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FLOW INDUCED BY A ROTATING MICROCHANNEL-A NUMERICAL STUDY

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Abstract

In this paper, a mathematical foundation has been developed for the primary understanding of complex interaction of the wall slip with the Coriolis and Lorentz forces acting orthogonally on the Electromagnetohydrodynamic (EMHD) flow of a power-law fluid in a microchannel. Modified Navier Stokes equations are solved numerically by incorporating the fully implicit computational scheme with suitable initial and boundary conditions, which generates numerical results in excellent comparison with the literature for a certain limiting case. An extensive effort has been made to understand how the Hartmann number, fluid behavior index, rotating Reynolds number, and slip parameter affects the flow. Results show the velocity of the power-law fluid depends strongly on flow parameters. Critical Hartmann number can be obtained for the power-law fluid in presence of uniform electric and magnetic fields. As a promising phenomenon, existence of a cross over point (which depends upon the fluid behavior index) for the centerline flow velocity, has also been predicted. Reduction in the shear stress and fluid viscosity can be controlled effectively by incorporating a slippery film of lubricant on the periphery of the microchannel. This work is useful to meet the upcoming challenges of future generation, like improvement in bio-magnetic-sensor technologies as well as electrical and mechanical mechanisms.

Keywords: Coriolis and Lorentz forces, Hartmann number, Rotating Reynolds number, Fluid behavior index, Slip condition.

1. Introduction

Microfluidics plays a fundamental role in various technological processes and their applications, concerning the examination of biological and chemical samples, detection, separation, and the professional design for transfer of mass and heat. In recent years, microfluidic transport based lab-on-a-chip has fascinated the researchers and attained extraordinary awareness due to its applications (Holger et

al. 2000; Ken-ichi et al. 2008; George et al. 2005; Van Lintel et al. 1988). Magnetohydrodynamics (MHD) micropumps have been paid great attention by the researchers, as it is one of the most significant non-mechanical micro-pumps. In MHD micro-pumps, numerous experimental and theoretical investigations have been made. MHD micropumps are much beneficial in generating continuous flow (Farideh et al. 2012), lower voltage of operation (Mehdi et al. 2016), various applications in biochemical and biomedical fields (Takashima 1976; An-Cheng et al. 2010).

Shit *et al.* (2016) studied rotating Electro-osmotic flow (EOF) in a varying microchannel with slip velocity. They adopted the Crank-Nicolson technique to resolve the Poisson-Boltzmann equation for acquiring velocity distribution. In controlling the EOF, the rotating Reynolds has a significant role to play. The problem was designed without applying a pressure gradient and considering rotating Reynolds number very small in the application of microfluidic. Chiu-On *et al.* (2015) have presented EOF in a rotating duct through a rectangular geometry for low-Rossby number. Problem was determined with the collective action of pressure, viscous, Coriolis, and electric forces. It was found that channel rotation affect the fluid flow, depends on the distribution of the zeta potential function and the width of the channel. Taking into consideration the inertial property, numerical learning may have worth in future to glance at the roll-cell instabilities in a revolving channel with rectangular or curved cross-section due to EOF. The significance of the flows in the revolving frame has fascinated the interest of numerous scholars (Yongjun 2015). Ching-Cheng *et al.* (2011) investigated the EOF over a rotating plate created by two parallel surfaces. The study was based on the charge distributions in the rotating frame of reference for Debye-Huckel approximation and the Navier-Stokes equation for a moving electrolyte. Influence of rotary motion has constantly reduced the rate of axial flow along the electric field direction by the transverse flow.

Recently, the involvement of micro-scale transport mechanisms in carrying the essential technological modification towards miniaturization has been initiated such as micro-electro-mechanical systems, and micro-electronic appliances of thermal management (Quan-sheng et al. 2011, 2011). In the literature, a variety of theoretical and numerical learning are available for EOF in microchannels over various geometries and physical situations. Moreover, EOF system may be positioned in a rotary situation like flow control separation in centrifuges. Armand *et al.* (2001) studied the creation of vortices with a potential sinusoidal surface in a microchannel. Takashima *et al.* (1976) inspected under the influence of an

alternating electric field restricted between two parallel plates rotating in a dielectric fluid by ignoring centrifugal force. Chien-Cheng *et al.* (2011) illustrated that electro-osmotic flow may be positioned in a revolving arrangement like centrifuges for calculating flow separation. They presented the EOF restricted by two electrically charged rotating plates in a microchannel. In their findings, it is generally scrutinized that with the increasing the rotating parameter, the maximal velocity vacillated towards the boundary. The numerical investigation by using Poisson–Boltzmann's potential distribution for a power-law fluid model in a slit microchannel, by rotating EOF in the existence of high zeta function studied by Zhi-Yong and Yong-Jun (2010). They found that with the existence of huge angular velocity, direction of velocity considerably alters because Coriolis force affects the fluid. Shun-Xiang *et al.* (2015) have demonstrated the rotating EOF in a microchannel of an incompressible third-grade fluid. They adopted the perturbation procedure for the linearized Poisson–Boltzmann equation for finding an appropriate analytical solution to the flow velocity and flow rate. Most of these learnings are limited to the uniform cross-section in the microchannels with constant zeta potential function and with no-slip conditions. However, a limited research has been carried out on EOF in a channel with asymmetric charged walls. Cheng and Chiu-On (2015) studied the mutual effect of EOF and pressure-driven flow with zeta potential of a power-law fluid model and periodic deviation through a slit microchannel.

Interfacial slip due to hydromagnetic contacts is commonly expressed by a Navier slip conditions, that's why velocity slip is represented in terms of the normal gradient of tangential velocity component. Velocity slip at the liquid-solid crossing point in the depletion layer is important in EOF through microchannel geometry. In depletion layer, velocity yields the noticeable hydrodynamic slip. An observable fact is seen in polymer solutions (Zhi-Yuan *et al.* 2010; Dayong *et al.* 2008), colloidal particles (Claudio *et al.* 2008), and considerably in blood cell suspension (Remco *et al.* 2005; Guillaume *et al.* 2006; Hartman *et al.* 2004). Theoretical investigation on EOF with the condition of an interfacial slip in a microchannel carried out by Suman (2005). Cunlu *et al.* (2011) have demonstrated the influence of hydrodynamics Navier slip in microchannel on pressure-driven flows. Utilizing the Debye-Huckel linearization, they presented an analytical solution and a critical length slip formula for graphing the hydrophobic surfaces. Still, no investigation has yet been made for evaluating the results of hydrodynamic slip on the EMHD power-law

fluid model for Newtonian fluid in microfluidic systems (Yves 1971; Brunn 1975; Prakash et al. 2011, Cunlu et al. 2011).

