Geometry of the fusiform excision for skin lesions

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Geometry of the fusiform excision for skin lesions

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Short running head: Geometry of the fusiform excision

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ABSTRACT

Objective: The fusiform excision technique is commonly used by surgeons to remove round skin lesions to minimize "dog-ears" at the ends of the incision. We propose a geometric analysis to easily design the fusiform incision and consequently standardize the surgical procedure.

Background: The classic ellipse is formed by tracing 2 arcs of a circle on the skin. The arcs, which are symmetrical with respect to the midline axis separating them, intersect at their ends to form a convex shape and classically result in a 1:3 width-length ratio between the short and long axes of the ellipse.

Methods: Using basic geometry rules, namely Pythagorean theorem and the ratios of the angles of right triangles, we first calculated the ratio between the radius of the lesion and the radius of the arcs of the circle of the fusiform incision and then the distance between the center of the lesion and the intersection between the line perpendicular to the axis of the fusiform excision and the tangent to the arcs of the circle.

Results: The ratio between the radius of the lesion and the radius of the arcs of the circle of the fusiform incision is 5 and the distance between the center of the lesion and the intersection between the vertical axis and the tangent is 2.25 for a fusiform incision with a width-length ratio of 1:3. We then generalized the formulas.

Conclusions: Our approach provides an introduction to the geometry of dermatologic surgery to students in order to standardize the surgical procedure.

BACKGROUND

The fusiform excision technique is commonly used by surgeons to remove skin and subcutaneous lesions that are roughly round or oval. Primary linear closure of a fusiform excision is a crucial tool in skin reconstruction, as this closure provides excellent cosmetic results by minimizing tissue removal, skin mobilization, and incision length.[1,2] The classic ellipse is formed by tracing 2 arcs of a circle on the skin (Figure 1). The arcs, which are symmetrical with respect to the midline axis separating them, intersect at their ends to form a convex shape and classically result in a 1:3 width-length ratio between the short and long axes of the ellipse.[3-5] The tapered ends of the resulting defect allow primary closure of the wound, while minimizing "standing cone deformities" or "dog-ears" at the ends of the incision. The fusiform excision technique is probably the most commonly performed surgical technique, but the geometry of this technique has been poorly analyzed. Using basic geometry, we propose a geometric analysis to more clearly understand and more easily design this incision in order to standardize the surgical procedure.

METHODS

Surgical procedure

The surgeon performs fusiform excision by incising 2 arcs of a circle on the skin. The arcs, which are symmetrical with respect to the midline axis separating them, intersect at their ends to form a convex shape that includes the circle representing the lesion. The centers of the arcs are located on the line drawn perpendicular to the long axis of the ellipse bisecting the long axis. It is also useful to draw the tangent to the circle at the end of the fusiform incision. The points and distances around the lesion (C1) are defined in the Figure legend.

Figure description and definitions

Geometric approach to fusiform incision. The grey disc (circle C1, center A) represents the lesion; B is the right extremity of the fusiform incision, C is the top of the lesion, O is the center of the circle C2, defining the superior limit of the fusiform incision (arc-of-circle of C2) and I is the intersection of the AC straight line and the tangent of circle C2 at B. *r = AC* is the radius of the lesion (Circle C1). *R* is the radius of circle C2, *h* is the distance between B and C, *t* is the distance between B and I, *h'* is the distance between C and I. We also define several angles that are used in the formulas: α, β, γ and δ. $\alpha = \hat{ACB} = \hat{CBO}; \beta = \hat{ABC}; \gamma = \hat{ABI}; \delta = \hat{AIB}$

Geometric approaches

Two approaches based on the geometry of right triangles can be used to draw the fusiform excision with arcs of a circle with radius R, by expressing *R* as a function of *r*.

1/ The first approach is based on the Pythagorean theorem.

2/ The second approach is based on the ratios of the angles of right triangles.

RESULTS

Drawing the circle

A / Approach based on the Pythagorean theorem

A 3:1 relationship between *R* and r can be used to draw the arcs of a circle that define the elliptic excision.

We know that ABC and AOB are right-angled triangles (Fig. 1).

