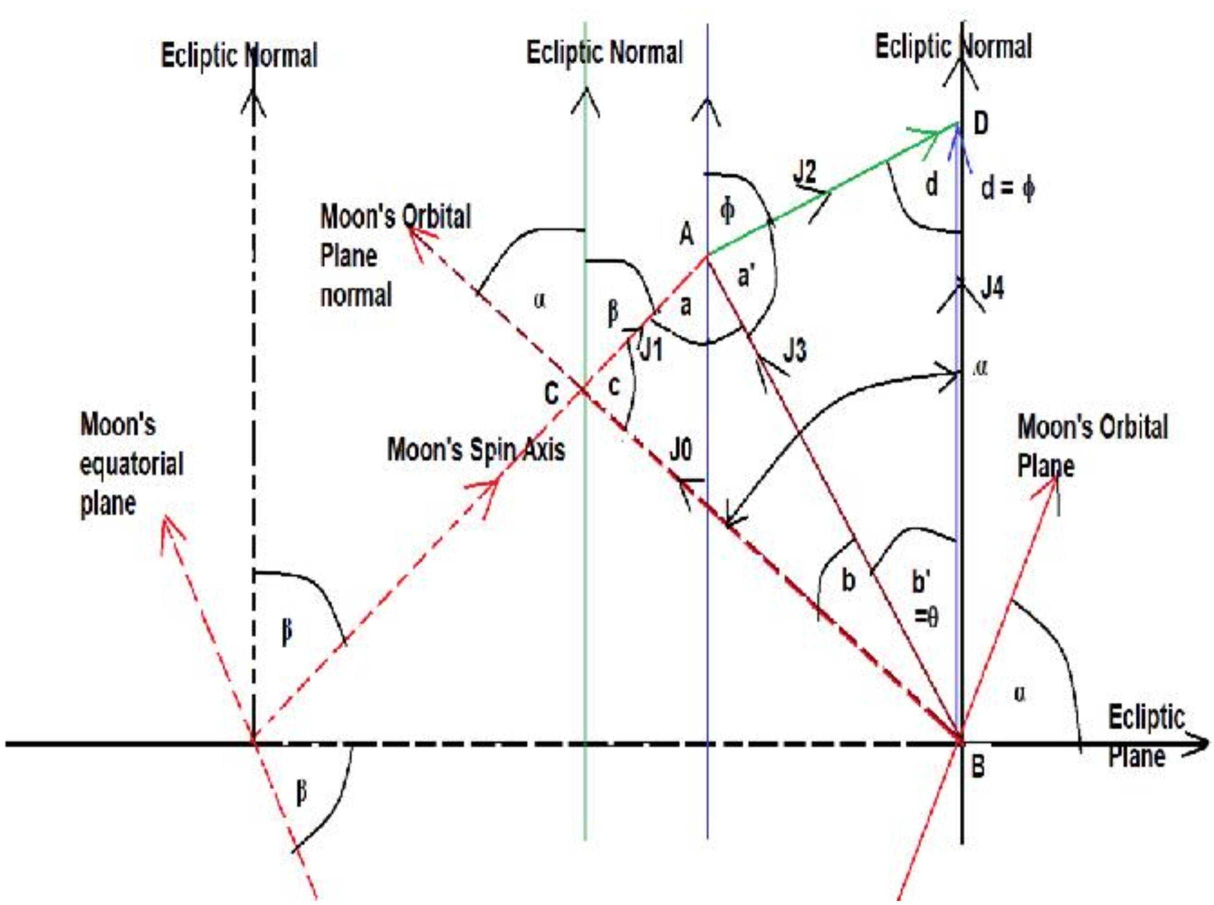

Figure 1. Spin-Orbital configuration of Earth-Moon System.

In Figure 1, J_0, J_1, J_2, J_3, J_4, symbols are defined in List of Abbreviations.

J_0, J_1 and ecliptic normal are coplanar according to Cassini Law 3.

Before Cassini State transition E-M sysem was in Cassini State I when Moon's spin axis (J_1) and Moon's Orbital plane Normal (J_0) were on the same side of Ecliptic Normal.

Presently E-M system is in Cassini State II hence J_0 vector and J_1 are on the two sides of Ecliptic Normal as shown in the Figure 1.

J_3, J_2, J_4 and Ecliptic normal are coplanar.

But the plane containing J_0 and J_1 and Ecliptic Normal and the plane containing J_2, J_3, J_4 and Ecliptic normal are two separate planes hence J_0 and J_1 are shown by dotted lines.

Definitions of Earth's Obliquity(ϕ), Moon's orbital plane inclination (α) and Moon's Obliquity (β):

Axial tilt of Earth's spin axis with respect to (w.r.t.) Ecliptic Normal = ϕ = 23.44^0 = 0.4091051767 radians;

Axial tilt of Moon's spin axis w.r.t. Ecliptic Normal = β = 1.54^0 = 0.02687807 radians;

Angle between Moon's equatorial plane and ecliptic plane = β ;

Total axial tilt of Moon's spin axis w.r.t. E-M orbital AM vector = $\alpha+\beta$ = 6.68^0 =0.11658 radians.

All these are observational Astronomy data in the current era and illustrated in Figure 1 and Figure S1.2 in S1 of supplementary materials..

The total resultant angular momentum vector (J_4) of Earth-Moon system is calculated in S1.1. of the supplementary materials.

