

# Stability and Time-Delay Effect of Rainfall-induced Landslide Considering Air Entrapment

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## Research Letter

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1 **Stability and time-delay effect of rainfall-induced landslide**  
2 **considering air entrapment**

3

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## Abstract

10  
11 Rainfall-induced landslide is a typical geological disaster in the Three Gorges reservoir area.  
12 The air entrapment in the pores of soils has a hindrance to the infiltration of the slope. It is  
13 mainly reflected in the hydraulic hysteresis after rainfall and the decrease of the slope anti-  
14 sliding force. A method considered the air entrapment of the closed gas in soil particles' pores  
15 is developed to study the time-delay effect and slope stability under the rainfall process. The  
16 Green-Ampt infiltration model is used to obtain the explicit analytical solution of the slope  
17 infiltration considering air entrapment. Moreover, the relationship between the safety factor,  
18 the rainfall duration, and the depth of the wetting front under the three rainfall conditions  
19 ( $q_{rain}=12, 26, 51$  mm/h) is discussed. The results show that the air entrapment causes a  
20 significant time-delay effect of the landslide, and the hydraulic hysteresis is the strongest under  
21 the condition of heavy rainfall ( $q_{rain}= 51$ mm/h). The time-delay effect lasts longer than low  
22 rainfall and heavy rainfall when the rainfall intensity ( $q_{rain}= 26$  mm/h) is slightly greater than  
23 saturated hydraulic conductivity  $K_s$ . Parameter analysis shows that when air entrapment is  
24 considered, the smaller the slope angle and the effective internal friction angle, the more  
25 significant the air entrapment has on the slope stability; the smaller the effective cohesion, the  
26 longer the air resistance lasts. Finally, the application of the Bay Area landslide is consistent  
27 with the actual state of the landslide.

28 **Keywords:** rainfall-induced landslides; infinite slope; Green-Ampt infiltration model; air  
29 entrapment; hydraulic hysteresis

30

## 31 **1 Introduction**

32         Rainfall-induced landslides are the most common type of landslide disaster (Higgitt et al.  
33 [2014](#); Li et al. [2020a](#); [2020b](#)). The rainfall-induced landslide caused devastating disasters to  
34 mountain residents, such as many economic losses and casualties (Saito et al. [2017](#); Fustos et  
35 al. [2020](#); Yang et al. [2020](#)).

36         The Green-Ampt (GA) infiltration model is similar to Darcy's law (Huo et al. [2020](#)), so  
37 the model has a clear physical meaning and good scalability. Since it was proposed, it has been  
38 widely used in infiltration research. Many studies have made various amendments and  
39 supplements on the application scope and conditions of the GA model, such as determining  
40 hydraulic parameters, the approximate solution of the theory, and the error analysis. The  
41 modified GA infiltration model was applied to the infiltration of stable rainfall conditions (Mein  
42 and Larson [1973](#)), one of the widely recognized modified GA models. The recommended  
43 extension model is also applied in this paper. It is worth noting that the GA infiltration model  
44 in slope infiltration needs to overcome the defect which is not suitable for slope. Therefore, an  
45 improved GA model suitable for slope rainfall infiltration was proposed (Chen and Young  
46 [2006](#)), and the sensitivity analysis of geometric parameters was carried out. The optimal  
47 parameters of the GA model for the sand, sandy clay, and clay under the rainfall were obtained  
48 by solving the objective function of the GA optimization model and Richards' equation, which  
49 provided an essential reference for the application of the GA infiltration model in slopes (Chen  
50 et al. [2015](#)). In general, the GA model is used with the infinite slope model. The application  
51 range of the improved method has been extended to landslides all over the world (e.g.,

52 [Muntohar and Liao 2009, 2010; Zhang et al. 2017; Wu et al. 2018](#)). The rainfall threshold is  
53 used as one of the indicators of rainfall-induced landslides, and it is also a standard tool for  
54 expressing the time portion of landslide hazard analysis ([Martinović et al. 2018](#)). Therefore, the  
55 saturated hydraulic conductivity is usually used as the critical rainfall intensity threshold for the  
56 ponding water on the slope. In this paper, three rainfall intensity conditions are designed to  
57 study the influence of air entrapment on the stability of infinite slopes during rainfall.

