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Vibration Characteristics of Spur Gear Under Tooth Fatigue Wear

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Abstract: Tooth surface wear is one of the most common faults of gearboxes which increases the fatigue load on the gearbox and endangers its stable output characteristics and service life. Gear meshing characteristics vary significantly with the wear location and severity. Thus, the wear fault model considering tooth profile deviation, friction and time varying mesh stiffness (TVMS) was established in order to quantitatively study the vibration characteristics of tooth fatigue wear (TFW). Tooth profile modification and friction coefficient coupling were applied to characterize TFW, and the maximum wear depth was introduced to characterize TFW severity. The gear-pair simulation models under multiple wear severities were established and imported to a single-stage parallel gearbox whose dynamic differential equation was established under multiple torque conditions. The dynamic simulation results showed good agreement with the experimental data, which validated the method's effectiveness. The vibration characteristics under different wear severities and simulation conditions were then obtained and compared.

Keywords: Wear; Tooth profile modification; Friction; Time varying meshing stiffness; Dynamics simulation; Vibration

1. Introduction

Gearbox is one of the most important components in the field of dynamic transmission which is widely paid attention to for its high failure rate. Tooth surface wear is one of the most common gear faults, including normal wear, fatigue wear and adhesion wear, while this paper mainly refers to fatigue wear. TFW is a relative sliding phenomenon between the two tooth profiles under alternating stress, which will greatly reduce the service life of the gearbox and harm the overall stable power generation characteristics of the wind turbine. Thus, scholars have conducted some in-depth studies on tooth surface wear which can basically be divided into simulations [1-4] and experiments [5-7], despite of the complicity in precise tooth wear modeling and the difficulty in obtaining experimental samples.

Plenty of wear methodologies and theories are introduced into TFW study, in which the most popular is Archard wear theory. Chen W [8] and Zhang J [9] proposed a modified analytical time-varying mesh stiffness (TVMS) model considering the worn tooth profile and Feng K [10] represented the deviation of the profile from an ideal involute curve and obtained the wear coefficient by comparing experimental and simulation data. Wu S [11] applied the cantilever beam theory to study gear's time varying meshing stiffness by consideration of bending, shearing, compressing and contacting deformation and Feng S [12] developed it by introducing the Timoshenko beam theory which additionally takes fillet-foundation deformation into account, but the results of the theories mentioned are similar. Both adopted the potential energy method to calculate the gear's nonlinear deformation for its accuracy and analyticity. Unfortunately, both of them ignored an important parameter-the gear's friction in their literatures. Furthermore, Zhang H [13] and Prabhu Sekar R [14] conducted a detailed investigation of gear body-induced tooth deflections which included the gear's fillet-foundation deformation through the finite element analysis and the results showed good agreement with experimental data. The lubrication conditions were emphatically considered in gear wear characteristics research [15-18]. Hu B [15] predicted the time-varying friction coefficient under loss-of-lubrication condition on the basis of computational inverse technique. Kotia A [16] proposed a new model has been for SiO₂/gear oil Nano lubricants. Zhang J [17] proposed a rapid prediction method of tooth surface wear rate under

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the condition of mixed thermal electrohydrodynamic lubrication. Wang H [18] compared the tooth surface wear status with or without lubricating oil and modified Archard's wear model. Some scholars have deeply explored the mechanism [7,19-21] of gear wear and fully considered the influence of friction [22-24], which provided significant reference value for later researches. Chin Z Y [19] introduced a new signal processing methodology that enables the calculation of absolute transmission error. Kotia A [20] established a purely torsional dynamic model of planetary gear with gear wear. Zhou C [21] compared the theory of adhesive wear, fatigue wear and energy wear. Yong Y [23] studied the TVMS of spur gears with wear defect considering friction, but the friction coefficient and wear severity were modeled independently, ignoring the potential coupling relationship between them. Thus, researches on tooth surface fatigue wear considering wear and friction coupling remains insufficient.

Tooth surface wear caused by fatigue load almost occurs on all meshing side tooth surfaces in the practical operation of gears, thus it is necessary to establish a full tooth surface wear model in order to quantitatively analyze the wear failure. The author proposed a tooth surface fatigue wear simulation method based on tooth profile modification and friction coupling (TPMFC) which took the time varying mesh length & wear depth & friction into account and used relevant experimental data to validate its accuracy. The wear failure pinion was imported to a 8DOF gearbox whose vibrational characteristics were obtained after being simulated under different rotating conditions and wear severities. The main research process was arranged as Fig.1. In section 2, the tooth fatigue wear model considering friction and tooth shape deviation coupling was established where the TVMS under TFW was obtained. In section 3, the TFW model was imported to the pinion and the dynamic equations were derived by lumped mass method. In section 4, simulation results under multiple sets of wear severity and rotating conditions were accessed, with which the TFW vibrational characteristics were analyzed. In section 5, the conclusions of the literature and some relevant future work were discussed.