First J_3 vector = J_0 vector + J_1 vector is calculated as shown in S1.2.

$$J_3 = 2.8498 \times 10^{34} \frac{Kg - m^2}{s} \qquad 6$$

188 The angle of inclination of J_3 w.r.t. ecliptic normal and left to normal = θ =α –b =

189 0.08970905087 radians = 5.13995^0 ~ α ;

190 Next J_4 vector = J_3 vector + J_2 vector =Total AM of E-M system.

$$J_{Total} = J_4 = 3.37492 \times 10^{34} \frac{Kg - m^2}{s} \angle(\theta - b')$$

$$= 3.37492 \times 10^{34} \frac{Kg - m^2}{s} \angle\gamma = 0.39^0 \qquad 7$$

191 Next we determine the LOM/LOD equation , the cardinal equation of the Advanced Kinematic

192 Model.(The details of this derivation is given in S1 of the supplementary materials.

$$(N)^2 \times a^3 = X^2 + (F\sqrt{1-k^2})^2 \times (a^2)^2 + G^2 +$$

$$2\left(F\sqrt{1-k^2} \times a^2\right)(G)\left\{\sqrt{1-D^2}\sqrt{1-A^2} - AD\right\} + 2 \times X$$

$$\times \sqrt{\left(F\sqrt{1-k^2} \times a^2\right)^2 + (G)^2 + 2\left(F\sqrt{1-k^2} \times a^2\right)(G)\ \{\sqrt{1-D^2}\sqrt{1-A^2} - AD\}}$$

$$\times \left\{\sqrt{1-A^2}\sqrt{1-B^2} - A.B\right\} \qquad 8$$

$$X = \frac{\omega}{\Omega} = \frac{LOM}{LOD};\ B = \sqrt{GM + Gm} = \sqrt{0.39860 \times 10^6 + 0.00490 \times 10^6}$$

$$= 2.00873 \times 10^7 \frac{m^{3/2}}{s}$$

$$N = \frac{J_4}{B \times C} = 2.09517 \times 10^{-11}\left(\frac{1}{m^{\frac{3}{2}}}\right), where\ J_4 = 3.37492 \times 10^{34} \frac{Kg - m^2}{s},$$

$$C = 8.01906 \times 10^{37} Kg - m^2,$$

$$G = \frac{I}{C} = 0.00108949, \qquad F = \frac{m^*}{C} = 9..04936 \times 10^{-16}\left(\frac{1}{m^2}\right) \qquad 9$$

193 Here we define the following Trignometric Identities:

$$Sin[\alpha] = A;\ Sin[\beta] = D;\ Sin[\phi] = B$$

These Trignometric Identities have been utilized in (8)

3. Calculation of Earth Moon system parameters.

3.1. Calculation of inner and outer geo-synchronous orbits in KM framework.

Our Moon has been born at inner geo-synchronous orbit , a_{G1} . E-M system has tidally evolved from inner geo-synchronous orbit , a_{G1} , to the outer geo-synchronous orbit , a_{G2}. It is midway in this evolutionary path at a = 384,400Km from the center of the Earth. At both the geo-synchronous orbit E-M system is in triple synchrony state when X =1.

Equating X to Unity we obtain the two geo-synchronous orbits which has been done in S1 supplementary materials.

$$a_{G1} = 1.48646 \times 10^7 m;$$
$$a_{G2} = 5.33505 \times 10^8 m \qquad 10$$

Using Kepler's third law:

$$\frac{1}{\Omega^2} = \frac{a^3}{B^2}$$

We obtain triple synchrony time period at a_{G1} and a_{G2} as 5 hours and 44.6 solar days respectively.

3.2. Calculation of modern times theoretical LOM/LOD in AKM framework

In E-M system LOM (length of month) = sidereal lunar month and LOD (length of day) = the sidereal day.

$$\frac{LOM}{LOD} = \frac{\omega}{\Omega} = 27.3217\ in\ modern\ times \qquad 11$$

In Section 2,(Methods and Materials) LOM/LOD equation has been formulated in (8) and it is being restated here:

$$(N)^2 \times a^3 = X^2 + (F\sqrt{1-k^2})^2 \times (a^2)^2 + G^2 +$$
$$2\left(F\sqrt{1-k^2} \times a^2\right)(G)\left\{\sqrt{1-D^2}\,\sqrt{1-A^2} - AD\right\} + 2 \times X$$
$$\times \sqrt{\left(F\sqrt{1-k^2} \times a^2\right)^2 + (G)^2 + 2\left(F\sqrt{1-k^2} \times a^2\right)(G)\ \{\sqrt{1-D^2}\,\sqrt{1-A^2} - AD\}}$$
$$\times \left\{\sqrt{1-A^2}\sqrt{1-B^2} - A.B\right\} \qquad 8$$

If real world is considered then:

Using current Earth's obliquity (Φ = 23.44°), current Moon's orbital inclination (α = 5.14°) and current Moon's obliquity (β=1.54°) and k= 0.0549

We obtain the following trigonometric identities:

Sin[α] = A=0.0895897 and Cos[α] =$\sqrt{(1\text{-}A^2)}$= 0.995979;

Sin[β] = D=0.0268768 and Cos[β] =$\sqrt{(1\text{-}D^2)}$= 0.999639;

Sin[Φ] = B=0.397784 and Cos[Φ] =$\sqrt{(1\text{-}B^2)}$=0.917479;

$$\left\{\sqrt{1-D^2}\,\sqrt{1-A^2} - AD\right\} = 0.993211 \ and\ \left\{\sqrt{1-A^2}\sqrt{1-B^2} - A.B\right\} = 0.87815$$

Rewriting (8) and substituting the numerical values of the trigonometric identities in Modern Times we get:

$$(N)^2 \times a^3 = X^2 + (F \times 0.99849)^2 \times (a^2)^2 + G^2 +$$
$$2(F \times 0.99849 \times a^2)(G)\{0.993211\} + 2 \times X$$

$$\times \sqrt{(F \times 0.99849 \times a^2)^2 \ + (G)^2 \ + 2\,(F \times 0.99849 \times a^2)\,(G)\ \{0.993211\}} \times \{0.87815\} \qquad 9$$

Solving (9) with numerical values of N, F and G and 'a' (the current semi-major axis) substituted we get the following quadratic equations:

$$17826.5 + 234.495X + X^2 - 24979.225 = 0 \qquad 10$$

The two roots of (10) are: -261.815 and 27.3199.

The negative root is rejected since both the spin of Earth and Moon and orbital motion are retrograde . Hence only 27.3199 is tenable.

We are having a perfect match with observed LOM/LOD

(8) is satisfied for the current epoch ω/Ω (LOM/lOD), α (Inclination angle) , β (lunar obliquity), Φ (terrestrial obliquity) and e (eccentricity).