In recent years, rotating EMHD flow has been a scorching topic as it plays a vital role in improving the equality of mixing liquids of the microfluidic system with a substantial reduction in the time of mixing. The shape of the microchannel is an essential aspect of generating vortices in EMHD. Employing the separation of variables technique, Yong-jun et al. (2015) investigated transient rotating EMHD flow between two slit micro parallel plates with incompressible, electrically conducting, and viscous fluid. Analytical solutions were presented in the existence of magnetic and electric fields for both alternating and direct currents. Zhi-Yong *et al.* (2017) investigated rotating EMHD flow of a power-law fluid via microchannel. Finite difference scheme was adopted in a rotating frame for the explanation of the Navier-Stoke equations and resolved through Crank-Nicolson implicit technique with suitable initial and boundary conditions. Effect of different constraints like rotating Reynolds number, Hartmann number, and fluid behavior index on distribution of velocity was presented through graphs. Current outcomes could be employed as an initial proposal for the development of well-organized applications for electromagnetic procedures concerning flow control separation. Rajib *et al.* (2013) critically found heat transfer in fully thermally developed flow in a microchannel combined with EMHD, by considering the influence of electro-kinetics, and viscous dissipation, for steady heat flux. Fundamental features of EMHD, and characteristics of heat aspects through the thin cavity, were highlighted by the deviation in velocity, and temperature profiles.

Purpose of this work is to learn numerically the impact of slip velocity in rotating EMHD flow of a power-law fluid model in a microchannel. Navier-Stoke equations are modified under a revolving microchannel, we get the profiles of the velocity distribution of EMHD flow by incorporating the fully implicit computational scheme for a range of fluid behavior index. Complete study is carried out to scrutinize the various flow parameter effects on the rotating EMHD flow of the power-law fluid. Effort of current investigation has a significant impact on the improvement of an electromagnetic gadget in a rotary system, which can be beneficial in the area of drug delivery and species control separation.

2. Mathematical formulation and Power-law fluid model

Physical demonstration of the rotating electromagnetohydrodynamic flow of a power-law fluid model can be seen in Fig. 1(a). At the center of the microchannel, Cartesian coordinates (x, y, z) are developed, since

the rotary motion of the entire system is about z-axis so, the system is assumed in non-inertial frame. Height of the microchannel is $2H$. Microchannel is filled with fluid and produces a Lorentz force in the x-direction, induces uniform electric field $\vec{E}_0$ in y-direction, and uniform magnetic field $\vec{B}_0$ in z-direction. We assume that the microchannel is rectangular having length L and the breadth W in x and y-directions, respectively. It is considered that the microchannel aspect ratio is too small, *i.e.* $W \ll L$, $2H \ll W$. Cross section outlook of a microchannel can be noticed in Fig. 1(b). With the constant angular velocity Ω , the entire microchannel spins about z-axis. In a rotating frame of reference, and flow velocity is called the relative velocity.

Fig. 1(a). 3D view of rotating EMHD microchannel, **1(b).** Ducts cross-section of rotating EMHD microchannel.

An incompressible fluid is packed in the microchannel and continuity equation is

$$\nabla \cdot \vec{V} = 0, \quad (1)$$

Here, $\vec{V}$ is velocity in vector form. Velocity can be obtained by resolving the Navier-Stokes momentum equation of the form [34]:

$$\rho \left(\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \nabla \right) \vec{V} + 2\vec{\Omega} \times \vec{V} \right) = -\nabla P + \vec{F}_b + \nabla \cdot \vec{\tau}. \quad (2)$$

Here, t is time, ρ is fluid density, dynamic pressure is P , i.e. $P = p - \rho \left| \vec{\Omega} \times \vec{r} \right|^2 / 2 + p_m$, with magnetic pressure is p_m and $\vec{r} = \vec{r}(x, y, z)$ is modified flow pressure because of centrifugal force, body force in vector form is $\vec{F}_b$, rotating angular velocity is $\vec{\Omega}$ and $\vec{\tau}$ is the stress tensor. In the current study, we have employed the power-law fluid model. A few dilute polymer solutions and complex biofluids with shear dependent viscosity signify as the power-law fluid model. The velocity vector $\vec{V} = (\tilde{u}, \tilde{v}, 0)$ has two components that are parallel to the plate. The mainstream velocity is denoted by $\tilde{u}$ along x-direction and secondary flow which is in the y-direction can be expressed as $\tilde{v}$. Angular velocity $\vec{\Omega} = (0, 0, \Omega)$ in a rotating system is taken in the z-direction. Electric field strength $\vec{E} = (0, E_0, 0)$ and magnetic field strength $\vec{B} = (0, 0, B_0)$ are imposed in the y and z directions, respectively. Uniform electric and magnetic fields produced a Lorentz force that acted as a body force on the entire system. It is considered that magnetic Reynolds number is negligible, and illustrated as $Re_m = \tilde{\sigma}_e \tilde{u}_e U_{ref} L_{ref}$, where the reference velocity, the reference length, and the magnetic diffusivity are U_{nf} , L_{ref} , and $\tilde{\sigma}_e \tilde{u}_e$ respectively. Order of rotating Reynolds number is about 10^{-5} , so, it implies that magnetic field is independent of the flow velocities. Lorentz force is expressed as (Zhi-Yong et al. 2017)

$$\vec{F} = \tilde{\sigma}_e \left(\vec{E} + \vec{V} \times \vec{B} \right) \times \vec{B}, \quad (3)$$

where $\tilde{\sigma}_e$, $\vec{E}$, and $\vec{B}$ are the electrical conductivity, electrical field strength, and magnetic field strength respectively. Dynamic viscosity μ in a power-law fluid model is:

$$\mu = \eta \Delta^{n-1}, \quad (4)$$

where η and n are represented as consistency index and fluid behavior index. Shear-thinning fluid viscosity ($n < 1$ also known as pseudo-plastic fluid) reduces with an enhancement in shear rate. Pseudo-plasticity can be verified by shaking a ketchup bottle, causes an irregular alteration in viscosity. The shear-thinning fluid viscosity ($n > 1$ also known as dilatants fluid) boost up with the shear rate. The result of

dilatants fluid can be observed with a cornstarch and water combination when thrown adjacent to a surface. When $n = 1$ fluid behaves like a Newtonian fluid and this is a reason, we recognize the model of a power-law fluid as the Newtonian model. Distinction between Newtonian and non-Newtonian fluids is that the viscous stress of non-Newtonian fluids does not have a linear relationship with strain rate tensor. Strain rate tensor in magnitude is denoted by Δ and can be demonstrated as

$$\Delta = \left[\frac{1}{2} (\Gamma : \Gamma) \right]^{1/2}. \quad (5)$$

Here, Γ is strain rate tensor. Using the relationship of the strain rate tensor and viscous stress rate tensor, the equation for power-law fluid model is signifying as

$$\tau = \mu(\Delta) \left[\nabla \vec{V} + \left(\nabla \vec{V} \right)^T \right]. \quad (6)$$

