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A study of buckling phenomenon in dielectric elastomeric actuator with different potential energy functions

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Abstract. The present study deals with the buckling phenomenon modeling in a dome-shaped dielectric elastomeric (DE) actuator used in different aerodynamic and fluid power system applications. The DE actuator is a circular membrane type of actuator, which shows a large out-of-plane axial-symmetric deformation with an electrically induced loading condition. A classical continuum mechanics-based analytical model is developed to predict the electrically induced buckling deformation in the actuator. A detailed parametric study has been performed to see the influence of standard Neo-Hookean and Mooney-Rivlin types of potential energies on the geometrical and physical parameters. The findings show that the present model successfully links the sensitivity of different potential energies concerning the actuator's initial dome height and radius.

INTRODUCTION

Dielectric Elastomer (DE) is a particular class of non-conducting material with almost constant dielectric properties, usually incompressible and hyperelastic [1, 2, 3, 4]. The commonly used examples of DE material for making smart actuators are polydimethylsiloxane (silicone) and VHB-4910 (very high bond). The DE-based system is coupled between two compliant electrodes such that it behaves like a parallel plate deformable capacitor unit. Electrostriction is the characteristic of DEs that causes them to deform their shape when subjected to an electric field. On applying an electrical voltage, they reduced in thickness and stretched in area. This phenomenon is known as electrically induced mechanical actuation. DE-based systems are the smart systems [5, 6, 7, 8] suited for the development of novel mechatronic actuators producing large strains. The current study focuses on a diaphragm (dome-shaped) DE actuator, a circular membrane type of actuator shown in Figure 1, offering a large out-of-plane axial-symmetric deformation with an electrically induced loading condition. The circular membrane is constrained here through a fixed, rigid ring to remain curved even during its inactive state. Potential applications of dome-shaped DE material-based spherical actuator are pumps, loud-speakers, electric generators, tactile displays, variable-curvature mirrors and tunable lenses etc.

In the literature, various researchers [9, 10, 11] proposed the dome-shaped geometry of the actuator. They also discussed the actuator's operating principle that can work either in inflation mode [9, 10] or in buckled mode [11]. The actuator's buckled mode that is also focussed here does not require any membrane pre-stretch [12, 13] or external force than the inflation mode [14]. Goulbourne et al. [10] and He et al. [15] studied the out-of-plane electroelastic deformation of the dome-shaped actuator by solving the systems of non-linear differential equations. Specifically, Goulbourne et al. [10] solved one algebraic and two ordinary differential non-linear equations compared to the work of He et al. [15]. Later, Zhao and Suo [16] and O'Brien et al. [17] simulated the out-of-plane electroelastic deformation of the dome-shaped actuator using the finite element method. Recently, Chang et al. [18] fabricated spherical dome-shaped pneumatic valves and performed a finite element method-based structural analysis. Their findings [18] favored the dome-shaped valve geometry over the typical rectangular cross-section. Alturki et al. [19] studied the snap-through instability phenomenon for the shallow domes with cosine-curved profile and observed hysteretic and elastic energy dissipation in series combination. Further, Piranda and Bourgeois [20] developed a quasi-spherical micro-robot, called catoms for Claytronics atoms could stick and move around each other. Furthermore, Tang et al. [21] silicon material-based dome-shaped transducer produced a sound pressure level as high as 88.21 dB in between 10 and 40 kHz.

From the literature, we may conclude that various modeling and simulational studies focusing on the out-of-plane electroelastic deformation of the dome-shaped actuator was performed. In the context of the current status of the problem, existing studies need a concise parametric study for the variation of the actuator's different parameters.

This paper performs an extensive parametric study to observe the influence of base radius values, initial dome height, initial thickness, and applied voltage on a cap tip deformation output of the dome-shaped actuator. Also, we

develop a physics-based actuation force model in line with the literature to capture the performance of a dome-shaped DE actuator used in different aerodynamic and fluid power system applications. The formulated actuation force model connects the cap tip deformation (spherical shaped) output for different base radius values, initial dome height, and initial thickness with an applied electrical voltage difference.

