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Research Article

Keywords: Group decision making, Information fusion, Evidence theory, Comprehensive grey relational analysis, Explainable AI

Posted Date: June 7th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-551277/v1>

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Group Decision-Making Method Based on Grey Relational Analysis and Evidence Theory for XAI

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Abstract: For the uncertainty and complexity ingroup decision making and the differences of decision makers' reliabilities, a group decision making method based on grey relational analysis and evidence theory is proposed. Combining grey relational analysis with evidence theory, a novel decision-making method extracting the degree of ignorance for individual decision makers' information and constructing the Mass function is presented based on the comprehensive grey relational analysis (CGRA) method. We should also address how AI systems make their black box decisions, which calls for research into Explainable AI (XAI) by pursuing reverse engineering and self-explainability in AI. Considering the differences of decision makers' reliabilities, the Mass function is modified by the evidence weight, and the group decision information is fused by the Dempster's combination rule. On this basis, the Mass function is further transformed into the probability by the Pignistic probability transformation, which issued for ranking analysis of group decision making. Finally, the proposed method is applied to the green supplier selection, and the comparative analysis is further performed to verify the rationality and effectiveness of the proposed method.

Keywords: Group decision making; Information fusion; Evidence theory;

1 Introduction

Multi-attribute group decision making is widely used in economy (Tang et al. 2019), management (Cali and Balaman 2019), engineering (Peng 2019) and other fields because it can extract the cognitive judgment information of multiple decision makers and carry out a comprehensive evaluation to alternatives. In group decision making, how to effectively fuse the individual decision-making information is the key of decision-making analysis, which has an important influence on the ranking of alternatives.

In reality, when making group decision, besides the weighted sum of evaluation values of alternatives under group decision makers for ranking analysis, in more cases, it uses the individual experience and the collected relevant facts and data as evidence of judgment and reasoning, and then finds out the optimal alternative. Here, it is called the group decision making of evidence reasoning. Evidence theory (Chemoun et al. 2018), as an effective information fusion technology, has been widely used in uncertain reasoning, information fusion and group decision-making analysis. At present, some scholars have integrated evidence theory into group decision making, and discussed the group decision making methods of evidence reasoning. Wang et al. (2015) proposed a multi-objective group decision-making CBM model based on D-S evidence theory. Leung et al. (2013) combined group decision making with evidence theory and developed a comprehensive method for automatically identifying and eliminating unreliable evidence. Gupta et al. (2016) utilized evidence theory, Bayes approximation and VIKOR method to present a multi-attribute group decision-making method. Wan et al. (2013) proposed a multi-attribute group decision-making method, in which the decision makers' weights were determined by evidence theory and Bayes

approximation. The introduction and application of these methods have promoted the further development of evidence theory and group decision making theory.

In group decision making, evidence theory can effectively fuse Mass functions of evidence bodies from individual decision makers. Because the quality of Mass function construction is directly related to whether the uncertainty characteristics of the problem can be described reasonably, and it has a decisive impact on the subsequent combination reasoning and group decision-making effect. Hence, accurately obtaining Mass functions is a key link in applying evidence theory (Zhou et al. 2018). At present, the determination of Mass function is generally a digital representation of support degree of different propositions in the frame of discernment based on the obtained evidence and personal experience, which has a certain subjectivity (Sun et al. 2018). In addition, different experts may give different Mass functions for the same proposition with the same evidence. Hence, how to determine the Mass function scientifically to reduce the group decision-making uncertainty is an urgent problem.

The Mass function of alternative under each individual decision maker is affected by the decision maker's reliability and its own measurement value, in which the measurement value can be reflected by the relational degree between alternative sequence and positive (negative) ideal point sequence. The greater the relational degree, the higher the event probability, and the larger the Mass function corresponding to the alternative. Grey relational analysis (GRA) is an effective method for describing the relational degree (Li et al. 2015). Its basic idea is to judge the relational degree of factors according to the similarity degree of sequence curves, and it has the advantage of no special requirements for the sample size and statistical regularity (Wang and Yao 2018). Therefore, GRA can be adopted to construct Mass

function objectively. However, the existing GRA focuses on measuring the proximity of geometric shapes between curves, while the similarity of speed and acceleration and the volatility of curve development trend are less considered. It is necessary to measure GRA degree more objectively and comprehensively.

In addition, when using evidence theory to solve group decision making problems, the reliability of evidence information of individual decisionmakers is usually no significant difference (Wang and Dang 2009). But in actual decision-making, the reliabilities of different decision makers are often different because of their professional background, social status, knowledge, and other reasons. Especially in the case of many alternatives and attributes, if decision makers' evaluation information is simply weighted average or the differences are ignored, it will inevitably affect the final evaluation result. Therefore, it is necessary to consider the differences of decision makers' reliabilities. However, the current research on group decision-making seldom considers the influence of the differences. If these differences are introduced into evidence reasoning and modified the evidence information, it will undoubtedly promote the effect of group decision information fusion.

According to the above analysis, starting from the idea of information fusion, the paper combines GRA with evidence theory, a method of extracting the degree of ignorance (DOI) for individual decision makers' information and constructing Mass function based on CGRA is proposed. Then, a weighted evidence theory is adopted to fuse the group decision information, and alternative ranking analysis is performed according to Pignistic probability transformation. Finally, the proposed method is used to deal with the problem of green supplier selection, and achieve a satisfactory result.

The main structure of this paper is as follows: Section 2 starts with a brief presentation of necessary related concepts. Section 3 proposes a group decision making method of relational evidence reasoning based on information fusion. In Section 4 the proposed method is used to evaluate the green suppliers. Finally, some conclusions are presented in Section 5.