58 The rainfall infiltration process and the slope failure are not entirely synchronized for the  
59 rainfall-induced landslide. Compared with rainfall, landslide hazards often have hysteresis and  
60 have a noticeable time-delay effect. The time-delay effect is mainly caused by the air  
61 entrapment during the rainfall process ([Lu and Likos 2006; Lu et al. 2013; Chen et al. 2015](#)).  
62 The effect of air pressure in the soil on the infiltration when the pores are closed was verified  
63 by Wang et al. ([1997, 1998](#)) through laboratory seepage tests. Test results pointed out that when  
64 the air overcomes the critical air pressure and breaks through the saturated soil layer and escapes  
65 the atmosphere, the pore air pressure of the unsaturated soil layer decreases. Unfortunately,  
66 Wang's method is mainly used in agricultural irrigation ([Vereecken et al. 2019; Gonçalves et  
67 al. 2020](#)). A laboratory test suitable for the Masa sandy loam soil from weathered granite rock  
68 and a TUAT light clay and soils from volcanic ash considering the influence of air entrapment  
69 on hydraulic conductivity was performed ([Sakaguchi et al. 2005](#)). Exhausted the air between  
70 pores in a vacuum environment, and then immerse in the water again to trap the air. The  
71 hydraulic conductivity measured by the falling head method decreases as the air content  
72 increases. An approach to combine the generalized stress framework with the suction stress

73 retention and the suction stress characteristic curve was proposed (Lu and Godt 2008; Chen et  
74 al. 2017). The modified method indicated that it is necessary to consider the hydraulic hysteresis  
75 caused by air entrapment in rainfall-induced landslides. Otherwise, the failure possibility of the  
76 infinite slope will often be underestimated. The hydraulic hysteresis of the wetting front is  
77 verified by field and laboratory experiments (Ebel et al. 2018), and the difference in soil  
78 between dry and wet conditions is critical for rainfall-induced landslides.

79 The existed method does not quantify the effect of air entrapment in the strength failure  
80 criteria. Therefore, the objective of this paper is to develop a novel method based on Wang's  
81 air entrapment theory and the infinite slope model to simulate the infiltration in slopes and  
82 analyze the stability of rainfall-induced landslides. The rest of this paper is organized as follows:

83 Section 2 introduced the improved GA model considering the air entrapment and the  
84 stability analysis of an infinite slope. The infiltration stage is divided into the unsaturated  
85 infiltration stage before ponding water and air entrapment infiltration after ponding water. In  
86 Section 3, verification and parameter analysis of the proposed method were carried out through  
87 an infinite slope example. The proposed method was applied to a landslide located in Alameda  
88 County, California, in Section 4. Finally, the conclusions were drawn in Section 5.

## 89 **2 Method**

### 90 **2.1 Assumptions**

91 Rainfall-induced landslides have been an essential issue that geotechnical engineers have  
92 been paying attention to for a long time in geological disasters. However, the actual slope is  
93 often complicated. Therefore, in order to facilitate the solution, the infinite slope is usually used

94 as the study object.

95 There is a prominent wetting front in the slope in the rainfall infiltration. The known  
96 conditions parameters are: the rainfall intensity  $q_{rain}$ ; saturated hydraulic conductivity  $K_s$ ; soil  
97 suction  $\psi_f$ ; slope angle  $\alpha$ ; atmospheric pressure  $P_0$ . And soil mechanical parameters are: the  
98 saturated unit weight  $\gamma_s$ , effective cohesion  $c'$ ; effective internal friction  $\phi'$ ; saturated moisture  
99 content  $\theta_s$ ; initial moisture content  $\theta_i$ .

100 Some reasonable assumptions about the infiltration model were used in previous studies,  
101 and their rationality had proved (Chen and Young 2006; Huang et al. 2008; Cevalco et al. 2014;  
102 Dolojan et al. 2021; Zhang et al. 2021). Therefore, before deriving the model, the following  
103 assumptions are made in this paper:

104 (1) The rainfall is uniform, that is, the total amount of the rainfall is constant;

105 (2) When  $q_{rain} > K_s$ , the soil above the wetting front is saturated. Unsaturated soil with initial  
106 moisture content below the wetting front. In contrast, when  $q_{rain} < K_s$ , the soil is unsaturated.

107 (3) The bottom of the slope is regarded as impermeable bedrock, and the impact of  
108 groundwater on infiltration is not considered.

109 (4) The modified GA model considering the air entrapment and proposed stability analysis  
110 method is suitable for soil slopes or accumulative slopes.