3.3. Evolution of inclination of Lunar orbital plane, eccentricity of Lunar orbit and obliquity of Moon's spin axis based on the information in Cuk et.al.(2016)

The empirical relation describing the evolution of Moon's orbital plane inclination with respect to the ecliptic is (Supplementary-Information S2):.

$$Inclination\ angle\ \alpha = \frac{1.18751 \times 10^{25}}{a^3} - \frac{7.1812 \times 10^{16}}{a^2} + \frac{1.44103 \times 10^{8}}{a} - 8.250567342 \times 10^{-3} \qquad 11$$

The empirical relation describing the evolution of Moon's obliquity angle (β) is given as below (Supplementary-Information S2):

$$Moon's\ Obliquity\ angle\ \beta = 3.36402 - 1.37638 \times 10^{-8} a + 1.32216 \times 10^{-17} a^2 \qquad 12$$

The empirical relation describing the evolution of Moon's orbit eccentricity is (Supplementary-Information S2):.

$$e = 0.210252 + 8.38285 \times 10^{-10} a - 3.23212 \times 10^{-18} a^2 \qquad 13$$

3.4. The Determination of the evolutionary history of Earth's Obliquity from Advanced Kinematic Model of tidally interacting E-M system

From a previous personal communication arXiv: http://arXiv.org/abs/0805.0100 LOM/LOD of Earth Moon system is known over the tidal evolutionary history. It is tabulated in Table 1 of the Results.

Using (8)

$$(N)^2 \times a^3 = X^2 + (F\sqrt{1-k^2})^2 \times (a^2)^2 + G^2 + 2\left(F\sqrt{1-k^2} \times a^2\right)(G)\left\{\sqrt{1-D^2}\sqrt{1-A^2} - AD\right\} + 2 \times X \times \sqrt{\left(F\sqrt{1-k^2} \times a^2\right)^2 + (G)^2 + 2\left(F\sqrt{1-k^2} \times a^2\right)(G)\ \{\sqrt{1-D^2}\sqrt{1-A^2} - AD\}} \times \left\{\sqrt{1-A^2}\sqrt{1-B^2} - A.B\right\} \qquad 8$$

Obliquity angle is determined.

In (8) all constant and all spatial functions are known except the obliquity angle Φ.

For a given lunar orbit , LOM/LOD is known. Using this information Sin[Φ] is determined and hence Φ and tabulated in Table 1.

We have six set of data from a =30R_E to the present day semi-major axis.

In Supplementary-Information S1, the evolutionary history expression have been derived for LOM/LOD and Earth's obliquity Φ (radians). They are as follows:

$$\frac{LOM}{LOD} = \frac{\omega}{\Omega} = -12.0501 + 2.6677 \times 10^{-7} \times a - 4.27538 \times 10^{-16} \times a^2 \qquad 14$$

$$\phi = -0.732299 + 2.97166 \times 10^{-9} \times a \qquad 15$$

4. Results: The tidally evolving Earth-Moon System and its evolving astrometric parameters.

Table 1 gives the evolutionary history of LOM/LOD and Earth's Obliquity.

Table 1. LOM/LOD and Earth's Obliquity for past geological epochs.

$a(\times R_E)$	$a(\times 10^8 m)$	LOM/LOD	Sin[Φ]	Φ (radians)	Φ°
30	1.9113	23.3752	-0.464076	unstable	unstable
35	2.22985	26.1194	-0.216896	unstable	unstable
40	2.5484	28.1147	0.02137757	0.0213773	1.22483
45	2.86695	29.2938	0.113547	0.113792	6.51
50	3.1855	29.5965	0.218451	0.220227	12.6
55	3.50405	28.9877	0.309749	0.314929	18
60	3.8226	27.4	0.388198	0.398676	22.84
60.335897	3.844	27.32	0.397788	0.409105	23.44

We clearly see that at Cassini State Transition, Earth's obliquity is indeterminate. From $40R_E$ to $60.336R_E$ it is well behaved and obliquity is increasing. It increases from 1.22483° to 23.44°. This means that during angular momentum conservative phase reduction in inclination is accompanied with increase in obliquity by necessity.

Table 2 gives the evolutionary history of ω/Ω (LOM/lOD),α (Inclination angle) , β (lunar obliquity), Φ (terrestrial obliquity) and e (eccentricity)

Table 2. evolutionary history of ω/Ω (LOM/lOD),α (Inclination angle) , β (lunar obliquity), Φ (terrestrial obliquity) and e (eccentricity)

a ($\times R_E$)	a ($\times 10^8$m)	ω/Ω	α radians	β	e	Φ(rad)	Sin[Φ]
30	1.9113	23.3752	0.480685 (27.4°)	1.21635 (69.69°)	0.2524	unstable	-0.464076
35	2.22985	26.1194	0.26478 (15.17°)	0.952317 (54.56°)	0.236	unstable	-0.216896
40	2.5484	28.1147	0.168969 (9.68°)	0.71512 (40.97°)	0.214	0.0213773	0.0213757
45	2.86695	29.2938	0.124631 (7.1408°)	0.504756 (28.92°)	0.1849	0.113792 (6.51°)	0.113547
50	3.1855	29.5965	0.103801 (5.04736°)	0.321225 (18.4°)	0.1493	0.220227 (12.6°)	0.218451
55	3.50405	28.9877	0.0941394 (5.39379°)	0.164527 (9.4267°)	0.10714	0.314929 (18°)	0.309749
60	3.8226	27.4	0.0898729 (5.149°)	0.03466 (1.986°)	0.0584	0.398676 (22.84°)	0.388198
60.336	3.844	27.32	0.08971 (5.14°)	0.0268 (1.54°)	0.0549	0.409105 (23.44°)	0.397788

In Figure 2, the evolution of Earth's obliquity (ϕ) based on AKM data (bold green) and based on Simulation data (dashed green) by Mutja Cuk et.al(2016) is given. We see the discontinuity at $45R_E$.

