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Spreading Dynamics Analysis of a Novel SEAIRS Epidemic Model with Protection and Hospital Quarantine under Government Pre-warning

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Spreading dynamics analysis of a novel *SEAIRS* epidemic model with protection and hospital quarantine under government pre-warning

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Abstract: To study the impact of protection and hospital quarantine measure, government pre-warning mechanism and heterogeneity of underlying networks on epidemic spreading, a novel *SEAIRS* epidemic model is proposed on scale-free networks. The spreading dynamics of the model is studied by means of the mean-field theory. Two equilibriums and the basic reproductive number R_0 of the model is analyzed in detail. The global asymptotic stability of the disease-free equilibrium, the permanence of the epidemic spreading and the global attractivity of the endemic equilibrium are proved. Sensitivity analysis shows that the basic reproductive number R_0 is dependent on the coverage rate of home quarantine (ω_Q, η_A, η_S), hospitalization rate η_I and government pre-warning intensity δ. Finally, the theoretical analysis results are confirmed by means of numerical simulations.

Keywords: Epidemic spreading; Protection quarantine; Hospital quarantine; Government pre-warning; Scale-free networks; Stability

1. Introduction

In recent years, despite the rapid advance and development of medical science and technology, infectious diseases still threaten public safety and human health [1]-[4]. What's worse, as the rapid development of transportation network around the world, contact between people becomes more frequent, which makes the new virus quickly spread to all regions of the world before the success of vaccine development [5]. Just as the current outbreak of coronavirus disease (COVID-19), since its discovery, it has rapidly spread all over the world, causing serious harm to people's health and social development [6]. The World Health Organization (WHO) has announced that COVID-19 constitutes the global pandemic "PHEIC".

Therefore, it is necessary to build an appropriate epidemic model to study epidemic spreading dynamics. It can not only understand the spread rule of epidemic, predict the development trend of epidemic and evaluate the effect of various prevention and control measures, but also has important guiding significance for the prevention and control of epidemic [7]-[13]. In epidemiology, the relatively representative models: the SIS (susceptible – infected – susceptible) model and the SIR (susceptible – infected – recovered) model. In 1999, researchers discovered that social networks have scale-free property [14]. Researchers considered the scale-free characteristics of the network to establish models, analyzed the spreading dynamic of epidemic in detail, so as to make it more truly reflect the spread of epidemic in the real world [15]-[18]. Marián Boguñá et al. [19] considered the correlation of node degree in the network, studied the SIS epidemic model on scale-free networks in detail. Based on the classical epidemic models and the characteristics of epidemic, some scholars have proposed practical epidemic models [20], [21]. For instance, Huang et al. [22] proposed an epidemic model in which an individual can be repeatedly infected by virus and

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discussed the change of infection rate with infection times. Liu et al. [23] proposed a SEIR epidemic model of diseases with age-dependent incubation period and recurrence.

However, protection and hospital quarantine measures, government pre-warning mechanism have not been considered in most of the research work mentioned above. Protection and hospital quarantine, government pre-warning play an important role in the prevention and control of epidemic [24]-[26]. Just as the COVID-19, we can find that the intensity of government pre-warning, protection and hospital quarantine in different regions are different, the degree of harm caused by COVID-19 to people's health and social development is completely different. So, taking protection and hospital quarantine measure, government pre-warning mechanism and heterogeneity of underlying networks into account, we propose and analyze the *SEAIRS* epidemic model with protection and hospital quarantine under government pre-warning.

The following is a breakdown of the paper's structure: Sect. 2 presents a novel *SEAIRS* epidemic model. In Sect. 3, the basic reproductive number R_0 and two equilibriums are obtained. Sect. 4 analyzes the stability of disease-free equilibrium and persistence of endemic equilibrium. In Sect. 5, the global attractivity of endemic equilibrium is proved. Sect. 6 obtains the sensitivity index of parameters by local sensitivity analysis. In Sect. 7, the main results are given by numerical simulations. Sect. 8 summarizes the paper.

2. System model

In order to describe the dynamic behavior of epidemic spreading on scale-free networks, considering government control, protection and hospital quarantine, we present a novel *SEAIRS* epidemic model. The whole population is divided into eight different categories: S, E, A, I, R, Q, H and P. The people who are healthy and at risk of infection (S, susceptible people); the people who do not show symptoms after infection and do not spread the virus (E, exposed people); the people who do not show symptoms after infection and spread the virus (A, asymptomatic infection people); the people who show symptoms after infection and spread the virus (I, symptomatic infection people); the people who produce antibodies to the virus (R, recovered people); the people who do not show symptoms after infection and are quarantined at home (Q, quarantined infected people); the people who show symptoms after infection and are treated in hospital quarantine (H, hospitalized people); the people who are healthy and are quarantined at home (P, protected susceptible people). The diagrammatic sketch of epidemic spreading model is given in Fig. 1.

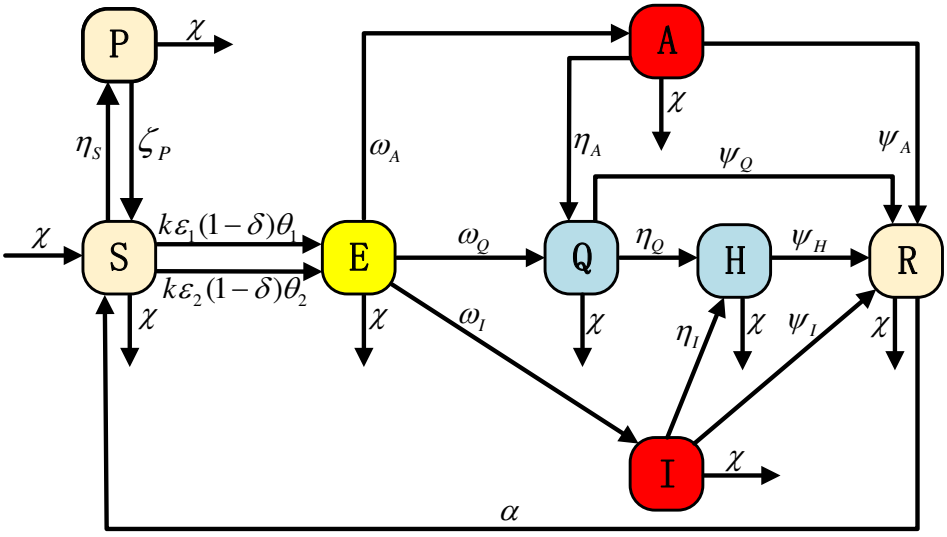


Fig. 1. The flow chart of the *SEAIRS* model.

The definition of parameters is shown in Table 1.

Table 1: Definition of parameters

Parameter	Definition
ε_1, ε_2	The probability of the susceptible being infected by asymptomatic and symptomatic infection people under government pre-warning
δ	Government pre-warning intensity
η_S, ω_Q, η_A	The coverage rate of home quarantine for susceptible, exposed and asymptomatic infection people
ζ_P	The rate of termination of quarantine for the protected susceptible people due to they were not identified as infected by the virus during quarantine
η_I	The probability of hospitalization for symptomatic infection people
α	The probability of virus antibody disappearance for recovered people
χ	Birth rate \ death rate

The model can be represented by the following ordinary differential equations based on mean-field theory and the above model.

$$\begin{cases} S_k'(t) = \chi + \alpha R_k(t) + \zeta_P P_k(t) - k(1-\delta)(\varepsilon_1\theta_1(t) + \varepsilon_2\theta_2(t))S_k(t) - (\eta_S + \chi)S_k(t), \\ E_k'(t) = k(1-\delta)(\varepsilon_1\theta_1(t) + \varepsilon_2\theta_2(t))S_k(t) - (\omega_A + \omega_Q + \omega_I + \chi)E_k(t), \\ A_k'(t) = \omega_A E_k(t) - (\eta_A + \psi_A + \chi)A_k(t), \\ I_k'(t) = \omega_I E_k(t) - (\eta_I + \psi_I + \chi)I_k(t), \\ Q_k'(t) = \omega_Q E_k(t) + \eta_A A_k(t) - (\eta_Q + \psi_Q + \chi)Q_k(t), \\ H_k'(t) = \eta_I A_k(t) + \eta_Q Q_k(t) - (\psi_H + \chi)H_k(t), \\ R_k'(t) = \psi_A A_k(t) + \psi_I I_k(t) + \psi_Q Q_k(t) + \psi_H H_k(t) - (\alpha + \chi)R_k(t), \\ P_k'(t) = \eta_S S_k(t) - (\zeta_P + \chi)P_k(t). \end{cases} \tag{1}$$

The possibility of connection between susceptible people and asymptomatic infection people at time t is given the symbol $\theta_1(t)$, where

$$\theta_1(t) = \frac{1}{\langle k \rangle}\sum_{i=1}^{n} kP(k)A_k(t), \tag{2}$$

The possibility of connection between susceptible people and asymptomatic infection people at time t is given the symbol $\theta_2(t)$, where

$$\theta_2(t) = \frac{1}{\langle k \rangle}\sum_{i=1}^{n} kP(k)I_k(t). \tag{3}$$

The average degree values are denoted by the symbol $\langle k \rangle$ in the network. The degree distribution is represented with $P(k)$. $A(t) = \sum_{k=1}^{n} P(k)A_k(t)$ denotes the density of all asymptomatic infection people on scale-free networks. $I(t) = \sum_{k=1}^{n} P(k)I_k(t)$ denotes the density of all symptomatic infection people on scale-free networks. Let $\varepsilon(t) = \varepsilon_1\theta_1(t) + \varepsilon_2\theta_2(t)$, for system (1), we can obtain the model shown below:

$$\begin{cases} S_k'(t) = \chi + \alpha R_k(t) + \zeta_P P_k(t) - k(1-\delta)\varepsilon(t)S_k(t) - (\eta_S + \chi)S_k(t), \\ E_k'(t) = k(1-\delta)\varepsilon(t)S_k(t) - (\omega_A + \omega_Q + \omega_I + \chi)E_k(t), \\ A_k'(t) = \omega_A E_k(t) - (\eta_A + \psi_A + \chi)A_k(t), \\ I_k'(t) = \omega_I E_k(t) - (\eta_I + \psi_I + \chi)I_k(t), \\ Q_k'(t) = \omega_Q E_k(t) + \eta_A A_k(t) - (\eta_Q + \psi_Q + \chi)Q_k(t), \\ H_k'(t) = \eta_I A_k(t) + \eta_Q Q_k(t) - (\psi_H + \chi)H_k(t), \\ R_k'(t) = \psi_A A_k(t) + \psi_I I_k(t) + \psi_Q Q_k(t) + \psi_H H_k(t) - (\alpha + \chi)R_k(t), \\ P_k'(t) = \eta_S S_k(t) - (\zeta_P + \chi)P_k(t). \end{cases} \tag{4}$$

According to the above description and the normalization conditions, we can obtain

$$S_k(t) + E_k(t) + A_k(t) + I_k(t) + Q_k(t) + H_k(t) + R_k(t) + P_k(t) = 1. \tag{5}$$

The system (1) satisfies the following initial conditions:

$$\begin{cases} S_k(0) + E_k(0) + A_k(0) + I_k(0) + Q_k(0) + H_k(0) + R_k(0) + P_k(0) = 1, \\ 0 \le S_k(0),\ E_k(0),\ A_k(0),\ I_k(0),\ Q_k(0),\ H_k(0),\ R_k(0),\ P_k(0) \le 1, \\ \varepsilon(0) > 0. \end{cases} \tag{6}$$

3. The basic reproductive number and existence of equilibriums

We obtain an analytic solution of the system (1) explaining the dynamics of *SEIARS* epidemic spreading on scale-free networks and two equilibriums of the system (1) are proved in this section.

Theorem 1. For system (1), define the basic reproductive number

$$R_0 = \frac{\langle k^2 \rangle}{\langle k \rangle} \cdot \frac{(1-\delta)(\zeta_P + \chi)\left[(\eta_I + \psi_I + \chi)\omega_A \varepsilon_1 + (\eta_A + \psi_A + \chi)\omega_I \varepsilon_2\right]}{(\eta_S + \zeta_P + \chi)(\omega_A + \omega_Q + \omega_I + \chi)(\eta_I + \psi_I + \chi)(\eta_A + \psi_A + \chi)}.$$

The system (1) has an endemic equilibrium $E^* = \left(S_k^*, E_k^*, A_k^*, I_k^*, Q_k^*, H_k^*, R_k^*, P_k^*\right)$ when $R_0 > 1$ and a disease-free equilibrium $E^0\left(\frac{\zeta_P + \chi}{\eta_S + \zeta_P + \chi}, 0, 0, 0, 0, 0, 0, \frac{\eta_S}{\eta_S + \zeta_P + \chi}\right)$ when $R_0 < 1$.

Proof. Taking $S_k(t) = 1 - E_k(t) - A_k(t) - I_k(t) - Q_k(t) - H_k(t) - R_k(t) - P_k(t)$ into account, the system (4) is equivalent to system (7):

$$\begin{cases} E_k'(t) = k(1-\delta)\varepsilon(t)(1 - E_k(t) - A_k(t) - I_k(t) - Q_k(t) - H_k(t) - R_k(t) - P_k(t)) \\ \qquad -(\omega_A + \omega_Q + \omega_I + \chi)E_k(t), \\ A_k'(t) = \omega_A E_k(t) - (\eta_A + \psi_A + \chi)A_k(t), \\ I_k'(t) = \omega_I E_k(t) - (\eta_I + \psi_I + \chi)I_k(t), \\ Q_k'(t) = \omega_Q E_k(t) + \eta_A A_k(t) - (\eta_Q + \psi_Q + \chi)Q_k(t), \\ H_k'(t) = \eta_I A_k(t) + \eta_Q Q_k(t) - (\psi_H + \chi)H_k(t), \\ R_k'(t) = \psi_A A_k(t) + \psi_I I_k(t) + \psi_Q Q_k(t) + \psi_H H_k(t) - (\alpha + \chi)R_k(t), \\ P_k'(t) = \eta_S(1 - E_k(t) - A_k(t) - I_k(t) - Q_k(t) - H_k(t) - R_k(t) - P_k(t)) - (\zeta_P + \chi)P_k(t). \end{cases} \tag{7}$$

It obvious that $E^0\left(\frac{\zeta_P + \chi}{\eta_S + \zeta_P + \chi}, 0, 0, 0, 0, 0, 0, \frac{\eta_S}{\eta_S + \zeta_P + \chi}\right)$ is a system equilibrium, also

known as disease-free equilibrium. Using the next-generation matrix method and the system (7), we can get

$$\frac{dy}{dt} = v(y) - b(y),$$

where

$$y = \left[E_k(t),\ A_k(t),\ I_k(t),\ Q_k(t),\ H_k(t),\ R_k(t),\ P_k(t)\right]^T,$$

$$v(x) = \begin{pmatrix} k(1-\delta)\varepsilon(t)(1-E_k(t)-A_k(t)-I_k(t)-Q_k(t)-H_k(t)-R_k(t)-P_k(t)) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$b(x) = \begin{pmatrix} (\omega_A+\omega_Q+\omega_I+\chi)E_k(t) \\ (\eta_A+\psi_A+\chi)A_k(t)-\omega_A E_k(t) \\ (\eta_I+\psi_I+\chi)I_k(t)-\omega_I E_k(t) \\ (\eta_Q+\psi_Q+\chi)Q_k(t)-\eta_I A_k(t)-\eta_Q Q_k(t) \\ (\psi_H+\chi)H_k(t)-\eta_I A_k(t)-\eta_Q Q_k(t) \\ (\alpha+\chi)R_k(t)-\psi_A A_k(t)-\psi_I I_k(t)-\psi_Q Q_k(t)-\psi_H H_k(t) \\ (\zeta_P+\chi+\eta_S)P_k(t)+\eta_S(E_k(t)+A_k(t)+I_k(t)+Q_k(t)+H_k(t)+R_k(t))-\eta_S \end{pmatrix},$$

