[[EQUATION]]-optimal Designs for Hierarchical Linear Models: an Equivalence Theorem and a Nature-inspired Meta-heuristic Algorithm

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Abstract

Hierarchical linear models are widely used in many research disciplines and estimation issues for such models are generally well addressed. Design issues are relatively much less discussed for hierarchical linear models but there is an increasing interest as these models grow in popularity. This paper discusses [[EQUATION]] -optimality for predicting individual parameters in such models and establishes an equivalence theorem for confirming the [[EQUATION]] -optimality of an approximate design. Because the criterion is non-differentiable and requires solving multiple nested optimization problems, it is much harder to find and study [[EQUATION]] -optimal designs analytically. We propose a nature-inspired meta-heuristic algorithm called competitive swarm optimizer (CSO) to generate [[EQUATION]] -optimal designs for linear mixed models with different means and covariance structures. We further demonstrate that CSO is flexible and generally effective for finding the widely used locally [[EQUATION]] -optimal designs for nonlinear models with multiple interacting factors and some of the random effects are correlated. Our numerical results for a few examples suggest that [[EQUATION]] and [[EQUATION]] -optimal designs may be equivalent and we establish that [[EQUATION]] and [[EQUATION]] -optimal designs for hierarchical linear models are equivalent when the models have only a random intercept only. The challenging mathematical question whether their equivalence applies more generally to other hierarchical models remains elusive.

Full Text

This preprint is available for download as a PDF.

