Novel Similarity Measures, Entropy of Intuitionistic Fuzzy Sets and their Application in Software Quality Evaluation

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Research Article

Keywords: Intuitionistic fuzzy set, similarity measure, software quality model

Posted Date: June 3rd, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-563745/v1>

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NOVEL SIMILARITY MEASURES, ENTROPY OF INTUITIONISTIC FUZZY SETS AND THEIR APPLICATION IN SOFTWARE QUALITY EVALUATION

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Abstract: Intuitionistic fuzzy sets (IFSs), including member and nonmember functions, have many applications in managing uncertain information. The similarity measures of IFSs proposed to represent the similarity between different types of sensitive fuzzy information. However, some existing similarity measures do not meet the axioms of similarity. Moreover, in some cases, they could not be applied appropriately. In this study, we proposed some novel similarity measures of IFSs constructed by combining the exponential function of membership functions and the negative function of non-membership functions. In this paper, we also proposed a new entropy measure as a stepping stone to calculate the weights of the criteria in the proposed multi-criteria decision making (MCDM) model. The similarity measures used to rank alternatives in the model. Finally, we used this MCDM model to evaluate the quality of software projects.

Keywords: Intuitionistic fuzzy set, similarity measure, software quality model.

1. Introduction

In 1986, Atanassov introduced intuitionistic fuzzy sets (IFSs) (Atanassov, 1986), which are a generalization of a fuzzy set (Zadeh, 1965). An IFS considers the information involving membership functions and non-membership functions. From their inception to the present day, IFSs have been proven to be a highly effective tool for processing uncertainties in real-world problems, including pattern recognition and decision-making. Later, Atanassov and Gargov introduced interval-valued IFSs (IVIFSs), in which membership functions and non-membership

functions are subintervals of a unit interval $[0, 1]$ (Atanassov and Gargov, 1989). Similar to fuzzy sets, IFSs have wide applications in processing uncertain data for various purposes, such as decision-making, medical diagnoses, and agriculture (Szmidt and Kacprzyk, 1996; Xu, 2010; Papakostas et al., 2013; Shidpour et al., 2013; Bharati and Singh, 2014; Li and Zeng, 2015; Liu et al., 2016; Xuan Thao, 2018; Thao and Duong, 2019; Thao et al., 2019; Joshi, 2020; Garg and Kumar, 2020; Xue & Deng, 2020; Thao^a, 2021). Along with distance and correlation measures, similarity measures of IFSs have been studied and widely used in many fields, such as decision-making, machine learning, and pattern recognition (Li and Cheng, 2002; Szmidt Kacprzyk, 2004; Xu, 2007; Ye, 2011; Hwang et al., 2012; Park et al., 2013; Rajarajeswari and Uma, 2013 Shi and Ye, 2013; Tian, 2013; Song et al., 2015; J. Ye, 2016). Today, they still are a hot topic attracting research attention. Liang and Shi proposed a similarity measure of IFSs and applied the measure for pattern recognition (Liang and Shi, 2003). Xu and Chen developed a similarity measure by using the distance measure in the multi-attribute decision-making process (Xu, 2007; Xu and Chen, 2008). Furthermore, Zhou proposed a similarity measure and applied it in multi-criteria decision-making (MCDM) process (Zhou, 2016). The aforementioned methods used either polynomial or fractional functions. In 2013, similarity measures of IFSs based on the cotangent function investigated and applied to medical diagnoses (Rajarajeswari and Uma, 2013; Tian, 2013). Ye introduced vector cosine similarity measures for IFSs and presented their applications (Ye, 2011). In 2016, Ye introduced similarity measures of IFSs based on the cosine function for decision-making in mechanical design schemes (Ye, 2016). These similarity measures of Ye alleviated the limitations of other existing measures. These limitations are discussed in section 3 (examples 1 and 2). These suggest that we continue to search for new measures to overcome the limitations of existing measures.

The International Organization for Standardization (ISO, 2017) declared that “The scope of the application of the quality models including supporting the specification and evaluation of software and software-intensive computer systems based on different perspectives by using the specifications associated with their acquisition, requirements, development, use, evaluation, support, maintenance,

quality assurance and control, and audit. For example, the models can be used by developers, acquirers, quality assurance and control staff, and independent evaluators, particularly those responsible for specifying and evaluating software product quality". Activities during product development that can benefit from the use of these quality models are as follows:

- Identifying software and system requirements
- Validating the comprehensiveness of requirement definitions
- Identifying software and system design objectives
- Identifying software and system testing objectives
- Identifying quality control criteria as part of quality assurance
- Identifying acceptance criteria for a software product and/or a software-intensive computer system
- Establishing measures of quality characteristics in support of these activities.

The quantification of these criteria to evaluate and rank software has practical significance. This will depend on human feelings and knowledge. Whereas IFS has proven to be an effective tool when dealing with this kind of knowledge. In this paper, we applied new measures on IFSs to the problem of pattern recognition and decision-making software quality assessment.

Contribution of this study as follows:

- + we introduced a new similarity measure of IFSs by combining exponential and negative functions.
- + Compare new proposed similarity measure to see that they are feasible.
- + To propose new entropy of IFS by combining exponential and negative functions.
- + Compare new proposed entropy measure to see that they are reasonable.

+ Recommend a method to rank and evaluate product quality. It is useful to help us consider to select, evaluate software quality. This is done through illustrative numerical examples.

+ Compare the new method to some other ranking method to demonstrate the performance of the new method.