The Pythagorean theorem can be applied to triangle AOB:

$$R^2 = (3r)^2 + (R-r)^2 = 9r^2 + R^2 + r^2 - 2rR$$

$$2rR = 10r^2$$

$$R = 5r$$

This can be generalized to any ratio between *r* and *R*

$$R^2 = (xr)^2 + (R-r)^2 = x^2r^2 + R^2 + r^2 - 2rR$$

$$2rR = (x^2+1)r^2$$

$$R = \frac{(x^2+1)}{2}r$$

B/ Approach based on the ratios of the angles of right triangles

The first step consists of calculating *h* (distance BC)

$$BC^2 = AC^2 + AB^2$$

$$h^2 = r^2 + (3r)^2$$

$$h = r\sqrt{10}$$

We then calculate angle a using the cosine formula

$$cos\ \alpha = \frac{r}{h} = \frac{r}{r\sqrt{10}} = \frac{1}{\sqrt{10}};$$

So, in the triangle defined by O, C, and the middle of [BC],

$$cos\ \alpha = \frac{\frac{r\sqrt{10}}{2}}{R}$$

$$R = \frac{r\frac{\sqrt{10}}{2}}{\cos\alpha} = \frac{r\frac{\sqrt{10}}{2}}{\frac{1}{\sqrt{10}}}$$

$$R = 5r$$

This can be generalized to any ratio between *r* and *R*

$$h^2 = r^2 + (xr)^2$$

$$h = r\sqrt{(x^2+1)}$$

$$\cos\alpha = \frac{r}{h} = \frac{r}{r\sqrt{(x^2+1)}} = \frac{1}{\sqrt{(x^2+1)}}$$

$$\cos\alpha = \frac{\frac{r\sqrt{(x^2+1)}}{2}}{R}$$

$$R = \frac{r\frac{\sqrt{(x^2+1)}}{2}}{\cos\alpha} = \frac{r\frac{\sqrt{(x^2+1)}}{2}}{\frac{1}{\sqrt{(x^2+1)}}}$$

$$R = \frac{(x^2+1)}{2}r$$

Drawing the tangent

A / Approach based on the Pythagorean theorem

The Pythagorean theorem can also be used for triangles ABI and OBI (Fig. 1):

$$t^2 = 9r^2 + (r + h^{'})^2$$

$$(R + h^{'})^2 = R^2 + t^2$$

$$R^2 + 2Rh^{'} + h^{'2} = R^2 + 9r^2 + r^2 + 2rh^{'} + h^{'2}$$

$$2(R - r)h' = 10r^2$$

$$h^{'} = \frac{10r^2}{2(5r - r)}$$

$$h^{'} = \frac{10}{8}r = \frac{5}{4}r$$

This can be generalized to any ratio *x* between *r* and *R*

$$t^2 = xr^2 + (r + h^{'})^2$$

$$(R + h^{'})^2 = R^2 + t^2$$

$$R^2 + 2Rh^{'} + h^{'2} = R^2 + x^2r^2 + r^2 + 2rh^{'} + h^{'2}$$

$$2(R - r)h' = 10r^2$$

$$h^{'} = \frac{(x^2 + 1)r^2}{2(\frac{(x^2 + 1)}{2}r - r)}$$

$$h^{'} = \frac{(x^2 + 1)r^2}{((x^2 + 1) - 2)r}$$

$$h^{'} = \frac{(x^2 + 1)}{(x^2 - 1)}r$$

We can also calculate γ

$$cos\,\gamma = \frac{3r}{\sqrt{(9r^2 + \frac{81}{16}r^2)}} = \frac{3}{\sqrt{\frac{225}{16}}} = \frac{3}{\frac{15}{4}} = \frac{12}{15}$$

$$\gamma = 36.87°$$

The apical angle for a 3:1 ratio is therefore 74°.