## 111 **2.2 Non-ponding water stage**

112 At the beginning of rainfall, unsaturated soils absorb water. According to the GA model,  
113 the rate of water infiltration  $q_{w1}$  in the slope determined by the rainfall intensity  $q_{rain}$  is:

$$114 \quad q_{w1} = q_{rain} \cos \alpha \quad (1)$$

115 During the infiltration process, the air in the soil escapes freely from the slope surface as  
116 the wetting front propagates downward. At this time, the pore air pressure in the saturated soil  
117 layer is not compressed, so there is no air entrapment. Therefore, when the  $q_{rain} < K_s$ , the  
118 traditional Green-Ampt infiltration model (del Vigo et al. 2021) can be used to obtain the  
119 amount of infiltration:

$$120 \quad F = \int_0^{z_w} [\theta_s - \theta_i] dy \quad (2)$$

121 where,  $F$  represents cumulative infiltration amount during rainfall;  $\theta_s$  and  $\theta_i$  represent the  
122 saturated moisture content and the initial moisture content, respectively.

123 The differentiation of rainfall concerning time  $t$  is the rainfall intensity (or infiltration rate),  
124 so the function of the depth of the wetting front and the rainfall intensity (or infiltration rate)  
125 can be obtained:

$$126 \quad \frac{dz_w}{dt} = \frac{q_{rain} \cos \alpha}{\theta_s - \theta_i} \quad (3)$$

127 In addition, a special infiltration situation which is no ponding water on the slope surface,  
128 also needs to be considered. When  $q_{rain} < K_s$ , the soil in the slope is always unsaturated. Therefore,  
129 it is necessary to solve the soil volumetric moisture content in the unsaturated state, and an  
130 improved solution based on the Mein-Larson rainfall infiltration model was used to obtain the  
131 volumetric water content by solving the pressure head above the wetting front (Almedeij and  
132 Esen 2014). Van Genuchten (VG model) soil-water characteristic curve is usually used to  
133 describe the unsaturated hydraulic characteristics of soil (van Genuchten 1980). The VG model  
134 can be written:

135 
$$\theta(h) = \begin{cases} \theta_r + \frac{\theta_s - \theta_r}{[1 + (\alpha|h|)^n]^m}, & h < 0 \\ \theta_s, & h > 0 \end{cases} \quad (4)$$

136 where,  $\theta(h)$  is the volumetric moisture content function;  $\theta_r$  is the residual moisture content;  $h$  is  
 137 the pressure water head;  $\alpha$  and  $n$  are suggested parameters (in this paper,  $\alpha=3.5 \text{ m}^{-1}$  and  $n=1.5$ ),  
 138 and  $m=1-1/n$ .

139 The rainfall intensity  $q_{rain}$  in the unsaturated infiltration can be obtained:

140 
$$q_{rain} = K_s \frac{\{1 - (\alpha|h_w|)^{n-1} [1 + (\alpha|h_w|)^n]^{-m}\}^2}{[1 + (\alpha|h_w|)^n]^{m/2}} \quad (5)$$

141 where,  $h_w$  is the pressure water head when the moisture content reaches  $\theta_w$ .

142 By solving Eq. (5), the volumetric moisture content when the pressure water head is  $h_w$   
 143 can be obtained. Then according to Eq. (2), the explicit expression of the infiltration depth  $z_w$   
 144 and rainfall duration when  $q_{rain} < K_s$  is:

145 
$$z_w = \frac{q_{rain} t}{\theta_w - \theta_i} \quad (6)$$

### 146 2.3 Ponding water stage

147 According to Darcy's law, the rate of water infiltration  $q_{w2}$  when ponding water on the  
 148 slope surface is:

149 
$$q_{w2} = K_s \frac{\psi_f + z_w \cos \alpha + h_0 - h_f}{z_w} \quad (7)$$

150 where,  $K_s$  represents the saturated hydraulic conductivity;  $h_{af}$  represents the air pressure below  
 151 the wetting front;  $\psi_f$  represents the suction.

152 When water is accumulated on the slope surface, the critical wetting front depth  $z_p$  is  
 153 expressed as:

154 
$$z_p = K_s \frac{\psi_f + h_0}{\cos \alpha (q_{rain} - K_s)} \quad (8)$$

155 For the vertical one-dimensional infiltration  $z_w-t$  of homogeneous soil, the formula can be  
 156 written:

157 
$$q_{w3} \cdot dt = dF = \Delta\theta \cdot dz_w \quad (9)$$

158 where,  $\Delta\theta = \theta_s - \theta_i$ ;  $F$  is the cumulative infiltration volume;  $q_{w3}$  is the infiltration rate after  
 159 ponding water.