At the $E^0\left(\frac{\zeta_P+\chi}{\eta_S+\zeta_P+\chi},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{\eta_S}{\eta_S+\zeta_P+\chi}\right)$, the Jacobian matrices of $v(x)$ and $b(x)$ are as follows:

$$V = Dv(E^0) = \begin{pmatrix} 0 & V_{12} & V_{13} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$B = Db(E^0) = \begin{pmatrix} B_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ B_{21} & B_{22} & 0 & 0 & 0 & 0 & 0 \\ B_{31} & 0 & B_{33} & 0 & 0 & 0 & 0 \\ B_{41} & B_{42} & 0 & B_{44} & 0 & 0 & 0 \\ 0 & B_{52} & 0 & B_{54} & B_{55} & 0 & 0 \\ 0 & B_{62} & B_{63} & B_{64} & B_{65} & B_{66} & 0 \\ B_{71} & B_{72} & B_{73} & B_{74} & B_{75} & B_{76} & B_{77} \end{pmatrix},$$

where

$$V_{12} = \langle k \rangle^{-1} \frac{(1-\delta)\eta_S \varepsilon_1}{(\eta_S+\zeta_P+\chi)} \begin{bmatrix} 1P(1) & 2P(2) & \mathrm{L} & nP(n) \\ 2P(1) & 4P(2) & \mathrm{L} & 2nP(n) \\ \mathrm{M} & \mathrm{M} & \mathrm{O} & \mathrm{M} \\ nP(1) & 2nP(2) & \mathrm{L} & n^2P(n) \end{bmatrix},$$

$$V_{13} = \langle k \rangle^{-1} \frac{(1-\delta)\eta_S \varepsilon_2}{(\eta_S+\zeta_P+\chi)} \begin{bmatrix} 1P(1) & 2P(2) & \mathrm{L} & nP(n) \\ 2P(1) & 4P(2) & \mathrm{L} & 2nP(n) \\ \mathrm{M} & \mathrm{M} & \mathrm{O} & \mathrm{M} \\ nP(1) & 2nP(2) & \mathrm{L} & n^2P(n) \end{bmatrix},$$

and

$B_{11}=(\omega_A+\omega_Q+\omega_I+\chi)I$, $B_{21}=(-\omega_A)I$, $B_{22}=(\eta_A+\psi_A+\chi)I$, $B_{31}=(-\omega_I)I$,

$B_{33}(\eta_I+\psi_I+\chi)I$, $B_{41}=(-\omega_Q)I$, $B_{42}=(-\eta_A)I$, $B_{44}=(\eta_Q+\psi_Q+\chi)I$, $B_{52}=(-\eta_I)I$,

$B_{55}=(\psi_H+\chi)I$, $B_{62}=(-\psi_A)I$, $B_{63}=(-\psi_I)I$, $B_{64}=(-\psi_Q)I$, $B_{65}=(-\psi_H)I$,

$B_{66}=(\alpha+\chi)I$, $B_{71}=\eta_S I$, $B_{72}=\eta_S I$, $B_{73}=\eta_S I$, $B_{74}=\eta_S I$, $B_{76}=\eta_S I$,

$B_{77}=(\zeta_P+\eta_S+\chi)I$, $B_{75}=\eta_S I$,

where I represents the identity matrix. The basic reproductive number can be got by theorem 2 in [27], namely

$$R_0=\rho(VB^{-1})=\frac{\langle k^2\rangle}{\langle k\rangle}\cdot\frac{(1-\delta)(\zeta_P+\chi)\left[(\eta_I+\psi_I+\chi)\omega_A\varepsilon_1+(\eta_A+\psi_A+\chi)\omega_I\varepsilon_2\right]}{(\eta_S+\zeta_P+\chi)(\omega_A+\omega_Q+\omega_I+\chi)(\eta_I+\psi_I+\chi)(\eta_A+\psi_A+\chi)},$$

where

$$\langle k^2\rangle=\sum\nolimits_k k^2P(k).$$

To get the endemic equilibrium $E^*=\left(S_k^*,\ E_k^*,A_k^*,I_k^*,Q_k^*,H_k^*,R_k^*,P_k^*\right)$, we let the system (7) equal to zero, it satisfies

$$\begin{cases}\chi+\alpha R_k^*+\zeta_P P_k^*-k(1-\delta)\varepsilon(t)S_k^*-(\eta_S+\chi)S_k^*=0,\\ k(1-\delta)\varepsilon(t)S_k^*-(\omega_A+\omega_Q+\omega_I+\chi)E_k^*=0,\\ \omega_A E_k^*-(\eta_A+\psi_A+\chi)A_k^*=0,\\ \omega_I E_k^*-(\eta_I+\psi_I+\chi)I_k^*=0,\\ \omega_Q E_k^*+\eta_A A_k^*-(\eta_Q+\psi_Q+\chi)Q_k^*=0,\\ \eta_I A_k^*+\eta_Q Q_k^*-(\psi_H+\chi)H_k^*=0,\\ \psi_A A_k^*+\psi_I I_k^*+\psi_Q Q_k^*+\psi_H H_k^*-(\alpha+\chi)R_k^*=0,\\ \eta_S S_k^*-(\zeta_P+\chi)P_k^*=0.\end{cases}\tag{8}$$

According to the system (8), we can obtain

$$\begin{cases}S_k^*=\dfrac{(\omega_A+\omega_Q+\omega_I+\chi)(\eta_A+\psi_A+\chi)}{k(1-\delta)\varepsilon\omega_A}A_k^*,\\ E_k^*=\dfrac{\eta_A+\psi_A+\chi}{\omega_A}A_k^*,\\ I_k^*=\dfrac{\omega_I(\eta_A+\psi_A+\chi)}{\omega_A(\eta_I+\psi_I+\chi)}A_k^*,\\ Q_k^*=\dfrac{\omega_Q(\eta_A+\psi_A+\chi)+\omega_A\eta_A}{\omega_A(\eta_Q+\psi_Q+\chi)}A_k^*,\\ H_k^*=\dfrac{\eta_I\omega_A(\eta_Q+\psi_Q+\chi)+\eta_Q\left[\omega_Q(\eta_A+\psi_A+\chi)+\omega_A\eta_A\right]}{(\psi_H+\chi)\omega_A(\eta_Q+\psi_Q+\chi)}A_k^*,\\ \qquad\dfrac{(\psi_H+\chi)(\eta_Q+\psi_Q+\chi)\left[\psi_A\omega_A(\eta_I+\psi_I+\chi)+\psi_I\omega_I(\eta_A+\psi_A+\chi)\right]}{+\left[\psi_Q(\psi_H+\chi)(\eta_I+\psi_I+\chi)+\psi_H\eta_Q\right]\left[\omega_Q(\eta_A+\psi_A+\chi)+\omega_A\eta_A\right]}\\ R_k^*=\dfrac{+\psi_H\eta_I\omega_A(\eta_Q+\psi_Q+\chi)}{(\alpha+\chi)(\psi_H+\chi)\omega_A(\eta_Q+\psi_Q+\chi)(\eta_I+\psi_I+\chi)}A_k^*,\\ P_k^*=\dfrac{\eta_S(\omega_A+\omega_Q+\omega_I+\chi)(\eta_A+\psi_A+\chi)}{(\zeta_P+\chi)k(1-\delta)\varepsilon\omega_A}A_k^*.\end{cases}\tag{9}$$

Because $S_k(t)+E_k(t)+A_k(t)+I_k(t)+Q_k(t)+H_k(t)+R_k(t)+P_k(t)=1$, we can calculate $A_k^*=\dfrac{(\zeta_P+\chi)k(1-\delta)\varepsilon\omega_A(\alpha+\chi)(\psi_H+\chi)\omega_A(\eta_Q+\psi_Q+\chi)(\eta_I+\psi_I+\chi)}{(\zeta_P+\eta_S+\chi)(\omega_A+\omega_Q+\omega_I+\chi)(\eta_A+\psi_A+\chi)(\alpha+\chi)(\psi_H+\chi)\omega_A(\eta_Q+\psi_Q+\chi)(\eta_I+\psi_I+\chi)+m(\varepsilon)}$. Here, $m(\varepsilon)$ is a function of ε.

Due to $\varepsilon(t)=\varepsilon_1\theta_1(t)+\varepsilon_2\theta_2(t)=\dfrac{\sum_k kP(k)}{\langle k\rangle}(\varepsilon_1 A_k(t)+\varepsilon_2 I_k(t))$, so

$$\varepsilon(\infty)=\frac{\sum_k kP(k)}{\langle k\rangle}(\varepsilon_1 A^*+\varepsilon_2 I^*)=\frac{\sum_k kP(k)}{\langle k\rangle}(\varepsilon_1+\frac{\varepsilon_2\omega_I(\eta_A+\psi_A+\chi)}{\omega_A(\eta_I+\psi_I+\chi)})A_k^*. \tag{10}$$

Let $\varepsilon(\infty)\ @f(\varepsilon(\infty))$, we can know $\varepsilon(\infty)=0$ is a solution of the system (10), it proves that the system (1) has disease-free equilibrium. To prove $\varepsilon(\infty)\ @f(\varepsilon(\infty))$ has a nontrivial solution, it must satisfy

$$\left.\frac{df(\varepsilon(\infty))}{\varepsilon(\infty)}\right|_{\varepsilon(\infty)=0}>1 \quad \text{and} \quad f(1)\le 1.$$

Therefore, we get

$$R_0=\frac{\langle k^2\rangle}{\langle k\rangle}\cdot\frac{(1-\delta)(\zeta_P+\chi)\left[(\eta_I+\psi_I+\chi)\omega_A\varepsilon_1+(\eta_A+\psi_A+\chi)\omega_I\varepsilon_2\right]}{(\eta_S+\zeta_P+\chi)(\omega_A+\omega_Q+\omega_I+\chi)(\eta_I+\psi_I+\chi)(\eta_A+\psi_A+\chi)}>1.$$

Thus, we can get $0\le S_k^*,\ E_k^*,\ A_k^*,\ I_k^*,\ Q_k^*,\ H_k^*,\ R_k^*,\ P_k^*\le 1$ is a nontrivial solution of the system (10) when $R_0>1$. Hence, the system (1) has an endemic equilibrium $E^*=\left(S_k^*,\ E_k^*,A_k^*,I_k^*,Q_k^*,H_k^*,R_k^*,P_k^*\right)$ when $R_0>1$. The proof is completed.

4. The stability of disease-free equilibrium and permanence of endemic equilibrium

Theorem 2. The disease-free equilibrium E_0 is globally asymptotically stable when $R_0<1$. The epidemic spread is permanent when $R_0>1$, which means there is a constant δ, so that $\liminf_{t\to\infty}\left(A(t)+I(t)\right)=\liminf_{t\to\infty}\sum_k P(k)(A_k(t)+I_k(t))\ge\delta$, where $A_k(0)>0$ and $I_k(0)>0$.

Proof. For disease-free equilibrium E_0 of system (7), the Jacobian matrix is $7n*7n$ matrix as follows:

$$N=\begin{pmatrix} M_{11} & \mathrm{L} & M_{1n} \\ \mathrm{M} & \mathrm{O} & \mathrm{M} \\ M_{n1} & \mathrm{L} & M_{nn} \end{pmatrix},$$

where

$$M_{11}=\begin{pmatrix} -(\omega_A+\omega_Q+\omega_I+\chi) & \frac{(\zeta_P+\chi)(1-\delta)\varepsilon_1}{\eta_S+\zeta_P+\chi}P_1 & \frac{(\zeta_P+\chi)(1-\delta)\varepsilon_2}{\eta_S+\zeta_P+\chi}P_1 & 0 & 0 & 0 & 0 \\ \omega_A & -(\eta_A+\psi_A+\chi) & 0 & 0 & 0 & 0 & 0 \\ \omega_I & 0 & -(\eta_I+\psi_I+\chi) & 0 & 0 & 0 & 0 \\ \omega_Q & \eta_A & 0 & -(\eta_Q+\psi_Q+\chi) & 0 & 0 & 0 \\ 0 & \eta_I & 0 & \eta_Q & -(\psi_H+\chi) & 0 & 0 \\ 0 & \psi_A & \psi_I & \psi_Q & \psi_H & -(\alpha+\chi) & 0 \\ -\eta_S & -\eta_S & -\eta_S & -\eta_S & -\eta_S & -\eta_S & -(\zeta_P+\eta_S+\chi) \end{pmatrix},$$

$$M_{1n}=\begin{pmatrix} 0 & \dfrac{(\zeta_P+\chi)(1-\delta)\varepsilon_1}{\eta_S+\zeta_P+\chi}P_n & \dfrac{(\zeta_P+\chi)(1-\delta)\varepsilon_2}{\eta_S+\zeta_P+\chi}P_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$M_{n1}=\begin{pmatrix} 0 & n\dfrac{(\zeta_P+\chi)(1-\delta)\varepsilon_1}{\eta_S+\zeta_P+\chi}P_1 & n\dfrac{(\zeta_P+\chi)(1-\delta)\varepsilon_2}{\eta_S+\zeta_P+\chi}P_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$M_{nn}=\begin{pmatrix} -(\omega_A+\omega_Q+\omega_I+\chi) & n\dfrac{(\zeta_P+\chi)(1-\delta)\varepsilon_1}{\eta_S+\zeta_P+\chi}P_n & n\dfrac{(\zeta_P+\chi)(1-\delta)\varepsilon_2}{\eta_S+\zeta_P+\chi}P_n & 0 & 0 & 0 & 0 \\ \omega_A & -(\eta_A+\psi_A+\chi) & 0 & 0 & 0 & 0 & 0 \\ \omega_I & 0 & -(\eta_I+\psi_I+\chi) & 0 & 0 & 0 & 0 \\ \omega_Q & \eta_A & 0 & -(\eta_Q+\psi_Q+\chi) & 0 & 0 & 0 \\ 0 & \eta_I & 0 & \eta_Q & -(\psi_H+\chi) & 0 & 0 \\ 0 & \psi_A & \psi_I & \psi_Q & \psi_H & -(\alpha+\chi) & 0 \\ -\eta_S & -\eta_S & -\eta_S & -\eta_S & -\eta_S & -\eta_S & -(\zeta_P+\eta_S+\chi) \end{pmatrix}.$$

