Stability of Soliton Solutions For A PT- Symmetric NLDC Considering High-Order Dispersion and Nonlinear Effects Simultaneously

Lida Safaei (✉ lidasafaei2015@gmail.com)

Yazd University   https://orcid.org/0000-0001-5564-3570

Mohsen Hatami

Shiraz University of Technology

Mahmood Borhani Zarandi

Yazd University

Research Article

Stability of soliton solutions for a $\mathcal{PT}$- Symmetric NLDC considering high-order dispersion and nonlinear effects simultaneously

**Lida Safaei · Mohsen Hatami ·
Mahmood Borhani Zarndi**

Abstract In this paper, we analytically solve the coupled equations of a $\mathcal{PT}$-Symmetric NLDC by considering high-order dispersion and nonlinear effects (Raman Scattering and self-steeping) simultaneously in normal dispersion regime. To the best of knowledge no works has been done in previous studies to decoupled these equations and obtain an exact analytical solution. The new exact bright solitary solutions are derived. In addition, to study the stability and instability of these propagated solitons in a $\mathcal{PT}$-Symmetric NLDC, perturbation theory is used. Numerical methods are applied to find perturbed eigenvalues and eigenfunctions. The Stability of obtained four perturbed eigenvalues and perturbed eigenfunctions for a $\mathcal{PT}$-Symmetric NLDC equations regard to high-order effects are examined. Using these results and simulating the propagation of perturbed temporal bright solitons through $\mathcal{PT}$-Symmetric NLDC show that perturbed solitons are mostly stable. This means that high-order dispersion and nonlinear effects canceled each other and do not affected the propagated solitons.
Furthermore, the evolution of perturbed solitons energies match well the previous results and confirmed the stability of these solitons in a $\mathcal{PT}$-Symmetric NLDC. As seen the energies of pulses in bar and cross behave in two manner 1) the exchange of energy is happened in some periods, but the shape of

L. Safaei
Faculty of Science, Department of Physics, Yazd University, Yazd, Iran
Tel.: +0098-9177140885
E-mail: lidasafaei2015@gmail.com

M. Hatami
Faculty of Physics, Shiraz University of Technology, Shiraz, Iran
Tel.: +0098-9131518991
E-mail: hatami@sutech.ac.ir

M. Borhani Zarandi
Faculty of Science, Department of Physics, Yazd University, Yazd, Iran
Tel.: +0098-9131520036
E-mail: mborhani@yazd.ac.ir

each pulse in bar and cross is preserved. Therefore, the solitons under this eigenfunction perturbation are mostly stable. 2) the evolution of energy in the bar and cross, demonstrate that there is no changes in their energies and they remain constant. It is straightforward to show that in spite of considering high-order effects, the perturbed soliton conserve the shape and it remain stable.

The deliverables of this article not only demonstrate a novel approach to ultrafast pulses, solitons and optical couplers, but more fundamentally, they could give insight for improving the new medical equipments technologies, enabling innovations in nonlinear optics and their usage in designing new communication systems and Photonic devices.

Keywords Fiber Optics · Nonlinear Optics · Third order dispersion · Ramman scattering · Self-Steepening · Optical communication system · $\mathcal{PT}$-Symmetric NLDC · Medical Technologies · Photonics.

1 Introduction

In 1998 Bender and Butcher found an unique remarkable phenomenon that even non-Hermitian Hamiltonians can still have completely real eigenvalue spectra if they respect Parity-Time ($\mathcal{PT}$) symmetry [1]. From then on, the unique properties of $\mathcal{PT}$-Symmetric systems have drawn considerable attention in both quantum mechanics and optics [2], [3], [4].

In optics $\mathcal{PT}$-Symmetric systems have attracted much attention both theoretically and experimentally. Ever since in this field $\mathcal{PT}$-Symmetric solitons have been widely studied [5], [6], [7], [8]. The importance of optical solitons for fundamental studies and technological applications in Photonic and Optics, such as bio-optical devices, all-optical switching, ultrafast communication systems, telecommunication engineering and all-optical gates is well known [9], [10], [11], [12], [13], [14].

The nonlinear Schrodinger equation (NLSE) is considered to be the key model to describes the dynamics of propagation of light-wave in an optical fiber [15], [16], [17], [18]. In the past few decades, many mathematician and scientist developed a powerful and direct techniques for the construction of analytical solutions of nonlinear evolution equations [19], [20], [21], [22]. The analytical and numerical study of nonlinear partial differential equations is one of the most fascinating and exciting areas of research for many researchers in recent years. The development of new mathematical techniques to find out a more compact and general form of exact solutions is one of the most important tasks to understand the complete dynamical process modeled by complex nonlinear partial differential equations from the past few decades. Extracting exact solutions of nonlinear partial differential equations is also important to check the stability of numerical solutions as well as to develop a wide range of new mathematical solvers to simplify the calculation. In recent time, an abundance of new more powerful and effective methods have been developed with

the help of different computer softwares like Mathematica, Maple and Matlab, such as the Kudryashov method, the truncated expansion method, the Boacklund transform method , the inverse scattering method, the extended Fan sub-equation method, the homogeneous balance method, the Jacobi elliptic function method, the tanh-function method, the BVI INIT Method and many more in several theoretical works about solitons and their applications [21], [22], [23], [24], [25], [26].

Nowadays, During studying the ultrashort pulses, the higher-order nonlinear effects and high-order dispersion cannot be neglected. In addition a class of optical systems consists of elements with gain and loss like nonlinear directional couplers (NLDC) arranged in a symmetric way have been receiving much attention these days [27], [28], [29], [30]. In particular, the third-order dispersion (TOD), Raman scattering (RS) and self-steepening (SS) affects the dynamics of solitons propagating in NLDC [31], [32].