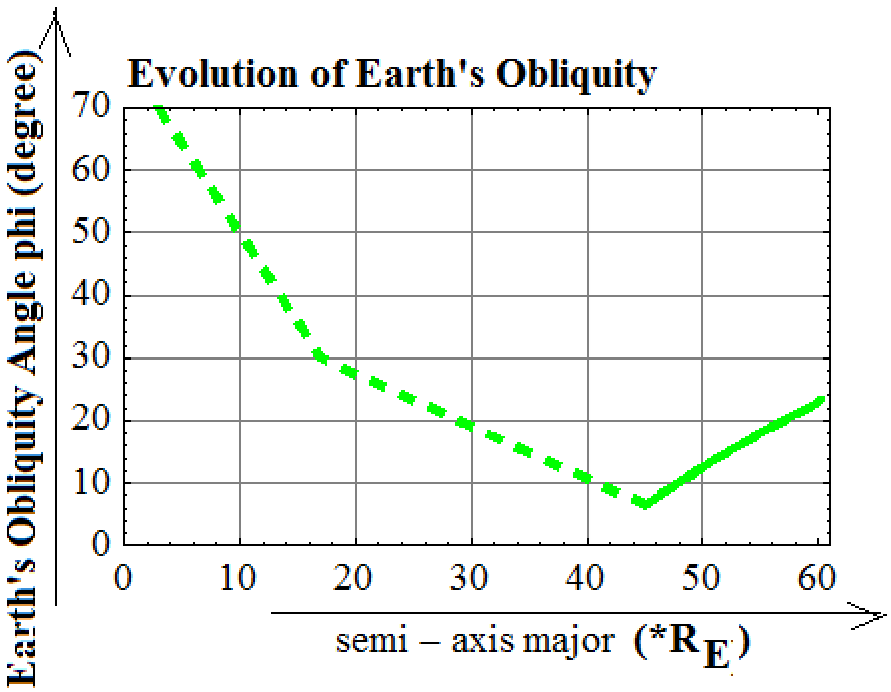

Figure 2. Earth's Obliquity angle (ϕ°) evolution according to AKM (bold green) and according to Simulation results(dash green) (Cuk et.al.2016).

In Figure 3, the evolution of Moon's orbital plane inclination (α) based on AKM (bold red) and based on Simulation done (dash red) by Matija Cuk et.al.(2016) is given. Here there is a continuity.

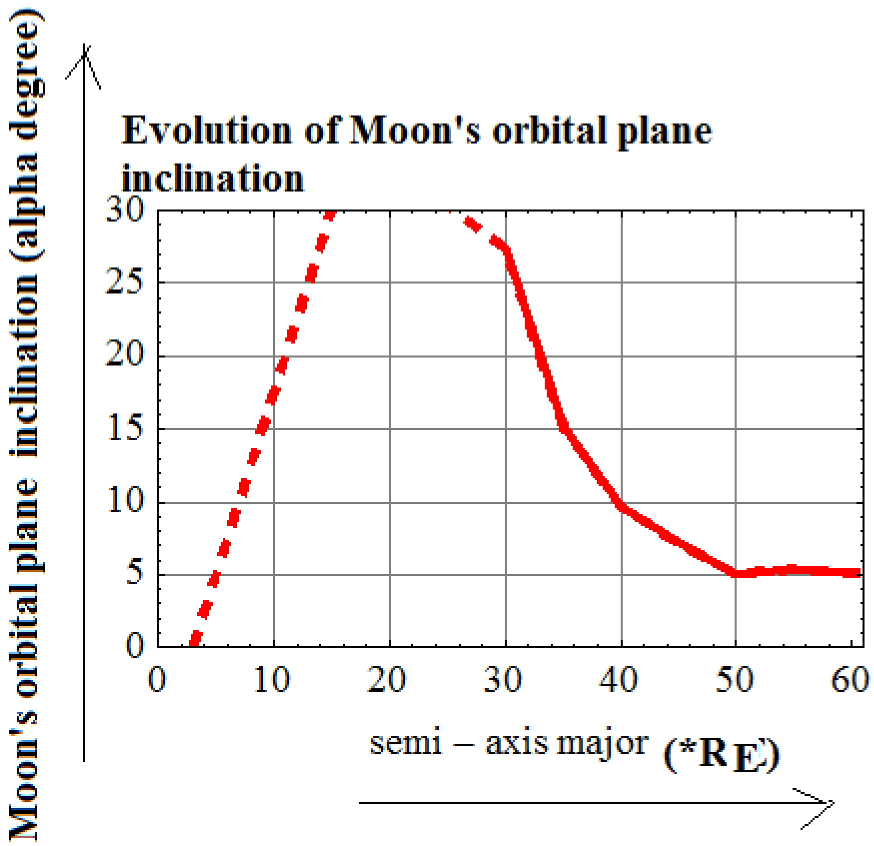

Figure 3.Moon's orbital plane inclination (α°) based on AKM (bold red) and based on Simulation results (dash red) (Cuk et.al.2016)

In Figure 4, the evolution of Moon's obliquity (β) based on AKM (bold blue) and based on Simulation (dashed blue) done by Matija Cuk et.al.(2016) is given.

