The Classification of Single Traveling Wave Solutions for the Fractional Coupled Nonlinear Schrödinger equation

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The classification of single traveling wave solutions for the fractional coupled nonlinear Schrödinger equation

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Abstract

The main purpose of this paper is to study the single traveling wave solutions of the fractional coupled nonlinear Schrödinger equation. By using the complete discriminant system method and computer algebra with symbolic computation, a series of new single traveling wave solutions are obtained, which include trigonometric function solutions, Jacobi elliptic function solutions, hyperbolic function solutions, solitary wave solutions and rational function solutions. In order to further explain the propagation of the fractional coupled nonlinear Schrödinger equation in nonlinear optics, two-dimensional and three-dimensional graphs are drawn.

Keywords: Fractional coupled nonlinear Schrödinger equation; Complete discriminant system method; Computer algebra; Traveling wave solution

1. Introduction

It is well known that nonlinear evolution equations (NLEEs) model various physical phenomena and play an important position in the investigation of numerous fields, such as combustion theory, fluid dynamics, ecological system, signal processing, nonlinear optics, engineering, statistical mechanics, and plasma physics. As a result, it is one of the critical problems to seek the exact solutions of these NLEEs in nonlinear science. However, due to the complexity of NLEEs, giving all the exact solutions of a NLEE with a unified technique seems to be impossible. Over the decades, a lot of efficient methods have been established and developed to fabricate exact solutions through the efforts of many mathematicians, such as the extended simplest equation method [1], the Riccati sub

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equation method [2], the Jacobi elliptic function method [3, 4], Painlevé analysis [5–9], $\exp_a$ -function method [10], Lax pairs [11], Bäcklund transformation [12], etc.

In the current study, the fractional coupled nonlinear Schrödinger equation (FCNLSE) is considered in the following form [13, 14]

$$\begin{cases} iD_t^\alpha \psi_1 + D_x^{2\beta} \psi_1 + \delta(|\psi_1|^2 + \gamma|\psi_2|^2)\psi_1 = 0, \\ iD_t^\alpha \psi_2 + D_x^{2\beta} \psi_2 + \delta(\gamma|\psi_1|^2 + |\psi_2|^2)\psi_2 = 0, \end{cases} \quad (1.1)$$

where i is imaginary unit, $i^2 = -1$, $\alpha, \beta \in (0, 1]$, $\psi_1 = \psi_1(x, t)$ and $\psi_2 = \psi_2(x, t)$ are complex functions, which represent the wave amplitudes in two polarizations, x represents the normalized propagation and t denotes the retard time. δ represents self-focusing and γ denotes cross-phase modulation are nonzero constants. When $\alpha = \beta = 1$, it is known to all that Equation (1.1) is the coupled nonlinear Schrödinger equation[15, 16].

The FCNLSE is a classical nonlinear model which can describe lots of physical nonlinear systems. The equation can be applied to many fields, such as biology, fluid mechanics, nonlinear optics, circulation system of chemical industry, heat pulses in solids and so on. Due to the importance of FCNLSE, the equation has been investigated by many researchers [17–22]. As a result, it is an important work to seek the exact solutions of the fractional differential equation. So far, lots of effective methods have been established about the traveling wave solutions of the FCNLSE [23–26]. In Ref. [27], Esen and his co-workers considered the space-time fractional $(1 + 1)$ -dimensional coupled nonlinear Schrödinger equation, a series of exact solutions including dark, mix dark-bright and mixed singular optical solitons are obtained via the extended sinh-Gordon equation expansion method. By applying the Kudryashov method, the traveling wave solutions to the time FCNLSE were derived by Eslami[28]. The traveling wave solutions of Equation (1.1) were obtained by Han and his collaborators [29] by using the bifurcation theory and planar portraits analysis method. Although there are lots of methods to construct the exact solutions of the FCNLSE, the discriminant system method to study the exact solutions of the FCNLSE, it seems as far as we know, not available in the literature. Especially in recent years, with the development of computer algebra theory, by using the mathematical software Maple or Mathematical, a series of traveling wave solutions can be obtained by solving complex algebraic equations. In 1996, with the help of computer algebra, a complete discrimination system of high-order polynomials has been derived by Yang and his co-workers [30]. As a matter of fact, it is a powerful tool to seek traveling wave solutions of NLEEs. Therefore, a range

of solutions of different forms are obtained [31].