2 Preliminaries

2.1 Evidence theory

Evidence theory is an uncertain reasoning method, which can fuse data from different sources. It has been widely used in the fields of information fusion (Seiti and Hafezalkotob 2018), fault diagnosis (Lin et al. 2018). In evidence theory, $\Theta = \{g_1, g_2, \dots, g_N\}$ is the frame of discernment, and $g_l \cap g_q = \emptyset (l \neq q)$. If there are N elements in Θ , the power set consisted by all subsets of Θ is $2^\Theta = \{\emptyset, \{g_1\}, \{g_2\}, \dots, \{g_N\}, \{g_1, g_2\}, \{g_1, g_3\}, \dots, \{g_1, g_2, g_N\}, \dots, \Theta\}$.

Definition 1 (Khojasteh and Jadid 2017) Let E be any subset of $\Theta = \{g_1, g_2, \dots, g_N\}$, if there exists a mapping $m: 2^\Theta \rightarrow [0, 1]$ which satisfies $m(\emptyset) = 0, m(E) \in [0, 1]$, $\sum_{E \subseteq \Theta} m(E) = 1$, then $m(E)$ is called the basic probability assignment (BPA) function of E on Θ , also called Mass function. For any $E \subseteq \Theta$, $m(E)$ represents the precise degree of trust in E .

Definition 2 For any $E \subseteq \Theta$, let $m_1, m_2, \dots, m_n$ be the Mass functions on Θ , then $m_1, m_2, \dots, m_n$ can be combined with Dempster's combination rules as follows

$$(m_1 \oplus m_2 \oplus \dots \oplus m_n)(E) = \begin{cases} 0, & E = \emptyset; \\ \frac{1}{K} \sum_{\substack{E_1 \cap E_2 \cap \dots \cap E_n = E}} m_1(E_1) m_2(E_2) \dots m_n(E_n), & E \neq \emptyset. \end{cases} \quad (1)$$

Where $K = \sum_{E_1 \cap \dots \cap E_n \neq \emptyset} m_1(E_1) \cdot m_2(E_2) \cdot \dots \cdot m_n(E_n)$. K is a normalized coefficient, and the

size of K reflects the degree of conflict among evidence.

2.2 Pignistic probability transformation

Pignistic probability was first proposed by Smith to solve system decision problems with uncertain information. Smets further analyzed its theoretical basis, and proposed to convert Mass function into Pignistic probability function, which provided an effective solution to uncertain decision-making problems (Smets 2005).

Definition 3 Let $m(G)$ be a Mass function on Θ , then its Pignistic probability function $BetP_m(\delta): \Theta \rightarrow [0,1]$ is defined as follows

$$BetP_m(\delta) = \sum_{G \subseteq \Theta, \delta \in G} \frac{1}{|G|} \frac{m(G)}{1 - m(\emptyset)} \quad (2)$$

Where $m(\emptyset) \neq 1$, $|G|$ is the cardinality of subset G . Pignistic transformation is the process of converting m to $BetP_m$, which uses to make a decision. If no otherwise

specified, it is considered that $m(\emptyset) = 0$, and $\frac{m(G)}{1 - m(\emptyset)}$ is simplified to $m(G)$.

3 The proposed group decision making method

In multi-attribute group decision making, let $A = \{a_1, a_2, \dots, a_n\}$ and $B = \{b_1, b_2, \dots, b_m\}$ be the alternative set and attribute set, respectively.

$DM = \{DM_1, DM_2, \dots, DM_h\}$ is a set of decision makers, and $\psi_k (\psi_k \in [0,1])$ is the

weight of $DM_k (k=1,2,\dots,h)$, where $\sum_{k=1}^h \psi_k = 1$. Let $c_{ij}^{(k)}$ be the attribute value

provided by DM_k on the assessment of a_i about b_j , then the decision matrix of

DM_k is $C^{(k)} = [c_{ij}^{(k)}]_{n \times m}$. The problem to be solved is how to effectively fuse group

decision making information and use decision making analysis method to rank alternatives.

In order to eliminate the differences of attributes in dimension, let $c_j^{(k)-} = \min_i \{c_{ij}^{(k)}\}$, $c_j^{(k)+} = \max_i \{c_{ij}^{(k)}\}$, the decision matrix $\mathbf{C}^{(k)} = [c_{ij}^{(k)}]_{n \times m}$ can be normalized as $\mathbf{X}^{(k)} = [x_{ij}^{(k)}]_{n \times m}$, where

$$x_{ij}^{(k)} = \begin{cases} \frac{c_{ij}^{(k)} - c_j^{(k)-}}{c_j^{(k)+} - c_j^{(k)-}}, & \text{for the benefit attribute } b_j \\ \frac{c_j^{(k)+} - c_{ij}^{(k)}}{c_j^{(k)+} - c_j^{(k)-}}, & \text{for the cost attribute } b_j \end{cases} \quad (3)$$

In information fusion decision making, the key to aggregating group information with evidence theory is to obtain the Mass function of the alternative (also known as the evidence) under individual decisionmakers. However, the existing Mass function is usually set subjectively. To obtain the Mass function more objectively and reasonably, a CGRA method is proposed to determine the Mass functions of alternatives. On this basis, the group evidence information is fused by weighted evidence theory and the alternatives are ranked by Pignistic probability transformation.

3.1 Construction of Mass Function Based on CGRA Method

Since the Mass functions under individual decision makers are the key to synthesizing group information with evidence theory, the premise of solving Mass function is to determine the degree of ignorance (DOI) for individual decision makers' information (Wang and Dang 2009). Hence, combining GRA with evidence theory, a CGRA method is proposed to determine the DOI and the Mass function.