Therefore, the characteristic polynomial of Jacobian matrix N can be determined as follows:

$$\begin{aligned}&(\lambda+\omega_A+\omega_Q+\omega_I+\chi)^{n-1}(\lambda+\eta_A+\psi_A+\chi)^{n-1}(\lambda+\eta_I+\psi_I+\chi)^{n-1}(\lambda+\eta_Q+\psi_Q+\chi)^n\\&\quad\times(\lambda+\psi_H+\chi)^n(\lambda+\alpha+\chi)^n(\lambda+\zeta_P+\eta_S+\chi)^n(\lambda^3+r\lambda^2+s\lambda+p)=0,\end{aligned}$$

where

$$r=\omega_A+\omega_Q+\omega_I+\eta_A+\psi_A+\eta_I+\psi_I+3\chi,$$

$$\begin{aligned}s=&(\omega_A+\omega_Q+\omega_I+\chi)(\eta_A+\psi_A+\eta_I+\psi_I+2\chi)+(\eta_A+\psi_A+\chi)(\eta_I+\psi_I+\chi)\\&-\omega_A\frac{(\zeta_P+\chi)(1-\delta)\varepsilon_1}{\eta_S+\zeta_P+\chi}\sum_{k=1}^{n}kP(k)-\omega_I\frac{(\zeta_P+\chi)(1-\delta)\varepsilon_2}{\eta_S+\zeta_P+\chi}\sum_{k=1}^{n}kP(k),\end{aligned}$$

$$\begin{aligned}p=&(\omega_A+\omega_Q+\omega_I+\chi)(\eta_A+\psi_A+\chi)(\eta_I+\psi_I+\chi)-\omega_A(\eta_I+\psi_I+\chi)\frac{(\zeta_P+\chi)(1-\delta)\varepsilon_1}{\eta_S+\zeta_P+\chi}\sum_{k=1}^{n}kP(k)\\&-\omega_I(\eta_A+\psi_A+\chi)\frac{(\zeta_P+\chi)(1-\delta)\varepsilon_2}{\eta_S+\zeta_P+\chi}\sum_{k=1}^{n}kP(k).\end{aligned}$$

We can find $r>0$. When $R_0<1$, we can get

$$\begin{aligned}&(\omega_A+\omega_Q+\omega_I+\chi)(\eta_A+\psi_A+\chi)(\eta_I+\psi_I+\chi)\\&>\left(\omega_A(\eta_I+\psi_I+\chi)\frac{(\zeta_P+\chi)(1-\delta)\varepsilon_1}{\eta_S+\zeta_P+\chi}+\omega_I(\eta_A+\psi_A+\chi)\frac{(\zeta_P+\chi)(1-\delta)\varepsilon_2}{\eta_S+\zeta_P+\chi}\right)\sum_{k=1}^{n}kP(k),\\&(\omega_A+\omega_Q+\omega_I+\chi)(\eta_A+\psi_A+\eta_I+\psi_I+2\chi)\\&>\left(\omega_A\frac{(\zeta_P+\chi)(1-\delta)\varepsilon_1}{\eta_S+\zeta_P+\chi}+\omega_I\frac{(\zeta_P+\chi)(1-\delta)\varepsilon_2}{\eta_S+\zeta_P+\chi}\right)\sum_{k=1}^{n}kP(k).\end{aligned}$$

It also means $s>0$, $p>0$ when $R_0<1$. Therefore, if $R_0<1$, the real eigenvalues λ of matrix N are all negative. In addition, N has a positive eigenvalue λ if $R_0>1$. By the Perron-Frobenius theorem, the maximum real part value of all eigenvalues of N is positive when $R_0>1$. Conclusively, it can get the results from the theorem of Lajmanovich and York [28]. The proof is completed.

5. The global attractivity of endemic equilibrium

We discuss the global attractivity of endemic equilibrium by means of a monotonic iterative algorithm in this section.

Theorem 3. Suppose that $(E_k(t), A_k(t), I_k(t), Q_k(t), H_k(t), R_k(t), P_k(t))$ is a solution of system (1) satisfying $A_k(0) > 0$ or $I_k(0) > 0$. If $R_0 > 1$, then

$$\lim_{t\to\infty}(E_k(t), A_k(t), I_k(t), Q_k(t), H_k(t), R_k(t), P_k(t)) = (E_k^*(t), A_k^*(t), I_k^*(t), Q_k^*(t), H_k^*(t), R_k^*(t), P_k^*(t))$$

where $(E_k^*(t), A_k^*(t), I_k^*(t), Q_k^*(t), H_k^*(t), R_k^*(t), P_k^*(t))$ is the endemic equilibrium of system (1) for $k = 1, 2, 3\mathrm{L}\ n$.

Proof. In this part, the value of k is {1, 2, 3, ⋯, n}. By Theorem 3, there is a constant ∂ $(0 < \partial < 1/7)$ and a large value $T > 0$, satisfying $I_k(t) > \partial$ and $A_k(t) > \partial$ for all $t > T$. Hence, $\varepsilon(t) > \partial(\varepsilon_1 + \varepsilon_2)$ for $t > T$. For the first equation of system (7), it gets

$$dE_k(t)/dt \le k(1-\delta)(\varepsilon_1+\varepsilon_2)(1-E_k(t)) - (\omega_A + \omega_Q + \omega_I + \chi),\ \ t > T.$$

According to the comparison theorem of differential equation theory [29], for given arbitrary value $0 < \partial_1 < \dfrac{\omega_A + \omega_Q + \omega_I + \chi}{2(k(1-\delta)(\varepsilon_1+\varepsilon_2) + (\omega_A + \omega_Q + \omega_I + \chi))}$, there is a $t_1 > T$ so that $E_k(t) \le X_k^{(1)} - \partial_1$ for $t > t_1$, where $X_k^{(1)} = \dfrac{k(1-\delta)(\varepsilon_1+\varepsilon_2)}{k(1-\delta)(\varepsilon_1+\varepsilon_2) + (\omega_A + \omega_Q + \omega_I + \chi)} + 2\partial_1 < 1$.

For the second equation of system (7), it gets

$$dA_k(t)/dt \le \omega_A(1 - A_k(t)) - (\eta_A + \psi_A + \chi)A_k(t),\ \ t > t_1.$$

Hence, for given arbitrary value $0 < \partial_2 < \dfrac{\eta_A + \psi_A + \chi}{2(\omega_A + \eta_A + \psi_A + \chi)}$, there is a $t_2 > t_1$ so that $A_k(t) \le Y_k^{(1)} - \partial_2$ for $t > t_2$, where $Y_k^{(1)} = \dfrac{\omega_A}{\omega_A + \eta_A + \psi_A + \chi} + 2\partial_2 < 1$.

For the third equation of system (7), it gets

$$dI_k(t)/dt \le \omega_I(1 - I_k(t)) - (\eta_I + \psi_I + \chi)I_k(t),\ \ t > t_2.$$

Hence, for given arbitrary value $0 < \partial_3 < \dfrac{\eta_I + \psi_I + \chi}{2(\omega_I + \eta_I + \psi_I + \chi)}$, there is a $t_3 > t_2$ so that $I_k(t) \le Z_k^{(1)} - \partial_3$ for $t > t_3$, where $Z_k^{(1)} = \dfrac{\omega_I}{\omega_I + \eta_I + \psi_I + \chi} + 2\partial_3 < 1$.

For the fourth equation of system (7), it gets

$$dQ_k(t)/dt \le (\omega_Q + \eta_A)(1 - Q_k(t)) - (\eta_Q + \psi_Q + \chi)Q_k(t),\ \ t > t_3.$$

Hence, for given arbitrary value $0 < \partial_4 < \dfrac{\eta_Q + \psi_Q + \chi}{2(\omega_Q + \eta_A + \eta_Q + \psi_Q + \chi)}$, there is a $t_4 > t_3$ so that $Q_k(t) \le O_k^{(1)} - \partial_4$ for $t > t_4$, where $O_k^{(1)} = \dfrac{\omega_Q + \eta_A}{\omega_Q + \eta_A + \eta_Q + \psi_Q + \chi} + 2\partial_4 < 1$.

For the fifth equation of system (7), it gets

$$dH_k(t)/dt \le (\eta_I + \eta_Q)(1 - H_k(t)) - (\psi_H + \chi)H_k(t),\ \ t > t_4.$$

Hence, for given arbitrary value $0 < \partial_5 < \dfrac{\psi_H + \chi}{2(\eta_I + \eta_Q + \psi_H + \chi)}$, there is a $t_5 > t_4$ so that

$H_k(t) \le W_k^{(1)} - \partial_5$ for $t > t_5$, where $W_k^{(1)} = \dfrac{\eta_I + \eta_Q}{\eta_I + \eta_Q + \psi_H + \chi} + 2\partial_5 < 1$.

For the sixth equation of system (7), it gets

$$dR_k(t)/dt \le (\psi_A + \psi_I + \psi_Q + \psi_H)(1 - R_k(t)) - (\alpha + \chi)R_k(t), \quad t > t_5.$$

Hence, for given arbitrary value $0 < \partial_6 < \dfrac{\alpha + \chi}{2(\psi_A + \psi_I + \psi_Q + \psi_H + \alpha + \chi)}$, there is a $t_6 > t_5$ so that

$R_k(t) \le V_k^{(1)} - \partial_6$ for $t > t_6$, where $V_k^{(1)} = \dfrac{\psi_A + \psi_I + \psi_Q + \psi_H}{\psi_A + \psi_I + \psi_Q + \psi_H + \alpha + \chi} + 2\partial_6 < 1$.

For the seventh equation of system (7), it gets

$$dP_k(t)/dt \le \eta_S(1 - P_k(t)) - (\zeta_P + \chi)P_k(t), \quad t > t_6.$$

Hence, for given arbitrary value $0 < \partial_7 < \dfrac{\zeta_P + \chi}{2(\eta_S + \zeta_P + \chi)}$, there is a $t_7 > t_6$ so that

$P_k(t) \le U_k^{(1)} - \partial_7$ for $t > t_7$, where $U_k^{(1)} = \dfrac{\eta_S}{\eta_S + \zeta_P + \chi} + 2\partial_7 < 1$.

Next, we replace $E_k(t) \le X_k^{(1)}$, $A_k(t) \le Y_k^{(1)}$, $I_k(t) \le Z_k^{(1)}$, $H_k(t) \le W_k^{(1)}$, $Q_k(t) \le O_k^{(1)}$, $R_k(t) \le V_k^{(1)}$, $P_k(t) \le U_k^{(1)}$ and $\varepsilon(t) > \partial(\varepsilon_1 + \varepsilon_2)$ into the first equation of the system (7), there is

$$\begin{aligned} dE_k(t)/dt \ge\ & k(1-\delta)\partial(\varepsilon_1 + \varepsilon_2)(1 - Y_k^{(1)} - Z_k^{(1)} - O_k^{(1)} - W_k^{(1)} - V_k^{(1)} - U_k^{(1)}) \\ & -(k(1-\delta)\partial(\varepsilon_1 + \varepsilon_2) + \omega_A + \omega_Q + \omega_I + \chi)E_k(t), \ t > T. \end{aligned}$$

Hence, for given arbitrary value $0 < \partial_8 < \min\left\{\dfrac{1}{8}, \partial_7, \dfrac{k(1-\delta)\partial(\varepsilon_1 + \varepsilon_2)(1 - Y_k^{(1)} - Z_k^{(1)} - O_k^{(1)} - W_k^{(1)} - V_k^{(1)} - U_k^{(1)})}{2(k(1-\delta)\partial(\varepsilon_1 + \varepsilon_2) + \omega_A + \omega_Q + \omega_I + \chi)}\right\}$, there is a $t_8 > t_7$ so that $E_k(t) \ge x_k^{(1)} + \partial_8$ for $t > t_8$, where

$$x_k^{(1)} = \frac{k(1-\delta)\partial(\varepsilon_1 + \varepsilon_2)(1 - Y_k^{(1)} - Z_k^{(1)} - O_k^{(1)} - W_k^{(1)} - V_k^{(1)} - U_k^{(1)})}{k(1-\delta)\partial(\varepsilon_1 + \varepsilon_2) + \omega_A + \omega_Q + \omega_I + \chi} - 2\partial_8 > 0.$$

For the second equation of system (7), it gets

$$dA_k(t)/dt \ge \omega_A x_k^{(1)} - (\eta_A + \psi_A + \chi)A_k(t), \quad t > t_8.$$

Hence, for given arbitrary value $0 < \partial_9 < \min\left\{\dfrac{1}{9}, \partial_8, \dfrac{\omega_A x_k^{(1)}}{2(\eta_A + \psi_A + \chi)}\right\}$, there is a $t_9 > t_8$ so that

$A_k(t) \ge y_k^{(1)} + \partial_9$ for $t > t_9$, where $y_k^{(1)} = \dfrac{\omega_A x_k^{(1)}}{\eta_A + \psi_A + \chi} - 2\partial_9 > 0$.

For the third equation of system (7), it gets

$$dI_k(t)/dt \ge \omega_I x_k^{(1)} - (\eta_I + \psi_I + \chi)I_k(t), \quad t > t_9.$$

Hence, for given arbitrary value $0 < \partial_{10} < \min\left\{\dfrac{1}{10}, \partial_9, \dfrac{\omega_I x_k^{(1)}}{2(\eta_I + \psi_I + \chi)}\right\}$, there is a $t_{10} > t_9$ so that $I_k(t) \ge z_k^{(1)} + \partial_{10}$ for $t > t_{10}$, where $z_k^{(1)} = \dfrac{\omega_I x_k^{(1)}}{\eta_I + \psi_I + \chi} - 2\partial_{10} > 0$.

For the fourth equation of system (7), it gets

$$dQ_k(t)/dt \ge \omega_Q x_k^{(1)} + \eta_A y_k^{(1)} - (\eta_Q + \psi_Q + \chi)Q_k(t),\ \ t > t_{10}.$$

Hence, for given arbitrary value $0 < \partial_{11} < \min\left\{\frac{1}{11},\ \partial_{10},\ \frac{\omega_Q x_k^{(1)} + \eta_A y_k^{(1)}}{2(\eta_Q + \psi_Q + \chi)}\right\}$, there is a $t_{11} > t_{10}$ so that $Q_k(t) \ge o_k^{(1)} + \partial_{11}$ for $t > t_{11}$, where $o_k^{(1)} = \frac{\omega_Q x_k^{(1)} + \eta_A y_k^{(1)}}{\eta_Q + \psi_Q + \chi} - 2\partial_{11} > 0$.

For the fifth equation of system (7), it gets

$$dH_k(t)/dt \ge \eta_I y_k^{(1)} + \eta_Q o_k^{(1)} - (\psi_H + \chi)H_k(t),\ \ t > t_{11}.$$

Hence, for given arbitrary value $0 < \partial_{12} < \min\left\{\frac{1}{12},\ \partial_{11},\ \frac{\eta_I y_k^{(1)} + \eta_Q o_k^{(1)}}{2(\psi_H + \chi)}\right\}$, there is a $t_{12} > t_{11}$ so that $H_k(t) \ge w_k^{(1)} + \partial_{12}$ for $t > t_{12}$, where $w_k^{(1)} = \frac{\eta_I y_k^{(1)} + \eta_Q o_k^{(1)}}{\psi_H + \chi} - 2\partial_{12} > 0$.