160 Therefore, the critical time  $t_p$  can be obtained is:

161 
$$t_p = K_s \frac{\Delta\theta(\psi_f - h_0)}{\cos^2 \alpha q_{rain} (q_{rain} - K_s)} \quad (10)$$

162 Substituting Eq. (10) into Eq. (9), the differentiation of the slope wetting front depth to the  
 163 rainfall duration can be obtained:

164 
$$\frac{dz_w}{dt} = \begin{cases} \frac{q_{rain} \cos \alpha}{\Delta\theta}, t \leq t_p \\ K_s \frac{\psi_f + \cos \alpha z_w + h_0 - h_{af}}{z_w \Delta\theta}, t \geq t_p \end{cases} \quad (11)$$

165 The analytical solution between the depth and duration of the wetting front is obtained by  
 166 solving Eq. (11):

167 
$$t = \begin{cases} \frac{z_w \Delta\theta}{q_{rain} \cos \alpha}, z_w \leq z_p \\ t_p + A - B, z_w > z_p \end{cases} \quad (12)$$

168 where,  $A$  and  $B$  can be drawn:

169 
$$\begin{cases} A = \frac{\Delta\theta(z_w - z_p)}{K_s \cos \alpha} \\ B = \frac{\Delta\theta(\psi_f + h_0 - h_{af}')}{K_s \cos \alpha} \ln\left(\frac{\psi_f + z_w \cos \alpha + h_0 - h_{af}'}{\psi_f + z_p \cos \alpha + h_0 - h_{af}'}\right) \end{cases} \quad (12a)$$

170 In the ponding water infiltration stage, due to the existed ponding water on the slope  
 171 surface, the porous air cannot escape from the slope surface during the downward movement  
 172 of the wetting front. Therefore, the air is compressed in the pores of soils under the wetting  
 173 front. When the air pressure in the pores exceeds the critical pressure, the air will break through  
 174 the water pressure and escape. Since the conductivity of air in porous media is better than that  
 175 of water, the pore pressure will decrease rapidly after the air escapes. The schematic diagram  
 176 of air compression and escape process is shown in Fig. 1.

177 The air-breaking value  $H_b$  and the air-closing value  $H_c$  are suggested to be used to describe  
 178 the pressure head  $h_{af}$  in the pore (Wang et al. 1997), and the  $H_b$  and  $H_c$  can be expressed as:

$$179 \quad H_b = h_0 + z_w + h_{ab} \quad (13)$$

$$180 \quad H_c = h_0 + z_w + h_{wb} \quad (14)$$

181 where,  $h_{ab}$  is the air-bubbling capillary pressure value;  $h_{wb}$  is the water-bubbling value.

182  $H_b$  and  $H_c$  represent the maximum value that is about to break through the slope area and  
 183 the minimum value of air pressure after the air escapes, respectively. In the air compression  
 184 process, when the air pressure is maximum, an air-flow barrier will be formed. The infiltration  
 185 rate  $q_{wmin}$  in this state is 0 m/h. According to Eq. (7), the infiltration depth  $z_0$  when the air  
 186 pressure reaches the maximum value can be obtained:

$$187 \quad z_0 = \left[ \frac{1}{2} (h_b^2 + h_0^2 + h_{wb}^2 + B^2) + h_b h_0 + h_b h_{wb} + h_0 h_b + h_{wb} B + h_0 B - h_b B \right]^{\frac{1}{2}} - \frac{1}{2} (h_b + h_0 + h_{wb} + B) + z_p \quad (15)$$

188 where,  $B$  is the depth of air-flow barrier below the soil surface, and it was measured by Wang  
 189 et al (1997).

190 The infiltration time at the minimum infiltration rate (maximum pore pressure) is:

191 
$$t_0 = \frac{z_0}{K_s} + t_p \quad (16)$$

192 After the air escapes, the pore pressure quickly drops to the minimum and the infiltration  
 193 rate increases to a maximum. The maximum infiltration rate  $q_{wmax}$  can be expressed as:

194 
$$q_{wmax} = K_s \frac{h_{ab} - h_{wb}}{z_w} \quad (17)$$

195 Regarding the infiltration rate  $q_{wair}$  under air entrapment as a linear change, it can be drawn:

196 
$$q_{wair} = \frac{q_{wmin} + q_{wmax}}{2} = \frac{K_s (h_{ab} - h_{wb})}{2z_w} \quad (18)$$

197 When the air breaks through the ponding water, the infiltration duration  $t$  can be expressed  
 198 as:

199 
$$t = t_0 + \frac{z_w^2 - z_0^2}{K_e (h_{ab} - h_{wb})} \quad (19)$$

200 where,  $K_e$  is the effective permeability coefficient,  $K_e = z_0/t_0$ .