Study the propagation of ultrashort pulses in a coupler is practically important in various areas of research; these are studied in nonlinear optics, plasma physics, nuclear physics, mathematical physics, bio-physics and many other physical sciences [33], [34], [35], [36]. In addition, recently practical applications of these solitons include high-order nonlinearities in couplers are applied in medical devices, such as new researches of Sydney University examine the usage of ultrashort pluses to improve the eye surgery devices [37].

In this paper the coupled higher-order nonlinear Schrodinger equation (HNLSE) with third-order dispersion (TOD), self-steepening (SS) and Raman scattering (RS) is considered simultaneously. Then we study the existence and stability of optical solitons in a $\mathcal{PT}$-Symmetric NLDC with gain in the bar (upper waveguide) and loss in the cross (lower waveguide). Perturbation Theory is used to find the perturbed eigenvalues and eigenfunctions. Stability of propagating such solitons is simulate by numerical methods in MATLAB program. Finite Difference, the BVP INIT and Shooting methods is used for numerical calculations. The evolution of their energies is examined, too.

2 Theory and Numerical results

In recent decades $\mathcal{PT}$-Symmetric nonlinear couplers with gain and loss has been studied theoretically and experimentally [8], [12], [17], [18], [27]. In these systems if the input pulses width were considered too small ($<$ps), the effects of higher-order, linear and nonlinear terms are so important. The higher-order terms should be considered, in the ultrashort pulse propagation equations through $\mathcal{PT}$-Symmetric NLDC.

Ultrashort pulses have been fundamental for the development of major photonic applications such as communications systems, nonlinear imaging and biomedical devices. Soliton effects, based on the balance of dispersion and nonlinearity, have allowed for the direct generation of optical pulses with duration below 10 fs. [30], [36].

Higher-order coupled equations with TOD, SS and RS terms are extended in

following forms [30]:

$$iu_z + \alpha_1 u_{tt} + \alpha_2 \mid u \mid^2 u + i[\alpha_3 u_{ttt} + \alpha_4 \mid u \mid^2 u_t + \alpha_5 u(\mid u \mid^2)_t] = -v + i\gamma u,$$
$$iv_z + \alpha_1 v_{tt} + \alpha_2 \mid v \mid^2 v + i[\alpha_3 v_{ttt} + \alpha_4 \mid v \mid^2 v_t + \alpha_5 v(\mid v \mid^2)_t] = -u + i\gamma v \quad (1)$$

These are the unperturbed nonlinear Schrodinger (NLS) coupled equation with three extra terms. These extra terms model the effects of third-order dispersion (3OD), Raman Scattering, and self-steepening. In a fiber, these terms have different strengths in different parameter regimes. The unperturbed NLS is a good model for pulse durations above 1 ps. However, the third-order dispersion term must be included for carrier wavelengths near the zero-dispersion wavelength, independently of the pulse duration. The Raman term is important for shorter pulses, of the order 1-0.05 ps. It is important to realize exactly what kind of approximations have lead to these equations and their regime of validity. For this purpose we use the slowly varying envelope approximation (SVEA).

In Eq. (1) u and v represent the slowly varying envelops, z and t are variables for propagation direction and retarded time respectively. α_1, α_2, α_3, α_4 and α_5 are group velocity dispersion (GVD), self-phase modulation (SPM), TOD, SS, and SRS coefficients respectively, which are real parameters.

We start to find soliton solution of nonlinear coupled equations by make a change of variables as traveling wave transformation:

$$u = U(z,t)e^{i(\omega t - \theta)} \qquad , v = V(z,t)e^{i(\omega t)}. \tag{2}$$

Where ω is a real parameter and θ is a constant angle and it defined $\gamma = \sin\theta$. By substituting into Eq.(1) we have:

$$iU_z + \alpha_1(U_{tt} - \Omega^2 U) + \alpha_2 \mid U \mid^2 U + i[\alpha_3(U_{ttt} + i\Omega U_{tt} - i\Omega^3 U)$$
$$+\alpha_4 \mid U \mid^2 (U_t + i\Omega U) + \alpha_5 U(\mid U \mid^2)_t] = -V\cos\theta + i\gamma(U - V),$$
$$iV_z + \alpha_1(V_{tt} - \Omega^2 V) + \alpha_2 \mid V \mid^2 V + i[\alpha_3(V_{ttt} + i\Omega V_{tt} - i\Omega^3 V)$$
$$+\alpha_4 \mid V \mid^2 (V_t + i\Omega V) + \alpha_5 V(\mid V \mid^2)_t] = -U\cos\theta + i\gamma(U - V). \tag{3}$$

We can decoupled these equations by applying $U = V = \phi$. Therefore Eqs.(3), reduce into:

$$i\phi_z + (\alpha_1 - \alpha_3\Omega)\phi_{tt} + (\alpha_2 - \alpha_4\Omega) \mid \phi \mid^2 \phi + i[\alpha_3\phi_{ttt} + \alpha_4 \mid \phi \mid^2 \phi_{tt}$$
$$+\alpha_5\phi(\mid \phi \mid^2)_t] + (-\alpha_1\Omega^2 + \alpha_3\Omega^3 + \Omega - a^2)\phi = 0. \tag{4}$$

For solving these equations, we try to change them into a form which used by Kodama and Hasegawa for HNLS [26], [30]. Due to the form of these equations we define the following parameters and taking the coefficient of ϕ to zero.