Fig.1 Tooth Fatigue Wear Research Steps

2. TFW modelling

2.1 Wear mechanism analysis

Roughness exists on the actual surface of any gear when gears mesh and the tooth surface contact is

discontinuous. Local stress and local deformation will be generated at the actual contact point under the action of normal force. When the two tooth surfaces slide relatively due to friction, the material properties of the contact area surface will change. At the same time, the fixed volume of the surface material will be exposed to repeated action of alternating stress, resulting in fatigue cracks in the microscopic volume, and finally cracks evolve into wear debris and fall off. The wear capacity Q is proportional to the normal load, inversely proportional to the hardness of the material, and proportional to the sliding distance according to Archard wear theory [8,20]. Although wear will lead to increased flank clearance, it is distinct from excessive flank clearance fault where a constant normal clearance value exists along the whole meshing surface. The pitch circle area of the tooth surface mainly bears the single-tooth meshing stress where wear is more evident in the primitive tooth wear stage, which is validated by some experiments [17]. Slight wear can reduce the roughness of the tooth surface and the sliding friction coefficient. However, the tooth surface meshing changes from line-contact to area-contact when wear depth reaches a certain value, resulting in more serious relative sliding between tooth surfaces. When the compressive stress exceeds the yield limit of the metal, the metal particles on the surface will peel off. Tooth surface wear is the most common failure in open gear transmission conditions or insufficient lubrication [18]. It is the vibration caused by periodical change of meshing stiffness that leads to tooth fatigue wear, thus the TVMS shall be investigated under wear mode.

2.2 Time varying mesh stiffness

Tooth thickness should be obtained first for the sake of establishing TFW expression to quantitatively characterize TFW severities. The tooth thickness expression at the meshing point of any involute tooth profile can be calculated as follows:

$$\begin{aligned} S_k &= S_p R_k / R_p - 2R_k (inv\alpha_k - inv\alpha) \\ inv\alpha_k &= \tan\alpha_k - \alpha_k \\ S_p &= \pi m / 2 \\ R_b &= R_k \cos\alpha_k \\ R_b &= R_p \cos\alpha \end{aligned} \quad (1)$$

Where α is standard pressure angle, $inv\alpha$ is the spreading angle corresponding to pitch circle, $inv\alpha_k$ is the spread angle of any point on the involute tooth profile, S_p is the tooth thickness at the pitch circle, R_p is the radius of the pitch circle, R_b is the radius of the base circle. When module and tooth numbers are determined (seen in appendix 1), R_p , R_b and S_p are all known. Tooth thickness is a function of the section radius or the corresponding spread angle which is a fault-dependent variable in the meshing area and tooth thickness in the meshing zone is transformed into a function of radial distance as follows:

$$S_k = \frac{S_p R_k}{R_p} - 2R_k \left(\tan \left(\arccos \left(\frac{R_b}{R_k} \right) \right) - \arccos \left(\frac{R_b}{R_k} \right) - \tan \left(\arccos \left(\frac{R_b}{R_p} \right) \right) + \arccos \left(\frac{R_b}{R_p} \right) \right) \quad (2)$$

The energy stored in a gear pair includes bending, shearing, radial strain energy and contact strain energy (The deformation of foundation fillet is ignored in the calculation of meshing stiffness), which is expressed as follows:

$$U_{\text{total}} = \frac{F^2}{2} \left(1/K_{bp} + 1/K_{bg} + 1/K_{sp} + 1/K_{sg} + 1/K_{rp} + 1/K_{rg} + 1/K_h \right) \quad (3)$$

Thus, the meshing stiffness K_e of a gear pair can be described as:

$$1/K_e = \left(1/K_{bp} + 1/K_{bg} + 1/K_{sp} + 1/K_{sg} + 1/K_{rp} + 1/K_{rg} + 1/K_h \right) \quad (4)$$

Where K_{bj} , K_{sj} , K_{rj} , K_h ($j = p, g$, respectively represent Pinion and Gear) represents bending stiffness, shear stiffness, radial compression stiffness and Hertzian contact stiffness, respectively. K_h is a constant value on the mesh line, which has no connection with the contact position of the tooth surface and penetration value, but is related to

the tooth width and gear material and defined as:

$$\frac{1}{K_h} = \frac{4(1-\nu^2)}{\pi E W_j} \quad (5)$$

The expressions of bending strain energy U_b , shear strain energy U_s and radial compression strain energy U_r are as follows:

$$U_b = \frac{F_n^2}{2K_b} = \int \frac{M^2}{2EI_x} dx, \quad U_s = \frac{F_n^2}{2K_s} = \int \frac{1.2F_t^2}{2GA_x} dx, \quad U_r = \frac{F_n^2}{2K_r} = \int \frac{F_r^2}{2EA_x} dx \quad (6)$$

Where the shear modulus is expressed as: $G = \frac{E}{2(1+\nu)}$

The friction force F_f and the torque were written in the form of normal force F_n . As experimental results are not available, the friction coefficient can be considered constant along the contact line according to the Niemann's equation [24], even if with the profile surface deviation.

$$\begin{aligned} F_t &= F_n \cos \alpha_k \\ F_r &= F_n \sin \alpha_k \\ F_f &= \mu F_n \end{aligned} \quad (7)$$

$$M = (F_t + F_f \sin \alpha_k)x - (F_r - F_f \cos \alpha_k)S_k/2 \quad (8)$$

Substitutes the above equation into (6) to obtain bending stiffness K_b , shear stiffness K_s and radial compression stiffness K_r whose expressions were as follows:

$$\frac{1}{K_b} = \int \frac{[2x \cdot (\cos \alpha_k + \mu \sin \alpha_k) - S_k \cdot (\sin \alpha_k - \mu \cos \alpha_k)]^2}{4EI_x} dx, \quad \frac{1}{K_s} = \int \frac{1.2 \cos^2 \alpha_k}{GA_x} dx, \quad \frac{1}{K_r} = \int \frac{\sin^2 \alpha_k}{EA_x} dx \quad (9)$$

The effective meshing stiffness is equal to the sum of two pairs of teeth meshing when two pairs of gears mesh simultaneously and can be described as:

$$K_t = \sum_{i=1}^2 \frac{1}{1/K_{bp,i} + 1/K_{bg,i} + 1/K_{sp,i} + 1/K_{sg,i} + 1/K_{rp,i} + 1/K_{rg,i} + 1/K_{h,i}} \quad (10)$$

Where $i=1$ represents the meshing period of one pair and $i=2$ represents two pair of gears meshing.