201 
$$z_w = [z_0^2 + K_e (h_{ab} - h_{wb})(t - t_0)]^{1/2} \quad (20)$$

202 In summary, three stages are mainly considered in the infiltration process: 1) non-ponding  
 203 water stage. When  $q_{rain} < K_s$ , the unsaturated soil is considered in the infiltration. The Eq. (6) is  
 204 used to solve the infiltration duration  $t$ ; 2) early ponding water stage. The air entrapment tends  
 205 to the maximum limit value. After the critical time  $t_p$ , the air entrapment is maximized after the  
 206 time  $t_0$ , and then the air escapes from the slope surface. Eq. (12) is used to obtain the depth of  
 207 the wetting front. 3) conventional ponding water stage. A linear function is used to describe the  
 208 change of air entrapment in the conventional ponding water stage and Eqs. (19)-(20) is used to  
 209 calculate the infiltration time and wetting front depth.

## 210 2.4 Safety factor of the infinite slope

211 For infinite slopes, the decrease of suction at the wetting front leads to a decrease in shear  
212 strength. The soil in the upper zone of the wetting front increases its unit weight due to the  
213 change in moisture content. Therefore, the critical slip surface is often the wetting front.  
214 According to the Mohr-Coulomb criterion and the shear strength theory of cohesive unsaturated  
215 soils (Duncan et al. 2014; Renani and Martin 2020):

$$216 \quad FOS = \frac{(\sigma_n - u) \tan \varphi' + c'}{\tau} \quad (21)$$

217 where,  $\sigma_n$  is the normal stress at the bottom of the slope;  $\tau$  is the sliding force;  $c'$  is the effective  
218 cohesion;  $\varphi'$  is the effective internal friction.

219 The normal stress  $\sigma_n$  and sliding force  $\tau$  can be calculated according to the following  
220 formulas:

$$221 \quad \sigma_n = \gamma_s z_w \cos^2 \alpha + \gamma_w h_0 \cos^2 \alpha \quad (22)$$

$$222 \quad \tau = \gamma_s z_w \cos \alpha \sin \alpha \quad (23)$$

223 In an infinite slope, the normal force and shear force between the two sides of the soil slice  
224 are both 0. The weight of soil slice  $W$  above the wetting front should be calculated using  
225 saturated unit weight  $\gamma_s$ :

$$226 \quad W = \gamma_s z_w \cos \alpha \quad (24)$$

227 After ponding water on the slope surface, the pressure on the slope surface  $G_w$  is:

$$228 \quad G_w = \gamma_w h_0 \cos^2 \alpha \quad (25)$$

229 During the downward movement of the wetting front, the pore air in the unsaturated soil  
230 layer below the wetting front is compressed. It can be seen from Fig. 1 that a water-air interface

231 is formed at the wetting front. The compressed pore air pressure supports the upper saturated  
 232 soil layer and slows down the tendency of infiltration. The supporting force of the water at the  
 233 wetting front can be expressed as:

$$234 \quad P_a = \gamma_w h_{af}' \cos^2 \alpha \quad (26)$$

235 where,  $h_{af}'$  is the average air pressure head,  $h_{af}' = 1/2(h_{ab} + h_{wb})$ .

236 When the air in the lower zone of the wetting front is restricted, Eq. (21) can be written as:

$$237 \quad FOS = \frac{c' + (\sigma_n - P_a) \tan \varphi' + \gamma_w \psi_f \tan \varphi^b}{\tau} \quad (27)$$

238 Substituting Eqs. (22)-(26) into Eq. (27), the slope safety factor considering air entrapment  
 239 can be obtained:

$$240 \quad FOS = F_1 + F_2 + F_3 \quad (28)$$

$$241 \quad \text{where, } F_1 = \frac{c'}{\gamma_s z_w \cos \alpha \sin \alpha} \square$$

$$242 \quad F_2 = \frac{(\gamma_s z_w - \gamma_w h_{af}') \tan \varphi'}{\gamma_s z_w \tan \alpha} \square$$

$$243 \quad F_3 = \frac{\gamma_w \psi_f \tan \varphi^b}{\gamma_s z_w \cos \alpha \sin \alpha}$$

244 It can be drawn from Eq. (28) that the safety factor of the infinite slope considering the air  
 245 entrapment during the infiltration process is mainly composed of three parts. The first part  $F_1$   
 246 is mainly determined by the effective cohesion, the second part  $F_2$  is mainly determined by the  
 247 effective internal friction angle and the air entrapment, and the third part  $F_3$  is mainly related to  
 248 the suction of the wetting front. Substituting Eqs. (6), (12), (19) into Eq. (28) can get the analytic  
 249 solution of the slope safety factor.