$$\eta_1 = \alpha_1 - \alpha_3\Omega,$$
$$\eta_2 = \alpha_2 - \alpha_4,$$

$\eta_3 = \alpha_3,$
$\eta_4 = \alpha_4,$
$\eta_5 = \alpha_5,$
$(-\alpha_1\Omega^2 + \alpha_3\Omega^3 + \Omega - a^2) = 0.$

By substituting these variables into Eq. (4) the following equation is obtained:

$$i\phi_z + \eta_1\phi_{tt} + \eta_2 \mid \phi \mid^2 \phi + i[\eta_3\phi_{ttt} + \eta_4 \mid \phi \mid^2 \phi_t + \eta_5\phi(\mid \phi \mid^2)_t] = 0. \quad (5)$$

Hence this equation is not integrable, so far no analytical answer has been provided. Therefore the following method is applied to obtain an analytical solution for this equation.

First, considering a traveling wave: $\phi(z,t) = r(\xi)e^{i\theta'}$ as a solution of Eq.(5), where $\theta' = kt - \omega'z$ and $\xi = b(t - cz)$, then we divide the equation into real and imaginary parts.

$$(b^2\eta_1 - 3b^2k\eta_3)r'' + (\omega' - k^2\eta_1 + k^3\eta_3)r + (\eta_2 - k\eta_4)r^3 = 0. \quad (6)$$

$$b^3\eta_3r''' + b(\eta_4 + 2\eta_5)r^2r' - (bc - 2b\eta_1 + 3bk^2\eta_3)r' = 0. \quad (7)$$

If taking integral from Eq.(7) and put the integral constant equal to zero, the following equation is obtained:

$$b^3\eta_3r'' + b(2\eta_1 - 3k^2\eta_3 - c)r + \frac{1}{3}b(\eta_4 + 2\eta_5)r^3 = 0. \quad (8)$$

Considering the conditions that equations (8) and (6) are compatible with each other and simultaneously have answer, the solution of $r(\xi)$ in these two equations should be the same and unique. Therefor, the coefficient of the same order terms are equated as below:

$$\begin{aligned}
(b^2\eta_1 - 3b^2k\eta_3) &= b^3\eta_3, \\
(\omega' - k^2\eta_1 + k^3\eta_3) &= b(2\eta_1 - 3k^2\eta_3 - c), \\
(\eta_2 - k\eta_4) &= \tfrac{1}{3}b(\eta_4 + 2\eta_5).
\end{aligned} \quad (9)$$

According to solve Eqs. (9), the following values are obtained for k and $\omega\prime$:

$$k = \frac{\eta_1\eta_4 - 3\eta_2\eta_3 + 2\eta_1\eta_5}{6\eta_3\eta_5}. \quad (10)$$

$$\omega' = \frac{2k\eta_1^2 - \eta_1(c + 8k^2\eta_3)}{\eta_3} + k(3c + 8k^2\eta_3). \quad (11)$$

Both equations (6) and (7) will be converted into the same equation as follow:

$$\beta_1r'' + \beta_2r + \beta_3r^3 = 0. \quad (12)$$

The parameter β_1, β_2, β_3 are defined as:

$$\beta_1 = \frac{b^2(3\eta_2\eta_3 - \eta_1\eta_4)}{2\eta_5}, \quad (13)$$

$$\beta_2 = -(3\eta_2\eta_3 - \eta_1\eta_4) \times \frac{(3\eta_2\eta_3 - \eta_1\eta_4)^2 + 4c\eta_5^2(3\epsilon\eta_3 - \eta_1^2)}{24\epsilon^2\eta_3^2\eta_5^3}, \qquad (14)$$

$$\beta_3 = \frac{(3\eta_2\eta_3 - \eta_1\eta_4)(\eta_4 + 2\eta_5)}{6\eta_{3\eta5}}. \qquad (15)$$

To find the solution of equation (12), the following boundary conditions are considered:

$$\begin{cases} r(\xi) > 0 \\ r'(\xi) \leq 0 \\ \lim_{|\xi|\to\infty} r(\xi) = 0 \end{cases} \qquad (16)$$

Using the boundary conditions, the equation (12) yields a bright soliton solution given by:

$$r(\xi) = \sqrt{\frac{2\beta_2}{\beta_3}} \times sech(\sqrt{\frac{\beta_2}{\beta_1}}\xi). \qquad (17)$$

Finally the analytic solutions for $\mathcal{PT}$-Symmetric higher-order nonlinear equations (1) are given as:

$$u = \sqrt{\frac{2\beta_2}{\beta_3}} \times sech(\sqrt{\frac{\beta_2}{\beta_1}} \times b(t - cz)) \exp^{i(kt - \omega'z)} \times \exp^{i(\omega t - \theta)},$$

$$v = \sqrt{\frac{2\beta_2}{\beta_3}} \times sech(\sqrt{\frac{\beta_2}{\beta_1}} \times b(t - cz)) \exp^{i(kt - \omega'z)} \times \exp^{i(\omega t)}. \qquad (18)$$

In the latter, it is important to note that the solutions obtained in Eq.(18), are for the perturbed NLDC while the solutions obtained in the previous works are for the Nonlinear Schroedinger Equation (NLSE) which can be easily obtained by setting the coefficients of dispersion terms and Raman scattering to zero, then the perturbed NLSE becomes NLSE with non-Kerr law nonlinearity. From the above comparison we can conclude that our obtained solutions are new and have not been solved before, which shows that our method is helpful, effective, straightforward and reliable to analytically study for the coupled nonlinear complex models.