B/ Approach based on the ratios of the angles of right triangles

$$cos\,\alpha = \frac{1}{\sqrt{10}}$$

which can be used to calculate numerical degrees:

$$\alpha = cos^{-1}\frac{1}{\sqrt{10}} = 71.57°$$

and then determine the other angles:

$$\beta = 180 - 90 - \alpha = 18.43°$$

and determine the γ angle:

$$\gamma = 90 - \alpha + \beta = 36.87°$$

$$sin\,\gamma = 0.6 = \frac{(r + h')}{\sqrt{(9r^2 + \frac{81}{16}r^2)}}$$

$$\frac{3}{5} * \frac{15}{4} * r = r + h'$$

$$h^{'} = \left(\frac{9}{4} - 1\right)r = \frac{5}{4}r$$

DISCUSSION

The fusiform excision technique is one of the most versatile and frequently used office surgery procedures. This technique is used to remove undetermined, benign and malignant lesions on or underneath the skin. Most surgeons perform freehand drawings. However, the design of the fusiform incision is composed of arcs of circles that can be fully described or more accurately approximated by the tangents of these circles at the extremities of the fusiform incision.

According to the literature,[1-6] the elliptic shape could be drawn with a ratio higher than 3:1, but would result in a longer skin incision. In this article, we propose generalization of the geometric formulas to help surgeons draw this incision. Although dog-ears less than 4 mm may not leave any permanent cosmetic sequelae at 6 months,[3] it is preferable to avoid dog-ears at the end of surgery. As a general rule of thumb, it is commonly admitted that a 3:1 length-to-width ratio often results in a defect with an angle of approximately 30 degrees without taking an excessive amount of tissue. However, Bennett[3] and Moody et al.[2] questioned the 30° angle. Bennett demonstrated that a 3.5:1 ellipse has an apical angle of 51°.[2] They demonstrated that the apical angle of a 3:1 elliptical excision is not 30 degrees, but ranges from 37 degrees to 74 degrees depending on excisional geometry. For a 3:1 ratio, they found similar results to ours with an apical angle of 74° using another geometric approach.

CONCLUSION

In clinical practice, it is not always possible to draw the incision by using a circle with radius R (5 times the radius of the lesion), especially when the lesion is located on a non-planar surface. Drawing the γ angle (37°) and the intersection between the perpendicular line to the axis of the fusiform excision and the tangent to the arc of the circle could be easier and more reliable than an intuitive approach. We propose the geometric approach as an exercise for our residents. We found that this exercise was appreciated differently by students, but it has the advantage of introducing the geometry of dermatologic surgery while many procedures are more complex than the fusiform excision. Compared to a more intuitive technique and demonstration-based teaching, we believe that, beyond the conceptualization of the fusiform incision, our study can be very constructive for residents.

DECLARATIONS

- *Ethics approval and consent to participate : Not applicable*
- *Consent to publish : Not applicable*
- *Availability of data and materials : Not applicable*
- *Competing interests : Not applicable*

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REFERENCES

1. Saun TJ, Truong J, Dev SP, et al. How to Perform an Elliptical Excision of the Skin. N Engl J Med. 2019 May 9;380(19):e34.

2. Moody BR, McCarthy JE, Sengelmann RD. The Apical Angle: A Mathematical Analysis of the Ellipse. *Dermatol Surg.* 2001;27:61-63.

2. Jennings TA, Keane JC, Varma R, et al. Observation of Dog-Ear Regression by Anatomical Location. *Dermatol Surg.* 2017;43:1367‐70.

3. Bennett RG. Fundamentals of cutaneous surgery. St. Louis: CV Mosby, 1988:370–72.

4. Dunlavey E, Leshin B. The simple excision. *Dermatol Clin.* 1998;16:49‐64.

5. Goldberg LH, Alam M. Elliptical Excisions: Variations and the Eccentric Parallelogram. *Arch Dermatol.* 2004;140:176-80.

6. Zitelli JA. TIPS for a better ellipse. *J Am Acad Dermatol.* 1990;22:101‐3.

FIGURE LEGENDS

Figure 1: Geometric approach to fusiform incision.