Single and double teeth meshing periodically occurs and tooth surface contact type is rolling line contact under theoretical conditions. A partial gap might appear when the tooth surface is worn, resulting in relative sliding and tooth surface friction, which cannot be ignored. The tooth is simplified as a variable section cantilever beam in order to study the change of meshing stiffness in TFW mode. As the tooth surface is not a slender rod, its shear strain energy cannot be ignored and some basic geometry parameters were marked in Fig.2.

Fig.2 Mesh force of a single tooth in Gear

Tooth fatigue wear (TFW) leads to its surface structural deformation which can be reflected by parameters as cross-sectional area, inertial moment and friction, which will have wear-varying effects on bending stiffness, shear stiffness and radial compressive stiffness. TFW is concentrated near the pitch circle, and the TFW depth curve is simplified as a quadratic curve along the radial direction with the maximum TFW depth fixed on the pitch circle. The TFW area was defined as a rectangle-like shape which is centered on the pitch circle and the modulus m as the diameter length, and the TFW depth remains constant along the direction of flank width, as shown in the Fig.3.

Fig.3 TFW Section with Radial Length

Maximum wear depth was regarded as the parameter of evaluating TFW severity, of which large values represent severe TFW condition. TFW depth with respect to the radial distance is expressed as:

$$\Delta S = Ax^2 + Bx + C = -\frac{4\Delta S_{max}}{m^2}x^2 + \frac{10\Delta S_{max}}{m}x - \frac{21\Delta S_{max}}{4} \quad (11)$$

Where, x is the distance between the projection of the contact point corresponding to the pressure angle α_k on the central axis and the tooth root circle. A , B , C are parameters for modification of wear depth curve. ΔS_{max} is the maximum wear depth and m is the module. TFW severity can be controlled by adjusting the maximum wear depth ΔS_{max} , and the wear depths at different radial positions can also be obtained from this expression. The friction coefficient was simplified as a proportional function of the maximum wear depth, shown in function (12).

Fig.4 Wear section in flank width

The tooth surface roughness is proportional to its TFW state in the fatigue wearing stage and so as the corresponding friction coefficient. It has little effect on the gear pair when the maximum wear depth is less than ΔS_0 where the friction can be neglected, but it might involve into a breakage when the maximum wear depth is higher than ΔS_1 . Five models were established with corresponding ΔS_{max} equal to 0.025mm, 0.05mm, 0.075mm, 0.1mm and 0.125mm respectively which were in the interval of fatigue wearing, and a group of non-wear model was set as reference. Thus, the relation between friction coefficient μ and maximum wear depth ΔS_{max} were described as follows:

$$\mu = k \Delta S_{max}, \Delta S_0 < \Delta S_{max} \leq \Delta S_1 \quad (12)$$

Wherein, k is defined as the friction-wear coupling coefficient, which is mainly related to gear material, and is taken as 2mm^{-1} according to Niemann's equation [24]. Since there is a large order of magnitude difference between the wear depth ΔS_k , as shown in Fig.3 and the tooth thickness S_k , the higher-order terms $(\Delta S_k)^2$ and $(\Delta S_k)^3$ were ignored to obtain the cross-section area at x under TFW state. In the case of TFW fault, the cross-section area A_{xwear} and the rotational inertia of the cross-section I_{xwear} at position x can be expressed as follows:

$$A_{xwear} = \begin{cases} S_x \cdot W, & \rho_1 < x \leq \rho_2 \\ (S_k - \Delta S_k) \cdot W, & \rho_2 < x \leq \rho_3 \\ S_x \cdot W, & \rho_3 < x \leq \rho_4 \end{cases} \quad (13)$$

$$I_{xwear} = \int y^2 dA_x = \begin{cases} \frac{1}{12} W S_k^3, & \rho_1 < x \leq \rho_2 \\ \frac{1}{12} W S_k^3 - \frac{1}{4} W S_k^2 \Delta S_k, & \rho_2 < x \leq \rho_3 \\ \frac{1}{12} W S_k^3, & \rho_3 < x \leq \rho_4 \end{cases} \quad (14)$$

Where $\rho_1 = l = [m(5 + 2z\cos\alpha - 2z)]/2$, $\rho_2 = 3m/4$, $\rho_3 = 7m/4$, $\rho_4 = L = 9m/4$.