2.1 Stability Analysis

Now, the stability of soliton propagating through the higher-order $\mathcal{PT}$-Symmetric NLDC is studied and obtained the equilibrium points for the propagating solitons in a nonlinear $\mathcal{PT}$-Symmetric coupler. In order to check the stability, we add small perturbation into the bright solitons solution:

$$\mathcal{U}(z,t) = u(z,t) + \delta u(z,t), \qquad \mathcal{V}(z,t) = v(z,t) + \delta v(z,t). \qquad (19)$$

Where u, v are soliton solutions which defined in Eq. (18). Applying perturbed solutions in Eq. (3) and linearized them with respect to δu and δv equations (3) can be written as:

$$i\delta u_z + (\alpha_1 - 3\alpha_3\Omega)u_{tt} + i\alpha_3\delta u_{ttt} + i(\alpha_4 + 2\alpha_5)r^2\delta u +$$
$$(-\Omega^2\alpha_1 + \alpha_3\Omega^3)\delta u + 3(\alpha_2 - \Omega\alpha_4)r^2\delta u = -\cos\theta\delta v + i\gamma(\delta u - \delta v),$$
$$i\delta v_z + (\alpha_1 - 3\alpha_3\Omega)v_{tt} + i\alpha_3\delta v_{ttt} + i(\alpha_4 + 2\alpha_5)r^2\delta v +$$
$$(-\Omega^2\alpha_1 + \alpha_3\Omega^3)\delta v + 3(\alpha_2 - \Omega\alpha_4)r^2\delta v = -\cos\theta\delta u + i\gamma(\delta u - \delta v).$$
$$(20)$$

To solve this pair of equations, two symmetric and asymmetric new variables are defined and substitute into Eqs. (20):

$$p = \frac{\delta u + \delta v}{\sqrt{2}}, \qquad q = \frac{\delta u - \delta v}{\sqrt{2}}. \qquad (21)$$

For solving the derived equation, separate solutions are considered as follows[17]:

$$p = \exp(\nu t)[(p_1' + ip_2')\cos\omega t + (p_1'' + ip_2'')\sin\omega t],$$
$$q = \exp(\nu t)[(q_1' + iq_2')\cos\omega t + (q_1'' + iq_2'')\sin\omega t]. \qquad (22)$$

Where

$$p_1 = p_1' + ip_1'' \qquad , \qquad p_2 = p_2' + ip_2'',$$
$$q_1 = q_1' + iq_1'' \qquad , \qquad q_2 = q_2' + iq_2''. \qquad (23)$$

Using the solutions (22), the relations and Eqs. (23), the following eigenvalue equations with respective eigenvalue are obtained λ:

$$\begin{pmatrix} L & N - \cos\theta \\ N - \cos\theta & -L \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} + 2\gamma \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \lambda \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}.$$
$$(24)$$

$$\begin{pmatrix} L & N - \cos\theta \\ N - \cos\theta & -L \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \lambda \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}. \qquad (25)$$

Where $\lambda = \nu - i\omega$ and the operators L and N are defined as:

$$L = \alpha_3\frac{d^3}{dt^3} + (\alpha_4 - 2\alpha_5)\frac{d}{dt},$$
$$(26)$$
$$N = (\alpha_1 - \alpha_3\Omega)\frac{d^2}{dt^2} + (3(\alpha_2 - \alpha_4\Omega)r^2) - \Omega^2(\alpha_1 - \alpha_3\Omega).$$

As These eigenvalue equations (25) can not be solved by the analytical methods, a numerical method is applied in order to solve it.

2.2 Numerical Analysis

To solve the perturbed equations Due to the accuracy of methods we use
Finite Difference, the BVP INIT and Shooting methods which embedded in
MATLAB software.
Furthermore, the parameters for a $50(fs)$ pulse with $1.55(\mu \text{ m})$ wavelength is
considered and the normalized coefficients of TOD, SS and RS are calculated
as: 0.03, 0.03 and 0.1, respectively.
Four eigenvalues are obtained as depicted in Fig.(1):

$$
\begin{aligned}
\lambda_1 &= 0.6, \\
\lambda_2 &= 0, \\
\lambda_3 &= -0.4, \\
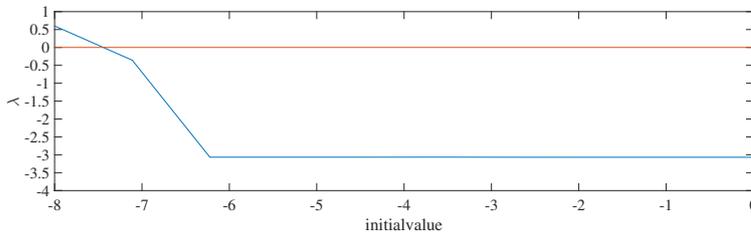\lambda_4 &= -3.
\end{aligned}
\tag{27}
$$

Fig. 1: Obtained eigenvalues corresponding to eigenequations associated with
solitons propagating in a $\mathcal{PT}$-Symmetric NLDC regard to the nonlinear effects
of Raman dispersion, self-steepening and third order dispersion.