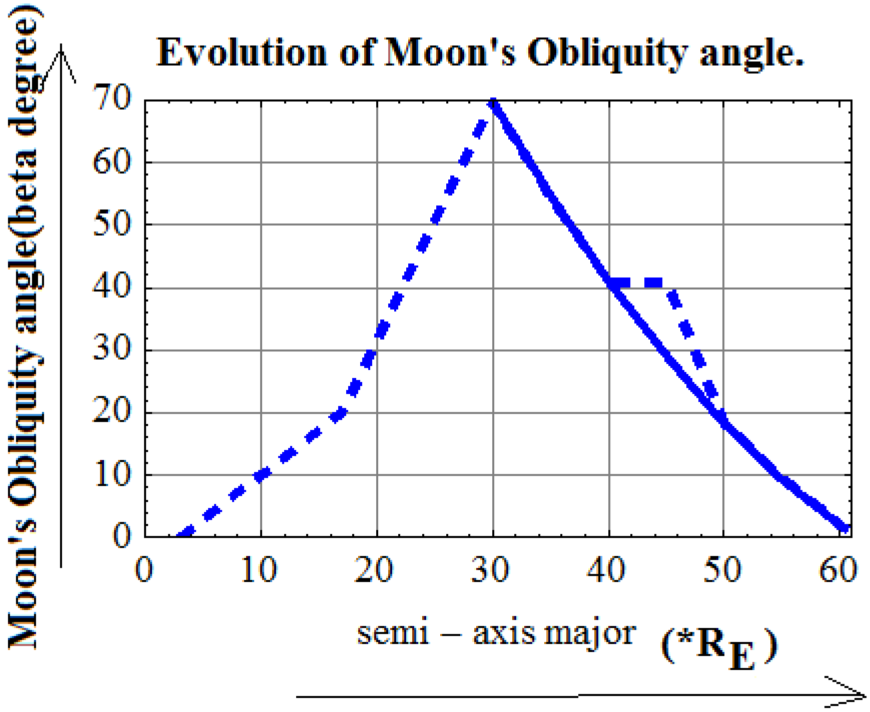

Figure 4. Moon's Obliquity (β°) based on AKM (bold blue) and based on Simulation results (dash blue) (Cuk et.al.2016)

5 Conclusions.

5.1. Discussion.

Examining the findings made by Cuk et.al and by this paper we see the following time line of tidal evolution as tabulated in Table 3.

Table 3. Four stages in tidal evolution of E-M system.

	Post-Impact	Laplace Plane Transition	Cassini State Transition	Cass. State2	Present
'a'	$3R_E$	$17R_E$	$33.3R_E$	$40R_E$	$60R_E$
Ecc.	Circular	0.5(excess J drained to	0.25	0.21	0.0549

	orbit	heliocentric orbit)			
α	0	35°	28°	27.54°	5.14°
β	0	?	69.69°	40.97°	1.54°
Φ	70°	30°	?	1.22°	23.44°

Inspection of Table 3 leads us to a definite conundrum. After Cassini State 2 is reached, Earth-Moon system enters angular momentum conservative phase. If lunar obliquity tides generated by Cassini state transition help reduce inclination angle from 27.54° to 5.14°. Then by necessity of angular momentum conservation, obliquity must increase. This implies that current obliquity of 23.44° can be achieved only if obliquity angle is zero after Cassini State Transition.

This is obvious by the inspection of Figure 3, Figure 4 and Figure 5. As we see Figure 4 and Figure 5 give a continuity between AKM results and Simulation results in the evolution of inclination and Moon's obliquity data. The two results smoothly merge. But Figure 3 shows a discontinuity near $40R_E$ for Earth's obliquity. To achieve 23.44° modern value of Earth's obliquity the Earth Moon system must achieve 0° Earth's obliquity just earlier than $40R_E$ when Moon settles down in Cassini state 2.

Since angular momentum conservation is not required from Laplace Plane Transition to Cassini State Transition it is quite possible that strong obliquity tides are reducing inclination angle as well as Obliquity angle. Then only the climate friendly low obliquity can be achieved.

At this point , Cuk et.al.(2016) are silent . This is a definite conundrum which needs to be addressed before we can assert that

"Our tidal evolutionary model supports high angular momentum, giant impact scenario to explain Moon's isotopic composition and provide a new pathway to reach Earth's climatically favourable low obliquity."

5.2. Final summing up.

This paper brings Kinematic Model renamed as advanced kinematic model (AKM) of tidally interacting binaries to a new level of maturity whereby it will prove to be more effective in dealing with real life scenario. There is dramatic improvement in the correlation of new model and the real world. In Section 2.4. an assumption has been made that:

$$in\ Figure\ 1, interior\ angle\ \widehat{a'} = \pi - \phi - \widehat{b'} = \pi - \phi - \alpha \qquad 16$$

How valid is this assumption over the entire permissible range ($40R_E$ to $60.33R_E$) of AKM will have to be critically examined in a sequel paper. Cuk Muteja et.al (2016) have proposed that Earth-Moon system while passing through Laplace plane transition and Cassini state transition pass through chaotic and turbulent phase and due to strong obliquity tides in Moon the tidal evolution gets stalled or even reversed for long periods of its existence. E-M system moves in 'Fits' from $3R_E$ to $17R_E$ and subsequently to $51.4R_E$ in 3.267Gy and then it 'Bounds' from $51.4R_E$ to $60.33R_E$ in 1.2Gy. At $17R_E$ Laplace plane transition occurs and at $33R_E$ Cassini state transition occurs. Cuk Matija have assumed that Moon is born from the Giant impact generated debris disk when Mars sized planetesimal made a glancing angle collision with proto Earth resulting in high obliquity and high Angular Momentum Earth. This resulted in isotopic identity of wide range of materials on Earth and Moon and the subsequent tidal evolution resulted in achieving climatically favorable Earth's obliquity of 23.44° . The application of AKM to this Fits and Bound model of E-M system gives a theoretical LOD curve which has precise match with observed LOD curve over last 1.2Gy as shown in SOM S6 using the Protocol Exchange algorithm http://org.1038/protex.2019.017 . In addition all the observed performance parameters are theoretically justified. The observed parameters are LOD = 24h, LOM/LOD= 27.32 and velocity of recession of Moon as 3.82±0,07cm/y all these are a natural corollary of AKM. Using the Protocol Exchange algorithim in Supplementary-On-line Materials S6 AKM is vindicated on every count. In effect AKM has helped arrive at the correct theoretical formalism of Observed LOD curve. This theoretical formalism will give the datum against which the real time fluctuations in LOD will be compared and the precursors of the impending Earth-quake and sudden volcanic eruptions will be identified and used to give Early Warning and Forecasting for terrestrial disasters triggered by plate tectonic movements.

List of Abbreviations:

J_0 – orbital angular momentum of Earth-Moon system;

J_1 – spin angular momentum of Moon;

J_2 – Earth's spin angular momentum of Earth-Moon system;

J_3 – vectorial sum of J_0 and J_1;

J_4 – J_{Total} - Resultant vectorial sum of angular momentums of Earth-Moon system;

KM – Kinematic Model;

AKM – Advanced Kinematic Model;

E-M system – Earth-Moon System;

'a' – semi-major xis;

'a_{G1}' – inner geo-synchronous orbit;

'a_{G2}' – outer geo-synchronous orbit;

'ω' – spin angular velocity of Earth or Moon;

'Ω' – orbital angular velocity of Earth-Moon;

NS – Neutron Star;

BH – Black Hole;

'α' – angle of inclination of Lunar Plane;

'β' – Moon's Obliquity;

'ϕ' – Earth's Obliquity;

Declarations:

Availability of Data:

Dataset for this Research has been generated in Section 3 and tabulated in Section 4 of the main Text as well as it is tabulated in S2 file of the Supplementary Information.