3.1.1 The proposed CGRA method

To measure the correlation accurately, the proximity, similarity and volatility of sequence evolution are comprehensively measured by considering the absolute level, dynamic level and variability level among sequences. On this basis, the comprehensive grey relational analysis (CGRA) method is constructed.

For the proximity between the alternative sequence and the positive (negative) ideal point sequence, their absolute level can be measured by the overall displacement change between the sequences.

Definition 4 Let $a_i^{(k)} = (x_{i1}^{(k)}, x_{i2}^{(k)}, \dots, x_{im}^{(k)}) (i=1, 2, \dots, n)$ and $a^{(k)+(-)} = (x_1^{(k)+(-)}, x_2^{(k)+(-)}, \dots, x_m^{(k)+(-)})$ be the alternative sequence and the positive (negative) ideal point sequence under decision maker DM_k respectively, then the overall displacement change of $a_i^{(k)}$ and $a^{(k)+(-)}$ is expressed as follows

$$d_i^{(k)+(-)} = \left[\sum_{j=1}^m \left(x_{ij}^{(k)} - x_j^{(k)+(-)} \right)^2 \right]^{1/2} \quad (4)$$

Where $x_j^{(k)+} = \max_{1 \leq i \leq n} x_{ij}^{(k)}$, $x_j^{(k)-} = \min_{1 \leq i \leq n} x_{ij}^{(k)}$. $d_i^{(k)+(-)}$ characterizes the proximity of the absolute level between $a_i^{(k)}$ and $a^{(k)+(-)}$ in development process, the positive ideal point sequence corresponds to the superscript +, and the negative ideal point sequence corresponds to the superscript -. When $d_i^{(k)+(-)} = 0$, it indicates that the development process of $a_i^{(k)}$ and $a^{(k)+(-)}$ is exactly the same.

To improve the distinguishing degree of relational results in traditional GRA method, the proximity relational degree is constructed based on Liu et al. (2014), which adopts the form of exponential function $f(x) = e^{-x}$ with faster decline rate and stronger distinguishability.

Definition 5 Let $d_i^{(k)+(-)}$ be the overall displacement change of sequences $a_i^{(k)}$ and $a^{(k)+(-)}$ under decision maker DM_k , then the proximity relational degree of $a_i^{(k)}$ and $a^{(k)+(-)}$ is expressed as follows

$$\alpha_i^{(k)+(-)} = \exp(-d_i^{(k)+(-)}) \quad (5)$$

Where $\alpha_i^{(k)+(-)}$ measures the proximity degree of absolute amount between $a_i^{(k)}$ and $a^{(k)+(-)}$ from the perspective of proximity. The larger the $\alpha_i^{(k)+(-)}$ is, the closer the change of overall absolute level between $a_i^{(k)}$ and $a^{(k)+(-)}$ in development process will be.

For the similarity of development trends between the alternative sequence $a_i^{(k)}$ and the positive (negative) ideal point sequence $a^{(k)+(-)}$, the dynamic level can be measured by the speed and acceleration of development trends of sequences.

Definition 6 For the alternative sequence $a_i^{(k)}$ and the positive (negative) ideal point sequence $a^{(k)+(-)}$ under decision maker DM_k , the similar speed relational coefficient of $a_i^{(k)}$ and $a^{(k)+(-)}$ from attribute b_{j-1} to attribute b_j is expressed as follows

$$\beta_{ij}^{(k)+(-)} = \text{sgn}((x_{ij}^{(k)})' \times (x_j^{(k)+(-)})') \cdot \frac{1 + |(x_{ij}^{(k)})'| + |(x_j^{(k)+(-)})'|}{1 + |(x_{ij}^{(k)})'| + |(x_j^{(k)+(-)})'| + |(x_{ij}^{(k)})' - (x_j^{(k)+(-)})'|} \quad (6)$$

Where $(x_{ij}^{(k)})'$ and $(x_j^{(k)+(-)})'$ are the speeds of sequences $a_i^{(k)}$ and $a^{(k)+(-)}$ from b_{j-1}

to b_j , respectively; $(x_{ij}^{(k)})' = \frac{x_{ij}^{(k)} - x_{i(j-1)}^{(k)}}{j - (j-1)} = x_{ij}^{(k)} - x_{i(j-1)}^{(k)}$; $j = 2, 3, \dots, m$. Similarly,

$$(x_j^{(k)+(-)})' = x_j^{(k)+(-)} - x_{j-1}^{(k)+(-)}; \text{sgn}((x_{ij}^{(k)})' \times (x_j^{(k)+(-)})') = \begin{cases} 1, & ((x_{ij}^{(k)})' \times (x_j^{(k)+(-)})') \geq 0 \\ -1, & ((x_{ij}^{(k)})' \times (x_j^{(k)+(-)})') < 0 \end{cases}.$$

According to the similar speed relational coefficient $\beta_{ij}^{(k)+(-)}$, the similar speed relational degree of $a_i^{(k)}$ and $a^{(k)+(-)}$ in the whole attribute set is expressed as follows

$$\beta_i^{(k)+(-)} = \frac{1}{(m-1)} \sum_{j=2}^m \beta_{ij}^{(k)+(-)} \quad (7)$$

Where $\beta_i^{(k)+(-)}$ measures the similarity degree of development speed between $a_i^{(k)}$ and $a^{(k)+(-)}$ from the perspective of similarity. The larger the $\beta_i^{(k)+(-)}$ is, the more similar the overall development speed between $a_i^{(k)}$ and $a^{(k)+(-)}$ will be.