In this paper, the complete discriminant system method is employed to seek exact solutions of the FCNLSE, with the assistance of computer algebra and symbolic computation, according to the root-classifications, a series of new traveling wave solutions are obtained.

40 The organization of this paper is as follows. In section 2, we review the definition of conformable fractional derivatives. In section 3, the description of the complete discriminant system method is given. In section 4, by applying the complete discriminant system method, the new traveling wave solutions to Equation (1.1) are obtained by adapting the inverse transformation. The last section summarizes the results of the current study.

45 2. Conformable fractional derivative

First of all, the conformable fractional derivatives can be defined as follows [32–35].

Definition 2.1. Let $u : [0, +\infty) \rightarrow \mathbf{R}$, $\alpha \in (0, 1]$. The conformable fractional derivative of u of order α is defined as

$$T_\alpha(u)(t) = \lim_{\varepsilon \rightarrow 0} \frac{u(t + \varepsilon t^{1-\alpha}) - u(t)}{\varepsilon}, \quad \forall t \geq 0, \quad (2.1)$$

the function u is α -conformable differentiable at a point t if the limit in Equation (2.1) exists.

Theorem 2.2. Assume that $u, v : (0, \infty) \rightarrow \mathbf{R}$ be differentiable and also α differentiable functions, then chain rule holds

$$T_\alpha(u \circ v)(t) = t^{1-\alpha} v(t)^{\alpha-1} v'(t) T_\alpha(u(t))|_{t=v(t)}. \quad (2.2)$$

3. Analysis of the method

Considering the nonlinear partial differential equation in the form:

$$G(u, D_t^\alpha u, D_x^\beta u, D_t^\alpha D_t^\alpha u, D_t^\alpha D_x^\beta u, D_x^\beta D_x^\beta u, \dots) = 0, \quad 0 < \alpha, \beta < 1. \quad (3.1)$$

First of all, by traveling wave transformations and some other suitable transformations, the Equation (3.1) can be transferred into an nonlinear ordinary differential equation as follows:

$$u_\xi^2 = F(u), \quad (3.2)$$

50 where $u_\xi := \frac{d}{d\xi}u$ and $F(u) = a_2u^2 + a_1u + a_0$ is the double degree polynomial with the parameters a_2, a_1, a_0 . Then integrating the above (3.2), we can obtain

$$\xi - \xi_0 = \int \frac{du}{\sqrt{F(u)}}, \quad (3.3)$$

where ξ_0 is the integration constant. Therefore, we need to solve Equation (3.3). However, it is a challenging work to decide the range of the parameters, which can be accomplished by complete discrimination system functions. we can derive its complete discrimination system

$$\Delta = a_1^2 - 4a_2a_0. \quad (3.4)$$

Finally, according to the root-classifications, the parameters mentioned above can be obtained from the integral Equation (3.3). And then by an inverse transformation, we can obtain the exact solutions of the original partial differential equation.

55 4. Traveling Wave solutions for FCNLSE

In this section, we consider the traveling wave solutions for the Equation (1.1), we assume that Equation (1.1) has the following traveling wave transformation

$$\psi_1(x, t) = Z_1(\xi)e^{i\eta}, \quad \psi_2(x, t) = Z_2(\xi)e^{i\eta}, \quad \xi = m\left(\frac{x^\beta}{\beta} - c\frac{t^\alpha}{\alpha}\right), \quad \eta = -\lambda\frac{x^\beta}{\beta} + \mu\frac{t^\alpha}{\alpha} + \eta_0, \quad (4.1)$$

where m, c, λ and μ are undetermined real constants, η_0 is an arbitrary constant.