The corresponding expressions of bending, shearing and radial compression stiffness under pinion wear fault can be described as follows, respectively:

$$\begin{aligned} \frac{1}{K_{bpwear}} = & \int_{\rho_1}^{\rho_2} \frac{12 \left[x \cdot \left(\frac{R_{bp}}{R_{kp}} + \mu_0 \frac{\sqrt{R_{kp}^2 - R_{bp}^2}}{R_{kp}} \right) - \frac{S_{kp}}{2} \cdot \left(\frac{\sqrt{R_{kp}^2 - R_{bp}^2}}{R_{kp}} - \frac{R_{bp}\mu_0}{R_{kp}} \right) \right]^2}{EW_p S_{kp}^2} dx \\ & + \int_{\rho_2}^{\rho_3} \frac{12 \left[x \cdot \left(\frac{R_{bp}}{R_{kp}} + k\Delta S_{max} \frac{\sqrt{R_{kp}^2 - R_{bp}^2}}{R_{kp}} \right) - \frac{S_{kp}}{2} \cdot \left(\frac{\sqrt{R_{kp}^2 - R_{bp}^2}}{R_{kp}} - \frac{R_{bp}k\Delta S_{max}}{R_{kp}} \right) \right]^2}{EW_1 S_{kp}^2 \left[S_{kp} + \Delta S_{max} \left(\frac{12x^2}{m^2} - \frac{30x}{m} + \frac{63}{4} \right) \right]} dx \end{aligned} \quad (15)$$

$$\begin{aligned} & + \int_{\rho_3}^{\rho_4} \frac{12 \left[x \cdot \left(\frac{R_{bp}}{R_{kp}} + \mu_0 \frac{\sqrt{R_{kp}^2 - R_{bp}^2}}{R_{kp}} \right) - \frac{S_{kp}}{2} \cdot \left(\frac{\sqrt{R_{kp}^2 - R_{bp}^2}}{R_{kp}} - \frac{R_{bp}\mu_0}{R_{kp}} \right) \right]^2}{EW_1 S_{kp}^2} dx \\ \frac{1}{K_{spwear}} = & \int_{\rho_1}^{\rho_2} \frac{1.2R_{bp}^2}{GW_1 S_{kp} R_{kp}^2} dx + \int_{\rho_3}^{\rho_4} \frac{1.2R_{bp}^2}{GW_1 S_{kp} R_{kp}^2} dx + \int_{\rho_2}^{\rho_3} \frac{1.2R_{bp}^2}{GW_p \left(S_{kp} + \frac{4x^2\Delta S_{max}}{m^2} - \frac{10x\Delta S_{max}}{m} + \frac{21\Delta S_{max}}{4} \right) R_{kp}^2} dx \end{aligned} \quad (16)$$

$$\frac{1}{K_{rpwear}} = \int_{\rho_1}^{\rho_2} \frac{R_{kp}^2 - R_{bp}^2}{EW_p S_{kp} R_{kp}^2} dx + \int_{\rho_3}^{\rho_4} \frac{R_{kp}^2 - R_{bp}^2}{EW_p S_{kp} R_{kp}^2} dx + \int_{\rho_2}^{\rho_3} \frac{R_{kp}^2 - R_{bp}^2}{EW_p R_{kp}^2 \left(S_{kp} + \frac{4x^2\Delta S_{max}}{m^2} - \frac{10x\Delta S_{max}}{m} + \frac{21\Delta S_{max}}{4} \right)} dx \quad (17)$$

There is a certain geometric relationship between R_{kp} , S_{kp} and x :

$$(x + R_{dp})^2 + (S_{kp}/2)^2 = R_{kp}^2 \quad (18)$$

Combined with the expression of tooth thickness (2), R_{kp} and S_{kp} can be transformed into the expression of x , and the total gear meshing stiffness in one rotation period under different TFW severity can be obtained, shown in Fig.5.

Fig.5 TVMS of Different TFW Severities in A Cycle

3. Dynamics analysis under TFW

Cylindrical spur gear pair was taken as the research object to fully amplify and better analyze the influence of TFW on the vibration of gearbox. Gear transmission model was shown in Fig.6 which mainly included shaft1, Gear, shaft2, Pinion, Shaft1's Bearing1, Shaft2's Bearing2, HSG and its Bearing.

Fig.6 Gearbox Transmission System

The cylindrical spur gear was taken as the research object with X axis taken as the rotation axis, so the translational DOF in X corresponding to the axial thrust is ignored. Gear was fixed on the input shaft1 and they both formed Block1, sharing the same rotational DOF in X axis and translational DOF in Y and Z directions. Pinion was fixed on Shaft2 and the both components formed Block2, sharing the same rotational DOF around the X axis and translational DOF in Y and Z directions. The gearbox shell HSG held translational DOF in Y and Z directions. All the outer rings of the bearing were fixed on HSG. There were 8 DOFs in the transmission system and 8 sets of equations should be established to study the dynamic characteristics of the gear meshing process by the lumped

parameter method [25,26], which were rotation equation of Block1 around X axis, rotation equation of Block2 around X axis, translation equation of Block1 in Z, translational equation of Block1 in Y, translational equation of Block2 in Z, translational equation of Block2 in Y, translational equation of HSG in Z, and translational equation of HSG in Y, respectively:

$$I_{block1}\ddot{\theta}_g + R_{bp}F_{pg} = T_{in} \quad (19)$$

$$I_{block2}\ddot{\theta}_p + C_{damp}\dot{\theta}_p - R_{bp}F_{pg} = -T_{out} \quad (20)$$

$$m_{block1}\ddot{z}_g + C_{b1}(\dot{z}_g - \dot{z}_H) + K_{b1}(z_g - z_H) + F_f \cos \alpha - F_{pg} \sin \alpha = 0 \quad (21)$$