The remainder of this paper is organized as follows: Section 2 reviews the concepts of IFSs and IVIFSs and their similarity measures. Section 3 describes how we constructed a new similarity measure between different IFSs and provides some examples in which the results obtained using our method are compared with those obtained using other methods. Section 4 presents the intuitionistic fuzzy software quality model and provides an example to illustrate the model. Moreover, the results of the model obtained using our similarity measure were compared with the results of a model obtained using the similarity measure proposed by (Ye, 2011; Ye, 2016). Finally, in section 5, we provide a conclusion about our method and present the future research.

2. Preliminary

Let X be a universal set. We reminder some concept related to IFS as follows.

Definition 1 (Atanassov, 1986) An intuitionistic fuzzy set on X is defined by form

$$A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$$

in which $\mu_A(x) \in [0,1]$ and $\nu_A(x) \in [0,1]$ are the membership and non-membership degree of the element x in X to A , respectively, such that

$$\mu_A(x) + \nu_A(x) \leq 1, \forall x \in X.$$

The degree of hesitant of element $x \in X$ in A is $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$.

We denote $IFS(X)$ is the collection of all IFSs on X .

For two intuitionistic fuzzy sets $A, B \in IFS(X)$, we have:

- Subset: $A \subset B$ if only if $\mu_A(x) < \mu_B(x)$ and $\nu_A(x) > \nu_B(x)$ for all $x \in X$.

- Equal: $A = B$ if only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for all $x \in X$.
- For all $\lambda > 0$, we have

$$A^\lambda = \left\{ \left(x, \mu_A^\lambda(x), 1 - (1 - \nu_A(x))^\lambda \right) \mid x \in X \right\} \quad (1)$$

Now, we recall the similarity measure in literal.

Given a finite universal set $X = \{x_1, x_2, \dots, x_n\}$. Let $A = \{(x_i, \mu_A(x_i), \nu_A(x_i)) \mid x_i \in X\}$, $B = \{(x_i, \mu_B(x_i), \nu_B(x_i)) \mid x_i \in X\}$ be two IFSs on X .

Definition 2. (Li and Cheng, 2002) A mapping $S: \text{IFS}(X) \times \text{IFS}(X) \rightarrow [0, 1]$ is the similarity measure of the intuitionistic fuzzy sets if it satisfies the following conditions:

- (S1) $0 \leq S(A, B) \leq 1$ for all $A, B \in \text{IFS}(X)$,
- (S2) $S(A, B) = S(B, A)$ for all $A, B \in \text{IFS}(X)$,
- (S3) $S(A, A) = 1$ for all $A \in \text{IFS}(X)$,
- (S4) For all $A, B, C \in \text{IFS}(X)$. If $A \subseteq B \subseteq C$ then $S(A, C) \leq \min\{S(A, B), S(B, C)\}$.

3. The new similarity measures of the IFSs

Let $X = \{x_1, x_2, \dots, x_n\}$ be a finite universal set, A and B are two arbitrary IFSs on X . We denote $S_i^\mu(A, B) = \frac{e^{-|\mu_A(x_i) - \mu_B(x_i)|} - e^{-1}}{1 - e^{-1}}$, $S_i^\nu(A, B) = 1 - |\nu_A(x_i) - \nu_B(x_i)|$ for all $i = 1, 2, \dots, n$.

Proposition 1. Let A and B be two arbitrary IFSs in X . Then $S_i^\mu(A, B)$ and $S_i^\nu(A, B)$ ($i = 1, 2, \dots, n$) satisfy the following conditions:

- (s1) $0 \leq S_i^\mu(A, B), S_i^\nu(A, B) \leq 1$ for all $A, B \in \text{IFS}(X)$.
- (s2) $S_i^\mu(A, B) = S_i^\mu(B, A)$ and $S_i^\nu(A, B) = S_i^\nu(B, A)$ for all $A, B \in \text{IFS}(X)$
- (s3) $S_i^\mu(A, A) = S_i^\nu(A, A) = 1$ for all $A \in \text{IFS}(X)$.
- (s4) For all A, B and $C \in \text{IFS}(X)$. If $A \subseteq B \subseteq C$ then $S_i^\mu(A, C) \leq \min\{S_i^\mu(A, B), S_i^\mu(B, C)\}$

and

$$S_i^\nu(A, C) \leq \min \{S_i^\nu(A, B), S_i^\nu(B, C)\}.$$

Proof.

(s1). For all $A \in \text{IFS}(X)$ and $B \in \text{IFS}(X)$, we have $-1 \leq -|\mu_A(x_i) - \mu_B(x_i)| \leq 0$

and

$$0 \leq |\nu_A(x_i) - \nu_B(x_i)| \leq 1$$

so that

$$e^{-1} \leq S_i^\mu(A, B) = e^{-|\mu_A(x_i) - \mu_B(x_i)|} \leq 1$$

and

$$0 \leq S_i^\nu(A, B) = 1 - |\nu_A(x_i) - \nu_B(x_i)| \leq 1.$$

(s2). It is obvious.

(s3). Consider two intuitionistic fuzzy sets A and B on X . If $A = B$ then $\mu_A(x_i) = \mu_B(x_i)$, $\nu_A(x_i) = \nu_B(x_i)$. This implies that $S_i^\mu(A, B) = 1$, $S_i^\nu(A, B) = 1$.