Figures

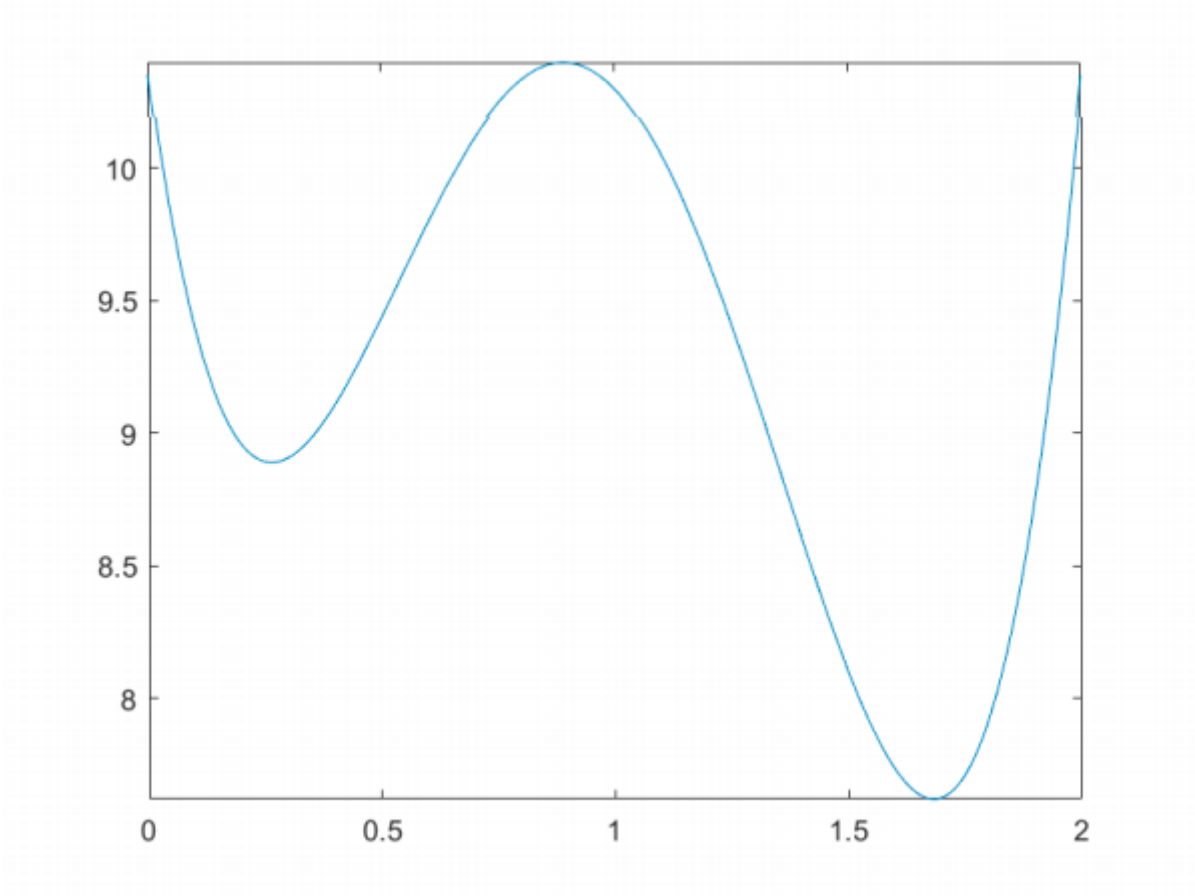

Figure 1

Sensitivity function of the CSO-generated for model (16) with uncorrelated random effects in Table 1.

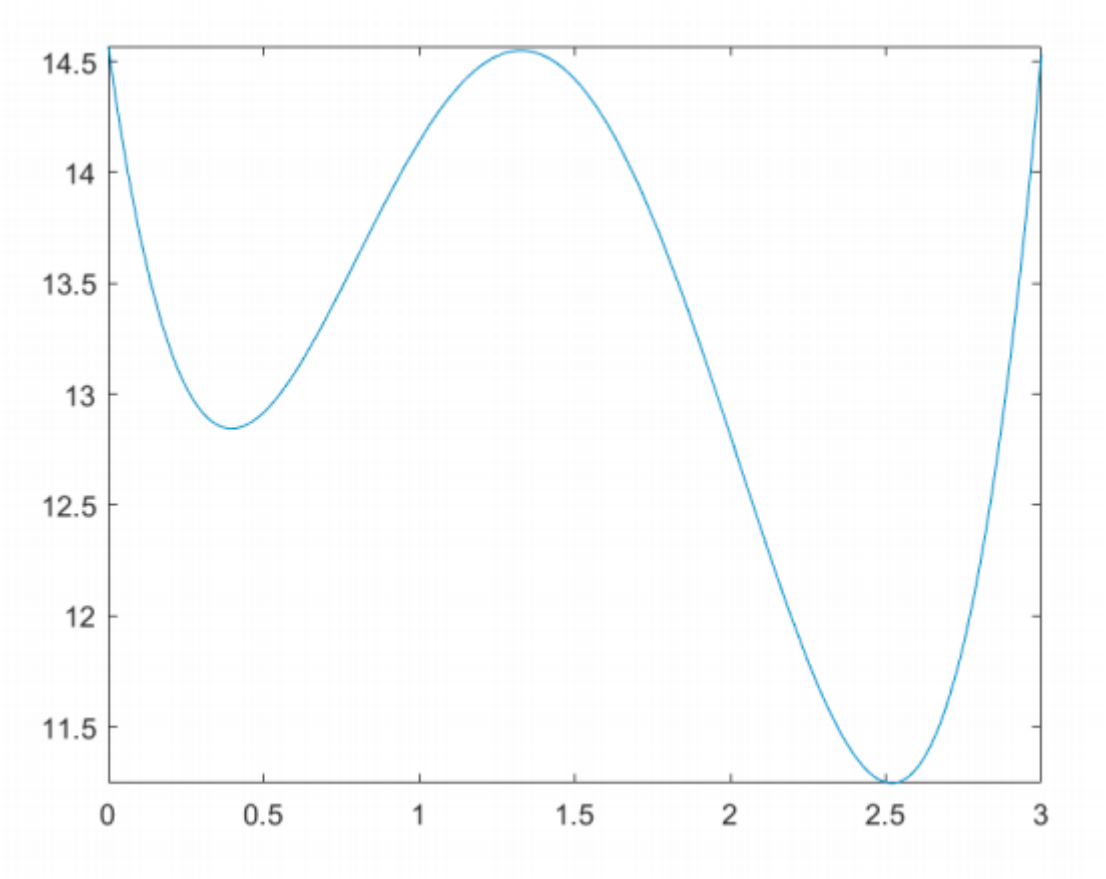

Figure 2

Sensitivity function of the CSO-generated for model (16) with correlated random effects in Table 2.