Here, the velocity gradient tensor is $\nabla \vec{V}$ and its transpose is $\left(\nabla \vec{V} \right)^T$. Regarding the above theoretical arguments, we summarize that the microchannel aspect ratio is very small and the electromagnetohydrodynamic velocity components are functions of z and t only *i.e.* $\tilde{u} = \tilde{u}(z, t)$, and $\tilde{v} = \tilde{v}(z, t)$. For incompressible fluid, the continuity equation can be easily satisfied. In power-law fluid model, dynamic viscosity is represented as

$$\mu(\Delta) = \eta \left[\left(\frac{\partial \tilde{u}}{\partial z} \right)^2 + \left(\frac{\partial \tilde{v}}{\partial z} \right)^2 \right]^{(n-1)/2}. \quad (7)$$

The shear stress in component form can be illustrated as

$$\begin{aligned} \tau_{zx} &= \mu(\Delta) \partial \tilde{u} / \partial z, \\ \tau_{zy} &= \mu(\Delta) \partial \tilde{v} / \partial z. \end{aligned} \quad (8)$$

Therefore, in the rotating coordinate system, the Navier-Stokes momentum equations are represented as

$$\rho \left(\frac{\partial \tilde{u}}{\partial t} - 2\Omega \tilde{v} \right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial z} \left(\mu(\Delta) \frac{\partial \tilde{u}}{\partial z} \right) + \sigma (B_0 E_0 - B_0^2 \tilde{u}), \quad (9)$$

$$\rho \left(\frac{\partial \tilde{v}}{\partial t} - 2\Omega \tilde{u} \right) = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial z} \left(\mu(\Delta) \frac{\partial \tilde{v}}{\partial z} \right) - \sigma B_0^2 \tilde{v}. \quad (10)$$

Concerning initial and boundary conditions are

$$\left[\begin{array}{l} \text{At } t=0, \quad \tilde{u} = \tilde{v} = 0, \\ \text{At } z=H, \quad \tilde{u} + \lambda \frac{\partial \tilde{u}}{\partial z} = 0, \quad \tilde{v} + \lambda \frac{\partial \tilde{v}}{\partial z} = 0, \\ \text{At } z=-H, \quad \tilde{u} = \tilde{v} = 0, \end{array} \right] \quad (11)$$

where λ is the slip length. Electromagnetohydrodynamic flow determined by the Lorentz force has already been investigated in Cunlu et al. (2011). Dimensionless variables that are suitable for resolving the above momentum equations are (Zhi-Yong et al. 2017),

$$u^* = \frac{\tilde{u}}{u_p}, \quad v^* = \frac{\tilde{v}}{v_p}, \quad z^* = \frac{z}{H}, \quad \mu^* = \frac{\mu}{\mu_c}, \quad \mu_c = \eta \left(\frac{u_p}{H} \right)^{n-1}, \quad \Gamma_x^* = -\frac{1}{\Gamma_c} \frac{\partial p}{\partial x}, \quad \Gamma_y^* = -\frac{1}{\Gamma_c} \frac{\partial p}{\partial y}, \quad \Gamma_c = -\frac{\eta u_p^{(n)}}{H^{(n+1)}}, \quad t^* = \frac{t}{1/\Omega}. \quad (12)$$

In the power-law fluid, the maximum velocity is denoted by u_p . It is obtained by pure pressure between the micro parallel plates in the flow problem.

$$u_p = \left(\frac{n}{n+1} \right) \left(-\frac{1}{\eta} \frac{\partial P}{\partial x} H^{(n+1)} \right)^{\frac{1}{n}}. \quad (13)$$

Here, the pressure gradient characteristic is Γ_c and the viscosity characteristic is μ_c , which are derived from momentum and viscous shear stress equations. Using the non-dimensional variables from eq.(12) into eqs. (7), (9), and (10) along with boundary conditions (11). Dynamic viscosity and Navier-Stokes equations in non-dimensional form are represented as

$$\mu = \left[\left(\frac{\partial \tilde{u}}{\partial z} \right)^2 + \left(\frac{\partial \tilde{v}}{\partial z} \right)^2 \right]^{(n-1)/2}, \quad (14)$$

$$\text{Re}_\Omega \left(\frac{\partial \tilde{u}}{\partial t} - 2\tilde{v} \right) = \Gamma_x + \frac{\partial}{\partial z} \left(\mu \frac{\partial \tilde{u}}{\partial z} \right) + Ha\beta - Ha^2 \tilde{u}, \quad (15)$$

$$\text{Re}_\Omega \left(\frac{\partial \tilde{v}}{\partial t} - 2\tilde{u} \right) = \Gamma_y + \frac{\partial}{\partial z} \left(\mu \frac{\partial \tilde{v}}{\partial z} \right) - Ha^2 \tilde{v}. \quad (16)$$

where the dimensionless pressure gradients are Γ_x and Γ_y in x and y directions, respectively. Converted initial and boundary conditions are given by:

$$\left[\begin{array}{l} \text{At } t=0, \quad \tilde{u} = \tilde{v} = 0, \\ \text{At } z=1, \quad \tilde{u} + S_L \frac{\partial \tilde{u}}{\partial z} = 0, \quad \tilde{v} + S_L \frac{\partial \tilde{v}}{\partial z} = 0, \\ \text{At } z=-1, \quad \tilde{u} = \tilde{v} = 0. \end{array} \right] \quad (17)$$

It is note that for our own convenience in eqs. (14)-(17), all dimensionless variables are shown by ignoring the star sign “*”. Many dimensionless factors related to governing problems are:

$$\text{Re}_\Omega = \frac{\rho \Omega H^{n+1} u_p^{1-n}}{\eta}, \quad \beta = \frac{E_0 H}{u_p} \sqrt{\frac{\sigma}{\mu_c}}, \quad Ha = B_0 H \sqrt{\frac{\sigma}{\mu_c}}, \quad S_L = \frac{\lambda}{H}. \quad (18)$$