(s4). For three intuitionistic fuzzy sets A , B and C such that $A \subseteq B \subseteq C$, then we have $\mu_A(x_i) \leq \mu_B(x_i) \leq \mu_C(x_i)$, and $\nu_C(x_i) \leq \nu_B(x_i) \leq \nu_A(x_i)$. So that, we have

$$\max \{|\mu_A(x_i) - \mu_B(x_i)|, |\mu_B(x_i) - \mu_C(x_i)|\} \leq |\mu_A(x_i) - \mu_C(x_i)|$$

and

$$\max \{|\nu_A(x_i) - \nu_B(x_i)|, |\nu_B(x_i) - \nu_C(x_i)|\} \leq |\nu_A(x_i) - \nu_C(x_i)|.$$

Then we have

$$\min \{-|\mu_A(x_i) - \mu_B(x_i)|, -|\mu_B(x_i) - \mu_C(x_i)|\} \geq -|\mu_A(x_i) - \mu_C(x_i)|,$$

and

$$\min \{|\nu_A(x_i) - \nu_B(x_i)|, |\nu_B(x_i) - \nu_C(x_i)|\} \geq |\nu_A(x_i) - \nu_C(x_i)|.$$

Since, we obtain

$$S_i^\mu(A, C) \leq \min \{S_i^\mu(A, B), S_i^\mu(B, C)\}$$

and

$$S_i^\nu(A, C) \leq \min \{S_i^\nu(A, B), S_i^\nu(B, C)\}. \square$$

Now, we can define the similarity measures of the intuitionistic fuzzy sets that assigned with the weight of each element in the universal set.

We assume that each element $x_i \in X$ assigned with a weight $\omega_i \in [0, 1]$ ($i = 1, 2, \dots, n$) such that $\sum_{i=1}^n \omega_i = 1$.

Definition 3. A mapping $S: \text{IFS}(X) \times \text{IFS}(X) \rightarrow [0, 1]$ is defined by

$$S_1(A, B) = \sum_{i=1}^n \omega_i \frac{S_i^\mu(A, B) + S_i^\nu(A, B)}{2} \quad (2)$$

Theorem 1. Let A and B be two arbitrary IFSs in X . The expression $S_1(A, B)$ in the eq. (2) satisfies the following conditions:

- (S1) $0 \leq S_1(A, B) \leq 1$ for all $A, B \in \text{IFS}(X)$.
- (S2) $S_1(A, B) = S_1(B, A)$ for all $A, B \in \text{IFS}(X)$,
- (S3) $S_1(A, A) = 1$ for all $A \in \text{IFS}(X)$,
- (S4) For all $A, B, C \in \text{IFS}(X)$. If $A \subseteq B \subseteq C$ then, we have

$$S_1(A, C) \leq \min \{S_1(A, B), S_1(B, C)\}.$$

It means that

$$S_1(A, B) = \sum_{i=1}^n \omega_i \frac{S_i^\mu(A, B) + S_i^\nu(A, B)}{2}$$

is a similarity measure of two intuitionistic fuzzy sets.

Proof.

From Proposition 1, we have:

- (S1) Since $0 \leq S_i^\mu(A, B), S_i^\nu(A, B) \leq 1$, we have

$$0 \leq S_1(A, B) = \sum_{i=1}^n \omega_i \frac{S_i^\mu(A, B) + S_i^\nu(A, B)}{2} \leq \sum_{i=1}^n \omega_i = 1.$$

(S2). It is obviously.

(S3). If $A=B$, then $\mu_A(x_i) = \mu_B(x_i)$, $\nu_A(x_i) = \nu_B(x_i)$, so that, we get

$$S_i^\mu(A, B) = 1, S_i^\nu(A, B) = 1. \text{ Hence } S_1(A, B) = \sum_{i=1}^n \omega_i \frac{S_i^\mu(A, B) + S_i^\nu(A, B)}{2} = \sum_{i=1}^n \omega_i = 1.$$

(S4). For all $A, B, C \in \text{IFS}(X)$ such that $A \subseteq B \subseteq C$, then we have

$$S_i^\mu(A, C) \leq \min\{S_i^\mu(A, B), S_i^\mu(B, C)\}$$

and

$$S_i^\nu(A, C) \leq \min\{S_i^\nu(A, B), S_i^\nu(B, C)\},$$

so that, we get

$$\begin{aligned} S_1(A, B) &= \sum_{i=1}^n \omega_i \frac{S_i^\mu(A, B) + S_i^\nu(A, B)}{2} \\ &\leq \sum_{i=1}^n \omega_i \min\left\{\frac{S_i^\mu(A, B) + S_i^\nu(A, B)}{2}, \frac{S_i^\mu(B, C) + S_i^\nu(B, C)}{2}\right\} \\ &= \min\{S_1(A, B), S_1(B, C)\}. \quad \square \end{aligned}$$

More generally, with $p \in \mathbb{N}^*$, we have the similarity measures determined in definition 5.

Definition 4. A mapping $S: \text{IFS}(X) \times \text{IFS}(X) \rightarrow [0, 1]$ defined by

$$S_p^\omega(A, B) = \sum_{i=1}^n \omega_i \sqrt[p]{\frac{(S_i^\mu(A, B))^p + (S_i^\nu(A, B))^p}{2}} \quad (3)$$

is the similarity measures of the intuitionistic fuzzy sets.

Comparison with some existing similarity measures of IFS.

Now, we remind some existing similarity measures on IFSs.

+ The cosine similarity measure proposed by (Ye, 2011)

$$C_Y(A,B) = \frac{1}{n} \sum_{i=1}^n \frac{\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i) + \nu_A^2(x_i)} \sqrt{\mu_B^2(x_i) + \nu_B^2(x_i)}}$$

+ The cosine similarity measure proposed by (Shi and Ye, 2013)

$$C_{SY}(A,B) = \frac{1}{n} \sum_{i=1}^n \frac{\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \pi_A(x_i)\pi_B(x_i)}{\sqrt{\mu_A^2(x_i) + \nu_A^2(x_i) + \pi_A^2(x_i)} \sqrt{\mu_B^2(x_i) + \nu_B^2(x_i) + \pi_B^2(x_i)}}$$

+ The similarity measure based on the cotangent function introduced by (Tian, 2013)

$$CT_1(A,B) = \frac{1}{n} \sum_{i=1}^n \cot \left[\frac{\pi}{4} + \frac{\pi}{4} \max \left(\left| \mu_A(x_i) - \mu_B(x_i) \right|, \left| \nu_A(x_i) - \nu_B(x_i) \right| \right) \right]$$