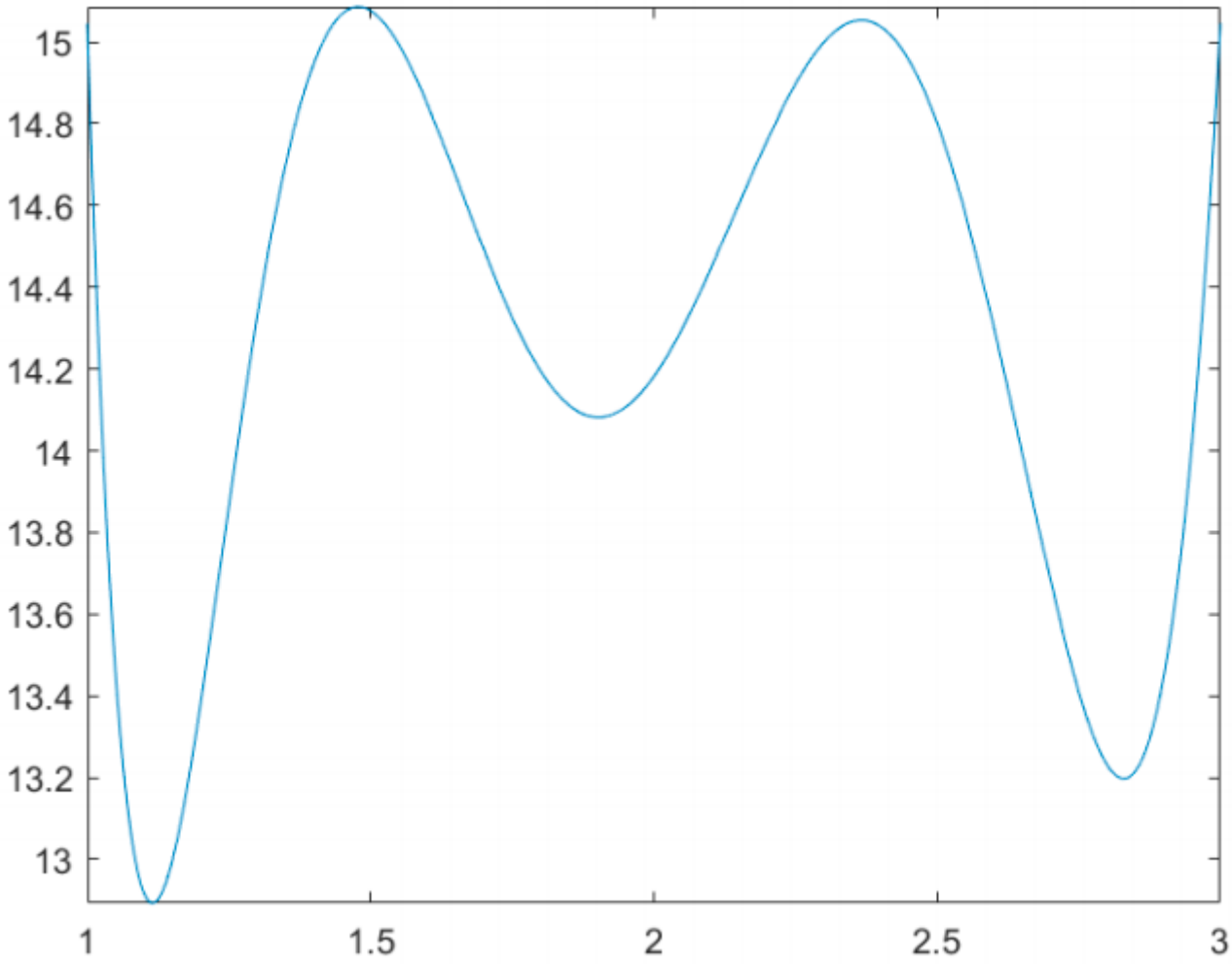

Figure 3

Sensitivity function of the CSO-generated design for model (17) with uncorrelated random effects in Table 3.