Definition 7 For the alternative sequence $a_i^{(k)}$ and the positive (negative) ideal point sequence $a^{(k)+(-)}$ under decision maker DM_k , the similar acceleration relational coefficient of $a_i^{(k)}$ and $a^{(k)+(-)}$ from attribute b_{j-1} to attribute b_j is expressed as follows

$$\beta_{ij}^{(k)+(-)} = \text{sgn}((x_{ij}^{(k)})'' \times (x_j^{(k)+(-)})'') \cdot \frac{1 + |(x_{ij}^{(k)})''| + |(x_j^{(k)+(-)})''|}{1 + |(x_{ij}^{(k)})''| + |(x_j^{(k)+(-)})''| + |(x_{ij}^{(k)})'' - (x_j^{(k)+(-)})''|} \quad (8)$$

Where $(x_{ij}^{(k)})''$ and $(x_j^{(k)+(-)})''$ are the accelerations of sequences $a_i^{(k)}$ and $a^{(k)+(-)}$

from b_{j-1} to b_j , respectively; $(x_{ij}^{(k)})'' = \frac{(x_{ij}^{(k)})' - (x_{i(j-1)}^{(k)})'}{2} = \frac{x_{ij}^{(k)} - 2x_{i(j-1)}^{(k)} + x_{i(j-2)}^{(k)}}{2}$;

$j = 3, 4, \dots, m$. Similarly, $(x_j^{(k)+(-)})'' = \frac{x_j^{(k)+(-)} - 2x_{(j-1)}^{(k)+(-)} + x_{(j-2)}^{(k)+(-)}}{2}$;

$$\text{sgn}((x_{ij}^{(k)})'' \times (x_j^{(k)+(-)})'') = \begin{cases} 1, & (x_{ij}^{(k)})'' \times (x_j^{(k)+(-)})'' \geq 0 \\ -1, & (x_{ij}^{(k)})'' \times (x_j^{(k)+(-)})'' < 0 \end{cases}.$$

According to the similar acceleration relational coefficient $\beta_{ij}^{(k)+(-)}$, the similar acceleration relational degree of $a_i^{(k)}$ and $a^{(k)+(-)}$ in the whole attribute set is

expressed as follows

$$\beta_i^{(k)^{+(-)}} = \frac{1}{m-2} \sum_{j=3}^m \beta_{ij}^{(k)^{+(-)}} \quad (9)$$

Where $\beta_i^{(k)^{+(-)}}$ measures the similarity degree of the change of development speed between $a_i^{(k)}$ and $a^{(k)^{+(-)}}$ from the perspective of similarity. The larger the $\beta_i^{(k)^{+(-)}}$ is, the more similar the level of the change of overall development speed between $a_i^{(k)}$ and $a^{(k)^{+(-)}}$ will be.

Definition 8 Let $\beta_i^{(k)^{+(-)}}$ and $\beta_i^{(k)^{+(-)}}$ be respectively the similar speed relational degree and acceleration relational degree of sequences $a_i^{(k)}$ and $a^{(k)^{+(-)}}$ under decision maker DM_k , then the similarity relational degree of $a_i^{(k)}$ and $a^{(k)^{+(-)}}$ is expressed as follows

$$\beta_i^{(k)^{+(-)}} = \frac{1}{2} (\beta_i^{(k)^{+(-)}} + \beta_i^{(k)^{+(-)}}) \quad (10)$$

Where $\beta_i^{(k)^{+(-)}}$ measures the similarity degree of the development speed and the change of development speed between $a_i^{(k)}$ and $a^{(k)^{+(-)}}$ from the perspective of similarity. The larger the $\beta_i^{(k)^{+(-)}}$ is, the more similar the overall development speed and the level of the change of overall development speed between $a_i^{(k)}$ and $a^{(k)^{+(-)}}$ will be.

For the volatility of development trends between the alternative sequence $a_i^{(k)}$ and the positive (negative) ideal point sequence $a^{(k)^{+(-)}}$, the variability level can be measured by the change of volatility distance between sequences.

Definition 9 For the alternative sequence $a_i^{(k)}$ and the positive (negative) ideal point

sequence $a^{(k)+(-)}$ under decision maker DM_k , the overall volatility distance of $a_i^{(k)}$ and $a^{(k)+(-)}$ is expressed as follows

$$f_i^{(k)+(-)} = \left| \frac{\sigma_i^{(k)}}{\bar{x}_i^{(k)}} - \frac{\sigma^{(k)+(-)}}{\bar{x}^{(k)+(-)}} \right| \quad (11)$$

Where $\bar{x}_i^{(k)} = \frac{1}{m} \sum_{j=1}^m x_{ij}^{(k)}$, $\sigma_i^{(k)} = \sqrt{\frac{1}{m-1} \sum_{j=1}^m (x_{ij}^{(k)} - \bar{x}_i^{(k)})^2}$. $\bar{x}_i^{(k)}$ and $\sigma_i^{(k)}$ denote the mean value and the standard deviation of variables about alternative a_i in attribute set B , respectively. The meanings of $\bar{x}^{(k)+(-)}$ and $\sigma^{(k)+(-)}$ are similar to those of $\bar{x}_i^{(k)}$ and $\sigma_i^{(k)}$. $f_i^{(k)+(-)}$ depicts the volatility difference between sequences $a_i^{(k)}$ and $a^{(k)+(-)}$ in development process. Obviously, if the proximity and similarity of the two sequences are greater, the degree of volatility should be similar. That is, $f_i^{(k)+(-)}$ should be smaller. Coefficient of variation is a representative of the degree of sequence volatility. Considering that the larger the coefficient of variation is, the bigger the degree of sequence volatility is, and the more unstable the development of sequence is. It can be seen that it is a negative index, so the reciprocal is used in calculating distance.