+ The similarity measure based on the cotangent function given by (Rajarajeswari and Uma, 2013)

$$CT_2(A,B) = \frac{1}{n} \sum_{i=1}^n \cot \left[\frac{\pi}{4} + \frac{\pi}{4} \max \left(\left| \mu_A(x_i) - \mu_B(x_i) \right|, \left| \nu_A(x_i) - \nu_B(x_i) \right|, \left| \pi_A(x_i) - \pi_B(x_i) \right| \right) \right]$$

+ The similarity measure based on the cosine function defined by (Ye, 2016).

$$CS_1(A,B) = \frac{1}{n} \sum_{i=1}^n \cos \left[\frac{\pi}{2} \max \left(\left| \mu_A(x_i) - \mu_B(x_i) \right|, \left| \nu_A(x_i) - \nu_B(x_i) \right|, \left| \pi_A(x_i) - \pi_B(x_i) \right| \right) \right]$$

+ The similarity measure based on the cosine function given by (Ye, 2016).

$$CS_2(A,B) = \frac{1}{n} \sum_{i=1}^n \cos \left[\frac{\pi}{4} \left(\left| \mu_A(x_i) - \mu_B(x_i) \right| + \left| \nu_A(x_i) - \nu_B(x_i) \right| + \left| \pi_A(x_i) - \pi_B(x_i) \right| \right) \right]$$

+ The similarity measure proposed by (Xu and Chen, 2008):

$$S_{XC}(A,B) = 1 - \frac{1}{n} \sum_{i=1}^n \left[\frac{\left| \mu_A(x_i) - \mu_B(x_i) \right| + \left| \nu_A(x_i) - \nu_B(x_i) \right|}{4} + \frac{1}{2} \max \left(\left| \mu_A(x_i) - \mu_B(x_i) \right|, \left| \nu_A(x_i) - \nu_B(x_i) \right| \right) \right]$$

+ The similarity measures given by (Papakostas et al., 2013)

$$S_{p1}(A, B) = 1 - \frac{1 - \exp(-\frac{1}{2} \sum_{i=1}^n |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|)}{1 - \exp(-n)}$$

and

$$S_{p2}(A, B) = 1 - \frac{1 - \exp(-\frac{1}{2} \sum_{i=1}^n |\sqrt{\mu_A(x_i)} - \sqrt{\mu_B(x_i)}| + |\sqrt{\nu_A(x_i)} - \sqrt{\nu_B(x_i)}|)}{1 - \exp(-n)}$$

+ The similarity measure introduced by of (Song et al., 2015)

$$S_s(A, B) = \frac{1}{2n} \sum_{i=1}^n [\sqrt{\mu_A(u_i)\mu_B(u_i)} + 2\sqrt{\nu_A(u_i)\nu_B(u_i)} + \sqrt{\pi_A(u_i)\pi_B(u_i)} + \sqrt{(1-\nu_A(u_i))(1-\nu_B(u_i))}]$$

+ The similarity measure introduced by of (Song et al., 2019)

$$S_{s1}(A, B) = \frac{1}{3n} \sum_{i=1}^n [2\sqrt{\mu_A(u_i)\mu_B(u_i)} + 2\sqrt{\nu_A(u_i)\nu_B(u_i)} + \sqrt{\pi_A(u_i)\pi_B(u_i)} + \sqrt{(1-\mu_A(u_i))(1-\mu_B(u_i))} + \sqrt{(1-\nu_A(u_i))(1-\nu_B(u_i))}]$$

We compare our proposed similarity measure with the above existing similarity measures with numerical examples of the recognition problems as follows.

Example 1. Suppose that A and B are two given patterns in form IFSs in $X = \{x\}$ as follows: $A = \{(x, 0, 1)\}$, $B = \{(x, 0, 0)\}$ and a new sample $C = \{(x, 1, 0)\}$.

Question: What pattern does we can put C belong to?

In this case, we can use the similarity measures to answer this question. If $S(A, C) < S(B, C)$ then we put C belongs to class of B , reverse, if $S(A, C) > S(B, C)$ then we put C belongs to class of A . Results in Table 1 indicate that our proposed measures and the similarity measures S_{p1} , S_{p2} , S_{xc} , S_s and S_{s1} show that C belongs to class of B , other similarity measures do not give out the classification results. It means, in this case, the similarity obtained is better than that of other measures $C_Y, C_{SY}, CT_1, CT_2, CS_1$ and CS_2 .

Table 1. The classification results in example 1.

Similarity measures	$S(A, C)$	$S(B, C)$	The classification results
C_Y (Ye, 2011)	0	Null	Null

C_{SY} (Shi and Ye, 2013)	0	0	Null
CT_1 (Tian, 2013)	0	0	Null
CT_2 (Rajarajeswari and Uma, 2013)	0	0	Null
CS_1 (Ye, 2016)	0	0	Null
CS_2 (Ye, 2016)	0	0	Null
S_{P1} (Papakostas et al., 2013)	0	0.3775	C belongs to class of B
S_{P2} (Papakostas et al. , 2013)	0	0.3775	C belongs to class of B
S_{XC} (Xu and Chen, 2008)	0	0.25	C belongs to class of B
S_S (Song et al., 2015)	0	0.5	C belongs to class of B
S_{S1} (Song et al., 2019)	0	0.3333	C belongs to class of B
S_1 (proposed measure)	0	0.5	C belongs to class of B
S_2^{ω} (proposed measure, in Eq.(2) with $p = 2$)	0	0.7071	C belongs to class of B

Null means that we can't determine where class C belong to

Next, we consider the benefits of using our proposed similarity measures in the example of pattern recognition.