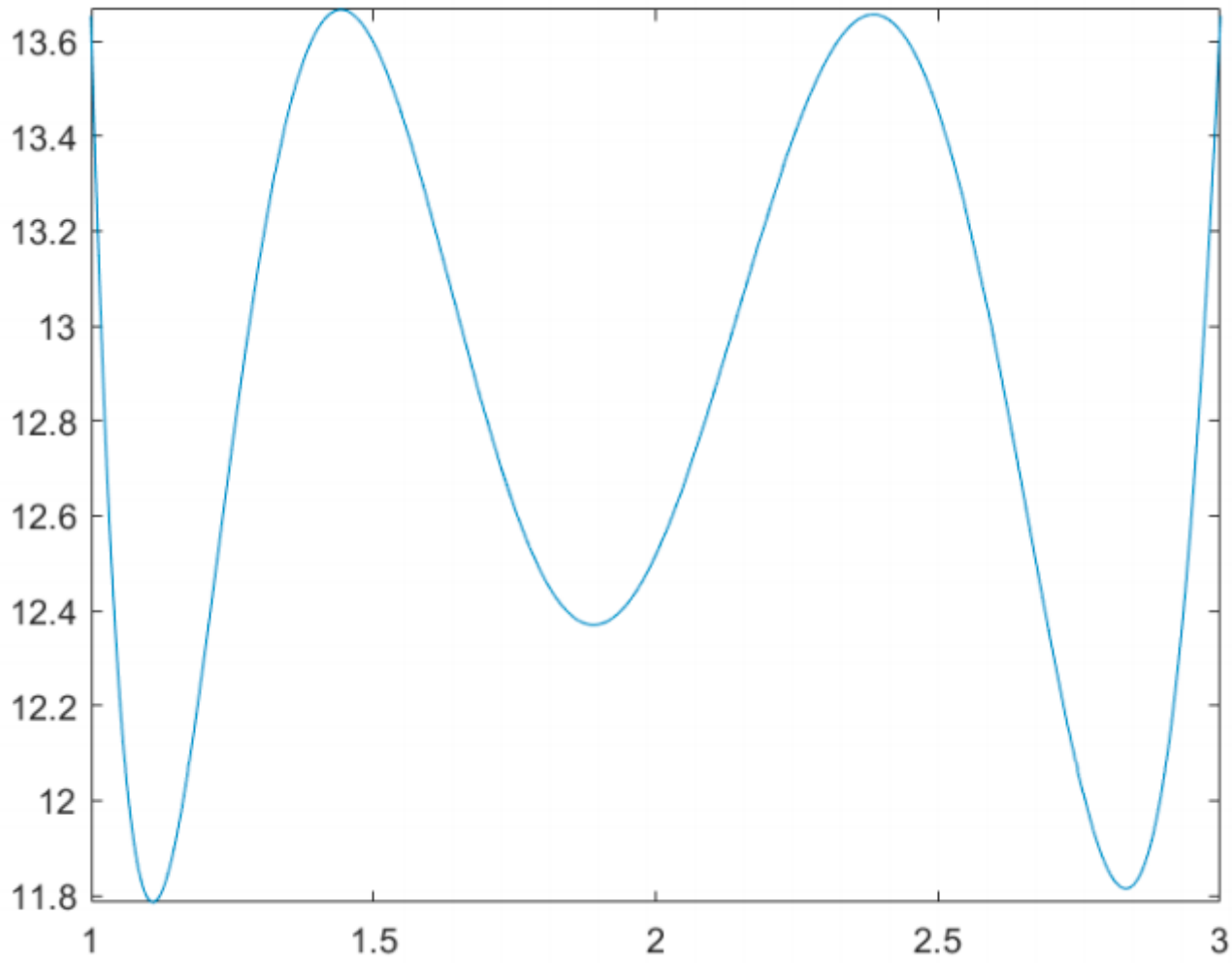

Figure 4

Sensitivity function of the CSO-generated design for model (19) with correlated random effects in Table 4.