$$m_{block1}\ddot{y}_g + C_{b1}(\dot{y}_g - \dot{y}_H) + K_{b1}(y_g - y_H) - F_{pg} \cos \alpha - F_f \sin \alpha = 0 \quad (22)$$

$$m_{block2}\ddot{z}_p + C_{b2}(\dot{z}_p - \dot{z}_H) + K_{b2}(z_p - z_H) - F_f \cos \alpha + F_{pg} \sin \alpha = 0 \quad (23)$$

$$m_{block2}\ddot{y}_p + C_{b2}(\dot{y}_p - \dot{y}_H) + K_{b2}(y_p - y_H) + F_f \sin \alpha + F_{pg} \cos \alpha = 0 \quad (24)$$

$$m_H\ddot{z}_H + C_{zh}\dot{z}_H + C_{b1}(\dot{z}_H - \dot{z}_g) + C_{b2}(\dot{z}_H - \dot{z}_p) + K_{zh}z_H + K_{b1}(z_H - z_g) + K_{b2}(z_H - z_p) = 0 \quad (25)$$

$$m_H\ddot{y}_H + C_{yh}\dot{y}_H + C_{b1}(\dot{y}_H - \dot{y}_g) + C_{b2}(\dot{y}_H - \dot{y}_p) + K_{yh}y_H + K_{b1}(y_H - y_g) + K_{b2}(y_H - y_p) = 0 \quad (26)$$

Where, θ_g and θ_p are rotation angles of the Gear and the Pinion, respectively; T_{in} , T_{out} and C_{damp} are input torque, load torque and load damping, respectively; I_{block1} , I_{block2} are inertial moment of Block1 and Block2, respectively; m_{block1} , m_{block2} are mass of Block1 and Block2; z_g , y_g , z_p , y_p , z_H and y_H are respectively the displacements of Gear, Pinion and HSG in Z and Y directions; C_{b1} , K_{b1} , C_{b2} and K_{b2} are bearing damping and bearing stiffness of shaft1 and shaft2, respectively; C_{zh} and K_{zh} are the translational damping and stiffness of HSG in Z direction, respectively; C_{yh} and K_{yh} are the translational damping and dynamic stiffness of HSG in Y direction, respectively. The Pinion was taken as the research object, and the meshing force was F_{pg} which was the sum of the viscous force corresponding to damping and the elastic force corresponding to stiffness, shown in (27) where C_m and K_m represented meshing damping and stiffness, respectively.

$$F_{pg} = C_m \cdot [R_{bg} \quad -R_{bp} \quad -\sin \alpha \quad \sin \alpha \quad 0 \quad -\cos \alpha \quad \cos \alpha \quad 0] \dot{X} \\ + K_m \cdot [R_{bg} \quad -R_{bp} \quad -\sin \alpha \quad \sin \alpha \quad 0 \quad -\cos \alpha \quad \cos \alpha \quad 0] X \quad (27)$$

The equations above could be transformed into Lagrange multi-body dynamical differential equation in the form of matrix.

$$M \cdot \ddot{X} + C \cdot \dot{X} + K \cdot X = Q \quad (28)$$

Where M was the mass matrix of the transmission system; X was the DOFs of the transmission system, expressed as $[\theta_g, \theta_p, z_g, z_p, z_H, y_g, y_p, y_H]^T$; C was the damping matrix; K was the stiffness matrix; Q was the outside force/torque, symbolled by $[T_{in}, -T_{out}, 0, 0, 0, 0, 0, 0]^T$; ξ_1 , ξ_2 , ξ_3 and ξ_4 are the coupling coefficients of friction coefficient and maximum wear depth and their expressions were as follows:

$$\xi_1 = k\Delta S_{max} \cos \alpha - \sin \alpha \quad (29)$$

$$\xi_2 = -k\Delta S_{max} \sin \alpha - \cos \alpha \quad (30)$$

$$\xi_3 = -k\Delta S_{max} \cos \alpha + \sin \alpha \quad (31)$$

$$\xi_4 = k\Delta S_{max} \sin \alpha + \cos \alpha \quad (32)$$

$$\mathbf{M} = \begin{Bmatrix} I_{block1} & & & & & & & & \\ & I_{block2} & & & & & & & \\ & & m_{block1} & & & & & & \\ & & & m_{block2} & & & & & \\ & & & & m_H & & & & \\ & & & & & m_{block1} & & & \\ & & & & & & m_{block2} & & \\ & & & & & & & m_H \end{Bmatrix} \quad (33)$$