### 250 **3 Infinite slope cases**

251 The shallow infinite slope is used as the object of the calculation example. The detailed  
252 slope geometric parameters and mechanical parameters are shown in [Table 1](#). The detailed  
253 hydrological parameters are shown in [Table 2](#). In this paper, three rainfall conditions ( $\alpha=40^\circ$ ,  
254  $\varphi'=36^\circ$ ,  $c'=3$  kPa) are designed based according to the Green-Ampt model. Condition 1 is  
255 rainfall intensity of 12 mm/h, simulating low rainfall; Condition 2 is the rainfall intensity of 26  
256 mm/h, which is slightly larger than the saturated permeability coefficient of the soil; Condition  
257 3 is the rainfall intensity of 51 mm/h, which simulates heavy rainfall and can reflect the time-  
258 delay effect. The infiltration process and slope stability analysis under three rainfall conditions  
259 to study the air entrapment were conducted. The calculation method based on the limit method  
260 proposed by Zhang et al. ([2011](#)) is used for the rainfall-induced slope without considering the  
261 air entrapment.

### 262 **3.1 Hydraulic hysteresis caused by air entrapment**

263 For the convenience of description, the five cases under the three conditions in this paper  
264 are recorded as Case 1 ( $q_{rain}=0.012$  m/h), Case 2 ( $q_{rain}=0.026$  m/h with air entrapment), Case 3  
265 ( $q_{rain}=0.026$  m/h without air entrapment), Case 4 ( $q_{rain}=0.051$  m/h with air entrapment), Case 5  
266 ( $q_{rain}=0.051$  m/h without air entrapment).

267 The change curves of the depth of the wetting front with the rainfall duration under the  
268 three rainfall conditions are shown in [Fig. 2](#). The rainfall intensity  $q_{rain}$  of Condition 1 (Case 1)  
269 is 12 mm/h, which is less than the saturated hydraulic conductivity  $K_s$ , so there will be no air  
270 entrapment in Condition 1. It can be concluded from [Fig. 2](#) that the wetting front of Condition  
271 3 (Cases 4 and 5) moves down the fastest. When the depth of the wetting front is less than the

272 critical depth of the wetting front, the migration curve of the wetting front almost overlaps in  
273 the two cases. This is because the rate of water infiltration is more significant in the early stage  
274 of infiltration than the rainfall intensity. No water is formed on the slope surface, and the  
275 rainwater is not affected by the air entrapment during the infiltration. When the rainfall duration  
276 exceeds  $t_p$ , air entrapment needs to be considered. Evident hydraulic hysteresis appears when  
277 considering the air entrapment in Condition 2 (Cases 2 and 3) and Condition 3 (Cases 4 and 5).  
278 The time to reach a specific infiltration depth when the infiltration depth increases from 2 m to  
279 3 m under Condition 2 (Cases 2 and 3) is given in [Table 3](#). The  $\Delta t$  at the wetting front is 2 m,  
280 2.25 m, 2.50 m, 2.75 m, and 3.00 m in the proposed model and the traditional infiltration model  
281 are 0.45 h, 29.27 h, 63.45 h, 101.69 h, and 143.99 h, respectively. It can be seen from [Table 3](#)  
282 that the air entrapment has a significant hydraulic hysteresis on the infiltration process.  
283 However, in Condition 2, the hydraulic hysteresis in the infiltration process is more significant  
284 than Condition 3. When the rainfall intensity is slightly larger than the saturated hydraulic  
285 conductivity, the air entrapment will last longer. In rainfall-induced landslides, short-duration,  
286 high-intensity rainfall is often more likely to cause shallow landslides ([Larsen and Simon 1993](#)).  
287 Therefore, attention should be paid to the impact of short-duration, high-intensity rainfall on  
288 landslides.

289 [Fig. 3](#) shows the change curves of safety factors and rainfall duration in five cases. It can  
290 be seen from [Fig. 3](#) that under Case 4 and Case 5 (heavy-rainfall), the decline rate of the slope  
291 safety factor is significantly greater than that under Case 1 (low-intensity rainfall) and Case 2  
292 and Case 3 (moderate-intensity rainfall). Moreover, there are apparent time-delay effects in

293 Case 4 and Case 5. When the safety factor is reduced to the same, it takes significantly longer  
294 to consider the slope of air entrapment. Furthermore, with the progress of the rainfall process,  
295 this time is still increasing. It is worth noting that when the safety factor of the slope without  
296 considering the air entrapment is lower than 1.0, the safety factor of the slope considering the  
297 air entrapment is still greater than 1.0. This time difference is critical for disaster decision-  
298 making.