ELECTRICALLY INDUCED BUCKLING DEFORMATION MODEL

In this section, a coupled nonlinear analytical model for a dome-shaped dielectric elastomeric (DE) actuator is presented in line with the literature [10, 14, 15]. The model is used to analyze a large out-of-plane axial-symmetric buckling deformation of the actuator for varying electrical voltage inputs.

The dome-shaped DE actuator is made of a thin-walled circular dielectric polymeric bladder-based membrane, as shown in Figure 1. The model is developed under the following assumptions:

1. The membrane material of the actuator is considered incompressible, isotropic, and hyperelastic.
2. The membrane is a perfect spherical cap with uniform thickness in both undeformed and deformed configurations.
3. The deformation is assumed to be axisymmetric.

FIGURE 1: Schematic diagram of a buckling dielectric elastomer actuator

The geometry of the actuator in both undeformed and deformed configurations is described in Figure 1. The geometry of the actuator holds a variable surface area $A(h) = \pi(r^2 + h^2)$ of any spherical cap with fixed base radius r and with varying height h for varying electrical voltage inputs. However, the volume of the actuator remains constant with the variation of a single longitudinal dimension and a fixed latitude dimension. Therefore, the principle stretches in principle directions for a given geometry of the actuator may be expressed as

$$\lambda_1 = \frac{r^2 + h^2}{r^2 + h_0^2}, \quad \lambda_2 = 1, \quad \lambda_3 = \frac{1}{\lambda_1 \lambda_2} \quad (\text{incompressibility}), \quad (1)$$

where h_0 denotes the initial dome height of the actuator. The volume V of the actuator is expressed as

$$V = \pi(r^2 + h_0^2)t_0 = \pi(r^2 + h^2)t. \quad (2)$$

As per the physical principle of the DE actuator, we may consider the DE membrane as a deformable parallel plate capacitor whose variable electrical capacitance $C(h)$ is defined as

$$C(h) = \frac{\epsilon_r \epsilon_0 (r^2 + h^2)}{t} = \frac{\epsilon_r \epsilon_0 (r^2 + h^2) \lambda_1}{t_0}, \quad (3)$$

where ε_r and ε_0 denote the dielectric constant of the material and the electrical permittivity of free space, respectively. On balancing the electromechanical energy for a dome-shaped like bubble deformation of the actuator in undeformed and deformed configurations, we may obtain an infinitesimal electrical work dL_{elec} given as

$$dL_{elec} = dW_{mech} + dW_{elec}, \quad (4)$$

where dW_{mech} and dW_{elec} denote the mechanical and electrical stores energies within the actuator system. An external energy supply source (i.e., a battery) is connected to provide the infinitesimal electrical work $dL_{elec} = \phi dq = \phi^2 dC$ for the movement of an infinitesimal charge dq from one compliant electrode to the other with an input voltage ϕ . Thus, the capacitance of the system membrane changes by the amount dC because of the electrically induced deformation. The mechanical W_{mech} and electrical W_{elec} energy expressions are obtained as

$$W_{mech}(h) = VU_{hyp}(I_1, I_2, I_3), \quad W_{elec}(h) = \frac{1}{2}C\phi^2, \quad (5)$$

where $U_{hyp}(I_1, I_2, I_3)$ denote the potential energy density of an incompressible isotropic hyperelastic material of DE actuator as a function of the invariants I_1 , I_2 , and I_3 . From the nonlinear theory of hyperelasticity, the invariants I_1 , I_2 , and I_3 may be expressed in terms of the principal stretches defined in equation (1) as

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2, \quad I_2 = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} + \frac{1}{\lambda_3^2}, \quad I_3 = \lambda_1^2 \lambda_2^2 \lambda_3^2. \quad (6)$$