The grey disc (circle C1, center A) represents the lesion; B is the right extremity of the fusiform incision, C is the top of the lesion, O is the center of the circle C2, defining the superior limit of the fusiform incision (arc-of-circle of C2) and I is the intersection of the AC straight line and the tangent of circle C2 at B. *r = AC* is the radius of the lesion (Circle C1). *R* is the radius of circle C2, *h* is the distance between B and C, *t* is the distance between B and I, *h'* is the distance between C and I. We also define several angles that are used in the formulas: α, β, γ and δ. $\alpha = A\hat{C}B = C\hat{B}O; \beta = A\hat{B}C; \gamma = A\hat{B}I; \delta = A\hat{I}B$

Figures

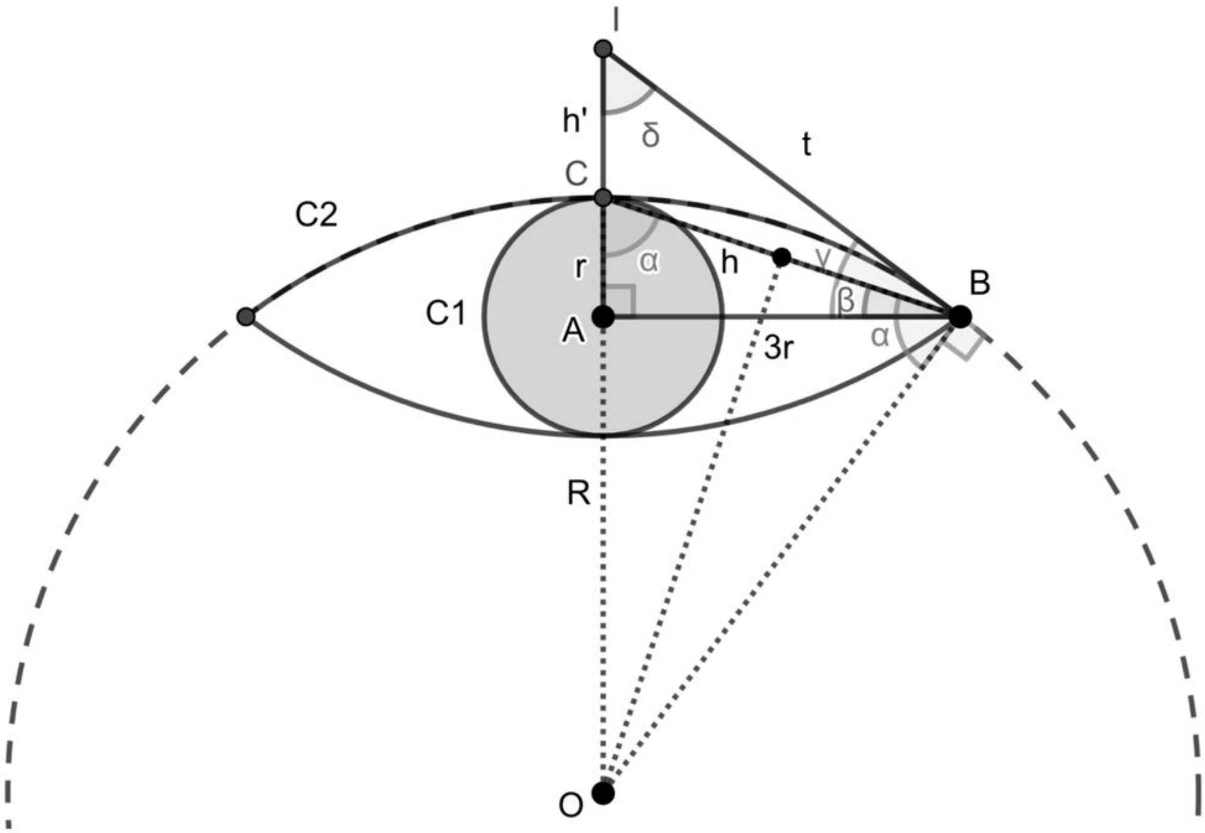

Figure 1

Geometric approach to fusiform incision. The grey disc (circle C1, center A) represents the lesion; B is the right extremity of the fusiform incision, C is the top of the lesion, O is the center of the circle C2, defining the superior limit of the fusiform incision (arc-of-circle of C2) and I is the intersection of the AC straight line and the tangent of circle C2 at B. r = AC is the radius of the lesion (Circle C1). R is the radius of circle C2, h is the distance between B and C, t is the distance between B and I, h' is the distance between C and I. We also define several angles that are used in the formulas