Substituting (4.1) into Equation (1.1), decomposes real parts and imaginary parts of Equation (1.1), the FCNLSE can be reduced into

$$\begin{cases} m^2 Z_1'' + \delta Z_1^3 + \delta\gamma Z_2^2 Z_1 - (\lambda^2 + \mu)Z_1 = 0, \\ m^2 Z_2'' + \delta Z_2^3 + \delta\gamma Z_1^2 Z_2 - (\lambda^2 + \mu)Z_2 = 0, \\ c = -2\lambda. \end{cases} \quad (4.2)$$

Suppose that there is a linear relationship between Z_1 and Z_2 , namely $Z_2 = kZ_1$ ($k \neq 0$), substituting $Z_2 = kZ_1$ ($k \neq 0$) into the first equation of (4.2), we can obtain the following form:

$$m^2 Z_1'' + (\delta + \delta\gamma k^2)Z_1^3 - (\lambda^2 + \mu)Z_1 = 0. \quad (4.3)$$

Thus,

$$Z_1'' = -\frac{\delta + \delta\gamma k^2}{m^2} Z_1^3 + \frac{\lambda^2 + \mu}{m^2} Z_1. \quad (4.4)$$

By multiplying (4.4) with Z_1' , we derive

$$Z_1' Z_1'' = -\frac{\delta + \delta\gamma k^2}{m^2} Z_1^3 Z_1' + \frac{\lambda^2 + \mu}{m^2} Z_1 Z_1'. \quad (4.5)$$

Integrating Equation (4.4) once, we obtain

$$(Z_1')^2 = a_4 Z_1^4 + a_2 Z_1^2 + a_0, \quad (4.6)$$

where $a_4 = -\frac{\delta + \delta\gamma k^2}{2m^2}$, $a_2 = \frac{\lambda^2 + \mu}{m^2}$, a_0 is a integral constant.

By a suitable transformation as follows

$$\begin{cases} Z_1 = \pm \sqrt{(4a_4)^{-\frac{1}{3}}} W, \\ b_1 = 4a_2(4a_4)^{-\frac{2}{3}}, \\ b_0 = 4a_0(4a_4)^{-\frac{1}{3}}, \\ \xi_1 = (4a_4)^{\frac{1}{3}} \xi. \end{cases} \quad (4.7)$$

Then the Equation (4.6) can be rewritten as

$$(W_{\xi_1})^2 = W(W^2 + b_1 W + b_0). \quad (4.8)$$

Integrating Equation (4.8) once, we derive

$$\pm(\xi_1 - \xi_0) = \int \frac{dW}{\sqrt{W(W^2 + b_1 W + b_0)}}, \quad (4.9)$$

where ξ_0 is the integration constant. Denoting $F(W) = W^2 + b_1 W + b_0$, thus we can establish the second order complete discrimination system as

$$\Delta = b_1^2 - 4b_0. \quad (4.10)$$

According to the root-classifications of (4.10), there are four cases to be discussed.

Case 1 Suppose that $\Delta = 0$. As for $W > 0$, we have

$$\pm(\xi_1 - \xi_0) = \int \frac{dW}{(W + \frac{b_1}{2})\sqrt{W}}. \quad (4.11)$$

If $b_1 > 0$, it follows from Equation (4.11), we can obtain

$$W = \frac{b_1}{2} \tan^2 \left[\frac{1}{2} \sqrt{\frac{b_1}{2}} (\xi_1 - \xi_0) \right]. \quad (4.12)$$

According to the Equation (4.7), (4.12) and $Z_2 = kZ_1$ ($k \neq 0$), the solution of Equation (1.1) can be obtained as follows (see Figure 1)

$$\begin{cases} \psi_{1,1}(x, t) = \pm \sqrt{\frac{\lambda^2 + \mu}{-\delta - \delta \gamma k^2}} \tan \left\{ 2^{-\frac{7}{6}} \sqrt{\frac{\lambda^2 + \mu}{m^2}} \left(\frac{2m^2}{-\delta - \delta \gamma k^2} \right)^{1/3} \left[\left(\frac{-2\delta - 2\delta \gamma k^2}{m^2} \right)^{1/3} \xi - \xi_0 \right] \right\} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)), \\ \psi_{2,1}(x, t) = \pm k \sqrt{\frac{\lambda^2 + \mu}{-\delta - \delta \gamma k^2}} \tan \left\{ 2^{-\frac{7}{6}} \sqrt{\frac{\lambda^2 + \mu}{m^2}} \left(\frac{2m^2}{-\delta - \delta \gamma k^2} \right)^{1/3} \left[\left(\frac{-2\delta - 2\delta \gamma k^2}{m^2} \right)^{1/3} \xi - \xi_0 \right] \right\} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)). \end{cases} \quad (4.13)$$

(a) Real 3D surface

(b) Real 2D Shape

Figure 1: The graphics of $\psi_{1,1}(x, t)$ in Equation(4.13) at $\lambda = \mu = \gamma = k = m = 1$, $c = 2$, $\delta = -1$, $\xi_0 = 0$, $\alpha = \beta = \frac{1}{2}$.