There are various forms of potential energy density functions for an incompressible isotropic hyperelastic material of the DE actuator. In the current study, we use the standard Neo-Hookean and Mooney-Rivlin types of potential energy density functions given as

$$U_{NH} = \mu(I_1 - 3), \quad U_{MR} = C_1(I_1 - 3) + C_2(I_2 - 3), \quad (7)$$

where μ , C_1 and C_2 are the constant parameters of the DE material of the actuator. On using the equations (3), (5) and (7), an explicit analytical model to obtain the variable dome height h of the DE actuator for the given electrical voltage ϕ and Neo-Hookean and Mooney-Rivlin potential energies material parameters C_1 and C_2 are expressed as

$$h(MR) = \sqrt{(r^2 + h_0^2) \left(1 - \frac{\varepsilon_r \varepsilon_0 \phi^2}{2(C_1 + C_2)t_0^2} \right)^{-1/4} - r^2}, \quad h(NH) = \sqrt{(r^2 + h_0^2) \left(1 - \frac{\varepsilon_r \varepsilon_0 \phi^2}{2\mu t_0^2} \right)^{-1/4} - r^2}. \quad (8)$$

The above relations represent a continuum mechanics-based analytical model to predict the electrically induced buckling deformation in the dome-shaped DE actuator.

RESULTS AND DISCUSSION

An extensive parametric study is carried out to obtain the cap tip deformation (spherical shaped) for different base radius values, initial dome height, and initial thickness process parameters of a dielectric elastomeric actuator (r, t_0, h_0). Neo-Hookean (NH) and Monney-Rivlin (MR) strain energies are considered. The parametric study presented here observes the influence of r , t_0 , and h_0 on a cap tip deformation. This cap tip deformation is estimated at different applied voltage, keeping other dielectric constant. To achieve a plane-axial symmetric deformation, a spherical shape of elastomer is considered. The material parameters for NH ($\mu = 11.12$ kPa) and MR ($C_1 = 11.13$ kPa, $C_2 = 0.350$ kPa) are obtained by fitting the uniaxial stress-stretch experimental data [14] correspond to the silicone dielectric elastomer under uniaxial compression test using least-square method.

Figure 2 shows the dependencies of cap tip deformation on radius r at different dielectric constant ε_r equals 4.6 and 7 using MR and NH strain energy density functions. The different values of ε_r correspond to the different sets of material properties of the dielectric elastomeric actuators. It is observed that with increasing the voltage, the cap tip deformation increases. This effect is also more pronounced at a higher voltage. It is seen from Figure 2 that the cap tip deformation is higher when ε_r is increased from 4.6 to 7 corresponds to the applied voltage. This is because the material properties are more pronounced towards the cap tip deformation while keeping the remaining process parameters of the actuators constant. To pick a range from 4.6 to 7 for dielectric constants, we may select the

same silicon dielectric elastomer at very high restoring force frequencies. The electrons inside the dielectric material experience a restoring force produced by an applied electric field. It is also observed that as r increased from 11.6 to 13.05 mm, the cap tip deformation increased by 15.49 % at $\epsilon_r = 4.6$ corresponding to the high voltage 12 kV whereas 13.63 % increased at $\epsilon_r = 7$. Similarly, when r is increased from 13.05 to 14.5 mm, the cap tip deformation increased by 16.67% at $\epsilon_r = 4.6$ and by 15.2% at $\epsilon_r = 7$. The cap tip deformation is the same at low voltage with MR and NH strain energy functions. However, the results deviate at a high voltage within the variation of less than 1% error in both MR and NH strain energy functions.