Here, Re_Ω recognized as rotating Reynolds number, that has an inverse relation with the Ekman number, which provides a frequency scale. The estimate electric field strength is β and the ratio of magnetic to the viscous forces is called Hartman number and denoted by Ha and the slip parameter is denoted by S_L . According to the literature survey (Zhi-Yong et al. 2017), the analysis of the rotating flow proves that the pressure gradient must be constant in x and y directions. By using this assumption in present study, the pressure gradients are neglected. Reinstating the dynamic viscosity in eqs. (15), (16) with eq. (14), we get

$$\text{Re}_\Omega \left(\frac{\partial \tilde{u}}{\partial t} - 2\tilde{v} \right) = K_1(z) \frac{\partial^2 \tilde{u}}{\partial z^2} + K_2(z) \frac{\partial^2 \tilde{v}}{\partial z^2} + Ha \beta - Ha^2 \tilde{u}, \quad (19)$$

$$\text{Re}_\Omega \left(\frac{\partial \tilde{v}}{\partial t} - 2\tilde{u} \right) = K_3(z) \frac{\partial^2 \tilde{v}}{\partial z^2} + K_4(z) \frac{\partial^2 \tilde{u}}{\partial z^2} - Ha^2 \tilde{v}, \quad (20)$$

where $K_1(z)$, $K_2(z)$, $K_3(z)$, and $K_4(z)$ are functions in non-dimensional form and written as

$$K_1(z) = \left[\left(\frac{\partial \tilde{u}}{\partial z} \right)^2 + \left(\frac{\partial \tilde{v}}{\partial z} \right)^2 \right]^{(n-3)/2} \left[n \left(\frac{\partial \tilde{u}}{\partial z} \right)^2 + \left(\frac{\partial \tilde{v}}{\partial z} \right)^2 \right], \quad (21)$$

$$K_2(z) = (n-1) \left[\left(\frac{\partial \tilde{u}}{\partial z} \right)^2 + \left(\frac{\partial \tilde{v}}{\partial z} \right)^2 \right]^{(n-3)/2} \left(\frac{\partial \tilde{v}}{\partial z} \right) \left(\frac{\partial \tilde{u}}{\partial z} \right) \quad (22)$$

$$K_3(z) = \left[\left(\frac{\partial \tilde{u}}{\partial z} \right)^2 + \left(\frac{\partial \tilde{v}}{\partial z} \right)^2 \right]^{(n-3)/2} \left[n \left(\frac{\partial \tilde{v}}{\partial z} \right)^2 + \left(\frac{\partial \tilde{u}}{\partial z} \right)^2 \right], \quad (23)$$

$$K_4(z) = (n-1) \left[\left(\frac{\partial \tilde{u}}{\partial z} \right)^2 + \left(\frac{\partial \tilde{v}}{\partial z} \right)^2 \right]^{(n-3)/2} \left(\frac{\partial \tilde{u}}{\partial z} \right) \left(\frac{\partial \tilde{v}}{\partial z} \right) \quad (24)$$

3. Numerical Method

The governing equations involve two types of derivatives, one is the time derivative whereas the other is spatial. For discretization purpose, we use the forward difference for the time derivatives in both the

equations. On the other hand, the central difference is employed for spatial derivatives because of its relatively better accuracy. Further, their spatial derivatives are treated implicitly which yield the following system of algebraic equations.

$$\text{Re}_\Omega \left(\frac{\tilde{u}_i^{(n+1)} - \tilde{u}_i^{(n)}}{\delta t} - 2\tilde{v}_i^{(n+1)} \right) = K_1^{n+1}(z) \left(\frac{\tilde{u}_{i-1}^{(n+1)} - 2\tilde{u}_i^{(n+1)} + \tilde{u}_{i+1}^{(n+1)}}{\delta z^2} \right) + K_2^{n+1}(z) \left(\frac{\tilde{v}_{i-1}^{(n)} - 2\tilde{v}_i^{(n)} + \tilde{v}_{i+1}^{(n)}}{\delta z^2} \right) + Ha\beta - Ha^2 \tilde{u}_i^{(n+1)}, \quad (25)$$

$$\text{Re}_\Omega \left(\frac{\tilde{v}_i^{(n+1)} - \tilde{v}_i^{(n)}}{\delta t} + 2\tilde{u}_i^{(n+1)} \right) = K_3^{n+1}(z) \left(\frac{\tilde{v}_{i-1}^{(n+1)} - 2\tilde{v}_i^{(n+1)} + \tilde{v}_{i+1}^{(n+1)}}{\delta z^2} \right) + K_4^{n+1}(z) \left(\frac{\tilde{u}_{i-1}^{(n+1)} - 2\tilde{u}_i^{(n+1)} + \tilde{u}_{i+1}^{(n+1)}}{\delta z^2} \right) - Ha^2 \tilde{v}_i^{(n+1)}, \quad (26)$$

where $K_1^{n+1}(z)$, $K_2^{n+1}(z)$, $K_3^{n+1}(z)$, and $K_4^{n+1}(z)$, are non-dimensional functions, written as

$$K_1^{n+1}(z) = \left[\left(\frac{\tilde{u}_{i+1}^{(n+1)} - \tilde{u}_{i-1}^{(n+1)}}{\delta z} \right)^2 + \left(\frac{\tilde{v}_{i+1}^{(n+1)} - \tilde{v}_{i-1}^{(n+1)}}{\delta z} \right)^2 \right]^{\frac{(n-3)}{2}} \left[n \left(\frac{\tilde{u}_{i+1}^{(n+1)} - \tilde{u}_{i-1}^{(n+1)}}{\delta z} \right)^2 + \left(\frac{\tilde{v}_{i+1}^{(n+1)} - \tilde{v}_{i-1}^{(n+1)}}{\delta z} \right)^2 \right], \quad (27)$$

$$K_2^{n+1}(z) = (n-1) \left[\left(\frac{\tilde{u}_{i+1}^{(n+1)} - \tilde{u}_{i-1}^{(n+1)}}{\delta z} \right)^2 + \left(\frac{\tilde{v}_{i+1}^{(n+1)} - \tilde{v}_{i-1}^{(n+1)}}{\delta z} \right)^2 \right]^{\frac{(n-3)}{2}} \left(\frac{\tilde{v}_{i+1}^{(n+1)} - \tilde{v}_{i-1}^{(n+1)}}{\delta z} \right) \left(\frac{\tilde{u}_{i+1}^{(n+1)} - \tilde{u}_{i-1}^{(n+1)}}{\delta z} \right), \quad (28)$$

$$K_3^{n+1}(z) = \left[\left(\frac{\tilde{u}_{i+1}^{(n+1)} - \tilde{u}_{i-1}^{(n+1)}}{\delta z} \right)^2 + \left(\frac{\tilde{v}_{i+1}^{(n+1)} - \tilde{v}_{i-1}^{(n+1)}}{\delta z} \right)^2 \right]^{\frac{(n-3)}{2}} \left[n \left(\frac{\tilde{v}_{i+1}^{(n+1)} - \tilde{v}_{i-1}^{(n+1)}}{\delta z} \right)^2 + \left(\frac{\tilde{u}_{i+1}^{(n+1)} - \tilde{u}_{i-1}^{(n+1)}}{\delta z} \right)^2 \right], \quad (29)$$

$$K_4^{n+1}(z) = (n-1) \left[\left(\frac{\tilde{u}_{i+1}^{(n+1)} - \tilde{u}_{i-1}^{(n+1)}}{\delta z} \right)^2 + \left(\frac{\tilde{v}_{i+1}^{(n+1)} - \tilde{v}_{i-1}^{(n+1)}}{\delta z} \right)^2 \right]^{\frac{(n-3)}{2}} \left(\frac{\tilde{u}_{i+1}^{(n+1)} - \tilde{u}_{i-1}^{(n+1)}}{\delta z} \right) \left(\frac{\tilde{v}_{i+1}^{(n+1)} - \tilde{v}_{i-1}^{(n+1)}}{\delta z} \right). \quad (30)$$