Example 2. Suppose that A and B are two patterns in form as IFSs in $X = \{x_1, x_2\}$ as follows:

$$A = \{(x_1, 0.4, 0.6), (x_2, 0.4, 0.2)\}, \quad B = \{(x_1, 0.2, 0.8), (x_2, 0.1, 0.5)\}.$$

We have a new sample $C = \{(x_1, 0.3, 0.7), (x_2, 0.2, 0.3)\}$.

Question: What pattern does C belong to?

Results in Table 2 indicate that our proposed measures and the similarity measures C_Y , C_{SY} show that C belongs to class of B , similarity measures S_S , S_{P2} show that C belongs to class of A , similarity measures in CT_1 , CT_2 , CS_1 , CS_2 , S_{P1} , S_{XC} do not give out the classification results. In this example, there are five measures show that C belongs to class of B , and only two measures show that C belongs to class of A . So that, we can put C belongs to class of B (see Table 2). It means, in this

case, the similarity obtained is better than that of other measures CT_1 , CT_2 , CS_1 , CS_2 , S_{P1} , S_{XC} .

Table 2. The classification results in example 2.

Similarity measures	$S(A, C)$	$S(B, C)$	The classification results
C_Y (Ye, 2011)	0.9258	0.9560	C belongs to class of B
C_{SY} (Shi and Ye, 2013)	0.8486	0.9567	C belongs to class of B
CT_1 (Tian, 2013)	0.7903	0.7903	Null
CT_2 (Rajarajeswari and Uma, 2013)	0.7903	0.7903	Null
CS_1 (Ye, 2016)	0.9694	0.9694	Null
CS_2 (Ye, 2016)	0.9694	0.9694	Null
S_{P1} (Papakostas et al., 2013)	0.8145	0.8145	Null
S_{P2} (Papakostas et al. , 2013)	0.7413	0.5841	C belongs to class of A
S_{XC} (Xu and Chen, 2008)	0.8625	0.8625	Null
S_S (Song et al., 2015)	0.9893	0.9854	C belongs to class of A
S_{S1} (Song et al., 2019)	0.9681	0.9714	C belongs to class of B
S_1 (proposed measure)	0.8407	0.8497	C belongs to class of B
S_2° (proposed measure, in Eq.(2) with $p = 2$)	0.8435	0.8501	C belongs to class of B

Null means that we can't determine where class C belong to

Here, we get the result that we can make C belong to pattern A . We find that the components $S_i^v(A, B) = 1 - |\nu_A(x_i) - \nu_B(x_i)|$ ($i = 1, 2$) in our proposed similarity measure have great influence on $S(A, B)$. It means that for three sets A , B and $C \in IFS(X)$, if $S_i^v(A, B) \geq S_i^v(A, C)$ then $S(A, B) \geq S(A, C)$. In this case, $S_1^v(A, C) = S_1^v(B, C) = 0.9$ and $S_2^v(B, C) = 0.8 < S_2^v(A, C) = 1$. These are also reasonable.

4. New entropy of intuitionistic fuzzy sets

Entropy of intuitionistic fuzzy set used to measure the intuition of the intuitionistic fuzzy sets. In 2001, Szmidt E and Kacprzyk introduce the concept of entropy of intuitionistic fuzzy set.

Definition 5 (Szmidt and Kacprzyk, 2001). An entropy on $\text{IFS}(X)$ is a function $E: \text{IFS}(X) \rightarrow [0,1]$, satisfying all following conditions

(E1) $E(A) = 0$ if $A \in \text{IFS}(X)$ has $\mu_A(x_i), \nu_A(x_i) \in \{0,1\}$ for all $x_i \in X$.

(E2) $E(A) = 1$ if only if $\mu_A(x_i) = \nu_A(x_i)$ for all $x_i \in X$,

(E3) $E(A) = E(A^c)$, for all $A \in \text{IFS}(X)$.

(E4) $E(A) \leq E(B)$ for all $A, B \in \text{IFS}(X)$ satisfy either if $\mu_A(x_i) \leq \mu_B(x_i) \leq \nu_B(x_i) \leq \nu_A(x_i)$ or $\mu_A(x_i) \geq \mu_B(x_i) \geq \nu_B(x_i) \geq \nu_A(x_i)$ for all $x_i \in X$.

Definition 7. A mapping $E: \text{IFS}(X) \rightarrow [0,1]$ is defined by

$$E_T(A) = \frac{1}{2n} \sum_{i=1}^n \left[\frac{e^{-|\mu_A(x_i) - \nu_A(x_i)|} - e^{-1}}{1 - e^{-1}} + (1 - |\mu_A(x_i) - \nu_A(x_i)|) \right] \quad (4)$$

Remark. We easy verify that $E_T(A) = S_1(A, A^c)$ for all $A \in \text{IFS}(X)$.

Theorem 2. Let A an arbitrary IFS in X . The function $E_T(A)$ in the eq. (4) is an entropy on $\text{IFS}(X)$.

Proof.

(E1) If $A \in \text{IFS}(X)$ having $\mu_A(x_i), \nu_A(x_i) \in \{0,1\}$ then $|\mu_A(x_i) - \nu_A(x_i)| = 1$, for all $x_i \in X$. It means that $E_T(A) = 0$.

(E2) It is obviously.

(E3) It is obviously.