### 299 **3.2 Parametric analysis**

300 The rainfall intensity has a significant effect on the stability of soil slopes. In Condition 3,  
301 the infinite slope stability during the rainfall process is analyzed in this paper. Furthermore, the  
302 impacts of different slope angles, effective cohesion, and effective internal friction angle on the  
303 safety factor are compared. In Condition 3, the critical infiltration depth is small when the air  
304 entrapment is exerted due to the greater rainfall intensity. In addition, the critical wetting front  
305 depth is 0.07 m, so it is not additionally marked in Fig. 3. It can be seen from Fig. 4 that when  
306 the depth of the wetting front exceeds 1.99 m and 2.13 m, respectively, the safety factors of  
307 Case 3 and Case 4 are lower than 1.0, and they are in an instability state. Furthermore, Case 5  
308 is in an unstable state on the slope, and its safety factor has fallen faster than Case 4. This is  
309 caused by the gradual weakening of the air entrapment in the later stage of infiltration.

#### 310 (a) Slope angle

311 Under Condition 3, the effect of different slope angles ( $\alpha=20^\circ$ ,  $40^\circ$ , and  $60^\circ$ ) on the safety  
312 factor is studied. It can be seen from Fig. 5 that the slope angle has a significant impact on the  
313 safety factor. With the increase of the slope angle, the decrease of the safety factor is pronounced.

314 The smaller the slope angle, the more obvious the hysteresis in the early stage of infiltration. It  
315 is worth noting that as the slope angle increases, the hysteresis caused by air entrapment  
316 gradually weakens. This is because the larger slope leads to a faster infiltration process.

317 (b) Effective cohesion

318 [Fig. 6](#) shows the effect of different effective cohesion ( $c' = 3, 8, 13$  kPa) on the safety factor  
319 in Condition 3. Other parameters are consistent with those given in [Section 3](#) and [Table 1](#) and  
320 [2](#). It can be seen from [Fig. 6](#) that the effective cohesion also has a significant impact on the  
321 safety factor. Considering the air entrapment in the early stage of infiltration will obtain a lower  
322 safety factor. At the same time, it will also have an obvious hydraulic hysteresis. By comparing  
323 [Fig. 5](#) and [Fig. 6](#), it can be found that the effective cohesion is more sensitive to the safety factor  
324 in the early stage of infiltration than the slope angle. This is because as the saturation unit weight  
325 of the upper soil layer increases, the role played by the effective cohesion gradually decreases.

326 (c) Effective internal friction angle

327 [Fig. 7](#) shows the effect of different effective internal friction angles ( $\varphi' = 24^\circ, 36^\circ, 48^\circ$ ) on  
328 the safety factor in Condition 3. Other parameters are consistent with those given in [Section 3](#)  
329 and [Table 1](#) and [2](#). It can be seen from [Fig. 7](#) that as the infiltration duration increases, the effect  
330 of the internal friction angle on the safety factor becomes more evident. The safety factor of  
331 slopes with low internal friction angles decreases with the increase of the depth of the wetting  
332 front slightly faster than slopes with high internal friction angles.

## 333 4 Discussion

334 In this section, an actual landslide located in Alameda County, California ([Fig. 8](#)) is used

335 to verify the results of this paper and compare with the method proposed by Chen et al. (2017).  
336 The detailed calculation parameters can be seen in Table 4. The Bay Area landslide is covered  
337 by the silt of approximately 1 m, derived from the weathering products of the underlying  
338 sandstone bedrock.

339 It can be seen from Table 5 that the proposed method is consistent with the actual state.  
340 Table 5 shows the corresponding relationship between the infiltration depth and the safety factor  
341 in the colluvial soil layer. Overall, the Bay Area landslide is still in a stable state. However,  
342 when the wetting front depth is 0.8 m, the landslide is in an under-stable state. This result is  
343 also consistent with the current state of the landslide (Chen et al. 2017). At the same time, the  
344 safety factor calculated by the proposed method is lower than Chen's method. The normal stress  
345 at the bottom of the soil slice and the anti-sliding force is reduced by the air entrapment. When  
346 the rainfall intensity is much greater than the saturated hydraulic conductivity of the soil, the  
347 air entrapment has a significant impact on slope stability. This is due to the rapid development  
348 of the rainfall infiltration process, and the air below the saturated soil layer is rapidly  
349 compressed, resulting in a significant hydraulic hysteresis in a short time. At the beginning of  
350 rainfall, the air entrapment is sufficient to affect the normal stress  $\sigma_n$  at the wetting front. As the  
351 infiltration progresses, the air entrapment caused by the air between the soil particles undergoes  
352 multiple compressed-escaped to a significant attenuation, and the impact on the slope stability  
353 is gradually weak. Therefore, it is suggested that Bay Area landslide should strengthen the  
354 supervision of the landslide when the rainfall intensity  $q_{rain}$  is  $2.52 \times 10^{-7}$  m/s.