In the above difference eqs. (25)-(30), n signifies the present time step, and $(n+1)$ stand for the next time step. To approximate first derivate of the velocity, the forward difference of first-order is used. Repeating this numerical procedure, we can find the velocity at any time step. In the presence of the uniform magnetic and electric fields, rotating microchannel flow is much fascinating than the transient flow. Effect of slip velocity in electromagnetohydrodynamic flow can be scrutinized as the steady status when the relative error at each spatial point of velocities between consecutive time steps is less than the pre-defined error 10^{-6} , i.e. $\left| \left(\tilde{u}_i^{(n+1)} - \tilde{u}_i^{(n)} \right) / \tilde{u}_i^{(n+1)} \right| \leq 10^{-6}$.

4. Results and Discussion

In the preceding part, Electromagnetohydrodynamic flow of a power-law fluid in a rotating microchannel has been examined and governing partial differential equations (PDEs) in dimensionless form have been solved numerically. Having confirmed the numerical technique, we are prepared to scrutinize our novel outcomes and concentrate on the influence of the power-law fluid model on the flow dynamics. Special influence of non-dimensional flow constraints are explored, such as the Hartman number, index of fluid behavior, rotating Reynolds number, and slip parameter on the preferred hydrodynamic features. To calculate the results for different constraints that are exposed in the description of every figure, the parametric values are projected in Table 1 based on the physical properties given in [7-9, 13]. Findings of current study may facilitate one to look for an appropriate configuration for the most favorable presentation of EMHD-based micropumps.

Table 1. Physical variable values for the flow problem

Microchannel height	H	100 – 200 μm
Electric field force	E	2×10^5 V/m
Microchannel length	L	0.005 m
Absolute average temperature	T_{av}	300 K
Permittivity of fluid	ε	5.3×10^{-10} C/V _m
Fluid Viscosity	ν	10^{-6} m ² /s
Fluid electrical conductivity	σ	$2.2 \times 10^{-4} - 10^6$ S/m
Fluid density	ρ	10^3 kg/m ³
Consistency index	η	10^{-3} kg/ms
Imposed magnetic field	B_0	0.02 – 1.0 T
Order of magnetic Hartman number	$O(Ha)$	0.01 to 7
Rotating angular velocity	Ω	0 – 15000 rpm
Rotating Reynolds number	Re_Ω	0.01 – 10
Range of non-dimensional parameter	β	0 – 10.

In order to judge the efficiency of the present code, a comparison is interrelated with the prior study of Jian *et al.* (2015) and (Zhi-Yong et al. (2017)). The assessment is found to be in excellent conformity, and it guarantees the precision of our numerical method. The detailed data for the current numerical and the analytical solution are presented in Table 2.

Table 2. Comparison of the analytical solution with current numerical solution for $Ha=0.3$, $S_L=0$.

u/u_p

η/H	Jian <i>et al.</i> (2015)	Z. Y. Xie <i>et al.</i> (2017)	Present results	Relative error
	Analytical solution (u_A)	Numerical solution	Numerical solution (u_N)	$ (u_A - u_N)/u_A $
0.0	0.26472	0.26442	0.26427	0.00169
0.1	0.26360	0.26340	0.26325	0.00132
0.2	0.26010	0.25997	0.25984	0.00049
0.3	0.25382	0.25372	0.25360	0.00086
0.4	0.24398	0.24392	0.24382	0.00065
0.5	0.22946	0.22942	0.22933	0.00056
0.6	0.20858	0.20856	0.20849	0.00043
0.7	0.17904	0.17902	0.17896	0.00044
0.8	0.13761	0.13760	0.13756	0.00036
0.9	0.07992	0.07991	0.07988	0.00050
1.0	0.00000	0.00000	0.00000	0.00000

v/u_p				
η/H	Jian <i>et al.</i> (2015)	Z. Y. Xie <i>et al.</i> (2017)	Present results	Relative error
	-0.09895	-0.09860	-0.09847	0.00485
0.1	-0.09810	-0.09784	-0.09771	0.00397
0.2	-0.09554	-0.09533	-0.09521	0.00345
0.3	-0.09117	-0.09101	-0.09089	0.00307
0.4	-0.08484	-0.08472	-0.08462	0.00259
0.5	-0.07639	-0.07630	-0.07622	0.00222
0.6	-0.06565	-0.06558	-0.06551	0.00213
0.7	-0.05247	-0.05243	-0.05237	0.00190
0.8	-0.03688	-0.03686	-0.03682	0.00162
0.9	-0.01915	-0.01913	-0.01911	0.00208
1.0	0.00000	0.00000	0.00000	0.00000

Fig. 2(a) and 2(b) exhibit velocity deviation for revolving electromagnetohydrodynamic flow for various fluid behavior indexes at low and high values of Hartman number. It is perceived from Fig. 2(a) that velocity profiles at the mid of the channel along x-direction are reduced and utmost velocity curves start shifting from the midpoint towards boundary in both cases of Newtonian fluid ($n=1$) and a dilatant fluid ($n>1$). Depression comes into view in the velocity profile at the middle portion of the channel. This observable fact can be due to the rotation of the channel. Coriolis force establishes at the microchannel diminishes the centerline velocity. On the other hand, in the pseudoplastic fluid ($n<1$) in the x-direction, the center velocity does not show a depression. Besides, we can still monitor from Fig. 2(a) that, the center

velocity is superior to the other two-fluid models. Motivation behind this observable fact is that fluid viscosity depends on location of the channel. Newtonian fluid viscosity at the midpoint of the channel is greater, compared to dilatant fluid but smaller than that of a pseudoplastic fluid. Due to negligible influence of rotating microchannel, the velocity u at the center of the channel for pseudo-plastic fluid becomes large. As a result, under the influence of the same electrical and magnetic fields, the depression for both Newtonian and dilatant fluids becomes visible.