For the sixth equation of system (7), it gets

$$dR_k(t)/dt \ge \psi_A y_k^{(1)} + \psi_I z_k^{(1)} + \psi_Q o_k^{(1)} + \psi_H w_k^{(1)} - (\alpha + \chi)R_k(t),\ \ t > t_{12}.$$

Hence, for given arbitrary value $0 < \partial_{13} < \min\left\{\frac{1}{13},\ \partial_{12},\ \frac{\psi_A y_k^{(1)} + \psi_I z_k^{(1)} + \psi_Q o_k^{(1)} + \psi_H w_k^{(1)}}{2(\alpha + \chi)}\right\}$, there is a $t_{13} > t_{12}$ so that $R_k(t) \ge v_k^{(1)} + \partial_{13}$ for $t > t_{13}$, where $v_k^{(1)} = \frac{\psi_A y_k^{(1)} + \psi_I z_k^{(1)} + \psi_Q o_k^{(1)} + \psi_H w_k^{(1)}}{\alpha + \chi} - 2\partial_{13} > 0$.

For the seventh equation of system (7), it gets

$$dP_k(t)/dt \ge \eta_S(1 - X_k^{(1)} - Y_k^{(1)} - Z_k^{(1)} - O_k^{(1)} - W_k^{(1)} - V_k^{(1)}) - (\eta_S + \zeta_P + \chi)P_k(t),\ \ t > t_{13}.$$

Hence, for given arbitrary value $0 < \partial_{14} < \min\left\{\frac{1}{14},\ \partial_{13},\ \frac{\eta_S(1 - X_k^{(1)} - Y_k^{(1)} - Z_k^{(1)} - O_k^{(1)} - W_k^{(1)} - V_k^{(1)})}{2(\eta_S + \zeta_P + \chi)}\right\}$, there is a $t_{14} > t_{13}$ so that $P_k(t) \ge u_k^{(1)} + \partial_{14}$ for $t > t_{14}$, where $u_k^{(1)} = \frac{\eta_S(1 - X_k^{(1)} - Y_k^{(1)} - Z_k^{(1)} - O_k^{(1)} - W_k^{(1)} - V_k^{(1)})}{\eta_S + \zeta_P + \chi} - 2\partial_{14} > 0$.

Because ∂ is a small value, so $0 < x_k^{(1)} < E_k(t) < X_k^{(1)} < 1$, $0 < y_k^{(1)} < A_k(t) < Y_k^{(1)} < 1$, $0 < z_k^{(1)} < I_k(t) < Z_k^{(1)} < 1$, $0 < o_k^{(1)} < Q_k(t) < O_k^{(1)} < 1$, $0 < w_k^{(1)} < H_k(t) < W_k^{(1)} < 1$, $0 < v_k^{(1)} < R_k(t) < V_k^{(1)} < 1$ and $0 < u_k^{(1)} < P_k(t) < U_k^{(1)} < 1$. Let

$$g^{(j)} = \sum\nolimits_k P_k(\varepsilon_1 y_k^{(j)} + \varepsilon_2 z_k^{(j)}),\ \ G^{(j)} = \sum\nolimits_k P_k(\varepsilon_1 Y_k^{(j)} + \varepsilon_2 Z_k^{(j)}),\ \ j = 1,\ 2,\ \mathrm{L}\ ,\ n.$$

We can get $0 < g^{(1)} \le \varepsilon(t) \le G^{(1)} < \varepsilon_1 + \varepsilon_2,\ \ t > t_{14}$.

Substitute $E_k(t) \ge x_k^{(1)} + \partial_8$, $A_k(t) \ge y_k^{(1)} + \partial_9$, $I_k(t) \ge z_k^{(1)} + \partial_{10}$, $Q(t) \ge o_k^{(1)} + \partial_{11}$, $H_k(t) \ge w_k^{(1)} + \partial_{12}$, $R_k(t) \ge v_k^{(1)} + \partial_{13}$ and $P_k(t) \ge u_k^{(1)} + \partial_{14}$ into the first equation of system (7), it gets

$$\begin{aligned} dE_k(t)/dt \le\ & k(1-\delta)\partial G^{(1)}(1 - y_k^{(1)} - z_k^{(1)} - o_k^{(1)} - w_k^{(1)} - v_k^{(1)} - u_k^{(1)}) \\ & -(k(1-\delta)\partial G^{(1)} + \omega_A + \omega_Q + \omega_I + \chi)E_k(t),\ t > t_{14}. \end{aligned}$$

For given arbitrary value $0<\partial_{15}<\min\left\{\frac{1}{15},\ \partial_{14}\right\}$, there is a $t_{15}>t_{14}$ so that

$$E_k(t)\le X_k^{(2)}\ @\min\left\{X_k^{(2)}-\partial_1,\ \frac{k(1-\delta)\partial G^{(1)}(1-y_k^{(1)}-z_k^{(1)}-o_k^{(1)}-w_k^{(1)}-v_k^{(1)}-u_k^{(1)})}{k(1-\delta)\partial G^{(1)}+\omega_A+\omega_Q+\omega_I+\chi}+\partial_{15}\right\}\ \text{for}\ t>t_{15},$$

For the second equation of system (7), it gets

$$dA_k(t)/dt\le\omega_A X_k^{(2)}-(\eta_A+\psi_A+\chi)A_k(t),\ \ t>t_{15}.$$

Hence, for given arbitrary value $0<\partial_{16}<\min\left\{\frac{1}{16},\ \partial_{15}\right\}$, there is a $t_{16}>t_{15}$ so that

$$A_k(t)\le Y_k^{(2)}\ @\min\left\{Y_k^{(1)}-\partial_2,\ \frac{\omega_A X_k^{(2)}}{\eta_A+\psi_A+\chi}+\partial_{16}\right\}\ \text{for}\ t>t_{16}.$$

For the third equation of system (7), it gets

$$dI_k(t)/dt\le\omega_I X_k^{(2)}-(\eta_I+\psi_I+\chi)I_k(t),\ \ t>t_{16}.$$

Hence, for given arbitrary value $0<\partial_{17}<\min\left\{\frac{1}{17},\ \partial_{16}\right\}$, there is a $t_{17}>t_{16}$ so that

$$I_k(t)\le Z_k^{(2)}\ @\min\left\{Z_k^{(1)}-\partial_3,\ \frac{\omega_I X_k^{(2)}}{\eta_I+\psi_I+\chi}+\partial_{17}\right\}\ \text{for}\ t>t_{17}.$$

For the fourth equation of system (7), it gets

$$dQ_k(t)/dt\le\omega_Q X_k^{(2)}+\eta_A Y_k^{(2)}-(\eta_Q+\psi_Q+\chi)Q_k(t),\ \ t>t_{17}.$$

Hence, for given arbitrary value $0<\partial_{18}<\min\left\{\frac{1}{18},\ \partial_{17}\right\}$, there is a $t_{18}>t_{17}$ so that

$$Q_k(t)\le O_k^{(2)}\ @\min\left\{O_k^{(1)}-\partial_4,\ \frac{\omega_Q X_k^{(2)}+\eta_A Y_k^{(2)}}{\eta_Q+\psi_Q+\chi}+\partial_{18}\right\}\ \text{for}\ t>t_{18}.$$

For the fifth equation of system (7), it gets

$$dH_k(t)/dt\le\eta_I Y_k^{(2)}+\eta_Q O_k^{(2)}-(\psi_H+\chi)H_k(t),\ \ t>t_{18}.$$

Hence, for given arbitrary value $0<\partial_{19}<\min\left\{\frac{1}{19},\ \partial_{18}\right\}$, there is a $t_{19}>t_{18}$ so that

$$H_k(t)\le W_k^{(2)}\ @\min\left\{V_k^{(1)}-\partial_5,\ \frac{\eta_I Y_k^{(2)}+\eta_Q O_k^{(2)}}{\psi_H+\chi}+\partial_{19}\right\}\ \text{for}\ t>t_{19}.$$

For the sixth equation of system (7), it gets

$$dR_k(t)/dt\le\eta_I Y_k^{(2)}+\eta_Q O_k^{(2)}-(\psi_H+\chi)H_k(t),\ \ t>t_{19}.$$

Hence, for given arbitrary value $0<\partial_{20}<\min\left\{\frac{1}{20},\ \partial_{19}\right\}$, there is a $t_{20}>t_{19}$ so that

$$R_k(t)\le V_k^{(2)}\ @\min\left\{V_k^{(1)}-\partial_6,\ \frac{\eta_I Y_k^{(2)}+\eta_Q O_k^{(2)}}{\psi_H+\chi}+\partial_{20}\right\}\ \text{for}\ t>t_{20}.$$

For the seventh equation of system (7), it gets

$$dP_k(t)/dt\le\eta_S(1-x_k^{(1)}-y_k^{(1)}-z_k^{(1)}-o_k^{(1)}-w_k^{(1)}-v_k^{(1)})-(\eta_S+\zeta_P+\chi)P_k(t),\ \ t>t_{20}.$$

Hence, for given arbitrary value $0<\partial_{21}<\min\left\{\frac{1}{21},\ \partial_{20}\right\}$, there is a $t_{21}>t_{20}$ so that $P_k(t)\le U_k^{(2)}$ @$\min\left\{U_k^{(1)}-\partial_7,\ \frac{\eta_S(1-x_k^{(1)}-y_k^{(1)}-z_k^{(1)}-o_k^{(1)}-w_k^{(1)}-v_k^{(1)})}{\eta_S+\zeta_P+\chi}+\partial_{21}\right\}$ for $t>t_{21}$.

Next, we replace $E_k(t)\le X_k^{(2)}$, $A_k(t)\le Y_k^{(2)}$, $I_k(t)\le Z_k^{(2)}$, $H_k(t)\le W_k^{(2)}$, $Q_k(t)\le O_k^{(2)}$, $R_k(t)\le V_k^{(2)}$, $P_k(t)\le U_k^{(2)}$ into the first equation of the system (7), there is

$$dE_k(t)/dt\ge k(1-\delta)\partial g^{(1)}(1-Y_k^{(2)}-Z_k^{(2)}-O_k^{(2)}-W_k^{(2)}-V_k^{(2)}-U_k^{(2)})$$
$$-(k(1-\delta)\partial g^{(1)}+\omega_A+\omega_Q+\omega_I+\chi)E_k(t),\ t>t_{21}.$$

Hence, for given arbitrary value $0<\partial_{22}<\min\left\{\frac{1}{22},\ \partial_{21},\ \frac{k(1-\delta)\partial g^{(1)}(1-Y_k^{(2)}-Z_k^{(2)}-O_k^{(2)}-W_k^{(2)}-V_k^{(2)}-U_k^{(2)})}{2(k(1-\delta)\partial g^{(1)}+\omega_A+\omega_Q+\omega_I+\chi)}\right\}$, there is a $t_{22}>t_{21}$ so that $E_k(t)\ge x_k^{(2)}+\partial_{22}$ for $t>t_{22}$, where $x_k^{(2)}=\max\left\{x_k^{(1)}+\partial_8,\ \frac{k(1-\delta)\partial g^{(1)}(1-Y_k^{(2)}-Z_k^{(2)}-O_k^{(2)}-W_k^{(2)}-V_k^{(2)}-U_k^{(2)})}{k(1-\delta)\partial g^{(1)}+\omega_A+\omega_Q+\omega_I+\chi}-2\partial_{22}\right\}$.

For the second equation of system (7), it gets

$$dA_k(t)/dt\ge\omega_A x_k^{(2)}-(\eta_A+\psi_A+\chi)A_k(t),\ t>t_{22}.$$

Hence, for given arbitrary value $0<\partial_{23}<\min\left\{\frac{1}{23},\ \partial_{22},\ \frac{\omega_A x_k^{(2)}}{2(\eta_A+\psi_A+\chi)}\right\}$, there is a $t_{23}>t_{22}$ so that $A_k(t)\ge y_k^{(2)}+\partial_{23}$ for $t>t_{23}$, where $y_k^{(2)}=\max\left\{y_k^{(1)}+\partial_9,\ \frac{\omega_A x_k^{(2)}}{\eta_A+\psi_A+\chi}-2\partial_{23}\right\}$.

For the third equation of system (7), it gets

$$dI_k(t)/dt\ge\omega_I x_k^{(2)}-(\eta_I+\psi_I+\chi)I_k(t),\ t>t_{23}.$$

Hence, for given arbitrary value $0<\partial_{24}<\min\left\{\frac{1}{24},\ \partial_{23},\ \frac{\omega_I x_k^{(2)}}{2(\eta_I+\psi_I+\chi)}\right\}$, there is a $t_{24}>t_{23}$ so that $I_k(t)\ge z_k^{(2)}+\partial_{24}$ for $t>t_{24}$, where $z_k^{(2)}=\max\left\{z_k^{(1)}+\partial_{10},\ \frac{\omega_I x_k^{(2)}}{\eta_I+\psi_I+\chi}-2\partial_{24}\right\}$.

For the fourth equation of system (7), it gets

$$dQ_k(t)/dt\ge\omega_Q x_k^{(2)}+\eta_A y_k^{(2)}-(\eta_Q+\psi_Q+\chi)Q_k(t),\ t>t_{24}.$$

Hence, for given arbitrary value $0<\partial_{25}<\min\left\{\frac{1}{25},\ \partial_{24},\ \frac{\omega_Q x_k^{(2)}+\eta_A y_k^{(2)}}{2(\eta_Q+\psi_Q+\chi)}\right\}$, there is a $t_{25}>t_{24}$ so that $Q_k(t)\ge o_k^{(2)}+\partial_{25}$ for $t>t_{25}$, where $o_k^{(2)}=\max\left\{o_k^{(1)}+\partial_{11},\ \frac{\omega_Q x_k^{(2)}+\eta_A y_k^{(2)}}{\eta_Q+\psi_Q+\chi}-2\partial_{25}\right\}$.

For the fifth equation of system (7), it gets

$$dH_k(t)/dt\ge\eta_I y_k^{(2)}+\eta_Q o_k^{(2)}-(\psi_H+\chi)H_k(t),\ t>t_{25}.$$

Hence, for given arbitrary value $0<\partial_{26}<\min\left\{\frac{1}{26},\ \partial_{25},\ \frac{\eta_I y_k^{(2)}+\eta_Q o_k^{(2)}}{2(\psi_H+\chi)}\right\}$, there is a $t_{26}>t_{25}$ so that $H_k(t)\geq w_k^{(2)}+\partial_{26}$ for $t>t_{26}$, where $w_k^{(2)}=\max\left\{w_k^{(1)}+\partial_{12},\ \frac{\eta_I y_k^{(2)}+\eta_Q o_k^{(2)}}{\psi_H+\chi}-2\partial_{26}\right\}$.