Definition 10 Let $f_i^{(k)+(-)}$ be the overall volatility distance of sequences $a_i^{(k)}$ and $a^{(k)+(-)}$ under decision maker DM_k , then the volatility relational degree of $a_i^{(k)}$ and $a^{(k)+(-)}$ is expressed as follows

$$\gamma_i^{(k)+(-)} = \exp(-f_i^{(k)+(-)}) \quad (12)$$

Where $\gamma_i^{(k)+(-)}$ measures the volatility degree of development trends between $a_i^{(k)}$ and $a^{(k)+(-)}$ from the perspective of volatility. The larger the $\gamma_i^{(k)+(-)}$ is, the more consistent the volatility degree between $a_i^{(k)}$ and $a^{(k)+(-)}$ in development process will

be.

According to the definitions of proximity relational degree $\alpha_i^{(k)+(-)}$, similarity relational degree $\beta_i^{(k)+(-)}$ and volatility relational degree $\gamma_i^{(k)+(-)}$ of $a_i^{(k)}$ and $a^{(k)+(-)}$ under decision maker DM_k , the larger the $\alpha_i^{(k)+}$, $\beta_i^{(k)+}$ and $\gamma_i^{(k)+}$ are, the closer the alternative a_i is to the positive ideal alternative; the smaller the $\alpha_i^{(k)-}$, $\beta_i^{(k)-}$ and $\gamma_i^{(k)-}$ are, the further the alternative a_i is from the negative ideal alternative. Based on this, the comprehensive grey relational analysis (CGRA) degree is constructed to measure the superiority and inferiority level between alternative and ideal alternatives under individual decision makers, which lays a foundation for further obtaining the Mass function of alternative (or call it evidence) under individual decision makers.

Definition 11 For the proximity relational degree $\alpha_i^{(k)+(-)}$, similarity relational degree $\beta_i^{(k)+(-)}$ and volatility relational degree $\gamma_i^{(k)+(-)}$ of $a_i^{(k)}$ and $a^{(k)+(-)}$ under decision maker DM_k , the CGRA degree of alternative a_i is expressed as follows

$$\phi_i^{(k)} = w_1 \frac{\alpha_i^{(k)+}}{\alpha_i^{(k)+} + \alpha_i^{(k)-}} + w_2 \frac{\beta_i^{(k)+}}{\beta_i^{(k)+} + \beta_i^{(k)-}} + w_3 \frac{\gamma_i^{(k)+}}{\gamma_i^{(k)+} + \gamma_i^{(k)-}} \quad (13)$$

Where w_1 , w_2 and w_3 are respectively the weights of three kinds of relational degrees $\alpha_i^{(k)+(-)}$, $\beta_i^{(k)+(-)}$ and $\gamma_i^{(k)+(-)}$, and satisfies $w_1 + w_2 + w_3 = 1$. The weight coefficient $w_v (v=1,2,3)$ can be determined subjectively or objectively according to the actual situation of the study. The bigger $\phi_i^{(k)}$ is, the better alternative a_i is under decision maker DM_k .

3.1.2 Determination of Mass Function using CGRA Method

On the basis of the proposed CGRA method in Section 3.1.1, the degree of

ignorance (DOI) for individual decision makers' information and the Mass function can be obtained. The detailed contents are presented as follows.

If the distribution of alternatives under a decision maker $DM_k (k=1,2,L,h)$ is more concentrated on a certain alternative, it shows that the decision-making environment under DM_k is more conducive to decision making. This is reflected in evidence theory, which indicates that the information DOI of DM_k is lower, and the alternative under DM_k should give a bigger Mass value than that of a decision maker with high DOI.

According to Definition 11, the CGRA degree is a measure of superiority and inferiority level among alternatives and ideal alternatives in individual decision-making environments. Based on this, we can obtain the following definitions.

Definition 12 For the CGRA degree $\phi_i^{(k)}$ of alternative a_i under decision maker $DM_k (k=1,2,L,h)$, the information DOI of DM_k is expressed as follows

$$DOI(DM_k) = \left(\prod_{i=1}^n \phi_i^{(k)} \right)^{\frac{1}{n}} \quad (14)$$

When using evidence theory to make fusion decision on group information, it is necessary to fuse the evidence information of individual decision makers in order to choose the ideal alternative among many alternatives. The determination of Mass function is the basis of information fusion. Based on the above analysis, the Mass function $m_i^{(k)}$ of alternative a_i under DM_k is expressed as follows

$$m_i^{(k)} = (1 - DOI(DM_k)) \phi_i^{(k)} \quad (15)$$

Due to the complexities of objective things and the limitations of human cognition, there is uncertainty in the overall understanding, the sum of Mass functions

of all the alternatives under DM_k is less than 1, that is, $\sum_{i=1}^n m_i^{(k)} < 1$. In order to make full use of the obtained information and reduce the uncertainty in the decision-making, the remaining part of Mass function is assigned to the frame of discernment Θ itself, which indicates the degree of support for all the alternatives and participates in the calculation of information fusion. Therefore, the Mass function of the overall uncertainty under DM_k can be obtained as follows

$$m_{\Theta}^{(k)} = 1 - \sum_{i=1}^n m_i^{(k)} \quad (16)$$

3.2 Fusion of Group Decision-making Information Based on Weighted Evidence Theory

In group decision-making, evidence theory usually treats the evidence information under individual decision makers equally. However, due to the differences of individual decision makers' own conditions and knowledge experience in practical application, the reliability of evidence provided by different individuals is often different, and the evidence information should be discounted according to the different reliabilities. Based on this, a modification method of evidence information based on evidence weight is proposed, and then the group decision-making information is fused.