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If $b_1 < 0$, it follows from Equation (4.11), we can obtain

$$\begin{cases} W = -\frac{b_1}{2} \tanh^2 \left[\sqrt{-\frac{b_1}{2}} \times \frac{1}{2} (\xi_1 - \xi_0) \right], \\ W = -\frac{b_1}{2} \coth^2 \left[\sqrt{-\frac{b_1}{2}} \times \frac{1}{2} (\xi_1 - \xi_0) \right]. \end{cases} \quad (4.14)$$

According to the Equation (4.7), (4.14) and $Z_2 = kZ_1$ ($k \neq 0$), the solution of Equation (1.1) can be obtained as follows (see Figure 2 and Figure 3)

$$\begin{cases} \psi_{1,2}(x, t) = \pm \sqrt{\frac{\lambda^2 + \mu}{\delta + \delta \gamma k^2}} \tanh \left\{ \sqrt{\frac{\lambda^2 + \mu}{-2m^2}} \left(\frac{-m^2}{-2\delta - 2\delta \gamma k^2} \right)^{1/3} \left[\left(\frac{-2\delta - 2\delta \gamma k^2}{m^2} \right)^{1/3} \xi - \xi_0 \right] \right\} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)), \\ \psi_{2,2}(x, t) = \pm k \sqrt{\frac{\lambda^2 + \mu}{\delta + \delta \gamma k^2}} \tanh \left\{ \sqrt{\frac{\lambda^2 + \mu}{-2m^2}} \left(\frac{-m^2}{-2\delta - 2\delta \gamma k^2} \right)^{1/3} \left[\left(\frac{-2\delta - 2\delta \gamma k^2}{m^2} \right)^{1/3} \xi - \xi_0 \right] \right\} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)). \end{cases} \quad (4.15)$$

(a) Real 3D surface

(b) Real 2D Shape

Figure 2: The graphics of $\psi_{1,2}(x, t)$ in Equation(4.15) at $\lambda = \gamma = k = m = 1$, $\mu = -2$, $\delta = -1$, $c = 2$, $\xi_0 = 0$, $\alpha = \beta = \frac{1}{2}$.

$$\begin{cases} \psi_{1,3}(x, t) = \pm \sqrt{\frac{\lambda^2 + \mu}{\delta + \delta \gamma k^2}} \coth \left\{ \sqrt{\frac{\lambda^2 + \mu}{-2m^2}} \left(\frac{-m^2}{-2\delta - 2\delta \gamma k^2} \right)^{1/3} \left[\left(\frac{-2\delta - 2\delta \gamma k^2}{m^2} \right)^{1/3} \xi - \xi_0 \right] \right\} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)), \\ \psi_{2,3}(x, t) = \pm k \sqrt{\frac{\lambda^2 + \mu}{\delta + \delta \gamma k^2}} \coth \left\{ \sqrt{\frac{\lambda^2 + \mu}{-2m^2}} \left(\frac{-m^2}{-2\delta - 2\delta \gamma k^2} \right)^{1/3} \left[\left(\frac{-2\delta - 2\delta \gamma k^2}{m^2} \right)^{1/3} \xi - \xi_0 \right] \right\} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)). \end{cases} \quad (4.16)$$

If $b_1 = 0$, it follows from Equation (4.11), we can obtain

$$W = \frac{4}{(\xi_1 - \xi_0)^2}. \quad (4.17)$$

Figure 3: The graphics of $\psi_{1,3}(x,t)$ in Equation(4.16) at $\lambda = \gamma = k = m = 1$, $\mu = -2$, $\delta = -1$, $c = 2$, $\xi_0 = 0$, $\alpha = \beta = \frac{1}{2}$.