For the sixth equation of system (7), it gets

$$dR_k(t)/dt\geq\psi_A y_k^{(2)}+\psi_I z_k^{(2)}+\psi_Q o_k^{(2)}+\psi_H w_k^{(2)}-(\alpha+\chi)R_k(t),\ \ t>t_{26}.$$

Hence, for given arbitrary value $0<\partial_{27}<\min\left\{\frac{1}{27},\ \partial_{26},\ \frac{\psi_A y_k^{(2)}+\psi_I z_k^{(2)}+\psi_Q o_k^{(2)}+\psi_H w_k^{(2)}}{2(\alpha+\chi)}\right\}$, there is a $t_{27}>t_{26}$ so that $R_k(t)\geq v_k^{(2)}+\partial_{27}$ for $t>t_{27}$, where $v_k^{(2)}=\max\left\{v_k^{(1)}+\partial_{13},\ \frac{\psi_A y_k^{(2)}+\psi_I z_k^{(2)}+\psi_Q o_k^{(2)}+\psi_H w_k^{(2)}}{\alpha+\chi}-2\partial_{27}\right\}$.

For the seventh equation of system (7), it gets

$$dP_k(t)/dt\geq\eta_S(1-X_k^{(2)}-Y_k^{(2)}-Z_k^{(2)}-O_k^{(2)}-W_k^{(2)}-V_k^{(2)})-(\eta_S+\zeta_P+\chi)P_k(t),\ \ t>t_{27}.$$

Hence, for given arbitrary value $0<\partial_{28}<\min\left\{\frac{1}{28},\ \partial_{27},\ \frac{\eta_S(1-X_k^{(2)}-Y_k^{(2)}-Z_k^{(2)}-O_k^{(2)}-W_k^{(2)}-V_k^{(2)})}{2(\eta_S+\zeta_P+\chi)}\right\}$, there is a $t_{28}>t_{27}$ so that $P_k(t)\geq u_k^{(2)}+\partial_{28}$ for $t>t_{28}$, where $u_k^{(2)}=\max\left\{u_k^{(1)}+\partial_{14},\ \frac{\eta_S(1-X_k^{(2)}-Y_k^{(2)}-Z_k^{(2)}-O_k^{(2)}-W_k^{(2)}-V_k^{(2)})}{\eta_S+\zeta_P+\chi}-2\partial_{28}\right\}$.

As a result of the above analysis, we obtain fourteen sequences: $\{x_k^{(1)}\}$, $\{y_k^{(1)}\}$, $\{z_k^{(1)}\}$, $\{o_k^{(1)}\}$, $\{w_k^{(1)}\}$, $\{v_k^{(1)}\}$, $\{u_k^{(1)}\}$, $\{X_k^{(1)}\}$, $\{Y_k^{(1)}\}$, $\{Z_k^{(1)}\}$, $\{O_k^{(1)}\}$, $\{W_k^{(1)}\}$, $\{V_k^{(1)}\}$ and $\{U_k^{(1)}\}$, where $1=1,\ 2,\ 3,\ \mathrm{L}\ ,\ n$. We find that the first half of the sequence increases monotonically, while the other half reduces monotonically. There is a large enough positive number r so that $1\geq r$:

$$
\begin{cases}
X_k^{(1)}=\dfrac{k(1-\delta)\partial G^{(1-1)}(1-y_k^{(1-1)}-z_k^{(1-1)}-o_k^{(1-1)}-w_k^{(1-1)}-v_k^{(1-1)}-u_k^{(1-1)})}{k(1-\delta)\partial G^{(1-1)}+\omega_A+\omega_Q+\omega_I+\chi}+\partial_{141-13},\\
Y_k^{(1)}=\dfrac{\omega_A X_k^{(1)}}{\eta_A+\psi_A+\chi}+\partial_{141-12},\ Z_k^{(1)}=\dfrac{\omega_I X_k^{(1)}}{\eta_I+\psi_I+\chi}+\partial_{141-11},\ O_k^{(1)}=\dfrac{\omega_Q X_k^{(1)}+\eta_A Y_k^{(1)}}{\eta_Q+\psi_Q+\chi}+\partial_{141-10},\\
W_k^{(1)}=\dfrac{\eta_I Y_k^{(1)}+\eta_Q O_k^{(1)}}{\psi_H+\chi}+\partial_{141-9},\ V_k^{(1)}=\dfrac{\eta_I Y_k^{(1)}+\eta_Q O_k^{(1)}}{\psi_H+\chi}+\partial_{141-8},\\
U_k^{(1)}=\dfrac{\eta_S(1-x_k^{(1-1)}-y_k^{(1-1)}-z_k^{(1-1)}-o_k^{(1-1)}-w_k^{(1-1)}-v_k^{(1-1)})}{\eta_S+\zeta_P+\chi}+\partial_{141-7},\\
x_k^{(1)}=\dfrac{k(1-\delta)\partial g^{(1-1)}(1-Y_k^{(1)}-Z_k^{(1)}-O_k^{(1)}-W_k^{(1)}-V_k^{(1)}-U_k^{(1)})}{k(1-\delta)\partial g^{(1-1)}+\omega_A+\omega_Q+\omega_I+\chi}-2\partial_{141-6},\\
y_k^{(r)}=\dfrac{\omega_A x_k^{(1)}}{\eta_A+\psi_A+\chi}-2\partial_{141-5},\ z_k^{(1)}=\dfrac{\omega_I x_k^{(1)}}{\eta_I+\psi_I+\chi}-2\partial_{141-4},\ o_k^{(1)}=\dfrac{\omega_Q x_k^{(1)}+\eta_A y_k^{(1)}}{\eta_Q+\psi_Q+\chi}-2\partial_{141-3},\\
w_k^{(1)}=\dfrac{\eta_I y_k^{(1)}+\eta_Q o_k^{(1)}}{\psi_H+\chi}-2\partial_{141-2},\ v_k^{(1)}=\dfrac{\psi_A y_k^{(1)}+\psi_I z_k^{(1)}+\psi_Q o_k^{(1)}+\psi_H w_k^{(1)}}{\alpha+\chi}-2\partial_{141-1},\\
u_k^{(1)}=\dfrac{\eta_S(1-X_k^{(1)}-Y_k^{(1)}-Z_k^{(1)}-O_k^{(1)}-W_k^{(1)}-V_k^{(1)})}{\eta_S+\zeta_P+\chi}-2\partial_{141}.
\end{cases}
\tag{11}
$$

Clearly, $x_k^{(1)}\le E_k(t)\le X_k^{(1)}$, $y_k^{(1)}\le A_k(t)\le Y_k^{(r)}$, $z_k^{(1)}\le I_k(t)\le Z_k^{(1)}$, $o_k^{(1)}\le Q_k(t)\le O_k^{(1)}$, $w_k^{(1)}\le E_k(t)\le W_k^{(1)}$, $v_k^{(1)}\le H_k(t)\le V_k^{(1)}$, $u_k^{(1)}\le R_k(t)\le U_k^{(1)}$ for $t>t_{141}$.

Due to the sequence limit of equation (11), let $\lim_{t\to\infty}\Omega_k^{(r)}=\Omega_k$, where

$$\Omega_k^{(1)}\in\{X_k^{(1)},\ x_k^{(1)},\ Y_k^{(1)},\ y_k^{(1)},\ Z_k^{(1)},\ z_k^{(1)},\ O_k^{(1)},\ o_k^{(1)},\ W_k^{(1)},\ w_k^{(1)},\ V_k^{(1)},\ v_k^{(1)},\ U_k^{(1)},\ u_k^{(1)},\ G_k^{(1)},\ g_k^{(1)}\},$$

$$\Omega_k\in\{X_k,\ x_k,\ Y_k,\ y_k,\ Z_k,\ z_k,\ O_k,\ o_k,\ W_k,\ w_k,\ V_k,\ v_k,\ U_k,\ u_k,\ G_k,\ g_k\}.$$

For $0<\gamma_1<1/1$, as $1\to\infty$, $\gamma_1\to 0$. When $1\to\infty$, it gets

$$
\begin{cases}
X_k=\dfrac{k(1-\delta)\partial G(1-y_k-z_k-o_k-w_k-v_k-u_k)}{k(1-\delta)\partial G+\omega_A+\omega_Q+\omega_I+\chi},\ Y_k=\dfrac{\omega_A X_k}{\eta_A+\psi_A+\chi},\\
Z_k=\dfrac{\omega_I X_k}{\eta_I+\psi_I+\chi},\ O_k=\dfrac{\omega_Q X_k+\eta_A Y_k}{\eta_Q+\psi_Q+\chi},\ W_k=\dfrac{\eta_I Y_k+\eta_Q O_k}{\psi_H+\chi},\\
V_k=\dfrac{\eta_I Y_k+\eta_Q O_k}{\psi_H+\chi},\ U_k=\dfrac{\eta_S(1-x_k-y_k-z_k-o_k-w_k-v_k)}{\eta_S+\zeta_P+\chi},\\
x_k=\dfrac{k(1-\delta)\partial g(1-Y_k-Z_k-O_k-W_k-V_k-U_k)}{k(1-\delta)\partial g+\omega_A+\omega_Q+\omega_I+\chi},\ y_k=\dfrac{\omega_A x_k}{\eta_A+\psi_A+\chi},\\
z_k=\dfrac{\omega_I x_k}{\eta_I+\psi_I+\chi},\ o_k=\dfrac{\omega_Q x_k+\eta_A y_k}{\eta_Q+\psi_Q+\chi},\ w_k=\dfrac{\eta_I y_k+\eta_Q o_k}{\psi_H+\chi},\\
v_k=\dfrac{\psi_A y_k+\psi_I z_k+\psi_Q o_k+\psi_H w_k}{\alpha+\chi},\ u_k=\dfrac{\eta_S(1-X_k-Y_k-Z_k-O_k-W_k-V_k)}{\eta_S+\zeta_P+\chi}.
\end{cases}
\tag{12}
$$

Where $g=\sum_k P_k(\varepsilon_1 y_k+\varepsilon_2 z_k)$, $G=\sum_k P_k(\varepsilon_1 Y_k+\varepsilon_2 Z_k)$.

The system (12) equations y_k , z_k , Y_k and Y_k are substituted into g and G respectively, we can get

$$g=\sum_k P_k\left(\frac{\varepsilon_1\omega_A}{\eta_A+\psi_A+\chi}+\frac{\varepsilon_2\omega_I}{\eta_I+\psi_I+\chi}\right)\frac{k(1-\delta)\partial g(1-Y_k-Z_k-O_k-W_k-V_k-U_k)}{k(1-\delta)\partial g+\omega_A+\omega_Q+\omega_I+\chi},$$

$$G=\sum_k P_k\left(\frac{\varepsilon_1\omega_A}{\eta_A+\psi_A+\chi}+\frac{\varepsilon_2\omega_I}{\eta_I+\psi_I+\chi}\right)\frac{k(1-\delta)\partial G(1-y_k-z_k-o_k-w_k-v_k-u_k)}{k(1-\delta)\partial G+\omega_A+\omega_Q+\omega_I+\chi},$$

So,

$$\frac{(1-Y_k-Z_k-O_k-W_k-V_k-U_k)}{k(1-\delta)\partial g+\omega_A+\omega_Q+\omega_I+\chi}=\frac{(1-y_k-z_k-o_k-w_k-v_k-u_k)}{k(1-\delta)\partial G+\omega_A+\omega_Q+\omega_I+\chi}. \tag{13}$$

We can obtain $G=g$, $Y_k=y_k$, $Z_k=z_k$, $O_k=o_k$, $W_k=w_k$, $V_k=v_k$, $U_k=u_k$ is the solution of system (13). Because $I_k(t)>\partial$ and $A_k(t)>\partial$ for all $t>T$, so the solution is positive. Therefore, $\lim_{t\to\infty}E_k(t)=X_k=x_k$, $\lim_{t\to\infty}A_k(t)=Y_k=y_k$, $\lim_{t\to\infty}I_k(t)=Z_k=z_k$, $\lim_{t\to\infty}Q_k(t)=O_k=o_k$, $\lim_{t\to\infty}H_k(t)=W_k=w_k$, $\lim_{t\to\infty}R_k(t)=V_k=v_k$, $\lim_{t\to\infty}P_k(t)=U_k=u_k$. The proof is completed.

6. Sensitivity analysis

In this section, we concentrate on local sensitivity analysis and obtain the sensitivity index of parameters δ, ω_Q, η_S, η_A and η_I with respect to the basic regenerative number R_0 as follows:

$$\Upsilon_{\delta}^{R_0}=\frac{\langle k^2\rangle}{\langle k\rangle}\cdot\frac{-(\zeta_P+\chi)\left[(\eta_I+\psi_I+\chi)\omega_A\varepsilon_1+(\eta_A+\psi_A+\chi)\omega_I\varepsilon_2\right]}{(\eta_S+\zeta_P+\chi)(\omega_A+\omega_Q+\omega_I+\chi)(\eta_I+\psi_I+\chi)(\eta_A+\psi_A+\chi)}<0,$$

$$\Upsilon_{\omega_Q}^{R_0}=\frac{\langle k^2\rangle}{\langle k\rangle}\cdot\frac{-(1-\delta)(\zeta_P+\chi)\left[(\eta_I+\psi_I+\chi)\omega_A\varepsilon_1+(\eta_A+\psi_A+\chi)\omega_I\varepsilon_2\right]}{(\eta_S+\zeta_P+\chi)(\omega_A+\omega_Q+\omega_I+\chi)^2(\eta_I+\psi_I+\chi)(\eta_A+\psi_A+\chi)}<0,$$

$$\Upsilon_{\eta_S}^{R_0}=\frac{\langle k^2\rangle}{\langle k\rangle}\cdot\frac{-(1-\delta)(\zeta_P+\chi)\left[(\eta_I+\psi_I+\chi)\omega_A\varepsilon_1+(\eta_A+\psi_A+\chi)\omega_I\varepsilon_2\right]}{\eta_S{}^2(\omega_A+\omega_Q+\omega_I+\chi)(\eta_I+\psi_I+\chi)(\eta_A+\psi_A+\chi)}<0,$$

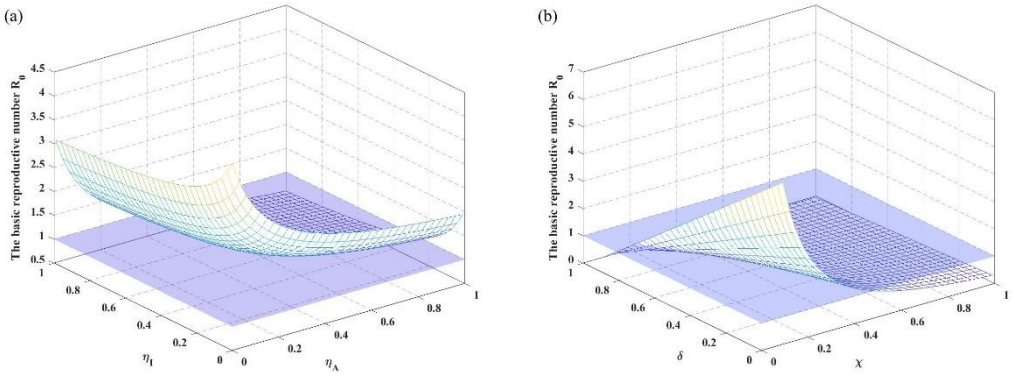
$$\Upsilon_{\eta_A}^{R_0}=\frac{\langle k^2\rangle}{\langle k\rangle}\cdot\frac{-(1-\delta)(\zeta_P+\chi)(\eta_I+\psi_I+\chi)\omega_A\varepsilon_1}{(\eta_S+\zeta_P+\chi)(\omega_A+\omega_Q+\omega_I+\chi)(\eta_I+\psi_I+\chi)(\eta_A+\psi_A+\chi)^2}<0,$$

$$\Upsilon_{\eta_I}^{R_0}=\frac{\langle k^2\rangle}{\langle k\rangle}\cdot\frac{-(1-\delta)(\zeta_P+\chi)(\eta_A+\psi_A+\chi)\omega_I\varepsilon_2}{(\eta_A+\zeta_P+\chi)(\omega_A+\omega_Q+\omega_I+\chi)(\eta_I+\psi_I+\chi)^2(\eta_A+\psi_A+\chi)}<0.$$

We can find that the basic regenerative number R_0 is negatively correlated with parameters δ, ω_Q, η_S, η_A and η_I.