3.2.1 Modification of Mass Function Based on Evidence Weight

Generally, the decision-making evidence under different individuals in system should have different weights. If the decision-making evidence under an individual is with the greatest comprehensive support or importance, the evidence information under the individual is more credible and the reliability is greater, and the impact on the final fusion conclusion is greater.

Definition 13 Let the frame of discernment Θ have u pieces of evidence $e_1, e_2, \dots, e_u$, and the corresponding Mass functions are $m_1, m_2, \dots, m_u$, $\eta(m_s)$ is the weight of evidence e_s , let $\eta(m_t) = \max_{1 \leq s \leq u} \{\eta(m_s)\}$, then e_t is called the key evidence, and the reliability of e_s about e_t is expressed as follows

$$\mathcal{G}_s = \frac{\eta(m_s)}{\eta(m_t)} (s = 1, 2, \dots, u)$$

According to the correlation algorithm of evidence weight, the Mass functions are modified and obtained as follows

$$\begin{cases} \hat{m}_s(F) = \mathcal{G}_s m_s(F) \\ \hat{m}_s(\Theta) = 1 - \sum_{s=1}^u \hat{m}_s(F) \\ \forall F \in 2^\Theta, F \neq \Theta \end{cases}$$

3.2.2 Synthesis of Evidence information

After obtaining the evidence information under individual decision makers, Dempster's combination rule is used to fuse it to obtain the integrated Mass function, and then to realize the subsequent reasoning decision-making. The specific synthesis process is as follows.

Firstly, according to the Mass function $m_i^{(k)}$ of alternative $a_i (i = 1, 2, \dots, n)$ under decision maker $DM_k (k = 1, 2, \dots, h)$, the modified Mass functions of a_i and the overall uncertainty are obtained by using the evidence weight, respectively;

$$\hat{m}_i^{(k)} = \mathcal{G}_k m_i^{(k)} \quad (17)$$

$$\hat{m}_\Theta^{(k)} = 1 - \sum_{i=1}^n \hat{m}_i^{(k)} \quad (18)$$

Where $\mathcal{G}_k = \frac{\psi_k}{\max_{1 \leq k \leq h} (\psi_k)}$ is the reliability of evidence information of DM_k ,

$\psi_k (\psi_k \in [0,1])$ is the weight of DM_k .

Then, by using Dempster's combination rule to fuse evidence information under individual decision makers, the integrated Mass functions of a_i and the overall uncertainty are obtained, denoted as $m(a_i)$ and $m(\Theta)$, respectively.

3.3 Optimization Decision-making of Alternatives Based on Pignistic Probability Value

When using evidence theory for group decision-making analysis, we often need to convert the Mass function into a probability. That is to say, the Mass assignments on subsets are aggregated into a single element. As can be seen from Definition 3, the Pignistic probability transformation is based on average distribution, and the information loss is smaller during transformation. Hence, on the basis of considering the overall uncertainty of the system, the Pignistic probability function $BetP(a_i)$ of alternative a_i is obtained as follows

$$BetP(a_i) = m(a_i) + \frac{m(\Theta)}{n} \quad (19)$$

Where $m(a_i)$ and $m(\Theta)$ are the integrated Mass functions of a_i and the overall uncertainty, respectively. n is the number of alternatives. The alternatives can be ranked according to the Pignistic probability values. The larger the $BetP(a_i)$, the better the alternative a_i will be.

3.4 The steps of the proposed group decision-making method

Step 1: Normalize decision matrices of individual decision makers with Equation (3);

Step 2: Calculate the CGRA degree of each alternative under individual decision makers with Equations (4)-(13);

Step 3: Construct the information DOI of individual decision makers with Equation (14), then construct the Mass functions of the alternatives and the overall uncertainty under individual decision makers with Equations (15)-(16);

Step 4: Calculate the integrated Mass functions of the alternatives and the overall uncertainty according to Equations (17)-(18) and Dempster's combination rule;

Step 5: Calculate the Pignistic probability value of each alternative with Equation (19);

Step 6: Rank alternatives. Sorting the Pignistic probability values in a descending sequence. The larger the value, the better the performance of the corresponding alternative.

4 Case study

4.1 Case background

With the improvement of public environmental protection consciousness, enterprises pay more attention to green practice and green strategy. Green supply chain is just to alleviate the pressure on resources and environment with economic development. In the process of implementing green supply chain, a key link is to select the appropriate green supplier. The green supplier selection needs the joint participation of quality, procurement, logistics and other departments, and the influence of many evaluation attributes should also be considered. Its essence is the sorting process of multi-attribute group decision-making problems. The following is an example of green supplier selection for a product component to illustrate the practicality of the proposed method.

A manufacturer plans to select the best supplier for production from the initially identified four suppliers (a_1, a_2, a_3, a_4) , which are evaluated by the decision-making

group consisting of quality department (DM_1), procurement department (DM_2) and logistics department (DM_3). The literature of green supplier selection is reviewed. Additionally, the enterprise investigation and expert consultation are performed. Then the attribute system of green supplier selection is established, which includes 5 criteria and 12 attributes, as shown in Table 1. The decision-making evaluation information provided by decision-making group (DM_1, DM_2, DM_3) is shown in Table 2, in which the attribute values of each supplier about attributes $b_1, b_3, b_6, b_8, b_{10}, b_{12}$ are given subjectively, and the attribute values about attributes $b_2, b_4, b_5, b_7, b_9, b_{11}$ are obtained by objective statistics.