Physically the higher-order nonlinearity and dispersion and coupling be-
tween two waveguides in NLDC cause to exchange energy and alert the shape
of input pulse. We find special pulses which can propagate without any ex-
change in energy and shape.
These eigenvalues are interpreted as equilibrium points but the stability and
instability of them should be studied. The perturbation method is applied as
used in Refs. [19], [20] to examine the stability and instability of each perturbed
eigenfunction. With respect to these eigenvalues the corresponding eigenfunc-
tions are obtained numerically.
The four obtained perturbed eigenfunctions for a $\mathcal{PT}$-Symmetric NLDC equa-
tions regard to high-order effects are depicted in Fig. (2) in which:
continues line (green) correspond to $\lambda_1 = 0.6$, dash line (–, red) correspond to
$\lambda_2 = 0$, dot line (..., purple) correspond to $\lambda_3 = -0.4$ and dash-dot line (-.-. ,
blue) correspond to $\lambda_4 = -3$.
By imposing the eigenfunctions as a perturbation to stable soliton solution
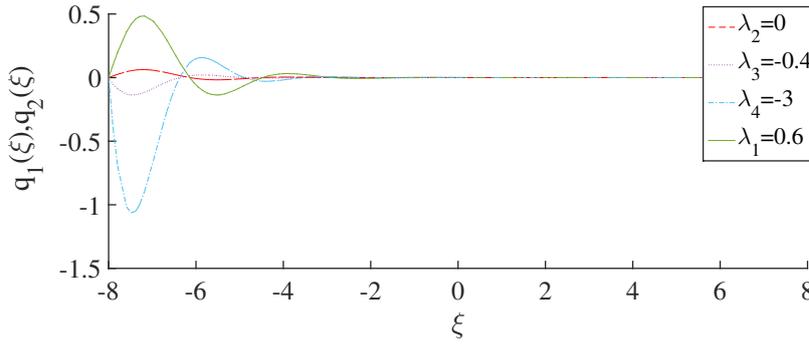
Fig. 2: the obtained eigenfunction: continues line (green) correspond to $\lambda_1 = 0.6$, dash line (–, red) correspond to $\lambda_2 = 0$, dot line (..., purple) correspond to $\lambda_3 = -0.4$ and dash-dot line (-.-. , blue) correspond to $\lambda_4 = -3$ for a $\mathcal{PT}$-Symmetric NLDC equations regard to high-order effects.

the stability and instability of bright solitons propagating in a high-order nonlinear $\mathcal{PT}$-Symmetric coupler are studied numerically.

For this purpose, Eq. (2) is solved numerically and the behavior of propagated soliton under perturbation in a nonlinear $\mathcal{PT}$-Symmetric coupler is simulated with the help of MATLAB program.

Fig. (3), shows the evaluation of solitons and corresponding energies in bar and cross under perturbation for $\lambda_1 = 0.6$.

As seen in this figure, the energies of pulses in bar and cross are exchanged despite unperturbed case which energy and the shape of each pulse in bar and cross remain constant. Also, the exchange of energy is happened but in some periods the shape of each pulse in bar and cross is preserved. So, the solitons under this eigenfunction perturbation are mostly stable. This behavior is like linear directional coupler which only energies are exchanged but the shape are preserved.

In Figs. (4) to (6) the propagation of perturbed soliton under perturbed eigenfunctions corresponding to λ_2, λ_3, λ_4 and respected eigenfunctions are depicted. In these three cases all perturbed solitons are stable, the energy of each soliton in bar and cross remains constant and also the solitons preserve their shapes unchanged.

The propagation of bright soliton in a nonlinear $\mathcal{PT}$-Symmetric coupler, taking into account the high-order effects, eigenvalue and the corresponding eigenfunction (dot line) has been investigated in Fig. (4). As it can be seen, the soliton remains unchanged in the bar and cross in this state and its energy remains constant in both waveguides. So the bright soliton is quite stable in this state.
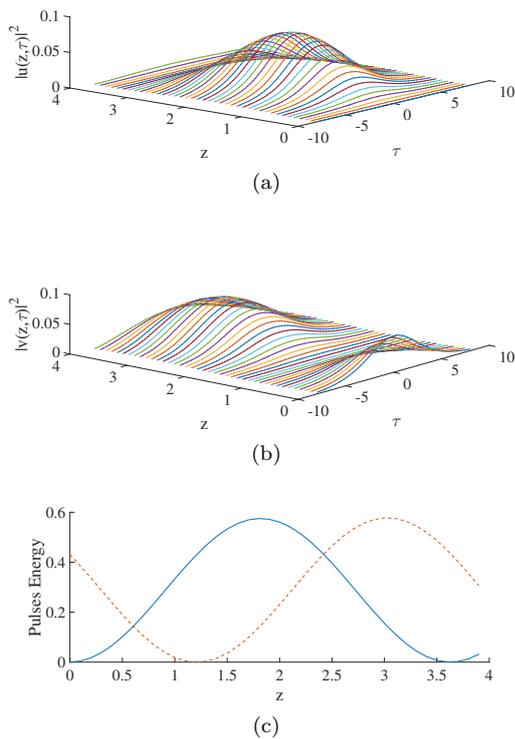
(a)

(b)

(c)