Fig. 2 Effect of fluid behavior index on rotating EMHD dimensionless velocities (a) u in x-direction (b) v in y-direction for ($\beta = 1, \text{Re}_\Omega = 4, S_L = 0.5, Ha = 0.5$)

Fig. 3 (a) and (b) Effect of fluid behavior index on rotating EMHD dimensionless velocities (a) u in x-direction (b) v in y-direction for ($\beta = 1, \text{Re}_\Omega = 4, S_L = 0.5, Ha = 4$)

Fig. 4 Effect of Hartmann number on rotating EMHD dimensionless velocities (a) u in x-direction (b) v in y-direction for ($\beta = 1, Re_{\Omega} = 4, S_L = 0.5, n = 0.8$)

Fig. 5 Effect of Hartmann number on rotating EMHD dimensionless velocities (a) u in x-direction (b) v in y-direction for ($\beta = 1, Re_{\Omega} = 4, S_L = 0.5, n = 1.2$)

It is clear from Fig. 3(a) that for both Newtonian and non-Newtonian fluids the velocity u does not show a depression, although the electric and magnetic field strength along with rotating Reynolds number remains the same. It means that for high values of Hartmann number the rotational microchannel system has a negligible effect on the center velocity u . The Lorentz force, which established because of mutual interaction of electrical and magnetic fields play a significant role as compared to the Coriolis force, that is why at the middle portion of the microchannel the profile of the velocity distribution may not become concave. A semi parabolic profile behavior of the lateral velocity distribution v is observed in Fig. 2(b) and

3(b), which persuades in the -y direction. The magnitude of the lateral velocity v depicts an increasing trend with n due to the rotational effect of the channel.

In Fig. (4-5), the velocity distribution in dimensionless form for different Hartmann numbers is illustrated. The related constraint is stated in the caption. From the analysis of the sketch, it is found that for low Hartmann number, the velocity profile distribution u in the x-direction raises for both pseudo-plastic fluid ($n=0.8$) and dilatant fluid ($n=1.2$). Moreover, a clear inconsistency between the sketches of Figs. 4(a) and 5(a) is that the depression in curves comes into sight, in a dilatant fluid model ($n>1$). For pseudo-plastic fluid, from the edges of the microchannel, the velocity profiles u are amplifying and reach its highest value in the middle part of the microchannel as reveals in Fig. 4(a). In a section, close to the microchannel wall, the dilatant fluid velocity u is boosting up and its maximum velocity is transferred towards the boundary layer. Based on the above parallel reason, in the middle of the microchannel, a depression occurs. In Figs. 4(b) and 5(b), the induced lateral velocity profiles of v turn into parabolic shape for both ($n<1$) and ($n>1$). Because of the rotation of the channel, Coriolis forces are induced, and thus lateral velocity v boost up from the wall to the middle part of the channel. For small Ha , the Lorentz force, which exhibit in the x-direction, can speed up the flow. Also, the axial velocity u and lateral velocity v in magnitude enhance with the growing values of Ha . Figs. 6 and 7 demonstrate the distribution velocity in dimensionless form for the high Hartmann number Ha . We examine that for a pseudoplastic and dilatant fluid, velocity profiles u along x-direction enlarges from the edges of the channel to the middle portion, and velocity magnitude rises with the declining values of Hartmann number. Induced magnetic field generates a Lorentz force that can obstruct the flow for high Hartmann number. Same argument in the case of the lateral velocity v can be given. It is noticeable that the velocity u in magnitude is superior to the lateral velocity v as can be seen in Figs. 6 and 7. The reason behind this phenomenon is the rotating EMHD fluid flows along the x-direction. Also, unlike Fig. 5(a), for further increase in Hartmann number Ha the depression in velocity profiles disappears as shown in Fig. 7(a). Due to the revolving effect of EMHD, influence of Coriolis force weakens as compared to the Lorentz force. However, from Fig. 4(a), it is concluded that due to large fluid viscosity in the pseudoplastic fluid model the depression cannot easily appear. For the high Ha , the depression vanishes as can be observed from Fig. 6(a).

Fig. 6 Effect of Hartmann number on rotating EMHD dimensionless velocities (a) u in x-direction (b) v in y-direction for $(\beta = 1, Re_{\Omega} = 4, n = 0.8)$

Fig. 7 Effect of Hartmann number on rotating EMHD dimensionless velocities (a) u in x-direction (a) v in y-direction for $(\beta = 1, Re_{\Omega} = 4, S_L = 0.5, n = 0.8)$

According to the prior study of the flow dynamics, because of mutual interaction of magnetic and electric fields, influence of the Ha seems to be very critical (Chien-Cheng et al. 2011; Quan-sheng et al. 2011). To demonstrate the effect of Ha , the centerline velocity profiles have been shown in Fig. 8 for various values of n . Fig 8(a) showed that the centerline velocity u_c enhances with Hartmann's number and reaches its

maximum $Ha = 3$. Also, with further enhancement in Ha , centerline velocity starts to show declining behavior for both $(n=1)$ and $(n \neq 1)$.

Fig. 8 Effect of centerline velocities with Ha by varying fluid behavior index on rotating EMHD dimensionless velocities (a) u_c in x-direction (b) v_c in y-direction for $(\beta = 1, Re_\Omega = 4, S_L = 0.5, \eta = 0)$

Presence of uniform electric and a magnetic field produces a Lorentz force that consists of two components, one is the aiding force, $(\sigma B_0 E_0)$ and the other is an opposing force $(\sigma B_0^2 u)$ (Chien-Cheng et al. 2011; Quan-sheng et al. 2011). The flow-aiding component becomes significant than the flow-opposing component for the smaller Hartmann number. Therefore, profiles of centerline velocity enhance with the enhancement of the Hartmann number. However, at the same time, the flow-opposing component signifies a leading role in further growth of the Hartmann number, triggering a diminution of the fluid velocity. For small Ha , centerline velocity is larger in the pseudo-plastic than that of a dilatant fluid and for large Ha an opposite behavior is observed. From Fig. 8(b), magnitude of centerline velocity for small Ha in EMHD flow confirms an increasing behavior and a further increase in Ha exhibits a declining behavior. This is happening because of the collective effect of the Lorentz force and the Coriolis force. Electrical and magnetic effect is not clear for small Hartmann number. Because of rotation of the rectangular channel, Coriolis forces are induced that affects the motion of the fluid easily and enhances the lateral velocity v in $-y$ direction. We know that an increase in Hartmann number produces a Lorentz force that shows a

dominant property as compared to the Coriolis force. Therefore, a declining fashion is observed in lateral velocity v_c .

Fig. 9 Effect of centerline velocities with Re_Ω by varying fluid behavior index on rotating EMHD dimensionless velocities **(a)** u_c in x-direction **(b)** v_c in y-direction for $(\beta = 1, Ha = 1, S_L = 0.5, \eta = 0)$

In Figs. 9-10 we have discussed two cases for small $Ha=1$ and large $Ha=4$ of centerline velocity in rotating EMHD flow with Re_Ω , respectively. It is clear from sketch 9(a) that with the growing values of rotating Reynolds number Re_Ω the centerline velocities u_c along x-direction start decreasing for both $(n = 1)$ and $(n \neq 1)$. Rotation of the channel is the major cause for the decrease in fluid mainstream velocity and an individual crossover point can be observed. In Newtonian fluid, when Re_Ω is smaller than the crossover point, the centerline velocity is greater in pseudoplastic and smaller in dilatant fluid. However, when Re_Ω goes over the crossover point the distribution of velocity profiles confirms the opposite trend. The reason behind this phenomenon is the mutual interaction of electrical and magnetic fields, rotation of the rectangular channel, and fluid viscosity. For small Re_Ω , the Lorentz force is showing a dominant property over the Coriolis force due to combined effect of uniform magnetic and electric fields. In dilatants fluid, the centerline velocity is the largest value in magnitude.