The protocal for determining LOD in the past epoch is deposited at http://doi.org/10.1038/protex.2019.017,

All the data and algorithm regarding LOM/LOD , LOD. Transit Time of Moon from higher Energy Orbit to a lower Energy Orbit and astrometric parameters of Sun-Earth-Moon system will be deposited in Open Topography Community Data Space once the access is obtained. Once the access is obtained the link of the Public Repository will be provided.

Conflict of Interest:

I have no conflict of interest financial or otherwise whatsoever with anybody.

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Author's contribution:

i. In 1995 at 82nd Session of India Science Congress I presented my first Research Paper titled 'Theoretical Formulation of Earth-Moon system revisited'.

ii. In 1998 I got registered as D.Sc candidate in Post Graduate Mathematics Department, BRA Bihar University, Muzaffarpur, Bihar, India, under Prof. Bhola Ishwar.
iii. Since then *this topic has been my valid primary research*.
iv. I have published my research as Conference Proceedings
v. Proceedings in 34th Scientific Assembly (2002) Houston, USA,
vi. Proceedings in 35th Scientific Assembly (2004),Paris France,
vii. Proceedings in 39th Scientific Assembly (2012) Mysore, India,
viii. in CELMEC V-2009 in Viterbo, Rome, Italy and
ix. in CELMEC VII-2017 in Viterbo, Rome, Italy
x. I also have two research articles in Advances in Space Research (43,460-466, 2009)
xi. and in Earth, Moon and Planets (108,#1,15-37,2011).

My three papers are under review:

A. in astronomy and computing: ASCOM-D-20-00005- precise theoretical formalism of observed LOD curve extending back to 1,2Gy.
B. ASCOM-D-20-00010- early demise of Phobos based on Architectural Design Rules.
C. 2020EF001728 – Past-Present and Future of Sun-Earth-Moon System in the Journal Earth's Future.

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Figures

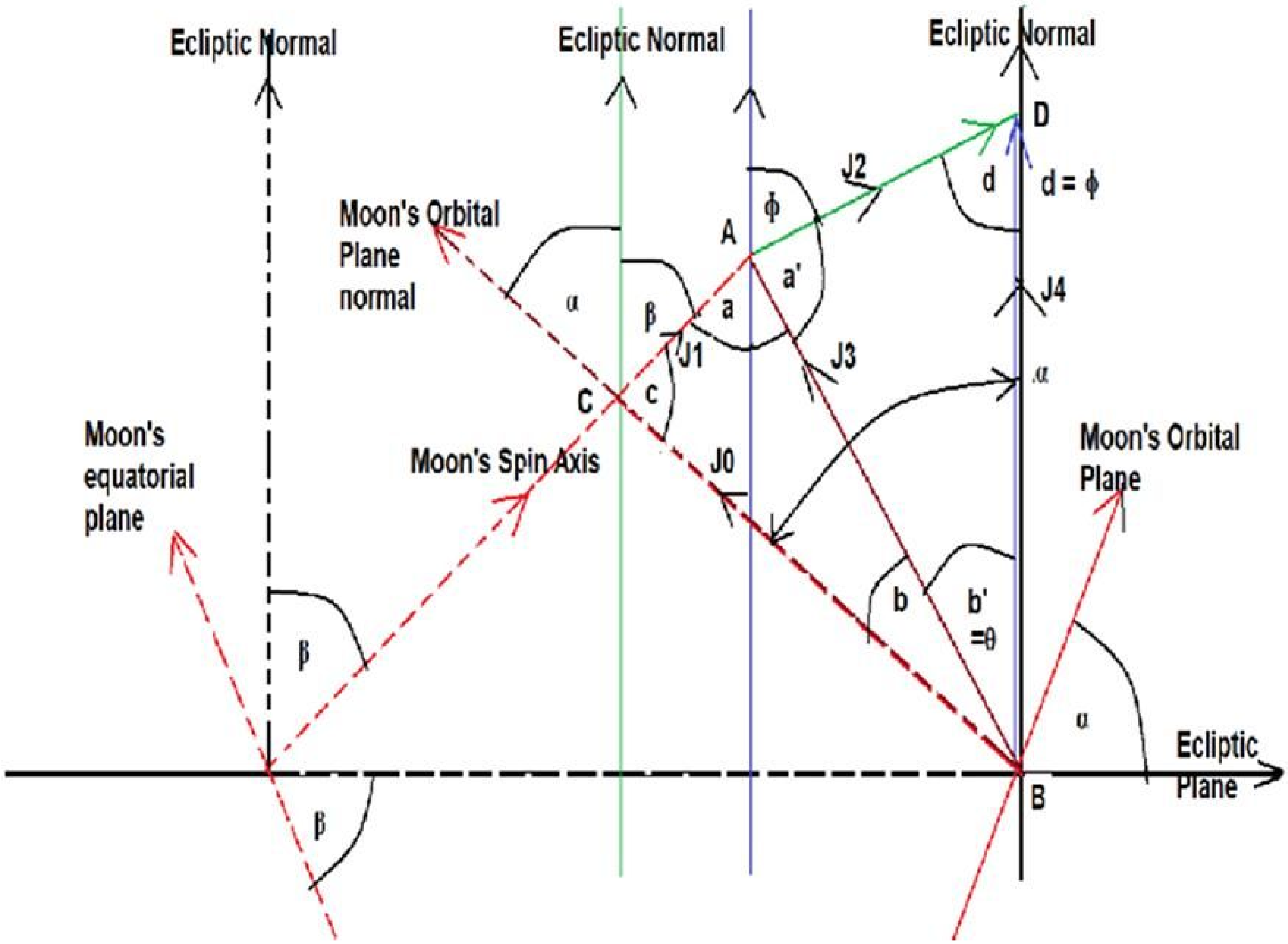

Figure 1

Spin-Orbital configuration of Earth-Moon System.