Fig. 3: The evolution of perturbed bright soliton correspond to $\lambda_1 = 0.6$ and it's eigenfunction continues line (green) in a nonlinear $\mathcal{PT}$-Symmetric coupler, regard to high-order effects (a) in bar, (b) in cross and (c) corresponding energies in bar and cross under perturbation.
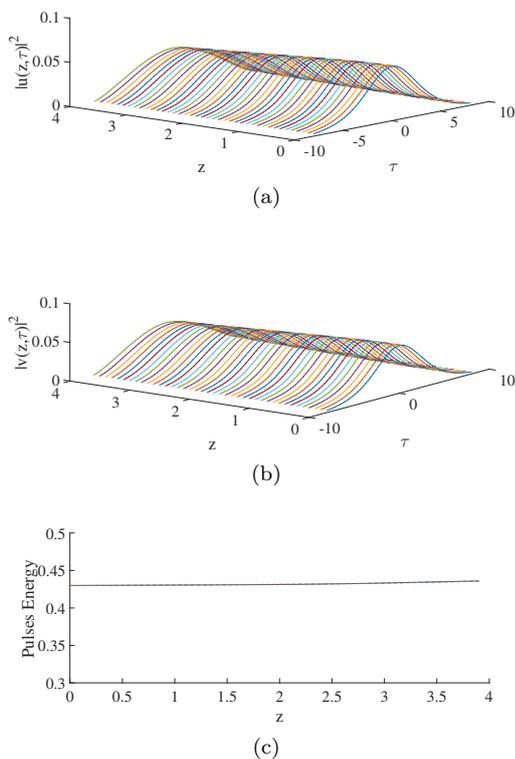

(a)

(b)

(c)

Fig. 4: The evolution of perturbed bright soliton correspond to $\lambda_2 = 0$ and it's eigenfunction dash line (–, red) in a nonlinear $\mathcal{PT}$-Symmetric coupler, regard to high-order effects (a) in bar , (b) in cross and (c) corresponding energies in bar and cross under perturbation.
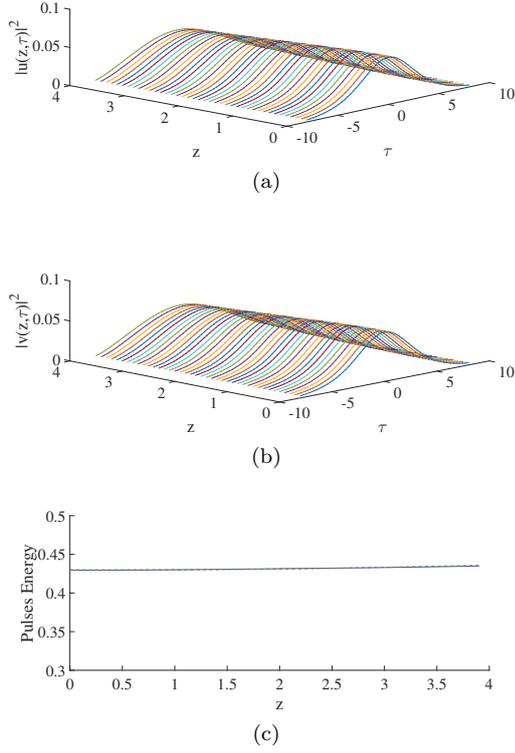
(a)

(b)

(c)

Fig. 5: The evolution of perturbed bright soliton correspond to $\lambda_3 = -0.4$ and it's eigenfunction dot line (..., purple) in a nonlinear $\mathcal{PT}$-Symmetric coupler, regard to high-order effects (a) in bar , (b) in cross and (c) corresponding energies in bar and cross under perturbation.

In Fig. (5), the stability and evolution of the propagating perturbed bright soliton and the corresponding energy, by using the perturbed eigenvalue correspond to its eigenfunction (dash-dot line) and considering higher order effects is examined. In this case, solitons remain in their shape and propagate without any changes in both waveguides. Energy change also indicate that the bright soliton with simultaneous consideration of higher order effects are stable in this case.
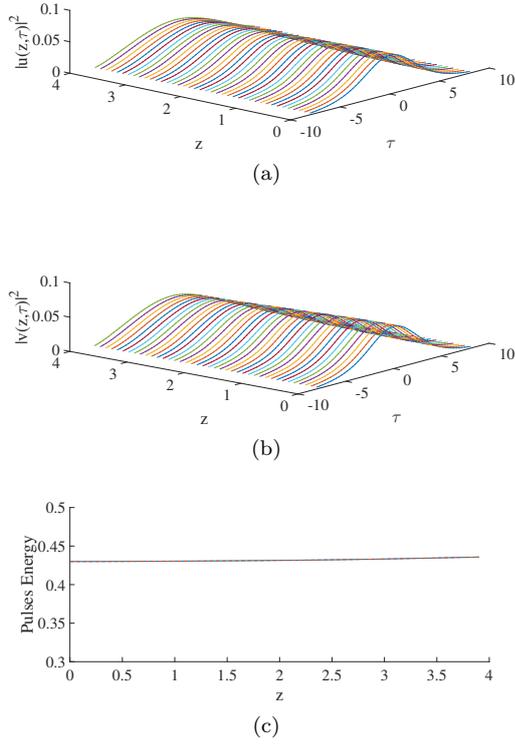
(a)

(b)

(c)

Fig. 6: The evolution of perturbed bright soliton correspond to $\lambda_3 = -0.4$ and it's eigenfunction dash-dot line (-.-. , blue)in a nonlinear $\mathcal{PT}$-Symmetric coupler, regard to high-order effects (a) in bar , (b) in cross and (c) corresponding energies in bar and cross under perturbation.

Finally, by adding the fourth eigenvalue, and the corresponding perturbed eigenfunction (the dash-dot line -.-) into the input bright soliton, in Fig. (6), the stability of this perturbed solitons are investigated.

The analysis of Fig. (6), shows that similar to the two previous states, despite considering higher-order effects, the perturbed bright soliton is stable in both waveguides and keep its solitary shape. For the evolution of energy in the bar and the cross, analysis demonstrate that there is no changes in their energies and they remain constant. It is straightforward to show that in spite of considering high-order effects, the perturbed soliton remain stable.