Fig. 10 Effect of centerline velocities with Re_Ω by varying fluid behavior index on rotating EMHD dimensionless velocities (a) u_c in x-direction (b) v_c in y-direction for ($\beta = 1, Ha = 4, S_L = 0.5, \eta = 0$)

Coriolis force performs a significant role as compared to the Lorentz force for large values Re_Ω . Therefore, for the dilatant fluid, the centerline velocity is smallest for the large Re_Ω due to the smaller viscosity of a dilatant fluid. Similar for small Ha in Fig. 9(a), an identical fashion of the velocity differences in the main flow can be initiated in Fig. 10(a) for large Ha . A clear examination of Fig. 10(a) reveals that for large Ha the crossover point moves towards right corresponding to bigger values of Re_Ω . The centerline velocity v_c in magnitude demonstrates an increasing-decreasing style for different values of n , depicted in Figs. 9(b) and 10(b). A lateral velocity always accompanies the direction of the main flow velocity when the entire system revolves around z-direction. Therefore, the velocity consistently decreases in the mainstream direction, and because of the Coriolis force, lateral velocity enhances in magnitude. However, Coriolis force no longer picks up the pace of the fluid flow with further enhancement in Re_Ω , and the lateral velocity shows a declining trend.

Deviation of both components of velocity along x and y directions for different values of the slip parameter S_L is demonstrated in Figs. 11 and 12. From these graphs, it is observed that with the increasing values of slip parameter S_L along x-direction velocity reasonably decreases but lateral velocity becomes more and more negative. In the middle part of the microchannel, velocity attains a constant value while closer to the wall, velocity profiles are strongly affected by S_L .

Fig. 11 Effect of slip parameter on rotating EMHD dimensionless velocities (a) u in x-direction (b) v in y-direction for ($\beta = 1, Re_\Omega = 3, Ha = 0.5, n = 0.8$)

Fig. 12 Effect of slip parameter on rotating EMHD dimensionless velocities (a) u in x-direction (b) v in y-direction for ($\beta = 1, Re_\Omega = 3, Ha = 0.5, n = 1.2$)

5. Conclusion

In the present work, a mathematical model has been formulated for incorporating the impact of wall slip in the rotating electromagnetohydrodynamic (EMHD) flow of power-law fluid in a microchannel. Following conclusions may be drawn:

1. Wall slip may be considered as an effective tool for controlling the flow and shear stress in a rotating channel. Therefore, the slip effects can not be simply ignored.

2. Under the same conditions in a rotating frame of reference, pseudoplastic flow is stronger compared to the dilatant fluid.
3. Under reasonably weaker magnetic field, fluid behavior index enhances the centerline velocity in the mainstream. For the dilatant fluids, a slight decrease in the velocity is observed in the middle of the microchannel. The trend, however, is overcome as the magnetic field is intensified.
4. The lateral velocity has been found to be an increasing function of the fluid behavior index, irrespective of the strength of the Lorentz force.
5. Finally, the rotating Reynolds number has been found to be the most relevant parameter for the flow dynamics of the present problem. No matter the nature of the fluid, there always existed a cross-over point for the centerline velocity.
6. The present study may be one of the primary studies in this area. However, any more would be required for understanding the complex characteristics of a Power-law fluids in rotating micro channel.

6. Declaration

Authors Contribution

Conceptualization, Shahzad Ahmad and Anique Ahmad; Methodology, Kashif Ali and Faial Ali; Software, Kashif Ali; Data curation, Kashif Ali and Anique Ahmad.; Writing—original draft preparation, Shahzad Ahmad and Anique Ahmad; Writing—review and editing, Kashif Ali and Faisal Ali; Supervision, Shahzad Ahmad and Kashif Ali;

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Data availability statement

All the relevant material is available upon request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Conflicts of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

7. References

- [1] Holger, B., Claudia, G. Polymer microfabrication methods for microfluidic analytical applications. *Electrophoresis*, **21**, 12-26 (2000). [https://doi.org/10.1002/\(SICI\)1522-2683\(20000101\)21:1<12::AID-ELPS12>3.0.CO;2-7](https://doi.org/10.1002/(SICI)1522-2683(20000101)21:1<12::AID-ELPS12>3.0.CO;2-7)
- [2] Ken-ichi, O., Kaoru T., & Andreas, M. Microfluidics: applications for analytical purposes in chemistry and biochemistry. *Electrophoresis*, **29**, 4443-4453 (2008). <https://doi.org/10.1002/elps.200800121>
- [3] George, K., Ali, B., & Narayan, A. *Microflows and Nanoflows Fundamentals and Simulation*. Springer Press, New York (2005). (Chapter 1).
- [4] Van Lintel, Van de Pol, S., & Bouwstra. A piezoelectric micropump based on micromachining of silicon. *Sensor and Actuators*, **15**, 153-167 (1988). [https://doi.org/10.1016/0250-6874\(88\)87005-7](https://doi.org/10.1016/0250-6874(88)87005-7)
- [5] Farideh, A., Haslina, J., & Nurul, A. M. Y. A comprehensive study of micropumps technologies. *International Journal of Electrochemical Science*, **7**, 9765-9780 (2012).
- [6] Mehdi, K., Nader, P. Laminar MHD flow and heat transfer of power-law fluids in square microchannels. *International Journal of Thermal Sciences*, **99**, 26-35 (2016). <https://doi.org/10.1016/j.ijthermalsci.2015.07.031>
- [7] Takashima, M. The effect of rotation on electrohydrodynamic instability. *Canadian Journal of Physics*, **54**, 3(1976). <https://doi.org/10.1139/p76-039>
- [8] An-Cheng, R., Min-Hsing, C., & Falin, C. Effect of rotation on the electrohydrodynamic instability of a fluid layer with an electrical conductivity gradient. *AIP Physics Fluids*, **22**, 024102 (2010). <https://doi.org/10.1063/1.3308542>
- [9] Shit, G. C., Mondal, A., Sinha, A., & Kundu. Effects of slip velocity on rotating electro-osmotic flow in a slowly varying micro-channel. *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, **489**, 249-255 (2016). <https://doi.org/10.1016/j.colsurfa.2015.10.036>
- [10] Chiu-On, Ng., Cheng, Qi. Electro-osmotic flow in a rotating rectangular Microchannel. *Proceedings of the Royal Society A*, **471**, 2179 (2015). <https://doi.org/10.1098/rspa.2015.0200>