Table 1 The attribute system of the green supplier selection

Criterion layer	Attribute layer	Reference	Unit	Explanation
Green ecology (B_1)	Green design of products (b_1)	Qin et al. 2017	Score	Environmental pollution degree throughout the life cycle
	Resources recovery (b_2)	Tsui and Wen 2014	%	Rate of recovery to input
	Carbon reduction measures (b_3)	Luthra et al. 2017	Score	Implementation of carbon reduction measures
Product information (B_2)	Product quality (b_4)	Roshandel et al. 2013	%	The product qualified rate
	Product price (b_5)	Govindan et al. 2017	Yuan	The price of purchased product
Enterprise competitiveness (B_3)	Financial situation (b_6)	Lin et al. 2015	Score	Sources and distribution of operating funds
	Market share (b_7)	Hamdan and Cheaitou 2017	%	Proportion of the enterprise sales volume in market like products
	Technical level (b_8)	Govindan et al. 2014	Score	The ability to support technical innovation
Service capability (B_4)	On time delivery (b_9)	Wu and Blackhurst 2009	%	The proportion of the number of punctual deliveries to the number of total deliveries in a given time interval
	After-sale service level (b_{10})	Amin and Zhang 2012	Score	Customer satisfaction with after-sale service
Innovative ability (B_5)	New product exploitation ratio (b_{11})	Azadi et al. 2015	%	Ratio of new product value to total product value
	Innovation input (b_{12})	Mafakheri et al. 2011	Score	Investments of innovative personnel, equipment and fund

Table 2 The decision matrices provided by DM_1, DM_2, DM_3

Attribute	DM_1				DM_2				DM_3			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
b_1	0.20	0.23	0.21	0.30	0.24	0.15	0.19	0.28	0.27	0.21	0.18	0.32
b_2	35	38	36	31	35	38	36	31	35	38	36	31
b_3	0.81	0.84	0.82	0.78	0.75	0.87	0.83	0.80	0.81	0.85	0.88	0.82
b_4	89	96	92	84	89	96	92	84	89	96	92	84

b_5	26	25	24	28	26	25	24	28	26	25	24	28
b_6	0.91	0.94	0.92	0.88	0.90	0.97	0.94	0.87	0.94	0.96	0.97	0.95
b_7	15	17	14	12	15	17	14	12	15	17	14	12
b_8	0.94	0.92	0.95	0.90	0.90	0.94	0.92	0.87	0.90	0.96	0.93	0.91
b_9	91	96	97	92	91	96	97	92	91	96	97	92
b_{10}	0.93	0.95	0.98	0.94	0.91	0.97	0.94	0.92	0.96	0.98	0.95	0.91
b_{11}	31	38	34	29	31	38	34	29	31	38	34	29
b_{12}	0.95	0.93	0.92	0.96	0.92	0.97	0.94	0.90	0.93	0.95	0.96	0.94

4.2 Calculations and results

Step 1: Normalize decision matrices provided by DM_1, DM_2, DM_3 with Equation (3);

Step 2: Calculate the CGRA degree of each alternative under DM_1, DM_2, DM_3 ;

Considering that there is no obvious preference for the three kinds of relational degrees, let $w_1 = w_2 = w_3 = 1/3$. The CGRA degree $\phi_i^{(k)}$ of each alternative $a_i (i=1,2,3,4)$ under each decision maker $DM_k (k=1,2,3)$ can be calculated by Equations (4)-(13), that are

$$\phi_1^{(1)} = 0.7413, \phi_2^{(1)} = 0.9189, \phi_3^{(1)} = 0.8896, \phi_4^{(1)} = 0.5507; \phi_1^{(2)} = 0.6113, \phi_2^{(2)} = 0.9775, \phi_3^{(2)} = 0.8829, \phi_4^{(2)} = 0.5287; \phi_1^{(3)} = 0.6061, \phi_2^{(3)} = 0.9535, \phi_3^{(3)} = 0.9254, \phi_4^{(3)} = 0.5340.$$

Step 3: Construct the information DOI of DM_1, DM_2, DM_3 and the Mass functions of the alternatives and the overall uncertainty under DM_1, DM_2, DM_3 ;

The information DOI of each $DM_k (k=1,2,3)$ can be calculated by Equation (14), namely, $DOI(DM_1) = 0.760043$, $DOI(DM_2) = 0.726746$, $DOI(DM_3) = 0.731031$.

The Mass functions of the alternatives and the overall uncertainty under DM_1, DM_2, DM_3 are as follows: $m_1^{(1)} = 0.1779$, $m_2^{(1)} = 0.2205$, $m_3^{(1)} = 0.2135$, $m_4^{(1)} = 0.1321$; $m_1^{(2)} = 0.1670$, $m_2^{(2)} = 0.2671$, $m_3^{(2)} = 0.2412$, $m_4^{(2)} = 0.1445$; $m_1^{(3)} = 0.1630$, $m_2^{(3)} = 0.2565$, $m_3^{(3)} = 0.2489$, $m_4^{(3)} = 0.1436$. $m_{\Theta}^{(1)} = 0.256024$,

$$m_{\Theta}^{(2)} = 0.180119, m_{\Theta}^{(3)} = 0.187978.$$

Step 4: Calculate the integrated Mass functions;

Firstly, let the weight vector of DM_1, DM_2, DM_3 be $\psi = (0.275, 0.318, 0.407)$, the modified Mass functions of $a_i (i=1, 2, 3, 4)$ and the overall uncertainty are as follows: $\hat{m}_1^{(1)} = 0.1202$, $\hat{m}_2^{(1)} = 0.1490$, $\hat{m}_3^{(1)} = 0.1442$, $\hat{m}_4^{(1)} = 0.0893$; $\hat{m}_1^{(2)} = 0.1305$, $\hat{m}_2^{(2)} = 0.2087$, $\hat{m}_3^{(2)} = 0.1885$, $\hat{m}_4^{(2)} = 0.1129$; $\hat{m}_1^{(3)} = 0.1630$, $\hat{m}_2^{(3)} = 0.2565$, $\hat{m}_3^{(3)} = 0.2489$, $\hat{m}_4^{(3)} = 0.1436$. $\hat{m}_{\Theta}^{(1)} = 0.4973$, $\hat{m}_{\Theta}^{(2)} = 0.3594$, $\hat{m}_{\Theta}^{(3)} = 0.1880$.