We verified that the behavior of figures (4), (5) and (6) are the same, as expected. Numerical simulation shows that the shape of pulses are not changed and act as solitons.

3 conclusion

In this paper, the higher-order coupled nonlinear equations of a $\mathcal{PT}$-Symmetric NLDC has been investigated. In these equations Raman scattering, Self-steepening and third-order dispersion effects are considered simultaneously. An analytical method have been employed on coupled equations to decoupled them and obtain new exact solitary solutions. Investigation shows that the exact solution of these equations is derived as a bright soliton. In the next step, to examine the stability of solitons propagating in a $\mathcal{PT}$-Symmetric NLDC, perturbation theory is used. As the coupled equations of a $\mathcal{PT}$-Symmetric NLDC regard to high-order effects and perturbation method are not solvable analytically, numerical methods are applied. Four perturbed eigenvalues and eigenfunctions are obtained by using Finite Difference, the BVP INIT and Shooting methods. Investigation of the stability of propagating bright solitons in a $\mathcal{PT}$-Symmetric NLDC by this method shows that perturbed solitons are almost stable and not affected by high-order effects.
Furthermore, the evolution of their energies are confirmed the stability of these solitons propagating through the $\mathcal{PT}$-Symmetric NLDC.

the energies of pulses in bar and cross behave in two manner: 1) For one of the perturbed propagated soliton, in some periods, energies are exchanged in bar and cross, but the shape of each pulse is preserved. Therefore, the solitons under this eigenfunction perturbation are mostly stable. 2) The other three perturbed solitons experience no changes in their energies and they remain constant. It is straightforward to show that in spite of considering high-order effects, the perturbed soliton conserve the shape and it remain stable.
Also, the deliverables of this paper can be used in new Bio-optical and medical devices, designing new optical communication systems, optical engineering and applied physics science.

References

1. Bender, C.M., Boettcher, S.: Real spectra in non- Hermitian Hamiltonians having PT-symmetry.Phys. Rev. Lett., 80, 5243-5246(1998).
2. Bender, C.M. , Brody, D.C., Jones, H. F.: Complex extension of quantum mechanics. Phys. Rev. Lett., 89, 270401–240404(2002).
3. Ruter, C. E, Mrkis, K. G, El-Gnainy, R, Christodoulides, D. N., Segev, M., Kip, D.: Observation of parity-time symmetry in optics. Nat.Phys.,6,192(2010).
4. Zyablovsky, A. A., Vinogradov, A. P., Pukhov, A. A., Dorofeenko, A. V., Lisyansky, A. A.: PT-symmetry in optics. Phys. Usp., 57, 1063–1082(2014).
5. Chen, X., Yang, J.: A direct perturbation theory for solitons of the derivative nonlinear Schrodinger equation and the modified nonlinear Schrodinger equation. Phys. Rev. E. 65, 066608(2002).
6. Lu, X., Ma, W. X.: The Inverse Cascade and Nonlinear Alpha-Effect in Simulations of Isotropic Helical Hydromagnetic Turbulence. Nonlinear Dyn.,2755-2758(2016). https://doi.org/10.1007/s11071
7. Dai, C. Q., Zhang, J. F.: Controllable dynamical behaviors for spatiotemporal bright solitons on continues wave background. Nonlinear Dyn., 2049-2057(2013).
8. Zhu, Y., Qin, W., Li, J., et al.: Recurrence behavior for controllable excitation of rogue waves in a two-dimensional PT-Symmetric couple. Nonlinear Dyn., 88, 1883(2017).

9. Sukhorukov, A. A., Xu, Z. Y., Kivshar, Y. S.: Nonlinear suppression of time reversals in PT-symmetric optical couplers. Phys. Rev. A., 82,043815–043818(2010).

10. Lupu, A., Benisty, H., Degiron, A.: Using optical PT-symmetry for switching applications. Photonics Nanostruct, Fundam. Appl., 12, 305–311(2014).

11. Suchkov, S. V., Sukhorukov, A. A., Huang, J., Dmitriev S. V., Lee, C., Kivshar, Y. S.: Nonlinear switching and solitons in PT-Symmetric photonic systems. Laser and Photonics Reviews, 14(2015).

12. Liu, W., Pang, L., Han, H., Shen, Z., Lei, M., Teng, H., Wei, Z.: Dark solitons in WS2 erbium-doped fiber lasers. Photon. Res., 4, 111-114(2016). https://doi.org/10.1364/PRJ.4.000111

13. Liu, W., Pang, L. ,Han, H., Bi, K. , Lei, M., Wei, Z.: Tungsten disulphide for ultrashort pulse generation in all-fiber lasers. Nanoscale,9, 5806-5811(2017). https://doi.org/10.1039/C7NR00971B

14. Liu, W., Pang, L., Han, H., Liu, M., Lei, M., Fang, S., Teng, H., Wei, Z.: Tungsten disulfide saturable absorbers for 67 fs mode-locked erbium-doped fiber lasers. Opt. Express 25, 2950-2959(2017). https://doi.org/10.1364/OE.25.002950

15. Nauman, R.,Saima, A., Ahmad, J.: Optical solitons and stability analysis for the generalized second-order nonlinear Schrödinger equation in an optical fiber. International Journal of Nonlinear Sciences and Numerical Simulation, 21, 7-8, 855-863(2020). https://doi.org/10.1515/ijnsns-2019-0287

16. Safaei, L., Hatami, M., Borhani Zarandi, M.: Numerical analysis of stability for temporal Bright solitons in a PT-symmetric NLDC. J. Optoelectronical Nanostructures, 2:2, 69-78(2017).