- [11] Yongjun, J. Transient MHD heat transfer and entropy generation in a micro parallel channel combined with pressure and electro-osmotic effects. *International Journal of Heat and Mass Transfer*, **89**, 193-205 (2015). <https://doi.org/10.1016/j.ijheatmasstransfer.2015.05.045>
- [12] Chien-Cheng, C., Chang, Y. W. Rotating electro-osmotic flow over a plate or between two plates. *Physical Review E*, **84**, 056320 (2011). <https://doi.org/10.1103/PhysRevE.84.056320>
- [13] Quan-sheng, L., Yong-jun, J., & Liangui, Y. Alternating current electroosmotic flow of the Jeffreys fluids through a slit microchannel. *AIP Physics of Fluids*, **23**, 102001 (2011). <https://doi.org/10.1063/1.3640082>
- [14] Quan-sheng, L., Yong-jun, J., & Liangui, Y. Time periodic electroosmotic flow of the generalized Maxwell fluids between two microparallel plates. *Journal of Non-Newton Fluid Mechanics*, **166**, 478-486 (2011). <https://doi.org/10.1016/j.jnnfm.2011.02.003>
- [15] Armand, A. Transverse electrokinetic and microfluidic effects in micropatterned channels: lubrication analysis for slab geometries. *Physical Review E*, **65**, 016301 (2001). <https://doi.org/10.1103/PhysRevE.65.016301>.
- [16] Takashima, M. The effect of rotation on electrohydrodynamic instability. *Canadian Journal of Physics*, **54**, 3 (1976). <https://doi.org/10.1139/p76-039>
- [17] Chien-Cheng, C., Chang, Y. W. Rotating electro-osmotic flow over a plate or between two plates. *Physical Review E*, **84**, 056320 (2011). <https://doi.org/10.1103/PhysRevE.84.056320>
- [18] Zhi-Yong, X., Yong-Jun, J. Rotating electroosmotic flow of power-law fluids at high zeta potentials. *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, **461**, 231-239 (2014). <https://doi.org/10.1016/j.colsurfa.2014.07.051>
- [19] Shun-Xiang, L., Yong-Jun, J., Zhi-Yong, X., Quan-Sheng, L., & Feng-Qin, Li. Rotating electro-osmotic flow of third-grade fluids between two microparallel plates. *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, **470**, 240-247 (2015). <https://doi.org/10.1016/j.colsurfa.2015.01.081>
- [20] Cheng, Q., Chiu-On, N. Electroosmotic flow of a power-law fluid through an asymmetrical slit microchannel with gradually varying wall shape and wall potential. *Colloids and Surfaces A:*

Physicochemical and Engineering Aspects, **472**, 26-37 (2015).
<https://doi.org/10.1016/j.colsurfa.2015.02.039>

[21] Zhi-Yuan, S., Yi-Tian, G., Xin, Y., & Ying, L. Formation of vortices in a combined pressure-driven electro-osmotic flow through the insulated sharp tips under finite Debye length effects. Colloids and Surf. A: Physicochemical and Engineering Aspects, **366**, 1-11 (2010).
<https://doi.org/10.1016/j.colsurfa.2010.04.038>

[22] Dayong, Y., Ying, L. Numerical simulation of electroosmotic flow in microchannels with sinusoidal roughness. Colloids and Surfaces A: Physicochemical and Engineering Aspects, **328**, 28-33 (2008).
<https://doi.org/10.1016/j.colsurfa.2008.06.029>

[23] Claudio, L. A. B., Maria, L. O. Electrokinetic flow of non-Newtonian fluids in microchannels. Journal of Colloid Interface Science, **320**, 582-589 (2008). <https://doi.org/10.1016/j.jcis.2007.12.032>

[24] Remco, T., Takashi, T. Polymer depletion-induced slip near an interface. Journal of Physics: Condense Matter, (2005). <https://iopscience.iop.org/article/10.1088/0953-8984/17>

[25] Guillaume, D., Pierre, J., & Patrick, T. Rheology of complex fluids by particle image velocimetry in microchannels. Applied Physics Letter, **89**, 024104 (2006). <https://doi.org/10.1063/1.2221501>

[26] Hartman, K., Kazarian, Briscoe, & Lawrence. Effects of particle size on near-wall depletion in mono-dispersed colloidal suspensions. Journal of Colloid and Interface Science, **280**, 511-517 (2004).
<https://doi.org/10.1016/j.jcis.2004.08.032>

[27] Suman, C. Dynamics of the capillary flow of blood into a microfluidic channel. Lab Chip, **5**, 421-430 (2005). <https://doi.org/10.1039/B414566F>

[28] Cunlu, Z., Chun, Y. On the competition between streaming potential effect and hydrodynamic slip effect in pressure-driven microchannel flows. Colloids and Surfaces A: Physicochemical and Engineering Aspects, **386**, 191-194 (2011). <https://doi.org/10.1016/j.colsurfa.2011.06.014>

[29] Yves, N. Blood flow slip and viscometry. Biophysical Journal, **11**, 252-264 (1971).
[https://doi.org/10.1016/S0006-3495\(71\)86212-4](https://doi.org/10.1016/S0006-3495(71)86212-4)

[30] Brunn, P. The velocity slip of polar fluids. Rheological Acta, **14**, 1039-1054 (1975).
<https://doi.org/10.1007/BF01515899>

- [31] Prakash, G., Suman, C. Semi-analytical solutions for electroosmotic flows with interfacial slip in microchannels of complex cross-sectional shapes. *Microfluid Nanofluid*, **11**, 255-267 (2011). <https://doi.org/10.1007/s10404-011-0793-6>
- [32] Cunlu, Z., Chun, Y. On the competition between streaming potential effect and hydrodynamic slip effect in pressure-driven microchannel flows. *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, **386**, 191-194 (2011). <https://doi.org/10.1016/j.colsurfa.2011.06.014>
- [33] Yong-Jun, J., Dongqing, Si., Long, C., & Quansheng, L. Transient rotating electromagnetohydrodynamic micropumps between two infinite microparallel plates. *Chemical Engineering Science*, **134**, 12-22 (2015). <https://doi.org/10.1016/j.ces.2015.04.036>
- [34] Zhi-Yong, X., Yong-Jun, J. Rotating electromagnetohydrodynamic flow of power-law fluids through a microparallel channel. *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, **529**, 334-345 (2017). <https://doi.org/10.1016/j.colsurfa.2017.05.062>
- [35] Rajib, C., Ranabir, D., & Suman, C. Thermal characteristics of electromagnetohydrodynamic flows in narrow channels with viscous dissipation and Joule heating under constant wall heat flux. *International Journal of Heat and Mass Transfer*, **67**, 1151-1162 (2013). <https://doi.org/10.1016/j.ijheatmasstransfer.2013.08.099>