7. Numerical simulations

We conduct theoretical simulation on the results of theoretical analysis by means of Runge-Kutta method in this section. The *SEAIRS* model is formulated on scale-free networks with degree distribution of $P(k)=\omega k^{-3}$, where the parameter ω satisfies $\sum_{k=0}^{n}\omega k^{-3}=1, n=1000$.



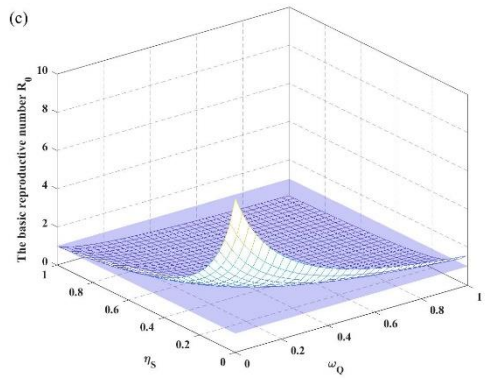


Fig. 2. The relation between the basic reproductive number R_0 and some parameters

Note that in the simulation, only one parameter of the basic reproductive number is changed each time, and the other parameters are fixed. In Fig. 2, we can find that R_0 is negatively correlated with parameters δ, ω_Q, η_S, η_A and η_I. The larger parameters δ, ω_Q, η_S, η_A and η_I, the larger R_0. This means that improving the coverage rate of home quarantine for susceptible, exposed, asymptomatic infection people, active hospitalization for symptomatic infection people and strengthen government pre-warning intensity can effectively reduce the spread intensity of epidemic.

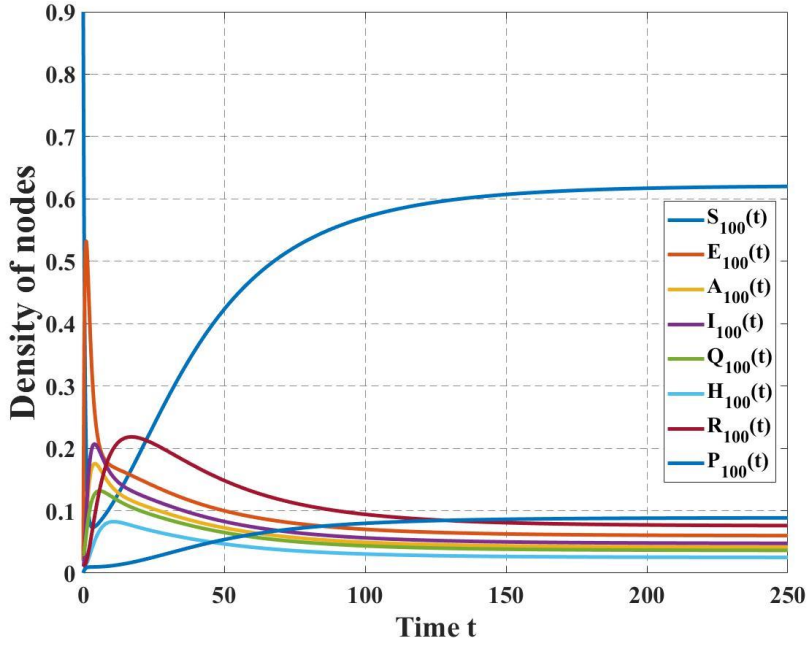


Fig. 3. Eight groups population density varies over time when $R_0 > 1$

In Fig. 3, the parameters $\chi = 0.12$, $\delta = 0.05$, $\varepsilon_1 = 0.95$, $\varepsilon_2 = 0.93$, $\omega_A = 0.18$, $\omega_I = 0.23$, $\omega_Q = 0.09$, $\eta_A = 0.08$, $\eta_I = 0.09$, $\eta_Q = 0.07$, $\eta_S = 0.08$, $\psi_A = 0.06$, $\psi_I = 0.08$, $\psi_Q = 0.05$, $\psi_H = 0.13$, $\alpha = 0.03$ and $\zeta_P = 0.43$ are chosen, thus $R_0 = 8.4923 > 1$. The density of nodes A_{100} and I_{100} tends to be a constant greater than zero when $R_0 > 1$, i.e., the asymptomatic and symptomatic infection people will always exist in scale-free networks and the epidemic spread is persistent.

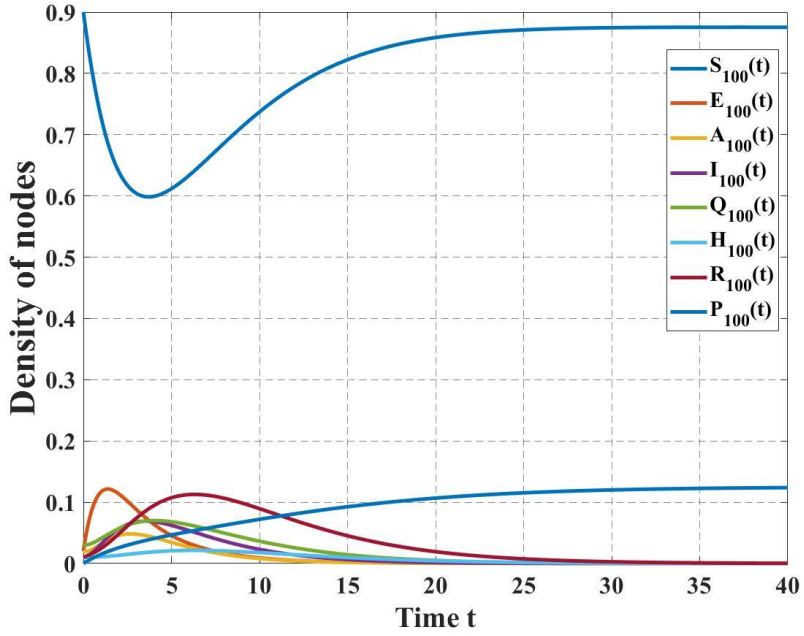


Fig. 4. Eight groups population density varies over time when $R_0 < 1$

In Fig. 4, the parameters $\chi = 0.12$, $\delta = 0.2$, $\varepsilon_1 = 0.13$, $\varepsilon_2 = 0.22$, $\omega_A = 0.35$, $\omega_I = 0.35$, $\omega_Q = 0.2$, $\eta_A = 0.1$, $\eta_I = 0.06$, $\eta_Q = 0.06$, $\eta_S = 0.02$, $\psi_A = 0.48$, $\psi_I = 0.2$, $\psi_Q = 0.06$, $\psi_H = 0.12$, $\alpha = 0.13$ and $\zeta_P = 0.02$ are chosen, thus $R_0 =$ 0.8357<1. The density of nodes A_{100} and I_{100} tends to be zero when $R_0 < 1$, i.e., the asymptomatic and symptomatic infection people will disappear in scale-free networks, the behavior of epidemic spreading will eventually stop and the epidemic will become extinct ultimately.

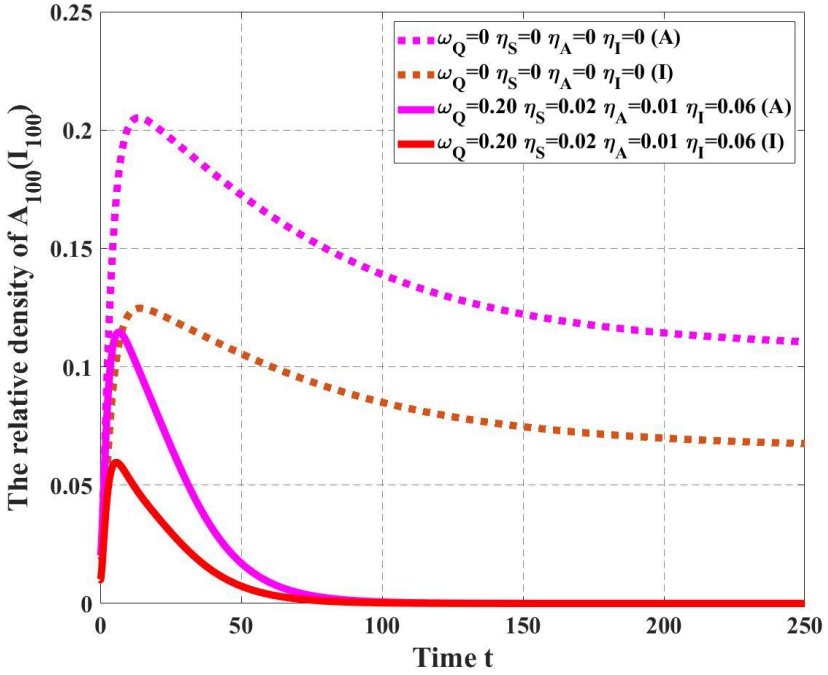


Fig. 5. The density of nodes A_{100} and I_{100} with or without quarantine measures

Fig. 5 depicts the density of nodes A_{100} and I_{100} with or without quarantine measures. In Fig.

5, we select the same parameters as $\chi = 0.12$, $\delta = 0.2$, $\varepsilon_1 = 0.89$, $\varepsilon_2 = 0.78$, $\omega_A = 0.075$, $\omega_I = 0.045$, $\eta_Q = 0.06$, $\psi_A = 0.048$, $\psi_I = 0.045$, $\psi_Q = 0.06$, $\psi_H = 0.12$, $\alpha = 0.13$ and $\zeta_P = 0.02$. The dotted line describes the density of nodes A_{100} and I_{100} without quarantine measures and the quarantine parameters are $\omega_Q = 0$, $\eta_A = 0$, $\eta_I = 0$ and $\eta_S = 0$; the solid line indicates the density of nodes A_{100} and I_{100} with quarantine measures and the quarantine parameters are $\omega_Q = 0.2$, $\eta_A = 0.01$, $\eta_I = 0.06$ and $\eta_S = 0.02$. We can find that after the implementation of quarantine measures, the density of nodes A_{100} and I_{100} at each time will be reduced. This shows that the implementation of quarantine measures can effectively reduce the number of asymptomatic and symptomatic infection people, reduce the spread intensity of epidemic and accelerate the disappearance of the epidemic.

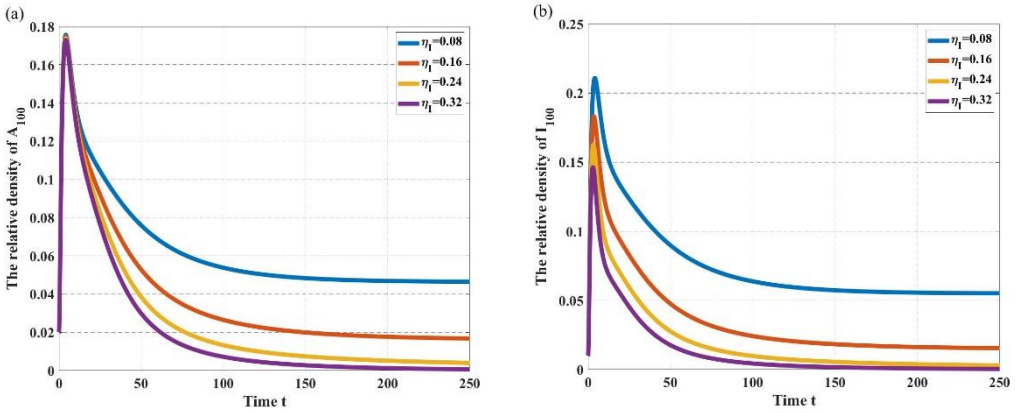


Fig. 6. The density of A_{100}, I_{100} with different η_I

In Fig. 6, the parameters $\chi = 0.12$, $\delta = 0.05$, $\varepsilon_1 = 0.95$, $\varepsilon_2 = 0.93$, $\omega_A = 0.18$, $\omega_I = 0.23$, $\omega_Q = 0.09$, $\eta_A = 0.08$, $\eta_Q = 0.07$, $\eta_S = 0.08$, $\psi_A = 0.06$, $\psi_I = 0.08$, $\psi_Q = 0.05$, $\psi_H = 0.13$, $\alpha = 0.03$ and $\zeta_P = 0.43$ are chosen, η_I is chosen as 0.08, 0.16, 0.24, 0.32. The corresponding density of nodes A_{100} and I_{100} decrease significantly as the parameter η_I increases, i.e., a larger hospitalization rate for symptomatic infection people can weaken the spread of epidemic.

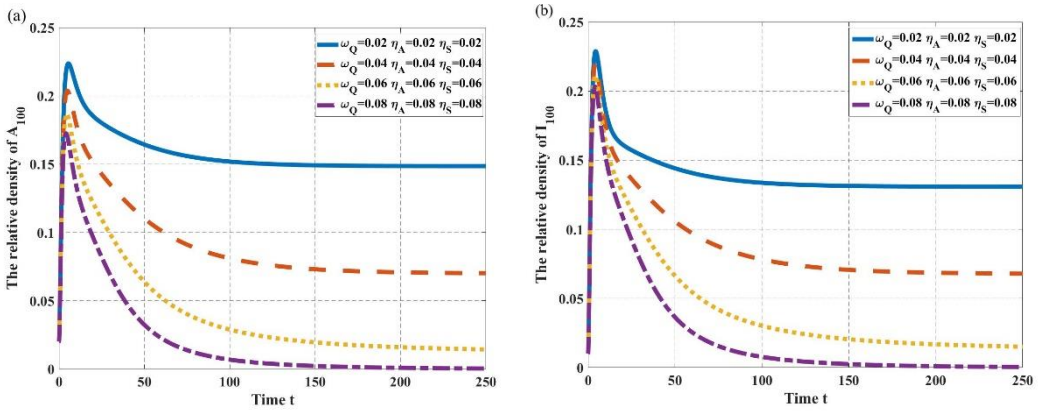


Fig. 7. The density of A_{100}, I_{100} with different coverage rate of home quarantine

In Fig. 7, the parameters $\chi = 0.12$, $\delta = 0.05$, $\varepsilon_1 = 0.95$, $\varepsilon_2 = 0.93$, $\omega_A = 0.18$, $\omega_I = 0.23$, $\eta_I = 0.09$, $\eta_Q = 0.07$, $\psi_A = 0.06$, $\psi_I = 0.08$, $\psi_Q = 0.05$, $\psi_H = 0.13$, $\alpha = 0.03$ and $\zeta_P = 0.43$ are chosen, the coverage rate of home quarantine for susceptible, exposed and

asymptomatic infection people (ω_Q , η_A , η_S) are chosen as 0.02, 0.04, 0.06, 0.08. With the increase of coverage rate of home quarantine, the density of nodes A_{100} and I_{100} decreases significantly, i.e., it means that strengthen the implementation of home quarantine measures can reduce the endemic level.