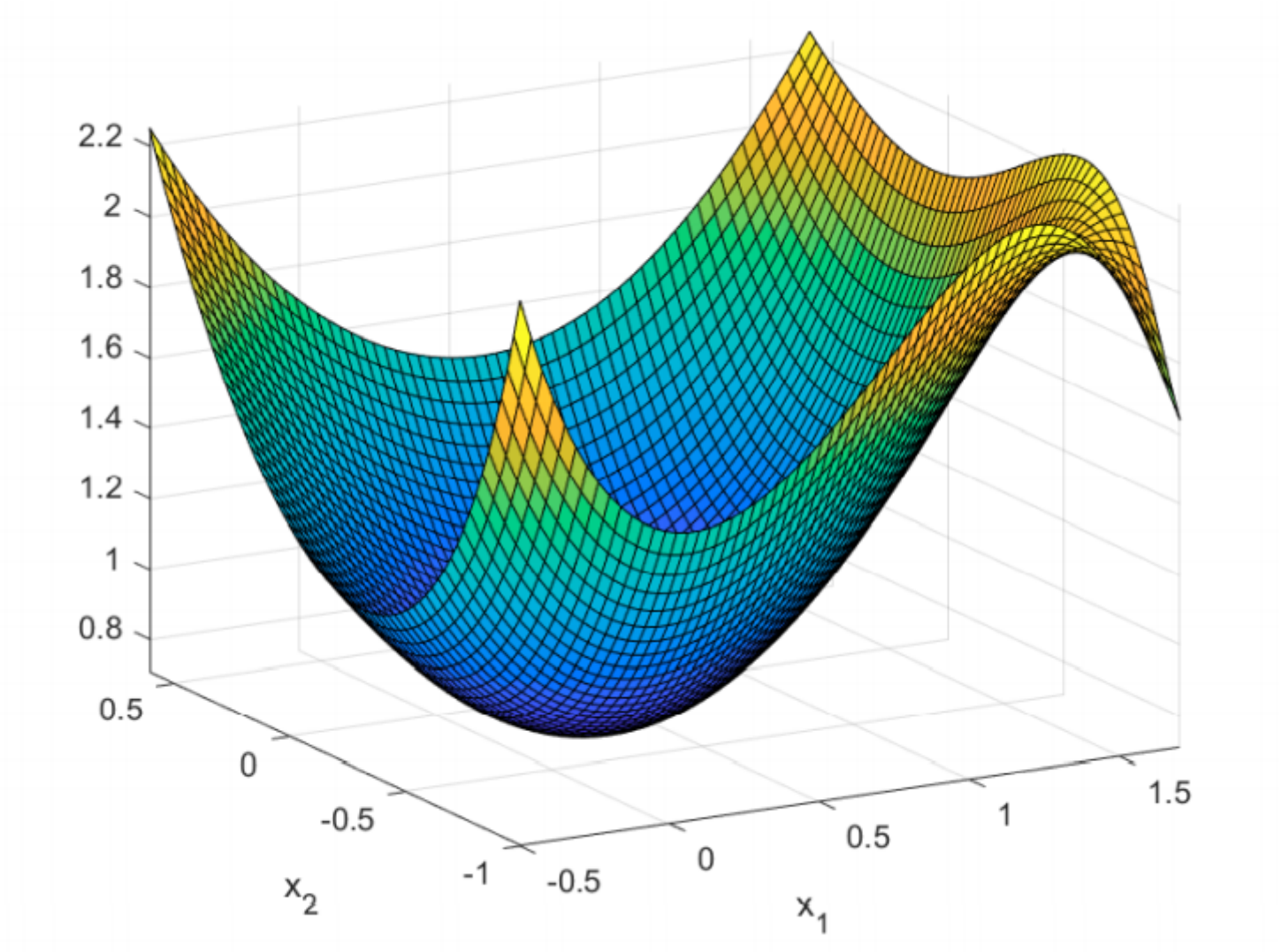

Figure 5

The sensitivity function of the CSO-generated design under the D-optimality criterion for the two-factor Poisson model with an interaction term and uncorrelated random effects shown in Table 5.