(E4) For all $A, B \in \text{IFS}(X)$ satisfy either if $\mu_A(x_i) \leq \mu_B(x_i) \leq \nu_B(x_i) \leq \nu_A(x_i)$ or $\mu_A(x_i) \geq \mu_B(x_i) \geq \nu_B(x_i) \geq \nu_A(x_i)$ for all $x_i \in X$, then $-|\mu_A(x_i) - \nu_A(x_i)| \leq -|\mu_B(x_i) - \nu_B(x_i)|$ for

all $x_i \in X$. This implies that $E(A) \leq E(B)$ for all $A, B \in IFS(X)$ satisfy either if $\mu_A(x_i) \leq \mu_B(x_i) \leq \nu_B(x_i) \leq \nu_A(x_i)$ or $\mu_A(x_i) \geq \mu_B(x_i) \geq \nu_B(x_i) \geq \nu_A(x_i)$ for all $x_i \in X$. $\square$

Compare to some existing other entropies.

We consider the comparison of six entropies via examples as follows.

Example 3. Let $A = \{(x_6, 0.2, 0.8), (x_7, 0.3, 0.5), (x_8, 0.6, 0.4), (x_9, 0.9, 0), (x_{10}, 1, 0)\}$ be a PFS on the universal set $X = \{x_6, x_7, x_8, x_9, x_{10}\}$, where x_i is an apartment having i rooms. In the characterizations of the linguistic variables, we regard A as “Large” then according to the operator law in the eq.(1) we have

$A^{0.5}$ may be regarded as “Quite Large”, A^2 may be regarded as “Very Large”, A^3 may be regarded as “Quite Very Large”, A^4 may be regarded as “Very Very Large”,
in which

$$A^{0.5} = \{(x_6, 0.4472, 0.5528), (x_7, 0.5477, 0.2929), (x_8, 0.7746, 0.2254), (x_9, 0.9486, 0), (x_{10}, 1, 0)\},$$

$$A^2 = \{(x_6, 0.04, 0.96), (x_7, 0.09, 0.75), (x_8, 0.36, 0.64), (x_9, 0.81, 0), (x_{10}, 1, 0)\},$$

$$A^3 = \{(x_6, 0.008, 0.992), (x_7, 0.027, 0.875), (x_8, 0.216, 0.784), (x_9, 0.729, 0), (x_{10}, 1, 0)\},$$

$$A^4 = \{(x_6, 0.0016, 0.9984), (x_7, 0.0081, 0.9375), (x_8, 0.1296, 0.8704), (x_9, 0.6561, 0), (x_{10}, 1, 0)\}$$

Based on the mathematical view and intuitive of human, we easy to see that $A^{m+1} \subset A^m (m > 0)$ and $\lim_{m \rightarrow +\infty} A^m = \emptyset = \{(x_1, 0, 1), (x_2, 0, 1)\}$. So that the entropy measure E must be satisfied $E(A) > E(A^2) > E(A^3) > E(A^4)$. According to Table 1, we find that six entropy measures of picture fuzzy sets satisfy this statement. To do this, we revisit some entropies on the fuzzy intuitionistic sets.

- Entropy of (Burillo and Bustince, 1996) $E_{BB}(A) = \frac{1}{n} \sum_{i=1}^n \pi_A(x_i)$
- Entropy of (Szmidt and Kacprzyk, 2001)

$$E_{SK}(A) = \frac{1}{n} \sum_{i=1}^n \frac{\max \text{count}(A(x_i) \cap A^c(x_i))}{\max \text{count}(A(x_i) \cup A^c(x_i))}$$

where $\max \text{count}(A(x_i)) = \mu_A(x_i) + \pi_A(x_i)$ for all $x_i \in X$.

- Entropy of (Hung and Yang, 2006)

$$E_H(A) = \frac{1}{n(1-\beta)} \sum_{i=1}^n \log [\mu_A^\beta(x_i) + \nu_A^\beta(x_i) + \pi_A^\beta(x_i)]$$

where $0 < \beta < 1$.

- Entropy of (Vlachos and Sergiagis, 2007)

$$E_{VS}(A) = -\frac{1}{n \ln 2} \sum_{i=1}^n \left[\mu_A(x_i) \ln \mu_A(x_i) + \nu_A(x_i) \ln \nu_A(x_i) - (1 - \pi_A(x_i)) \ln(1 - \pi_A(x_i)) - \pi_A(x_i) \ln 2 \right]$$

- Entropy of (Ye, 2010)

$$E_Y(A) = \frac{1}{n(\sqrt{2}-1)} \sum_{i=1}^n \left[\cos \frac{\pi(1-\mu_A(x_i)+\nu_A(x_i))}{4} + \cos \frac{\pi(1+\mu_A(x_i)-\nu_A(x_i))}{4} - 1 \right].$$

- Entropy of (Zhang and Jiang, 2008)

$$E_{ZJ}(A) = \frac{1}{n} \sum_{i=1}^n \frac{\mu_A(x_i) \wedge \nu_A(x_i)}{\mu_A(x_i) \vee \nu_A(x_i)}.$$

- Entropy of (Zhu and Li, 2016)

$$E_{ZL}(A) = \frac{1}{4n} \sum_{i=1}^n \left[(1 + \pi_A(x_i)) (1 - |\mu_A(x_i) - \nu_A(x_i)|) + \pi_A(x_i) \right].$$

The results of six entropy measures $E_T, E_{BB}, E_{SK}, E_H, E_Y, E_{ZJ}$ and E_{ZL} shown in the Table 3.