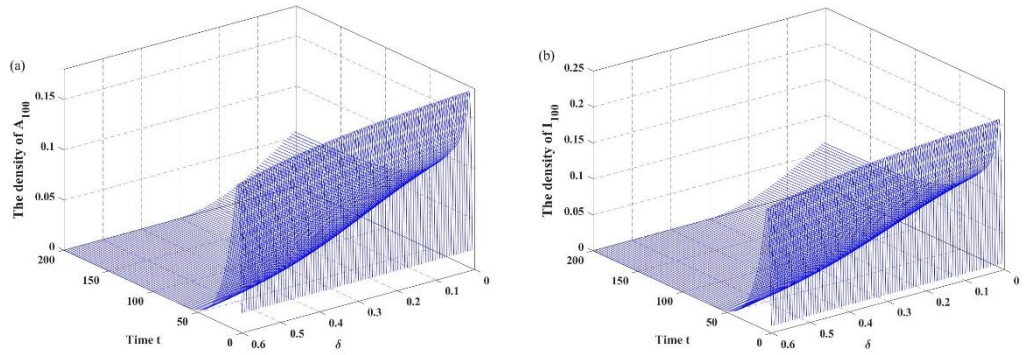


Fig. 8. The density of A_{100} , I_{100} with different government pre-warning intensity

In Fig. 8 (a) and (b), the density of asymptomatic and symptomatic infection people was regarded as a function of time and parameter δ . The parameters $\chi = 0.12$, $\varepsilon_1 = 0.95$, $\varepsilon_2 = 0.93$, $\omega_A = 0.18$, $\omega_I = 0.23$, $\omega_Q = 0.09$, $\eta_A = 0.08$, $\eta_I = 0.09$, $\eta_Q = 0.07$, $\eta_S = 0.08$, $\psi_A = 0.06$, $\psi_I = 0.08$, $\psi_Q = 0.05$, $\psi_H = 0.13$, $\alpha = 0.03$ and $\zeta_P = 0.43$ are chosen, δ varies from 0 to 0.6. The results show that the density of nodes A_{100} and I_{100} gradually becomes a constant, the constant eventually becomes zero as the parameter δ increases, i.e., strengthen government pre-warning intensity can weaken the spread of epidemic and effectively reduce the epidemic lever.

8. Conclusions

In this paper, a novel *SEAIRS* epidemic model with protection and hospital quarantine under government pre-warning has been presented and the epidemic spreading dynamics have been analyzed. The basic reproductive number R_0 and two equilibriums have been obtained by means of the next-generation matrix method and differential method. The endemic equilibrium is globally asymptotically stable when $R_0 > 1$ has been proved by means of comparison principle; the disease-free equilibrium is globally asymptotically stable when $R_0 < 1$, the permanence of the epidemic spreading has been proved. We draw conclusions that the epidemic will not disappear and the epidemic spread is persistent if $R_0 > 1$; the behavior of epidemic spreading will eventually stop and the epidemic will become extinct ultimately if $R_0 < 1$. Furthermore, the impact of protection and hospital quarantine measures, government pre-warning mechanism on epidemic spreading has been discussed. Strengthening government pre-warning intensity, improving the coverage rate home quarantine measures and active hospitalization can weaken the spread of epidemic and effectively reduce the epidemic lever. The study has a vital guiding significance for the prevention and control of epidemic.

Declarations

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Competing interests

Page: 21

The authors declare that they have no conflict of interest.

Availability of data and material

Page: 21

All data generated or analysed during this study are included in this published article.

Authors' contributions

Bingchuan Xue performed the analysis, and wrote the manuscript; Tao Li designed the study; Xinming Cheng developed the methodology; Yumiao Li, Yuanyuan Wu and Gaojun Shi helped perform the analysis with constructive discussions. All authors read and approved the final manuscript.

Ethical statement

The work has been undertaken to accepted standards of ethics and of professional standards.

Consent to participate

Page: 21

Not applicable.

Consent for publication

Not applicable.

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Figures

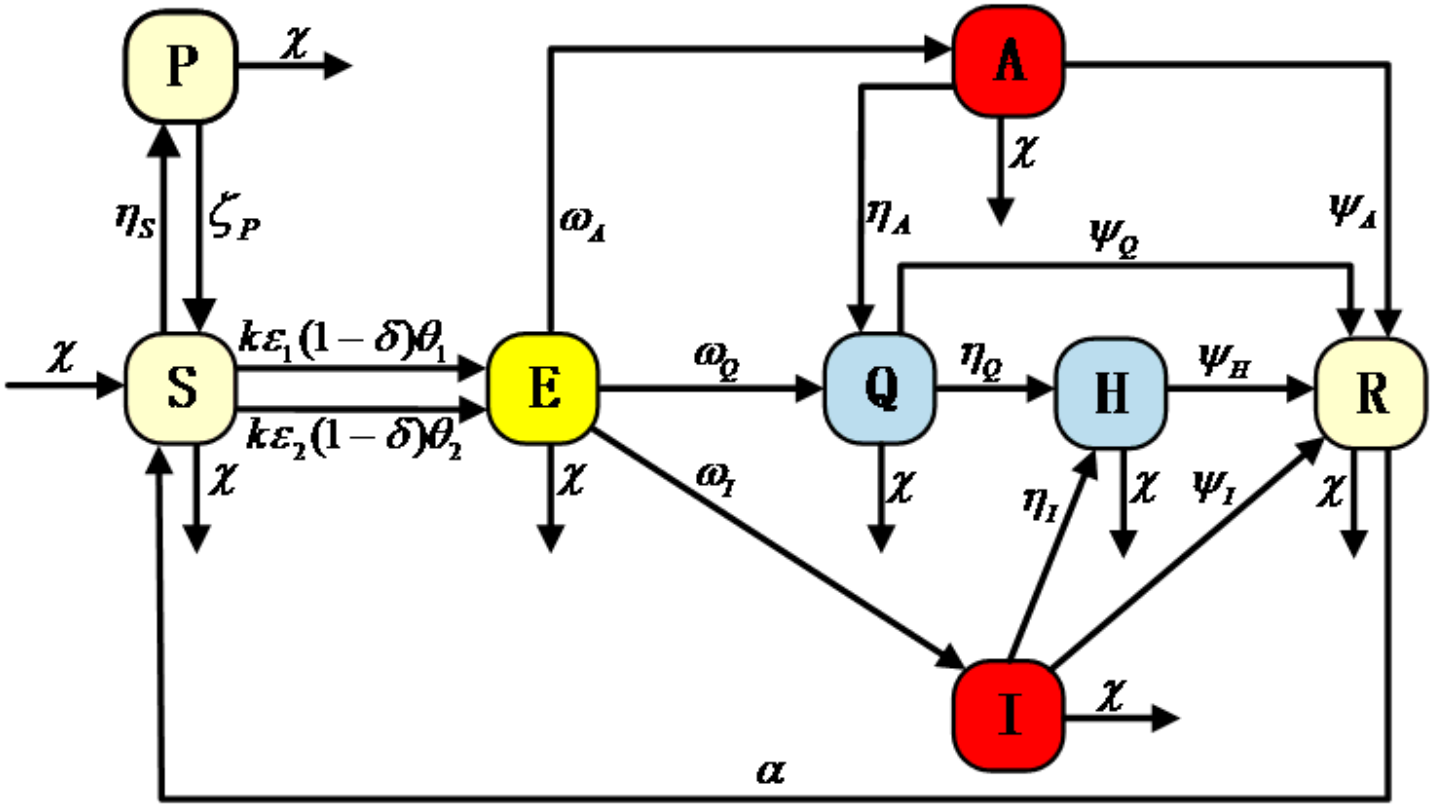

Figure 1

The flow chart of the SEAIRS model.

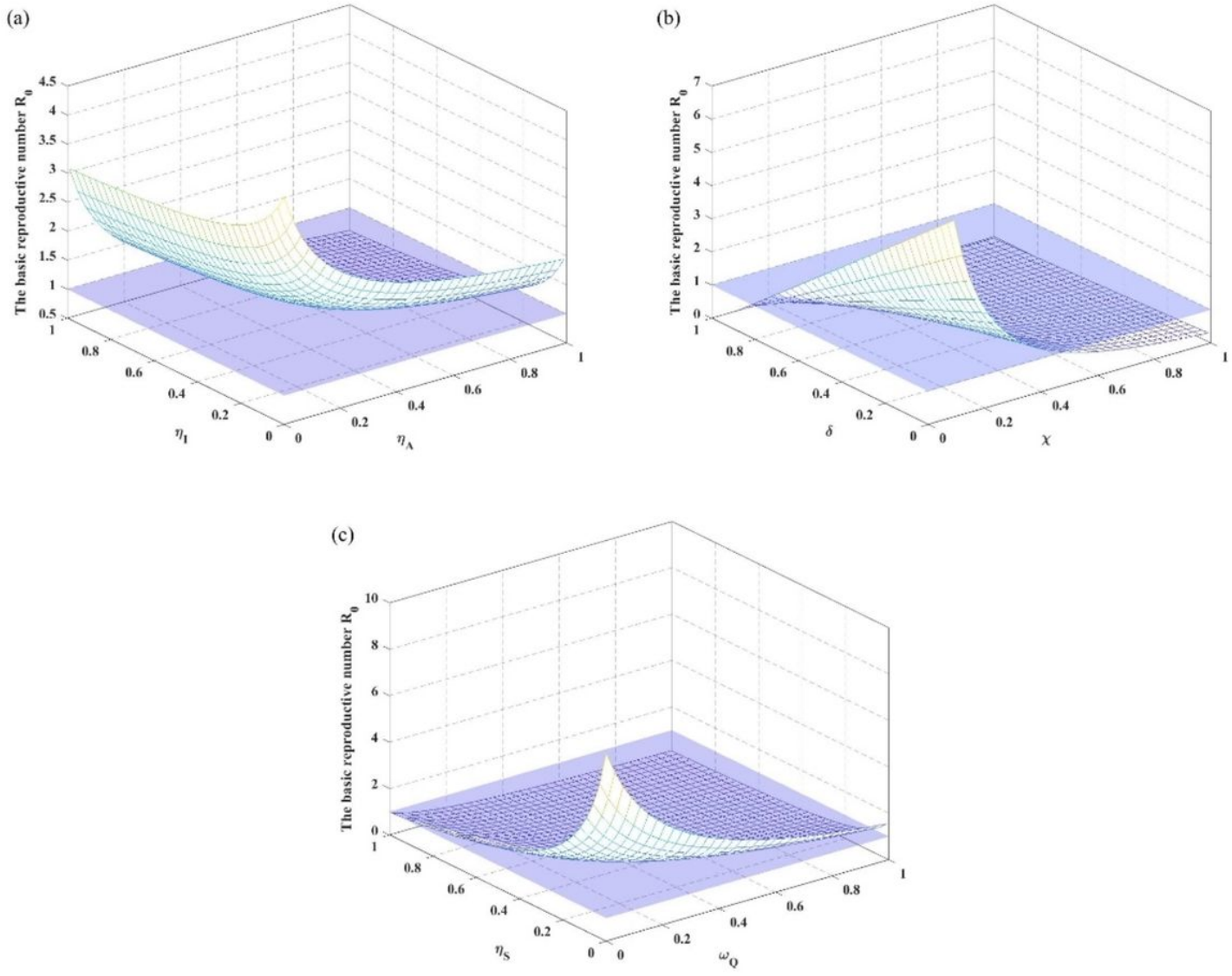

Figure 2

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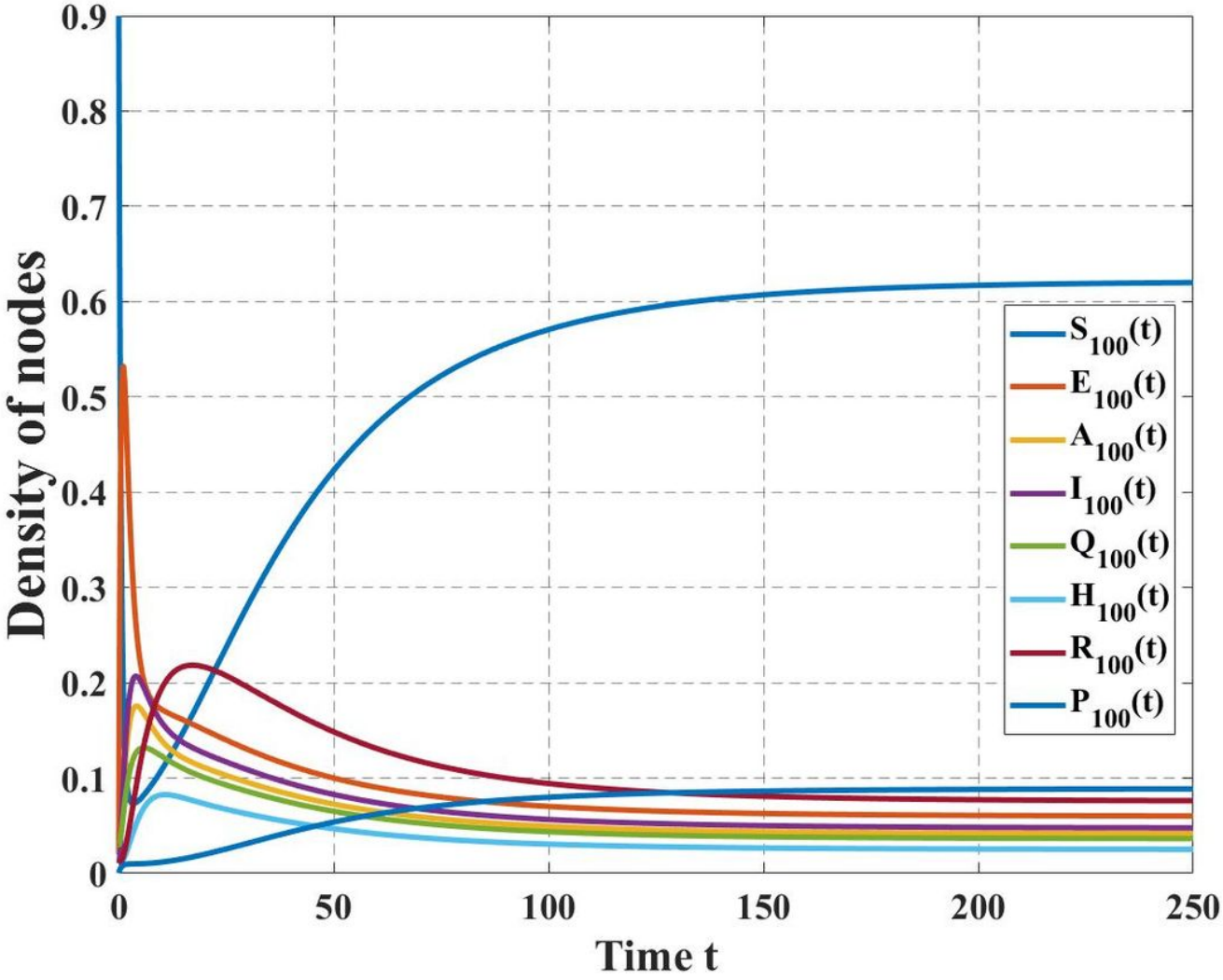