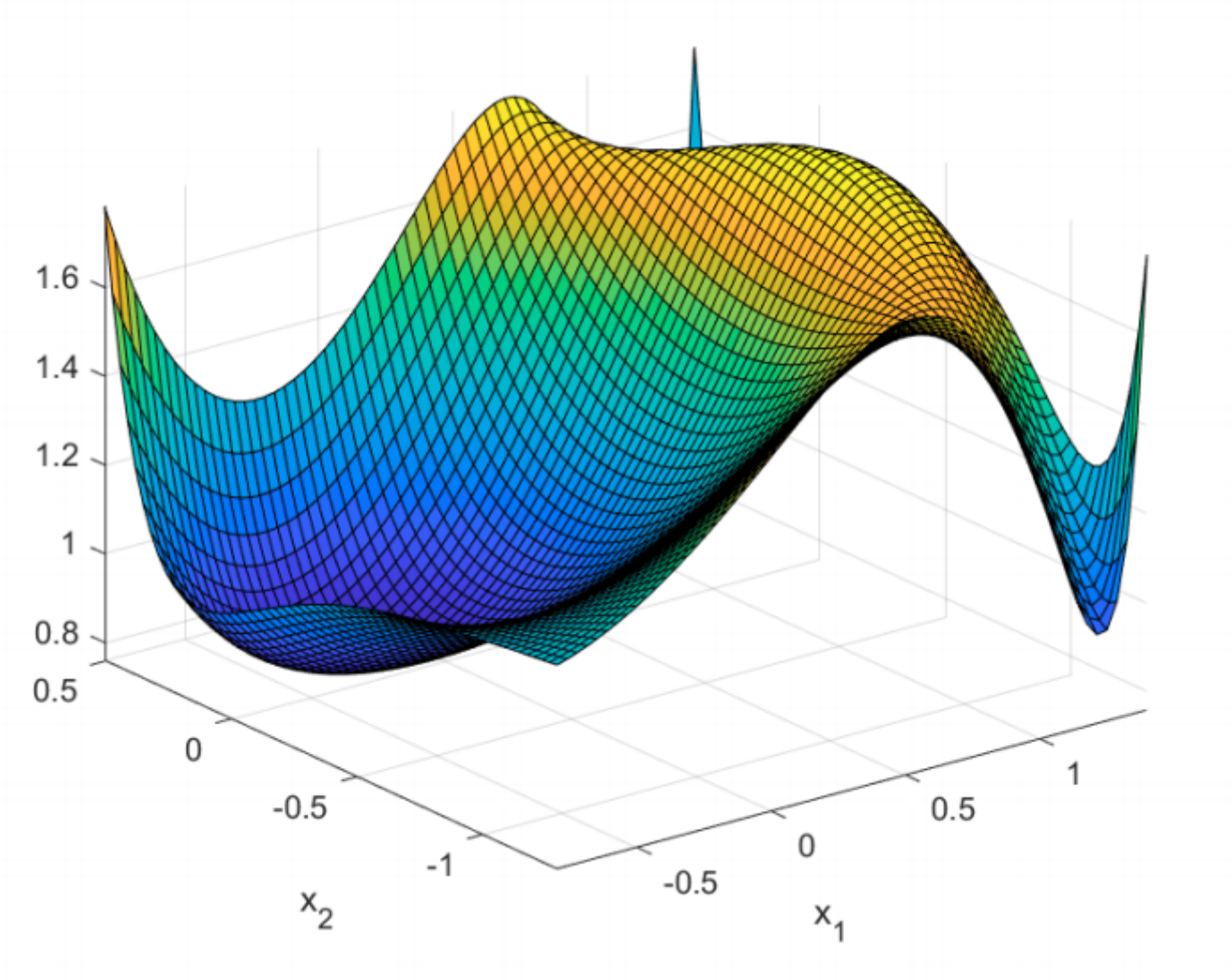

Figure 6

The sensitivity function of the CSO-generated design under the D-optimality criterion for a two-factor Poisson model with an interaction term and correlated random effects shown in Table 6.