FIGURE 2: Effect of radius r on $\epsilon_r = 4.6$ (a) and $\epsilon_r = 7$ (b) at different applied voltage

The influence of initial height h_0 on the cap tip deformation at different voltages is presented in Figure 3. The variation of cap tip deformation with h_0 follows a different pattern than that for r_0 . This can be ascertained by comparing Figure 2 and Figure 3. It is observed that with increasing the voltage, the cap tip deformation increases. Three different levels of h_0 with the variation of voltage are studied, and it can be seen from Figure 3 that the cap tip deformation pattern at h_0 equals 3 and 3.6 mm differs from that of $h_0 = 3.3$ mm. This may be due to the inconsistency of the spherical cap deformation's geometric parameters, and it can be further explored to understand the limitation of the deformation mechanism at $h_0 = 3.3$ mm. The rate of deformation is less sensitive, corresponding to the low voltage, and it becomes steeper with increasing the voltage at $h_0 = 3$ mm and $h_0 = 3.6$ mm (Figure 3). On the other hand the maximum cap tip deformation is up to 0.6 for $\epsilon_r = 4.6$ and $\epsilon_r = 7$. It is also observed that the variation of cap tip deformation with a voltage at different h_0 is consistent with MR as well as NH strain energy functions.

Figure 4 shows the dependencies of cap tip deformation on initial thickness t_0 at different dielectric constant ϵ_r equals 4.6 and 7. Three different levels of initial thickness 0.72, 0.81, and 0.9 mm are considered. It is observed that cap tip deformation variations with the voltage at different t_0 follow a similar pattern with r . This can be ascertained by comparing Figure 2 and Figure 4. However, the cap tip deformation with the variations of t_0 is more pronounced than that of r . It is seen from Figure 4 that the cap tip deformation is increasing with changing the ϵ_r from 4.6 to 7. When t_0 is increased from 0.72 to 0.81 mm, the cap tip deformation decreased by 22.22 % and 26.02 % at $\epsilon_r = 4.6$ and 7, respectively for $\phi = 12$ kV. Similarly, when t_0 is increased from 0.81 to 9 mm, the cap tip deformation decreased by 19.05% and 27.78% while ϵ_r and ϕ constant.

Figure 5 shows the influence of cap tip deformation with the variations of ϵ_r at different applied voltage ϕ . Here, the interpretation of ϵ_r refers to the different sets of dielectric elastomers-based actuators' material properties. It is observed that the pattern of cap tip deformation is linearly varied with ϵ_r at three levels of ϕ . However, the slope of the cap tip deformation pattern is changed. The slopes between the cap tip deformation and ϵ_r are obtained as follows: 1, 2, and 3 correspond to applied voltage 3, 6, and 12 kV, respectively. It is also found that the cap tip deformation is similar at ϕ equals to 3 and 6 kV for MR and NH strain energy functions. However, the MR and NH strain energy

(a) $\epsilon_r = 4.6$

(b) $\epsilon_r = 7$

FIGURE 3: Effect of initial height h_0 Effect of radius r on $\epsilon_r = 4.6$ (a) and $\epsilon_r = 7$ (b) at different applied voltage

(a) $\epsilon_r = 4.6$

(b) $\epsilon_r = 7$

FIGURE 4: Effect of initial thickness t_0 Effect of radius r on $\epsilon_r = 4.6$ (a) and $\epsilon_r = 7$ (b) at different applied voltage

functions' consistency becomes weaker at $V = 12$ kV (Figure 5).

Based on the parametric study, it is observed that strain energy function (MR and NH are employed in the present paper) is not very sensitive at the low applied voltage for analyzing the cap tip deformation. The cap tip deformation is more pronounced with the variations of r , h_0 , and t_0 at different applied voltage ϕ . These observations can be useful to design and control the dielectric elastomers-based actuators and assess their applicability to fabrication and manufacturing.

FIGURE 5: Effect of voltage ϕ at different dielectric constant of elastomers ϵ_r

CONCLUDING REMARKS

In the present work, an extensive parametric study has been performed to analyze the cap tip deformation of spherically/dome-shaped DE actuator by varying the base radius, initial dome height, and initial thickness with different applied voltage. Neo-Hookean and Monney-Rivlin strain energy functions are employed. We found that the cap tip deformation is highly sensitive at high applied voltage compared to the low applied voltage as expected from the previous studies. Hence, the paper results will be useful in the design and development of mechatronics based smart actuators. It is planned to simulate and demonstrate the experiments to realize the feasibility of the end product.

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