Figure 3

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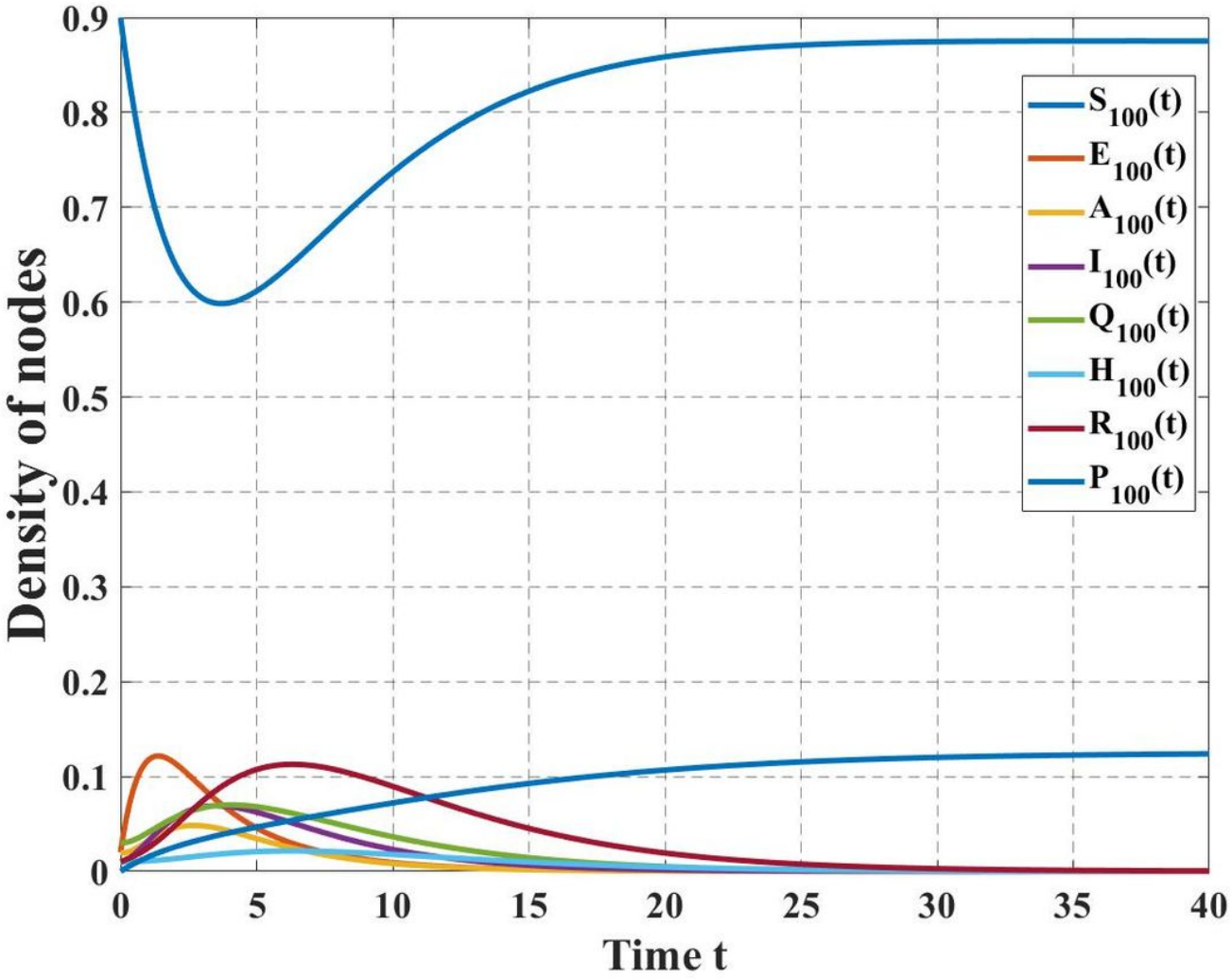

Figure 4

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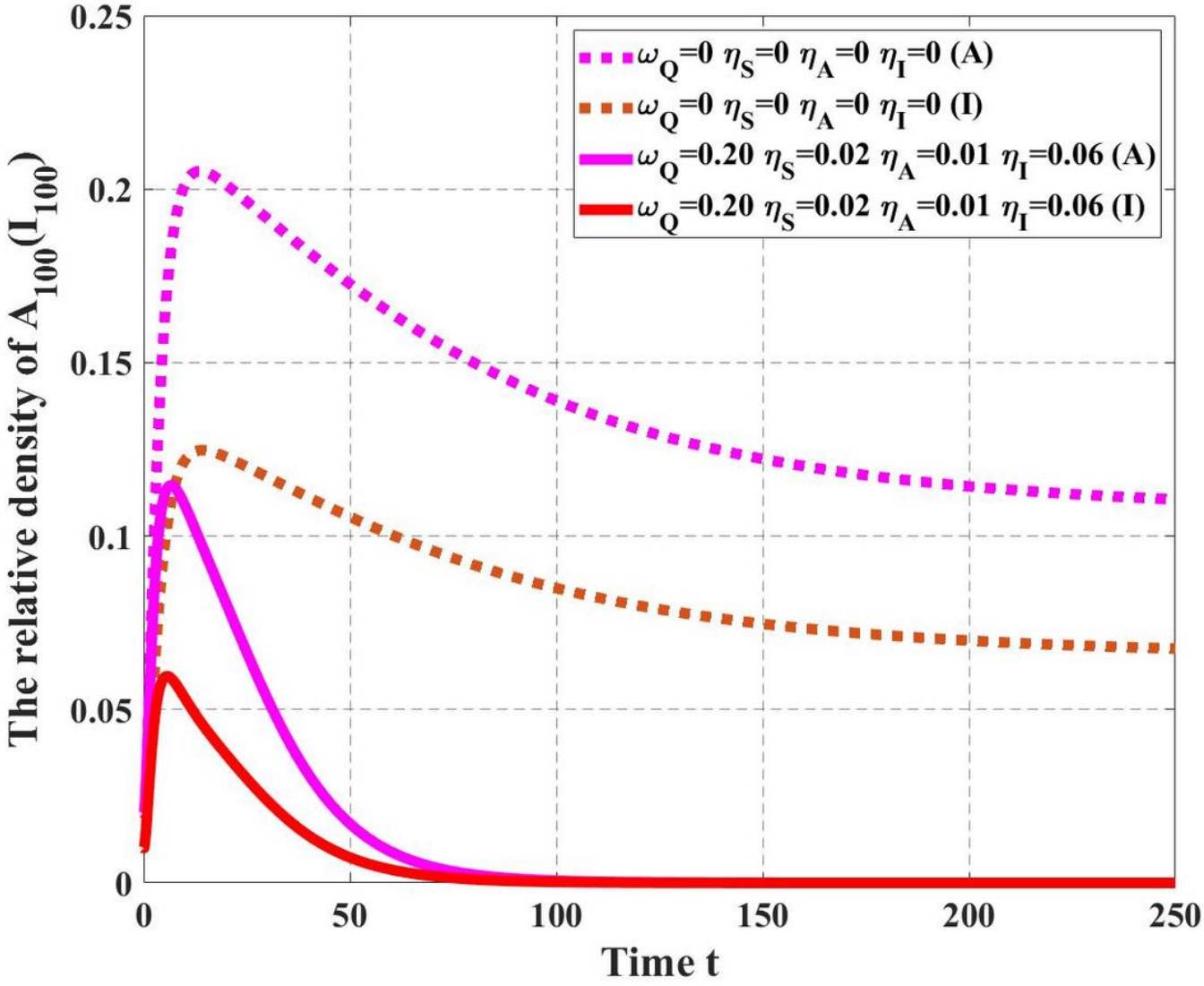

Figure 5

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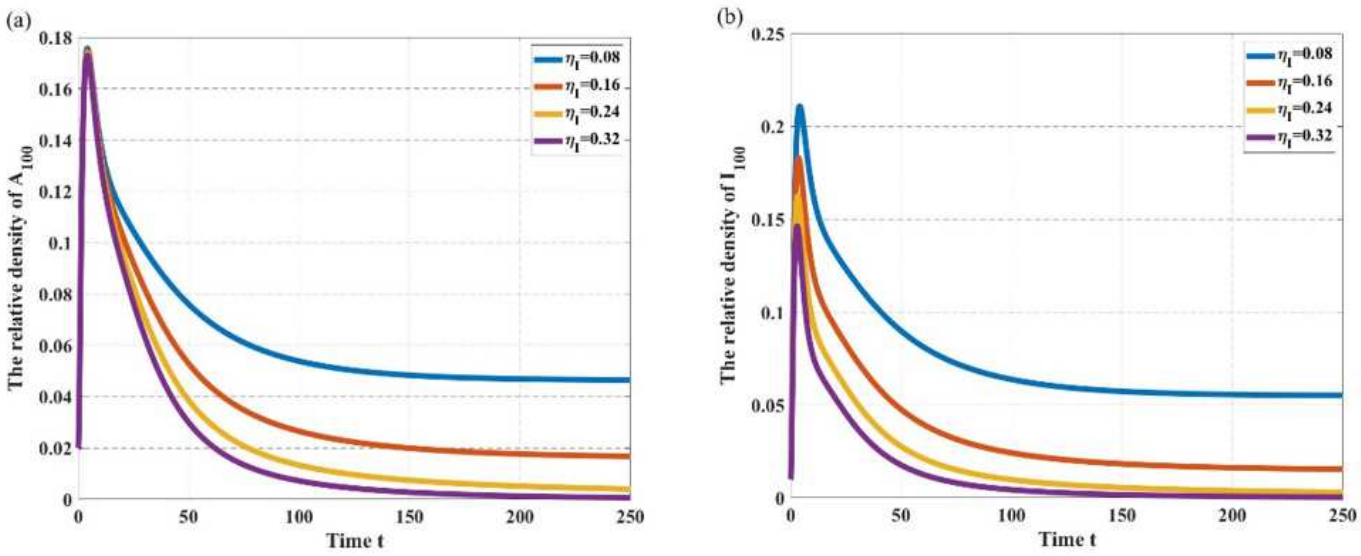

Figure 6

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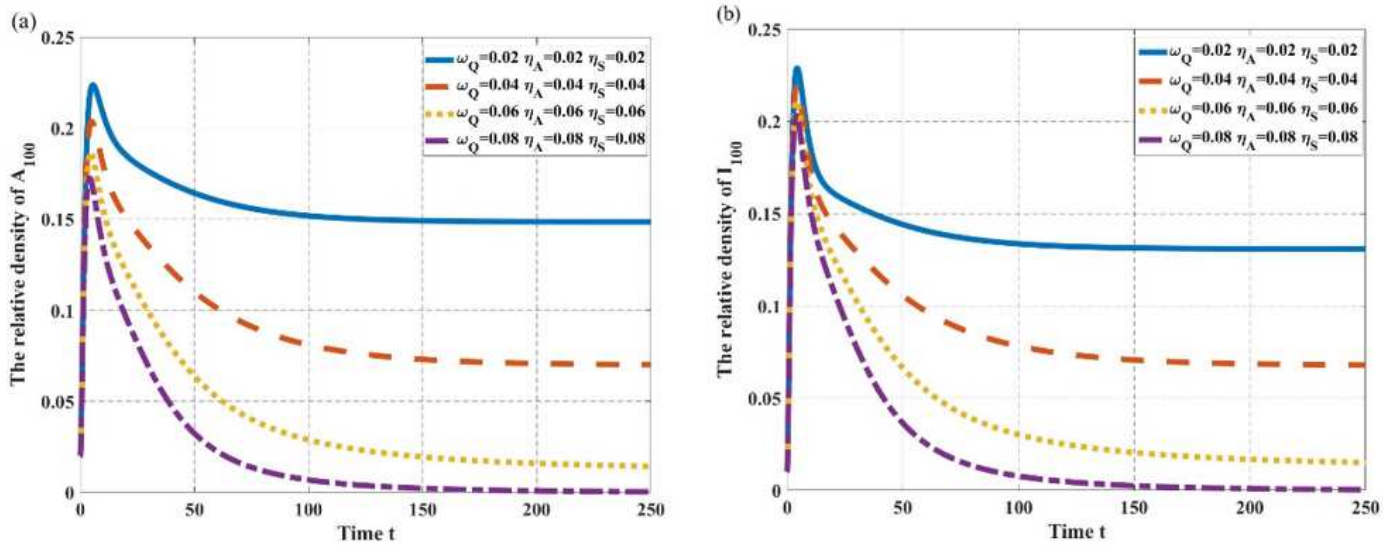

Figure 7

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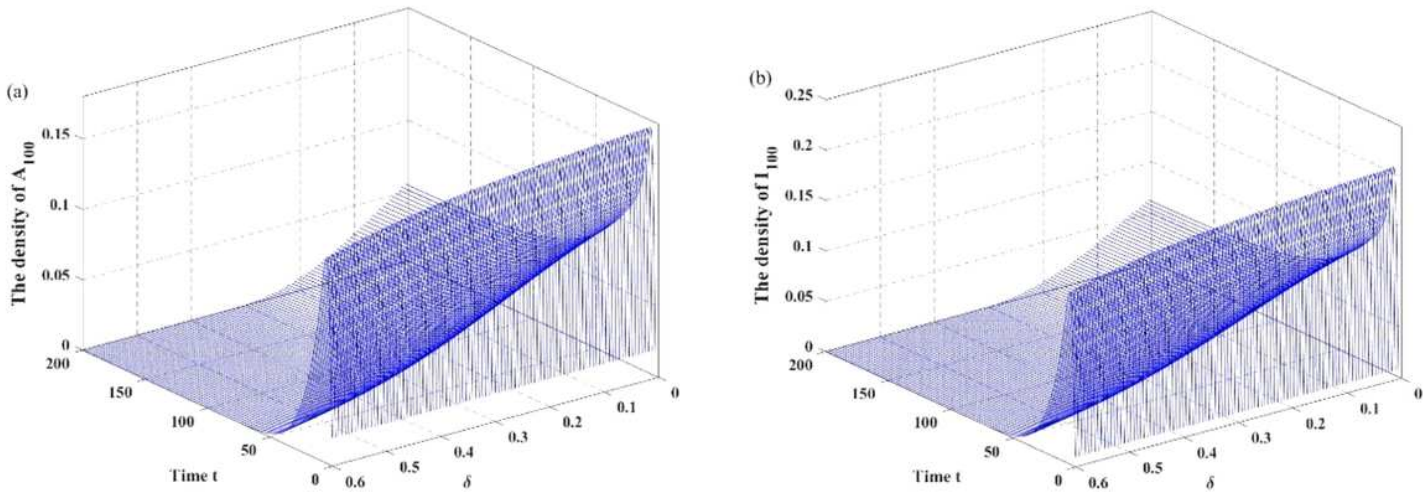

Figure 8

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