Then, according to Dempster's combination rule, the integrated Mass functions of $a_i (i=1, 2, 3, 4)$ and the overall uncertainty are as follows: $m(a_1) = 0.1753$, $m(a_2) = 0.3130$, $m(a_3) = 0.2897$, $m(a_4) = 0.1408$; $m(\Theta) = 0.0812$.

Step 5: Calculate the Pignistic probability value of each alternative;

The Pignistic probability values of alternatives are as follows: $BetP(a_1) = 0.1956$, $BetP(a_2) = 0.3333$, $BetP(a_3) = 0.3100$, $BetP(a_4) = 0.1610$.

Step 6: Rank alternatives.

According to the Pignistic probability values, the final ranking of alternative strategies is obtained as follows: $a_2 \succ a_3 \succ a_1 \succ a_4$, that is, a_2 is the best, and the enterprise may consider cooperating with supplier a_2 first.

4.3 Comparative analysis

In order to verify the feasibility and effectiveness of the proposed method, the above evaluation results are compared with those of other methods, as shown in Table 3.

Table 3 The Comparison Results of Relevant Methods

Methods	Pignistic probability values	Ranking orders	Decision results
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The method based on D-S	$BetP(a_1) = 0.1642, BetP(a_2) = 0.3745$ $BetP(a_3) = 0.3518, BetP(a_4) = 0.1963$	$a_2 \text{ f } a_3 \text{ f } a_4 \text{ f } a_1$	a_2
The method based on classical GRA	$BetP(a_1) = 0.2157, BetP(a_2) = 0.4386$ $BetP(a_3) = 0.4624, BetP(a_4) = 0.2031$	$a_3 \text{ f } a_2 \text{ f } a_1 \text{ f } a_4$	a_3
The proposed method	$BetP(a_1) = 0.1956, BetP(a_2) = 0.3333$ $BetP(a_3) = 0.3100, BetP(a_4) = 0.1610$	$a_2 \text{ f } a_3 \text{ f } a_1 \text{ f } a_4$	a_2

As can be seen from Table 3, the ranking orders based on D-S and classical GRA are not exactly the same as that of the proposed method. The reason for the difference between the method based on D-S and the proposed method may be that the method based on D-S does not distinguish the reliability of evidence information under individual decisionmakers, which ignores the differences among individuals. The proposed method not only considers the reliabilities of individuals, but also uses the evidence weight to modify the evidence information, which make the group decision-making fusion result more reasonable.

In addition, the reason for the different ranking results between the method based on classical GRA and the proposed method may be that the method based on classical GRA only considers the similarity of geometric shapes between the alternative sequence and the positive (negative) ideal point sequence to measure the relational degree. The proposed method uses CGRA to measure the proximity, similarity and volatility of the development and evolution between sequences comprehensively, which lays a foundation for obtaining Mass functions under individual decision makers more scientifically.

5 Conclusions

In this paper, a group decision making method based on GRA and evidence theory is proposed to solve the group decision making problems. Firstly, combining GRA with evidence theory, a CGRA method is proposed to determine the degree of ignorance for individual decision makers' information and the Mass function. Then,

the weighted evidence theory is adopted to fuse the group decision-making information. On this basis, the alternatives are ranked according to Pignistic probability values. Finally, an example of green supplier selection is given to illustrate the effectiveness of the proposed method. Meanwhile, the proposed method also provides a new way to solve the group decision making problems in reality.

The paper makes three important contributions. Firstly, a CGRA method is proposed to measure the proximity, similarity, and volatility between sequences synthetically, which makes the determination of Mass function more objective. Secondly, the reliability of decision makers' evidence information is introduced into evidence theory, and the individual evidence information is modified by the evidence weight, which lays a scientific basis for subsequent reasoning fusion decision-making. Thirdly, on the basis of considering the overall uncertainty of the system, the Mass function is transformed into probability by Pignistic probability transformation, which effectively reduces the information loss during the transformation.

In this paper, the group decision making problem is studied from the perspective of static information fusion, and the dynamic reasoning decision-making problem that the generation of reasoning information may require interactive discussions among decisionmakers is not considered. Hence, expanding this problem from the perspective of dynamic interaction is the focus of the next research. Additionally, the weights of proximity, similarity and volatility in CGRA degree are mainly determined subjectively, and the quantitative optimization method can be further studied to determine the weights objectively.

Acknowledgments

The authors are grateful to the anonymous referees for useful suggestions that have improved the quality of this paper. This research is supported by the National Natural

Science Foundation of China (51979106), Scientific and Technological Plan Fund Project of Henan Province (182102310014), Key Research Project Plan of Henan Universities (18A630030, 18A630032), and the Quality Curriculum of Postgraduate Education in Henan Province (HNYJS2015KC02).

Conflict of interest: The authors declare that they have no conflict of interest.

Data availability: The data that support the findings of this paper are available from the corresponding author.

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