Thus, the solution of Equation (1.1) can be obtained as follows (see Figure 4)

$$\begin{cases} \psi_{1,4}(t, x) = \pm 2^{\frac{2}{3}} \left(\frac{-\delta - \delta \gamma k^2}{2m^2} \right)^{-1/6} \left[\left(\frac{-2\delta - 2\delta \gamma k^2}{m^2} \right) \xi - \xi_0 \right] \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)), \\ \psi_{2,4}(t, x) = \pm 2^{\frac{2}{3}} k \left(\frac{-\delta - \delta \gamma k^2}{2m^2} \right)^{-1/6} \left[\left(\frac{-2\delta - 2\delta \gamma k^2}{m^2} \right) \xi - \xi_0 \right] \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)). \end{cases} \quad (4.18)$$

Case 2 Suppose that $\Delta > 0$ and $b_0 = 0$. As for $W > -b_1$, we have

$$\pm(\xi_1 - \xi_0) = \int \frac{dW}{W\sqrt{W+b_1}}. \quad (4.19)$$

If $b_1 > 0$, it follows from Equation (4.19), we can obtain

$$\begin{cases} W = \frac{b_1}{2} \tanh^2 \left[\sqrt{\frac{b_1}{2}} \times \frac{1}{2}(\xi_1 - \xi_0) \right] - b_1, \\ W = \frac{b_1}{2} \coth^2 \left[\sqrt{\frac{b_1}{2}} \times \frac{1}{2}(\xi_1 - \xi_0) \right] - b_1. \end{cases} \quad (4.20)$$

According to the Equation (4.7), (4.20) and $Z_2 = kZ_1$ ($k \neq 0$), the solution of Equation (1.1) can

(a) Real 3D surface

(b) Real 2D Shape

Figure 4: The graphics of $\psi_{1,4}(x, t)$ in Equation(4.18) at $\gamma = k = m = 1$, $\delta = -1, c = 2$, $\xi_0 = 0$, $\alpha = \beta = \frac{1}{2}$.

be obtained as follows:

$$\left\{ \begin{array}{l} \psi_{1,5}(x, t) = \pm \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-1/6} \left\{ \left(\frac{2\lambda^2 + 2\mu}{m^2} \right) \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{2}{3}} \right. \\ \quad \times \tanh^2 \left[\frac{\sqrt{2}}{2} \sqrt{\frac{\lambda^2 + \mu}{m^2}} \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-1/3} \xi - \xi_0 \right] - \left(\frac{4\lambda^2 + 4\mu}{m^2} \right) \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{2}{3}} \Big\}^{\frac{1}{2}} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)), \\ \psi_{2,5}(x, t) = \pm k \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-1/6} \left\{ \left(\frac{2\lambda^2 + 2\mu}{m^2} \right) \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{2}{3}} \right. \\ \quad \times \tanh^2 \left[\frac{\sqrt{2}}{2} \sqrt{\frac{\lambda^2 + \mu}{m^2}} \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-1/3} \xi - \xi_0 \right] - \left(\frac{4\lambda^2 + 4\mu}{m^2} \right) \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{2}{3}} \Big\}^{\frac{1}{2}} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)). \end{array} \right. \quad (4.21)$$

$$\left\{ \begin{array}{l} \psi_{1,6}(x, t) = \pm \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-1/6} \left\{ \left(\frac{2\lambda^2 + 2\mu}{m^2} \right) \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{2}{3}} \right. \\ \quad \times \coth^2 \left[\frac{\sqrt{2}}{2} \sqrt{\frac{\lambda^2 + \mu}{m^2}} \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-1/3} \xi - \xi_0 \right] - \left(\frac{4\lambda^2 + 4\mu}{m^2} \right) \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{2}{3}} \Big\}^{\frac{1}{2}} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)), \\ \psi_{2,6}(x, t) = \pm k \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-1/6} \left\{ \left(\frac{2\lambda^2 + 2\mu}{m^2} \right) \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{2}{3}} \right. \\ \quad \times \coth^2 \left[\frac{\sqrt{2}}{2} \sqrt{\frac{\lambda^2 + \mu}{m^2}} \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-1/3} \xi - \xi_0 \right] - \left(\frac{4\lambda^2 + 4\mu}{m^2} \right) \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{2}{3}} \Big\}^{\frac{1}{2}} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)). \end{array} \right. \quad (4.22)$$