Table 3. Comparison of the degree of fuzziness with entropy measures

	E_T	E_{BB}	E_{SK}	E_H	E_{VS}	E_Y	E_{ZJ}	E_{ZL}
$A^{0.5}$	0.3989	0.0421	0.3270	0.2346	0.4716	0.5406	0.4087	0.1237
A	0.3874	0.0600	0.3033	0.2436	0.4601	0.5452	0.3792	0.1285
A^2	0.2350	0.0700	0.1448	0.2154	0.2519	0.3896	0.1810	0.0885
A^3	0.1472	0.0738	0.0629	0.1817	0.1446	0.2844	0.0786	0.0664
A^4	0.1132	0.0797	0.0318	0.1538	0.1061	0.2262	0.0398	0.0599

“**bold**” means that the entropy values are sensible

5. Application of similarity measures and entropy in a software quality model

There are some models for evaluating software quality, such as the software quality evaluation model generated using the hesitant fuzzy uncertain linguistic information (Li et al., 2014) or the model based on the fuzzy analytic hierarchy process (Chang et al., 2008). These models are quite complex because they either require building linguistic variables (Li et al., 2014) or require using primitive ISO standards published in 2001 (Chang et al., 2008). In this section, we constructed a MCDM model based on the proposed similarity measures (in section 3) and entropy (in section 4) to evaluate the quality of software projects. Software is an important factor in the development of computers. Efficient orchestrated software enables applications solving real-world problems more effectively. Software engineers often focus on software production. Good-quality software that can keep up with budget constraints and time is considered to be effective software. In 2011, ISO defined a new standard ISO 25010 to evaluate system and software product quality. This standard confirmed in (IOS, 2017) and included the following key quality identifiers.

- Functional Suitability C_1
- Functional Correctness C_2
- Testability C_3
- Performance Efficiency C_4
- Compatibility C_5
- Usability C_6
- Appropriateness Recognizability C_7
- User Interface Aesthetics C_8
- Reliability C_9
- Security C_{10}
- Maintainability C_{11}
- Modifiability C_{12}
- Portability C_{13}

In the quality evaluation of software projects, the quality of software projects is compared with the best example based on a set of criteria (i.e., one that matches all criteria). For the criterion of benefits, being the best is often desirable. However, for the criterion of cost, having the lowest cost is usually desirable. Quality evaluation is an MCDM problem. Moreover, the digitization of qualitative criteria is highly dependent on human psychology. The theory of IFSs is more useful for solving uncertainty problems. Therefore, we proposed an MCDM model for evaluating software quality by using an IFS similarity measure, namely the intuitionistic fuzzy software quality model.

The IFSQM has seven steps as follows:

1. Determine the criteria for quality assessment C_j , $(j=1,2,\dots,n)$
2. Identify different projects for evaluation P_i , $(i=1,2,\dots,m)$
3. Determine the intuitionistic fuzzy sets $P_i = \{(C_j, \mu_{ij}, \nu_{ij}) | C_j \in U\}$ for $i=1,2,\dots,m$. We consider that P_i ($i=1,2,\dots,m$) is an intuitionistic fuzzy set on the universal set $U = \{C_1, C_2, \dots, C_n\}$
4. Determine the weight ω_j of the criteria C_j ($j=1,2,\dots,n$). In this step, we consider $C_j = \{(P_i, \mu_{ij}, \nu_{ij}) | P_i \in P\}$ is an intuitionistic set on $P = \{P_1, P_2, \dots, P_n\}$. We denote $e_j = e(C_j)$, then we can determine the weight

$$\omega_j = \frac{1 - e_j}{n - \sum_{j=1}^n e_j} \quad (5)$$

for all C_j ($j=1,2,\dots,n$).

5. Build the best project P_b : cost criteria are (0, 1) and benefit criteria are (1, 0). We consider that P_b is an intuitionistic fuzzy set on the universal set $U = \{C_1, C_2, \dots, C_n\}$
6. Calculate the similarity measure $S(P_i, P_b)$ of each project P_i ($i=1,2,\dots,m$) with the best project.

7. Rank projects based on the similarity measure. The project P_i is better than the project P_k (we denote $P_i \succ P_k$) if $S(P_i, P_b) > S(P_k, P_b)$, $i = 1, 2, \dots, m$ and $k = 1, 2, \dots, m$.

This algorithm consists of simple computation steps on fuzzy decision-making matrix IF with m rows and n columns (corresponding to m alternatives and n criteria). So we can define its computational complexity as just $O(mn)$ polynomial time.

Example 4. We consider the IFSQM to evaluate the software quality of five software projects: P_1, P_2, P_3, P_4 and P_5 . The software quality criteria are $U = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}\}$. In this model, we consider P_i ($i = 1, 2, \dots, m$) are the intuitionistic fuzzy sets on U . Then, we have an intuitionistic fuzzy decision matrix of software based on software quality criteria (Table 4). These are also step 1, step 2 and step 3 of IFSQM.

Step 4. We compute the weight of each C_j ($j = 1, 2, \dots, n$) based on the eq. (5), then the weigh vector of criteria as follows:

$$\omega = (0.062, 0.1, 0.097, 0.12, 0.063, 0.07, 0.072, 0.097, 0.063, 0.072, 0.06, 0.068, 0.065)$$

Step 5. Building the best project: In this example, all the software quality criteria used for evaluation are the criteria of benefit. Therefore, we chose the best project is $P_b = \{(C_i, 1, 0) | i = 1, 2, \dots, 13\}$.

Step 6. The similarity measures of each project with the best project is shown in Table 5 and Table 6.

Step 7. Ranking projects are shown in Table 5 and Table 6.

We denote IFSQM 1 and IFSQM 1 as versions of IFSQM when using the similarity measure in step 6 by equation (2) and equation (3) with $p = 2$, respectively.