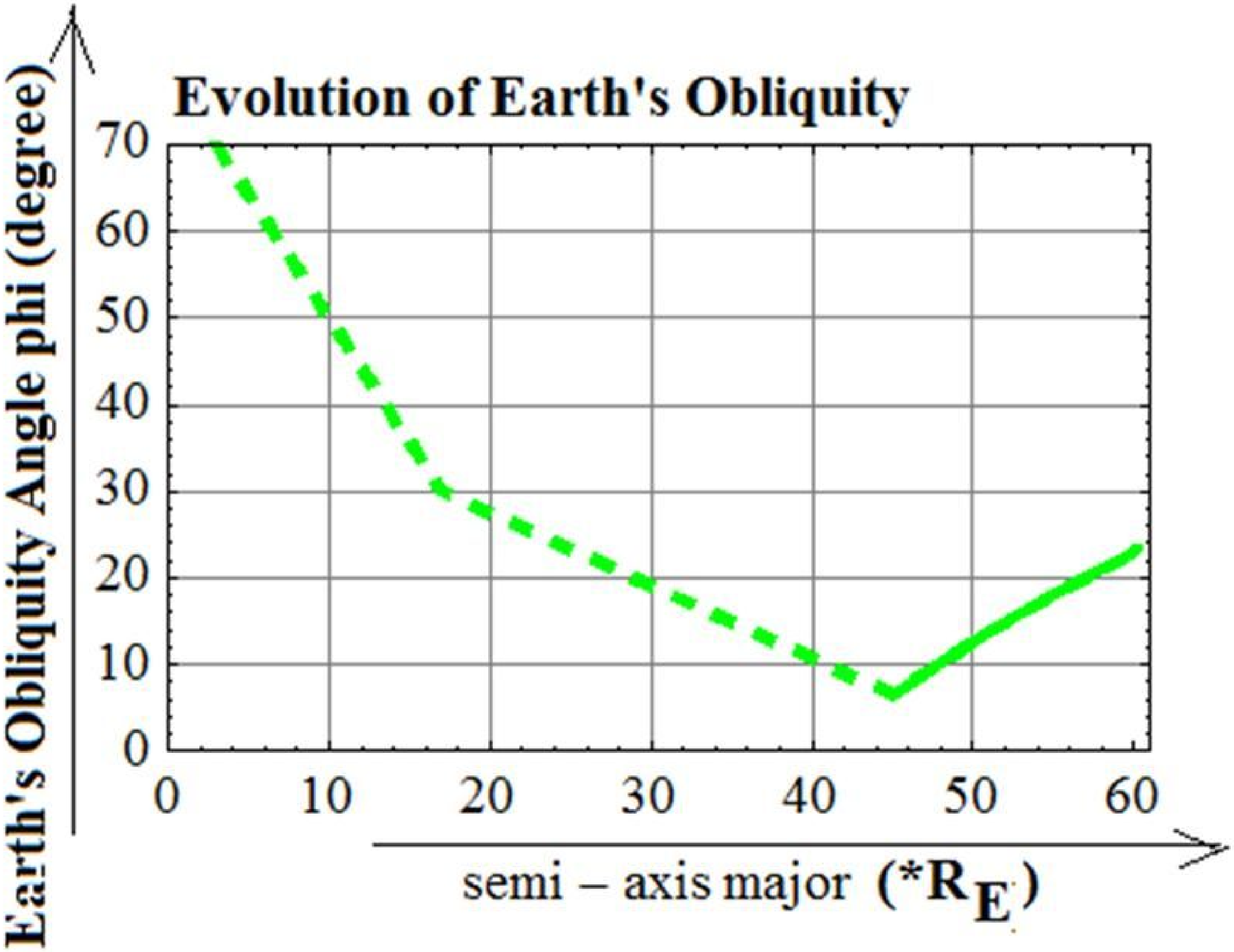

Figure 2

Earth’s Obliquity angle (□°) evolution according to AKM (bold green) and according to Simulation results(dash green) (Cuk et.al.2016).