If $b_1 < 0$, it follows from Equation (4.19), we can obtain

$$W = -\frac{b_1}{2} \tan^2[\sqrt{-\frac{b_1}{2}} \times \frac{1}{2}(\xi_1 - \xi_0)] - b_1. \quad (4.23)$$

According to the Equation (4.7), (4.23) and $Z_2 = kZ_1$ ($k \neq 0$), the solution of Equation (1.1) can be obtained as follows:

$$\left\{ \begin{array}{l} \psi_{1,7}(x, t) = \pm \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-1/6} \left\{ \left(\frac{-2\lambda^2 - 2\mu}{m^2} \right) \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{2}{3}} \right. \\ \quad \times \tan^2 \left[\frac{\sqrt{2}}{2} \sqrt{-\frac{\lambda^2 + \mu}{m^2}} \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-1/3} \xi - \xi_0 \right] - \left(\frac{4\lambda^2 + 4\mu}{m^2} \right) \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{2}{3}} \Big\}^{\frac{1}{2}} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)), \\ \psi_{2,7}(x, t) = \pm k \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-1/6} \left\{ \left(\frac{-2\lambda^2 - 2\mu}{m^2} \right) \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{2}{3}} \right. \\ \quad \times \tan^2 \left[\frac{\sqrt{2}}{2} \sqrt{-\frac{\lambda^2 + \mu}{m^2}} \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-1/3} \xi - \xi_0 \right] - \left(\frac{4\lambda^2 + 4\mu}{m^2} \right) \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{2}{3}} \Big\}^{\frac{1}{2}} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)). \end{array} \right. \quad (4.24)$$

Case 3 Suppose that $\Delta > 0$ and $b_0 \neq 0$ and $\omega_1 < \omega_2 < \omega_3$. Then, we adopt the assumption that one of $\omega_1, \omega_2, \omega_3$ is zero and the rest of them are two different real roots of $F(W) = 0$. Taking
70 the transformation $W = \omega_1 + (\omega_2 - \omega_1) \sin^2 \theta$, it is clear that

$$\pm(\xi_1 - \xi_0) = \frac{1}{\sqrt{\omega_3 - \omega_1}} \int \frac{d\theta}{\sqrt{1 - n_1^2 \sin^2 \theta}}, \quad (4.25)$$

where $n_1^2 = \frac{\omega_2 - \omega_1}{\omega_3 - \omega_1}$. It follows from Equation (4.25), we obtain

$$W = \omega_1 + (\omega_2 - \omega_1) sn^2(\sqrt{\omega_3 - \omega_1}(\frac{1}{2}(\xi_1 - \xi_0)), n_1). \quad (4.26)$$

According to the Equation (4.7), (4.26) and $Z_2 = kZ_1$ ($k \neq 0$), the solution of Equation (1.1) can be obtained as follows

$$\left\{ \begin{array}{l} \psi_{1,8}(x, t) = \pm \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{1}{6}} \left\{ \omega_1 + (\omega_3 - \omega_1) sn^2 \left[\frac{\sqrt{\omega_3 - \omega_1}}{2} \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right) \xi - \xi_0, n_1 \right] \right\}^{\frac{1}{2}} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)), \\ \psi_{2,8}(x, t) = \pm k \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{1}{6}} \left\{ \omega_1 + (\omega_3 - \omega_1) sn^2 \left[\frac{\sqrt{\omega_3 - \omega_1}}{2} \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right) \xi - \xi_0, n_1 \right] \right\}^{\frac{1}{2}} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)). \end{array} \right. \quad (4.27)$$

For another transformation $W = \frac{-\omega_2 \sin^2 \theta + \omega_3}{\cos^2 \theta}$, it follows from Equation (4.25), we obtain

$$W = \frac{-\omega_2 sn(\sqrt{\omega_3 - \omega_1}(\frac{1}{2}(\xi_1 - \xi_0)), n_1) + \omega_3}{cn(\sqrt{\omega_3 - \omega_1}(\frac{1}{2}(\xi_1 - \xi_0)), n_1)} \quad (4.28)$$