$$\mathbf{C} = \begin{Bmatrix} R_{bg}^2 C_m & -R_{bg} R_{bp} C_m & -\sin \alpha R_{bg} C_m & \sin \alpha R_{bg} C_m & 0 & -\cos \alpha R_{bg} C_m & \cos \alpha R_{bg} C_m & 0 \\ -R_{bg} R_{bp} C_m & C_{damp} + R_{bp}^2 C_m & \sin \alpha R_{bp} C_m & -\sin \alpha R_{bp} C_m & 0 & \cos \alpha R_{bp} C_m & \cos \alpha R_{bp} C_m & 0 \\ C_m R_{bg} \xi_1 & -C_m R_{bp} \xi_1 & C_{b1} - \sin \alpha C_m \xi_1 & \sin \alpha C_m \xi_1 & -C_{b1} & -\cos \alpha C_m \xi_1 & \cos \alpha C_m \xi_1 & 0 \\ C_m R_{bg} \xi_3 & -C_m R_{bp} \xi_3 & -\sin \alpha C_m \xi_3 & C_{b2} + \sin \alpha C_m \xi_3 & -C_{b2} & -\cos \alpha C_m \xi_3 & \cos \alpha C_m \xi_3 & 0 \\ 0 & 0 & -C_{b1} & -C_{b2} & 0 & 0 & 0 & 0 \\ R_{bg} C_m \xi_2 & -R_{bp} C_m \xi_2 & -\sin \alpha C_m \xi_2 & \sin \alpha C_m \xi_2 & C_{zh} + C_{b1} + C_{b2} & C_{b1} - \cos \alpha C_m \xi_2 & \cos \alpha C_m \xi_2 & -C_{b1} \\ R_{bg} C_m \xi_4 & -R_{bp} C_m \xi_4 & -\sin \alpha C_m \xi_4 & \sin \alpha C_m \xi_4 & 0 & -\cos \alpha C_m \xi_4 & C_{b2} + \cos \alpha C_m \xi_4 & -C_{b2} \\ 0 & 0 & 0 & 0 & 0 & -C_{b1} & -C_{b2} & C_{yh} + C_{b1} + C_{b2} \end{Bmatrix} \quad (34)$$

$$\mathbf{K} = \begin{Bmatrix} R_{bg}^2 K_m & -R_{bg} R_{bp} K_m & -\sin \alpha R_{bg} K_m & \sin \alpha R_{bg} K_m & 0 & -\cos \alpha R_{bg} K_m & \cos \alpha R_{bg} K_m & 0 \\ -R_{bg} R_{bp} K_m & R_{bp}^2 K_m & \sin \alpha R_{bp} K_m & -\sin \alpha R_{bp} K_m & 0 & \cos \alpha R_{bp} K_m & \cos \alpha R_{bp} K_m & 0 \\ R_{bg} K_m \xi_1 & -R_{bp} K_m \xi_1 & K_{b1} - \sin \alpha R_{bg} K_m \xi_1 & \sin \alpha R_{bg} K_m \xi_1 & -K_{b1} & -\cos \alpha R_{bg} K_m \xi_1 & \cos \alpha R_{bg} K_m \xi_1 & 0 \\ R_{bg} K_m \xi_3 & -R_{bp} K_m \xi_3 & -\sin \alpha R_{bg} K_m \xi_3 & C_{b2} + \sin \alpha R_{bg} K_m \xi_3 & -K_{b2} & -\cos \alpha R_{bg} K_m \xi_3 & \cos \alpha R_{bg} K_m \xi_3 & 0 \\ 0 & 0 & -K_{b1} & -K_{b2} & K_{zh} + K_{b1} + K_{b2} & 0 & 0 & 0 \\ R_{bg} K_m \xi_2 & -R_{bp} K_m \xi_2 & -\sin \alpha R_{bg} K_m \xi_2 & \sin \alpha R_{bg} K_m \xi_2 & 0 & K_{b1} - \cos \alpha R_{bg} K_m \xi_2 & \cos \alpha R_{bg} K_m \xi_2 & -K_{b1} \\ R_{bg} K_m \xi_4 & -R_{bp} K_m \xi_4 & -\sin \alpha R_{bg} K_m \xi_4 & \sin \alpha R_{bg} K_m \xi_4 & 0 & -\cos \alpha R_{bg} K_m \xi_4 & C_{b2} + \cos \alpha R_{bg} K_m \xi_4 & -C_{b2} \\ 0 & 0 & 0 & 0 & 0 & -K_{b1} & -K_{b2} & K_{yh} + K_{b1} + K_{b2} \end{Bmatrix} \quad (35)$$

4. Simulation results

6 × 5 groups ($\Delta S_{max} \times$ input torques) of simulation conditions were set and the vibration velocities of the HSG and Pinion were extracted. The influence of TFW severities on the vibration of gearbox was analyzed under the same speed condition. Vibration acceleration signals of gearbox and pinion were obtained to study the influence of TFW on the impact load of the gearbox under different speed conditions and the variation of meshing force amplitude under different TFW conditions were analyzed. Spectral characteristics of torsional vibration signals under different speed conditions and TFW faults were obtained and analyzed. The vibration signals in the Y and Z directions of the gearbox were similar in principle, so vibration in Y was selected as the original data. Vibration velocity signals is most related to TFW whose effective values were taken as the most critical signal to reveal TFW's vibration characteristics.

4.1 Model validation

The simulation results were compared with the relevant experimental data to verify the effectiveness of the proposed TPMFC method. Experimental test data of a parallel shaft gearbox under TFW fault whose corresponding vibration and spectrum analysis results were shown in Fig.7 was taken as a reference [27]. The Y-direction vibration signal and spectrum analysis were shown in Fig.8 where the maximum TFW depth ΔS_{max} was 0.05mm and the friction coefficient was 0.05.