Table 4. The intuitionistic fuzzy decision matrix of software based on software quality criteria

	P_1	P_2	P_3	P_4	P_5

C_1	(0.49,0.1)	(0.6,0.04)	(0.36,0.04)	(0.81,0.05)	(0.25,0.25)
C_2	(0.7,0.16)	(0.8,0.01)	(0.73,0.03)	(0.6,0.11)	(0.81,0.05)
C_3	(0.8,0.1)	(0.8,0.01)	(1,0)	(0.49,0.19)	(0.64,0.1)
C_4	(0.81,0.05)	(0.9,0.1)	(1,0)	(0.8,0.01)	(0.81,0.05)
C_5	(1,0)	(0.25,0.1)	(1,0)	(0.25,0.3)	(0.25,0.3)
C_6	(0.25,0.4)	(1,0)	(1,0)	(0.25,0.16)	(0.25,0.4)
C_7	(0.25,0.4)	(0.6,0.04)	(1,0)	(0.16,0.18)	(0.81,0.05)
C_8	(0.8,0.1)	(0.8,0)	(1,0)	(0.49,0.19)	(0.64,0.1)
C_9	(1,0)	(0.25,0.1)	(1,0)	(0.25,0.3)	(0.25,0.3)
C_{10}	(0.25,0.4)	(0.6,0.04)	(1,0)	(0.16,0.176)	(0.81,0.05)
C_{11}	(0.49,0.1)	(0.6,0.04)	(0.36,0.04)	(0.81,0.05)	(0.25,0.25)
C_{12}	(0.25,0.4)	(1,0)	(1,0)	(0.25,0.16)	(0.25,0.4)
C_{13}	(0.3,0.4)	(1,0)	(1,0)	(0.25,0.16)	(0.25,0.4)

Evaluation of the quality of the product based on similarity measures is carried out in accordance with the following principles. If $0 < S(P_i, P_b) < 0.5$ then the project P_i belongs to the class Low (L); if $0.5 \leq S(P_i, P_b) < 0.75$ then the project P_i belongs to the class Medium (M) ; if $0.75 \leq S(P_i, P_b) < 0.9$ then the project P_i belongs to the class High (H); if $0.9 \leq S(P_i, P_b) < 1$ then the project P_i belongs to the class Very High (VH).

Accordingly, we can see that the rank of project P_3 is the maximum and its quality is Very High. The second is project P_2 which has its quality High. Three projects P_1 , P_4 and P_5 have their quality being Medium, simultaneously ranked 3rd, 4th and 5th, respectively.

Table 5. The quality of software projects using IFSQM1

Software projects	P_1	P_2	P_3	P_4	P_5
Similarity values	0.6988	0.8541	0.9510	0.6954	0.6680
Ranking	3	2	1	4	5
Quality	M	H	VH	M	M

Table 6. The quality of software projects using IFSQM2

Software projects	P_1	P_2	P_3	P_4	P_5
Similarity values	0.6753	0.8305	0.9400	0.6543	0.6381
Ranking	3	2	1	4	5
Quality	M	H	VH	M	M

[illegible]

$\omega_3 = (0.062, 0.1, 0.096, 0.119, 0.063, 0.07, 0.072, 0.096, 0.063, 0.072, 0.06, 0.062, 0.065)$, respectively. In these cases, we see that having a small change in the ranking results of P4 and P5 when using the ranking methods of (Ye, 2016), (Zhou, 2016) and (Quynh et al., 2020). The rankings of the other options still remain. Meanwhile, the proposed method along with the remaining methods (Song et al., 2015; Song et al., 2019; Thao and Duong, 2019; Thao, 2021) all give stable ranking results.

Figure 1. Comparing results of software raking.

Figure 2. Comparing results of software raking with weight 2

Figure 3. Comparing results of software raking with weight 3

6. Conclusion

The evaluation of software quality based on ISO standards (IOS, 2017) associated with the difficulty is that most of the standards are qualitative. The evaluation considerably depends on the opinion of the decision-maker. When assessing software quality, the evaluation result is affected by the understanding and subjective psychology of decision-makers. Therefore, using the intuitionistic fuzzy set is an appropriate selection for this problem. In this study, we provided some formulas that define similarity measures between different IFSs. Another advantage of this IFSQM model over some other MCDM models is that it combines the entropy induced from the similarity measure to calculate the weights of the criteria. This helps us to avoid emotional mistakes made by decision-makers when assigning weights to criteria. The proposed similarity measures were judged to be superior to existing measures. The evidence for this statement was provided by comparing the proposed measures with other similarity measures. Finally, a software quality model based on the similarity measures of IFSs was introduced. This model is simpler and easier to use for evaluating software quality than some previous models. Moreover, we presented a numerical example to illustrate the proposed model. The proposed method is also limited as it has not mentioned the handling of linguistic variables or the integration of many expert opinions to assess product quality more comprehensively. Those are very meaningful expansion directions and we will continue to study them in the near future.

In the future, we will continue to find other methods to evaluate software quality and will apply similarity, entropy measures to other applied mathematical problems, such as the labeled classification problem (Quynh et al, 2020), clustering problems (Zhou et al., 2016; Thao et al., 2019), decision-making problems (Joshi, 2020; Thao^b, 2021), applications to supply chain management (Xiao et al., 2020; Thao^a, 2021), using IFS and other extended fuzzy sets to “buy online and pick up in store mode” (Wang et al., 2020), and/or other measures as dissimilarity measure (Dinh et al., 2017; Duong and Thao, 2021, Chou et al., 2021); or other methods (Garg and Kumar, 2020; Nguyen et al., 2014; Nguyen and Nguyen, 2015; Zhou et al., 2016; Thao et al., 2019; Thao^b, 2021).

Disclosure of potential conflicts of interest: The author declare that they do not have any conflict of interests

Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

Acknowledgement: This work was supported in part by the Center for Cyber-physical System Innovation from The Featured Areas Research Center Program within the framework of the Higher Education Sprout Project by the Ministry of Education in Taiwan.

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Figures

Figure 1

Comparing results of software raking.

Figure 2

Comparing results of software raking with weight 2

Comparing results of software raking with weight 2

Figure 3

Comparing results of software raking with weight 3