According to the Equation (4.7), (4.28) and $Z_2 = kZ_1$ ($k \neq 0$), the solution of Equation (1.1) can be obtained as follows:

$$\left\{ \begin{array}{l} \psi_{1,9}(x, t) = \pm \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{1}{6}} \left\{ \frac{-\omega_2 sn \left[\frac{\sqrt{\omega_3 - \omega_1}}{2} \left(\left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{1/3} \xi - \xi_0 \right), n_1 \right] + \omega_3}{cn \left[\frac{\sqrt{\omega_3 - \omega_1}}{2} \left(\left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{1/3} \xi - \xi_0 \right), n_1 \right]} \right\}^{\frac{1}{2}} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)). \\ \psi_{2,9}(x, t) = \pm k \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{1}{6}} \left\{ \frac{-\omega_2 sn \left[\frac{\sqrt{\omega_3 - \omega_1}}{2} \left(\left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{1/3} \xi - \xi_0 \right), n_1 \right] + \omega_3}{cn \left[\frac{\sqrt{\omega_3 - \omega_1}}{2} \left(\left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{1/3} \xi - \xi_0 \right), n_1 \right]} \right\}^{\frac{1}{2}} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)). \end{array} \right. \quad (4.29)$$

Case 4 Suppose that $\Delta < 0$. Taking the transformation $W = \sqrt{b_0} \tan^2 \frac{\theta}{2}$, it is clear that

$$\pm(\xi_1 - \xi_0) = \frac{1}{2}(b_0)^{-\frac{1}{4}} \int \frac{d\theta}{\sqrt{1 - n_2^2 \sin^2 \theta}}. \quad (4.30)$$

where $n_2^2 = \frac{2\sqrt{b_0 - b_1}}{4\sqrt{b_0}}$, it follows from Equation (4.30), we obtain

$$W = \frac{2\sqrt{b_0}}{1 + cn(b_0^{\frac{1}{4}}(\xi_1 - \xi_0), n_2)} - \sqrt{b_0}. \quad (4.31)$$

According to the Equation (4.7), (4.31) and $Z_2 = kZ_1$ ($k \neq 0$), the solution of Equation (1.1) can be obtained as follows

$$\left\{ \begin{array}{l} \psi_{1,10}(x, t) = \pm \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{1}{6}} \left\{ \frac{2\sqrt{b_0}}{1 + cn \left[b_0^{\frac{1}{4}} \left(\left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{\frac{1}{3}} \xi - \xi_0 \right), n_2 \right]} - \sqrt{b_0} \right\}^{\frac{1}{2}} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)), \\ \psi_{2,10}(x, t) = \pm k \left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{-\frac{1}{6}} \left\{ \frac{2\sqrt{b_0}}{1 + cn \left[b_0^{\frac{1}{4}} \left(\left(\frac{-2\delta - 2\delta\gamma k^2}{m^2} \right)^{\frac{1}{3}} \xi - \xi_0 \right), n_2 \right]} - \sqrt{b_0} \right\}^{\frac{1}{2}} \\ \quad \times \exp(i(-\lambda \frac{x^\beta}{\beta} + \mu \frac{t^\alpha}{\alpha} + \eta_0)). \end{array} \right. \quad (4.32)$$

5. Conclusion

It is known to all that the nonlinear Schrödinger equation is an example of a universal nonlinear model that describes many physical nonlinear systems. In this paper, the complete discriminant system method is employed to seek exact solutions of the FCNLSE, by using the mathematical

75 software Maple, combining computer algebra with symbolic computation, we obtain a series of
new traveling wave solutions, including trigonometric function solutions, Jacobi elliptic function
solutions, hyperbolic function solutions, solitary wave solutions, rational function solutions. The
complete discriminant system method is employed to seek traveling wave solutions of CFNLSE, it
seems as far as we know, not available in the literature. Therefore, the research in this paper has an
80 important application and scientific research value.

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Authors' contributions

85 All authors read and proved the final manuscript.

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Figures

(a) Real 3D surface

(b) Real 2D Shape

Figure 1

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(a) Real 3D surface

(b) Real 2D Shape

Figure 2

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Figure 3

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Figure 4

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