Fig.7 Experimental Test Data

Fig.8 TPMFC Simulation

It can be seen from Fig.7 & Fig.8 that entire TFW fault will not arouse fault sidebands at integer times of meshing frequency in the spectrum. The main vibration excitation of the gear comes from two groups of high meshing frequency and the nearby frequency domain. The simulation results and experimental data displayed distinction in the spectrum analysis due to variations in their basic gear parameters, wear severities and operating conditions, but their basic vibration signal characteristics showed good agreement. Therefore, this method can effectively reflect TFW, and can quickly realize the modeling of different TFW severities by adjusting ΔS_{max} and corresponding μ .

4.2 Vibration characteristic analysis of HSG

The time-domain vibration velocity signals under steady rotational speed were derived from the TPMFC models whose input torques and TFW severities were set in equal intervals. Thus, the corresponding effective values of vibration velocity were obtained, shown in Fig.9(a). The obtained data sets were all divided by that under none-wear status and the individual effects of ΔS_{max} and torque on fatigue vibration of the gearbox were illustrated in Fig.9(b) & Fig.9(c).

Fig.9 Effective Values of Vibration Velocity & Effective Values Ratio VS None-Wear State

As can be seen from Fig.9, the fatigue vibration of the gearbox increases with TFW severity up and input torque growing. The effective vibration velocity value reaches 1.65 times of that without TFW fault in 40% rated torque and the average effective value is about 1.5 times of those without TFW in all conditions when the maximum wear depth is 0.125mm. The ratio of the effective value of gearbox vibration under different torque conditions to the non-wear state increases approximately linearly with TFW increment, except for that at 20% torque where a exponential correlation occurs. The slope is nearly negatively related to torque, which means the gearbox is more sensitive to TFW faults under low torque conditions. The vibration signal increases most significantly when the torque is 40% of the rated torque. Therefore, operating vibration velocity data can be extracted under relatively lower speed conditions when detecting gearbox TFW faults.

The effective values of vibration acceleration of the gearbox corresponding to different torques and tooth surface TFW severities were obtained, shown in Fig.10(a). The obtained data sets were all divided by that under none-wear

status and the effects of ΔS_{max} and torque on shock vibration of the gearbox were illustrated in Fig.10(b) & Fig.10(c), respectively.

Fig.10 Effective Values of Vibration Acceleration & Effective Values Ratio VS None-Wear State

As can be seen from Fig.10, the shock vibration of the gearbox increases with TFW severity up and input torque growing and the shock vibration characteristics are basically consistent with fatigue vibration characteristics, but not entirely. The effective vibration acceleration value reaches 1.75 times of that without TFW fault in 40% rated torque and the average effective value is about 1.6 times of those without TFW in all conditions when the maximum TFW depth is 0.125mm. The ratio of the effective value of gearbox shock vibration under different torque conditions to the non-wear state increases approximately linearly with TFW increment. The slope is nearly negatively related to torque, which means the gearbox is more sensitive to TFW faults under low torque conditions. The ratio of effective vibration acceleration signals to that under fault-free model increase first and then decrease, and reach its maximum value at 40% torque. The results show that tooth fatigue TFW causes the shock vibration aggravation of the gearbox mainly in the medium rotate speed area, and has relatively little influence on the rated speed and low speed area.

The gear meshing force presents periodicity law whose period sections under different torques and TFW severities were extracted, as shown in Fig.11 and Fig.12.

Fig.11 Time Varying Mesh Force under Varied Torques

Fig.12 Time Varying Mesh Force under Varied TFW Severities

As can be seen from figures above, the maximum meshing force amplitude adds with the augment TFW severity, and the impact mesh force load under TFW is related to wear severity. The amplitude of maximum meshing force basically remains unchanged before the maximum wear depth of tooth surface reaches 0.025mm. The peak value of the meshing force is about 1.25 times of that in the case of none-wear state under the rated torque when the maximum wear depth reaches 0.125mm, thus the corresponding component's service life might be reduced by 20%, which means great danger to the reliability of gearbox. The peak value of meshing force is positively related to the input torque, so the TFW will be intensified under the condition of high rotation speed. When the maximum wear depth of tooth surface reaches 0.05mm, the frequency of peak meshing force is affected by the time-varying meshing stiffness which varies with TFW severity.

4.3 Spectrum analysis of Pinion's torsional vibration

The noise of the gearbox in motion mainly comes from gear-pair's torsional vibration, so it reasonable to find out the excitation source of the torsional vibration signal through spectrum analysis in order to process the fault as soon as possible and improve its reliability. Angular acceleration vibration signals of the pinion was taken as the primitive data. Parts of time-domain signals under TFW fault states were intercepted and the spectrum analysis was carried out to explore the influence of different TFW severities on torsional vibration of the pinion under 40% rated torque, as shown in Fig.13.

Fig.13 Torsional Vibration Analysis in Time/Frequency-domain under 40% Rated Torque

The time-domain and frequency-domain signals at 100% rated torque under varied TFW states were as follows:

Fig.14 Torsional Vibration Analysis in Time/Frequency-domain under 100% Rated Torque

It can be seen from Fig.13 & Fig.14 that the torsional vibration signal is positively related to TFW severities. Therefore, the greater TFW severity brings about stronger impact load on the gearbox. The meshing frequency changes slightly with the TFW severity according to the spectrum analysis, regardless of the rotational speed conditions. The meshing frequency decreases correspondingly with the increase of TFW severity, which is mainly caused by tooth surface material loss, leading to speed transient.