17. Safaei L., Hatami, M., Borhani Zarandi, M.: Stability of temporal dark soliton in PT-symmetric nonlinear fiber couplers in normal dispersion regime. J. Optoelectronical Nanostructures, 3,141(2016).

18. Safaei, L., Hatami, M., Borhani Zarandi, M.: PT-Symmetric Nonlinear Directional Fiber Couplers with Gain and Loss for Ultrashort Optical Pulses. J. Laser Opt. Photonics., 4, 155(2017).

19. Wang, Y. Y., Zhang, Y. P., Dai, C. Q.: Re-study on localized structures based on variable separation solutions from the modified tanh-function method. Nonlinear Dyn. 83, 1331(2016). https://doi.org/10.1007/s11071-015-2406-5

20. Liu, X., Triki, H., Zhou, Q. et al.: Analytic study on interactions between periodic solitons with controllable parameters. Nonlinear Dyn., 94, 703–709(2018). https://doi.org/10.1007/s11071-018-4387-7

21. Ahmad, H., Seadawy, A. R., Khana, T. A.: modified variational iteration algorithm to find approximate solutions of nonlinear Parabolic equation. Mathematics and Computers in Simulation, 177, 13-23(2020).

22. Iqbal, M., Seadawy, A. R., Khalil, O. H., Lu, D.: new solitary wave solutions of nonlinear Nizhnik-Novikov-Vesselov equation. Results in Phys., 16, 102838(2020). https://doi.org/10.1016/j.rinp.2019.102838

23. Cheemaa, N., Seadawy, A.R., Chen, S.: More general families of exact solitary wave solutions of the nonlinear Schrödinger equation with their applications in nonlinear optics. Eur. Phys. J. Plus 133, 547 (2018). https://doi.org/10.1140/epjp/i2018-12354-9

24. Cheemaa, N., Seadawy, A.R., Chen, S.: Some new families of solitary wave solutions of the generalized Schamel equation and their applications in plasma physics. Eur. Phys. J. Plus 134, 117 (2019). https://doi.org/10.1140/epjp/i2019-12467-7

25. Seadawy, A. R., Iqbal, M., Lu, D.: nonlinear wave solutions of the Kudryashov-Sinelshchikov dynamical equation. J. Taibah University for Sci., 13, 10601072(2019). https://doi.org/10.1080/16583655.2019.1680170

26. Seadawy, A. R.: Two-dimensional interaction of a shear flow with a free surface in a stratified fluid and its solitary-wave solutions via mathematical methods. European Physical Journal Plus, 132,518 (2017). https://doi.org/10.1140/epjp/i2017-11755-6

27. Alexeeva, N. V., Barashenkov, I. V., Sukhorukov, A. A., Kivshar, Y. S.: Optical solitons in PT-Symmetric nonlinear couplers with gain and loss. Phys. Rev. A.,85, 063837(2012).

28. Kivshar, Y. S., Agrawal, G. P.: Optical Solitons from fibers to photonic crystals. Academic Press, Sandiego(2003).

29. Safaei, L., Hatami, M., Borhani Zarandi, M.: The effect of relative phase on the stability of temporal dark soliton in PT-Symmetric nonlinear directional fiber coupler. Optical and Quantum Electronics, 50, 382(2018). https://doi.org/10.1007/s11082-018-1646-2

30. Arshad, M., Seadawy, A. R., Lu, D.: Modulation stability and optical soliton solutions of nonlinear Schrödinger equation with higher order dispersion and nonlinear terms and its applications. Superlattices and Microstructures, 112, 422-434(2017). https://doi.org/10.1016/j.spmi.2017.09.054

31. Govindarajia, A., Mahalingamb, A., Uthayakumar, A.: Femtosecond pulse switching in a fiber coupler with third order dispersion and self-steepening effects. Optik - International Journal for Light and Electron Optics, 125, 4135-4139(2014). https://doi.org/10.1016/j.ijleo.2014.01.098

32. Karlsson, M.: Nonlinear propagation of optical pulses and beams. Submitted to the School of Electrical Engineering, Chalmers University of Technology(1994).

33. Wang, C., Nie, Z., Xie, W., Gao, J., Zhou, Q., Liu, W.: Dark soliton control based on dispersion and nonlinearity for third-order nonlinear Schrodinger equation. Optik,184, 370-376(2019). https://doi.org/10.1016/j.ijleo.2019.04.020

34. Liu, W., Zhu, Y., Liu, M., Wen, B., Fang, S.,Te ng, H., Lei, M., Liu, L., Wei, Z.: Optical properties and applications for MoS2-Sb2Te3-MoS2 heterostructure materials. Photon. Res., 6, 220-227(2018).

35. Farah, N., Seadawy, A.R., Ahmad, S. et al.: Interaction properties of soliton molecules and Painleve analysis for nano bioelectronics transmission model. Opt. Quant. Electron 52, 329 (2020). https://doi.org/10.1007/s11082-020-02443-0

36. Seadawy, A. R: Ion acoustic solitary wave solutions of two-dimensional nonlinear Kadomtsev–Petviashvili–Burgers equation in quantum plasma. Mathematical methods and applied Sciences, 40, 5 1598-1607 (2017). https://doi.org/10.1002/mma.4081

37. Runge, A. F. J., Hudson, D. D., Tam, K. K., Martijin de Stereke, C., Blanco-Redono, A.: The pure-quartic soliton laser. Nature Photonics, 14, 492-497(2020). https://doi.org/10.1038/s41566-020-0629-6