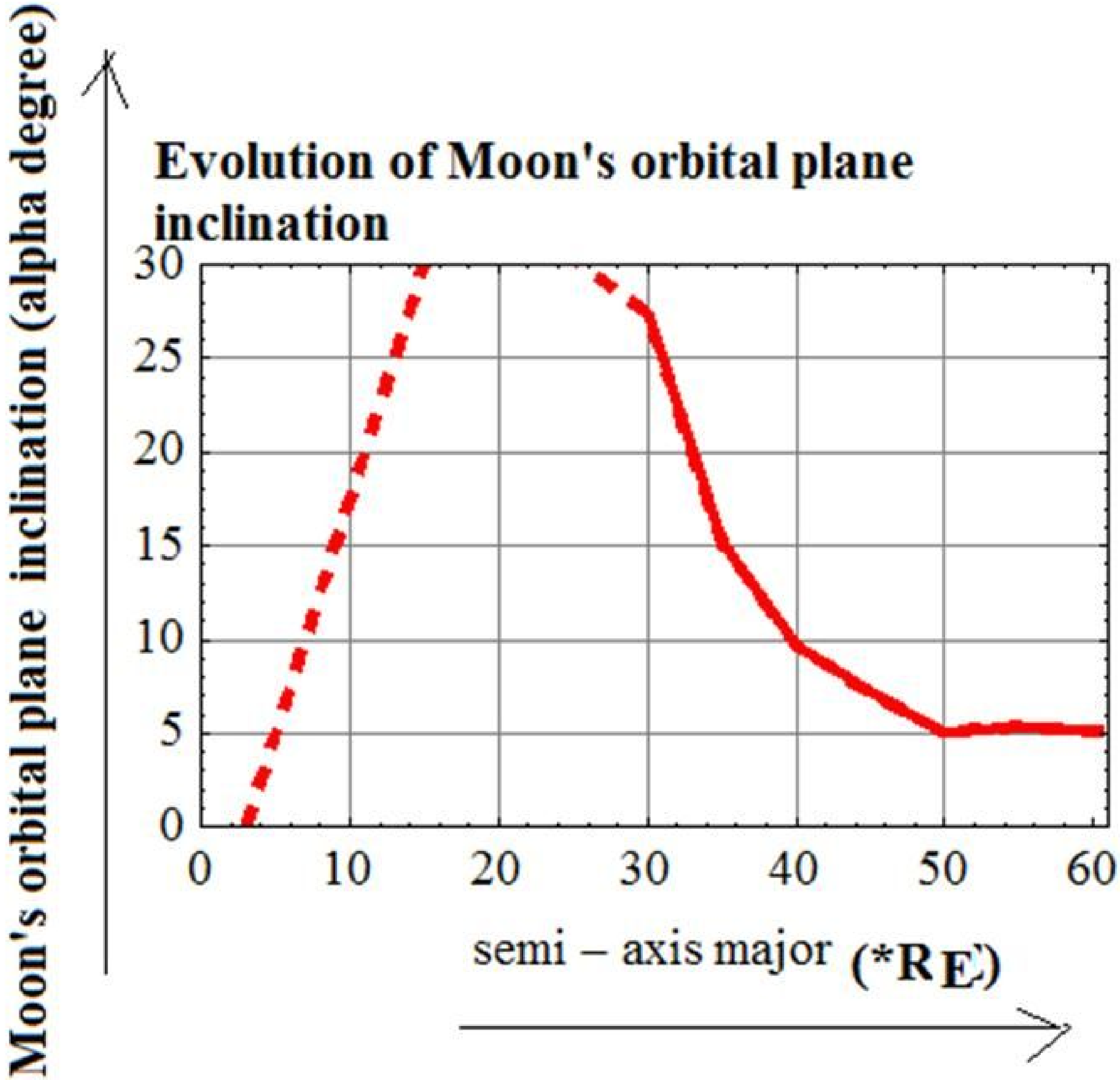

Figure 3

Moon’s orbital plane inclination (α°) based on AKM (bold red) and based on Simulation results (dash red) (Cuk et.al.2016)

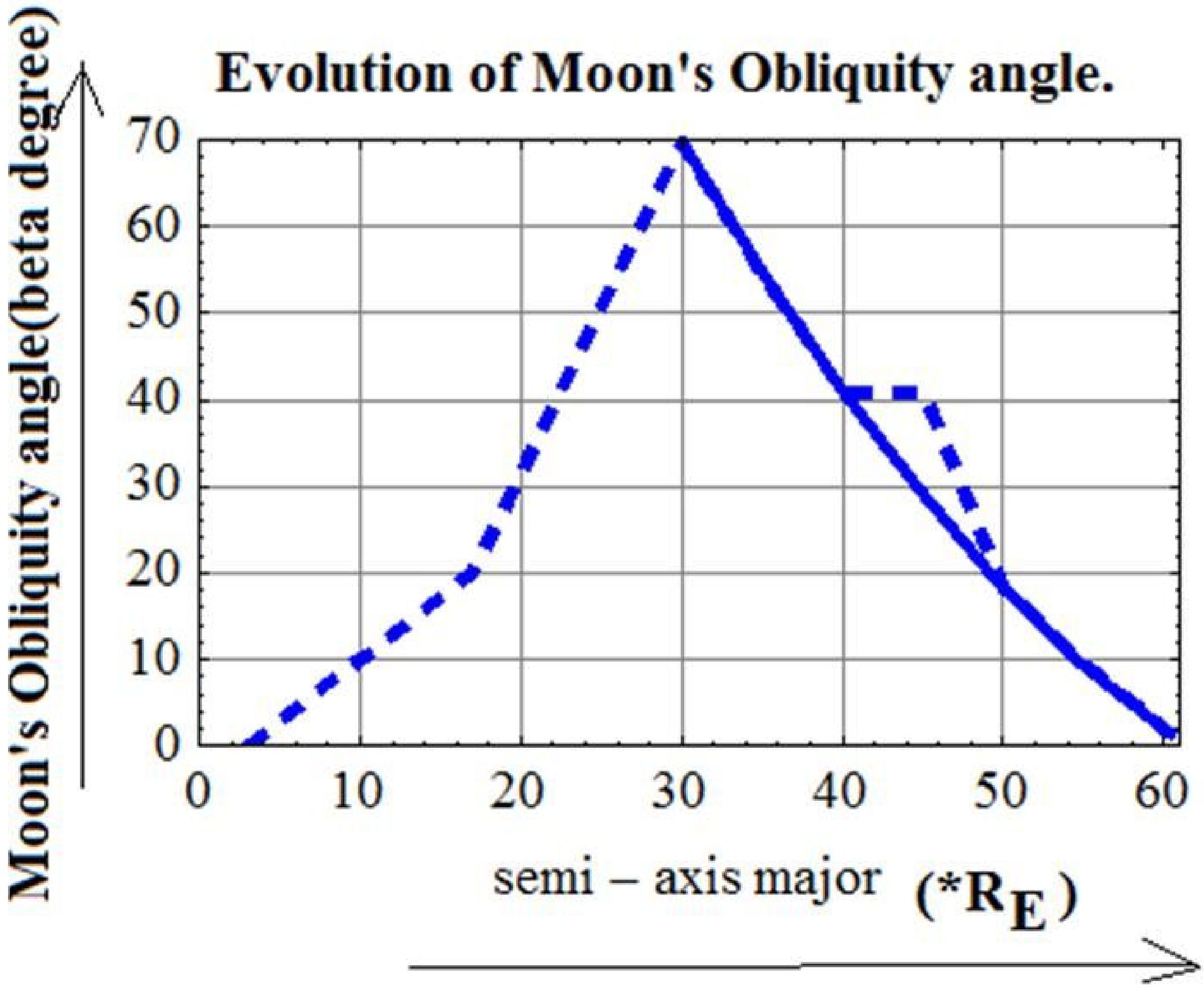

Figure 4

Moon's Obliquity (β°) based on AKM (bold blue) and based on Simulation results (dash blue) (Cuk et.al.2016)

Supplementary Files

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