The main frequencies arousing torsional vibration are $1f_m$, $2f_m$ and $4f_m$, which rarely changes with the TFW severity. However, the vibration amplitude at high-order excitation frequencies increases apparently with TFW severity. TFW does not form sidebands at meshing frequency multiplication when the wear fault exists on the entire flank, but forms a discrete fault excitation source near high meshing frequency multiplications. Taking $\Delta S_{max} = 0.125mm$ at low rotating speed as an example, shown in Fig.13(c), fault excitation frequency is generated near high meshing frequency, such as $5f_m$, $7f_m$ and $8f_m$. The transmission error motivates torsional vibration when ΔS_{max} is lower than 0.05mm, while fault characteristic dominates as ΔS_{max} exceeds 0.05mm, except for meshing frequency and its multiples.

5. Conclusion and discussion

A novel TFW simulation method based on gear modification and friction coupling was proposed whose effectiveness was verified by comparing the simulation results with relevant experimental data in this literature. This method can quickly accomplish modeling of different TFW severities by changing max wear depth ΔS_{max} and corresponding friction coefficient μ .

The gearbox's fatigue load is more sensitive to TFW faults under low torque conditions, thus operating vibration velocity data can be extracted under relatively lower speed conditions when detecting gearbox TFW conditions. TFW mainly causes the gearbox's shock vibration aggravation in medium rotate speeds, and has relatively little influence at high and low speeds. TFW does not form sidebands with equal fault frequency interval at meshing frequency multiplications but some discrete fault excitation sources at high meshing frequency multiplications.

There are still some shortages in the paper that the coupling relationship between TFW and friction coefficient was processed by linear simplification and transmission error was not fully studied in detail, which shall be explored in-depth in the future work.

6. Declarations

Availability of data and materials

The datasets used and analyzed during the current study are available from the corresponding author on reasonable request.

Competing interests

We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work. There is no professional or other personal interest of any nature or kind in any product, service or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled.

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Authors' contributions

De Tian: offered some suggestions and revised the literature. Lizhuang Tao: proposed the idea of the research; established the wear model; analyzed the simulation results; drafted the manuscript. Yue Hu: translated the literature. Bei Li: offered some relevant data and revised the literature. Xiaoxuan Wu: offered some relevant data and revised the literature. Shize Tang: offered some relevant data and revised the literature.

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Appendix 1

Module	$m=12$ mm	Mass of gearbox housing	$m_H=6000$ kg
Number of Gear teeth	$Z_g=51$	Moment inertia of block1	$I_{block1}=255\text{kg}\cdot\text{m}^2$
Number of Pinion teeth	$Z_p=17$	Moment inertia of block2	$I_{block2}=17$ kg·m ²
Pressure angle	$\alpha=20$ deg	Torque-in	$T_{in}=1.352\times 10^5$ N·m
Tooth width of Gear	$W_G=0.2448$ m	Torque of load	$T_{out}=1.352\times 10^5/3$ N·m
Tooth width of Pinion	$W_P=0.25$ m	Tortional damping of Shaft2	$C_{damp}=2000$ Nm/rad
Addendum circle radis of Gear	$R_{ag}=0.636$ m	Stiffness of shaft1 bearing	$K_{bl}=1\times 10^{11}$ N/m
Base circle radis of Gear	$R_{bg}=0.575$ m	Damping of shaft1 bearing	$C_{bl}=1\times 10^8$ N·s/m
Addendum circle radius of Pinion	$R_{ap}=0.228$ m	Stiffness of shaft2 bearing	$K_{b2}=4\times 10^{10}$ N/m
Base circle radius of Pinion	$R_{bp}=0.1917$ m	Damping of shaft2 bearing	$C_{b2}=4\times 10^7$ N·s/m
Young's Modulus	$E=2.09\times 10^{11}$ Pa	Trans stiffness of HSG in Z	$K_{zh}=3\times 10^8$ N/m
Poisson's Ratio	$\nu=0.3$	Trans damping of HSG in Z	$C_{zh}=3\times 10^5$ N·s/m
Mass of block1	$m_{block1}=3258$ kg	Trans stiffness of HSG in Y	$K_{yh}=2\times 10^8$ N/m
Mass of block2	$m_{block2}=1097$ kg	Trans damping of HSG in Y	$C_{yh}=2\times 10^5$ N·s/m

Figures

Figure 1

Tooth Fatigue Wear Research Steps

Figure 2

Mesh force of a single tooth in Gear

Figure 3

TFW Section with Radial Length

Figure 4

Wear section in flank width

Figure 5

TVMS of Different TFW Severities in A Cycle

Figure 6

Gearbox Transmission System

(a) Time-domain Vibration Signals

(b) Frequency-domain Amplitude

Figure 7

Experimental Test Data

(a) Time-domain Vibration signals

(b) Frequency-domain Amplitude

Figure 8

TPMFC Simulation

(a) RMS Values

(b) Variations on Max TFW Depth

(c) Variations on Torque

Figure 9

Effective Values of Vibration Velocity & Effective Values Ratio VS None-Wear State

Figure 10

Effective Values of Vibration Acceleration & Effective Values Ratio VS None-Wear State

Figure 11

Time Varying Mesh Force under Varied Torques

Figure 12

Time Varying Mesh Force under Varied TFW Severities

Figure 13

Torsional Vibration Analysis in Time/Frequency-domain under 40% Rated Torque

Figure 14

Torsional Vibration Analysis in Time/Frequency-domain under 100% Rated Torque