## 355 **5 Conclusions**

356 In this paper, an analytic solution of rainfall-induced landslide considering air entrapment  
357 is proposed by combining the Green-Ampt model and the unsaturated soil shear strength criterion.  
358 Furthermore, the rainfall-induced landslides stability and time-delay effect caused by hydraulic  
359 hysteresis are also studied. In summary, the following conclusions can be drawn:

360 1) According to the analytical expression form, the analytical solution of the safety factor  
361 of rainfall-induced landslides considering air entrapment is mainly composed of three parts.  
362 Therefore, the factors affecting the slope stability under the rainfall can be summarized as soil  
363 mechanical parameters, air entrapment, and suction in unsaturated soil.

364 2) Considering the air entrapment on the infiltration process, the time to reach the same  
365 depth of the wetting front is significantly later than without considering the air entrapment.  
366 Therefore, the time-delay effect of air entrapment should not be ignored in the slope stability  
367 analysis. As a result, the slope safety factor is postponed after reaching the dangerous point,  
368 providing valuable time for disaster prevention and mitigation.

369 3) For external factors, rainfall intensity determines the influence of air entrapment on  
370 slope stability. The air entrapment under heavy rainfall causes the normal stress at the wetting  
371 front to decrease rapidly at the beginning of the rainfall, and the safety factor of the slope  
372 decreases significantly. Whereby, the hydraulic hysteresis is the most evident. For rainfall  
373 intensity slightly greater than the saturated hydraulic conductivity soil, the air entrapment lasts  
374 longer. However, the impact on slope stability is lower than that of heavy rainfall.

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### 380 **Data availability statement**

381 All data generated or analyzed during this study are included within the article.

### 382 **Conflicts of interest**

383 The authors declare that they have no conflict of interest.

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476

477 **Captions of Tables and Figures**

478 **Table 1** Soil physical parameters and rainfall conditions

479 **Table 2** Hydraulic parameters

480 **Table 3** Hydraulic hysteresis comparison

481 **Table 4** Parameters for Bay Area landslide ([Chen et al. 2017](#))

482 **Table 5** The FOS of the colluvial soil layer

483 **Fig. 1** Porous air compressed-escaped process

484 **Fig. 2** Changes in the depth of the wetting front during rainfall

485 **Fig. 3** Safety factor of 5 cases

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487 **Fig. 5** Comparison of three different slope angles

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489 **Fig. 7** Comparison of three effective internal friction angles

490 **Fig. 8** Schematic diagram of the field monitoring hillslope in Alameda County, California ([Chen](#)  
491 [et al. 2017](#))

492

493

494 **Table 1** Soil physical parameters and rainfall conditions

$\gamma_d$ /(kN·m <sup>-3</sup> )	$\gamma_s$ /(kN·m <sup>-3</sup> )	$\phi^b$ /(°)	$\psi_f$ /m	Condition 1 $q_{rain}/(m \cdot h^{-1})$	Condition 2 $q_{rain}/(m \cdot h^{-1})$	Condition 3 $q_{rain}/(m \cdot h^{-1})$
16.5	21.7	6	0.06	0.012	0.026	0.051

495

496 **Table 2** Hydraulic parameters

$\gamma_w$ (kN·m <sup>-3</sup> )	$K_s$ (m·h <sup>-1</sup> )	$\theta_s$ %	$\theta_i$ %	$\theta_r$ %	$h_{ab}$ /m	$h_{wb}$ /m	$h_{af}$ /m
9.8	0.0248	45	10	1.5	1.3	0.6	0.35

497

498 **Table 3** Hydraulic hysteresis comparison

$z_w$ /m	Time required without air entrapment /h	Time required with air entrapment /h	$\Delta t$ /h
2.00	35.25	35.70	0.45
2.05	36.14	40.99	4.89
2.10	37.04	47.75	10.71
2.15	37.93	54.67	16.74
2.20	38.83	61.75	22.92
2.25	39.73	69.00	29.27
2.30	40.62	76.41	35.79
2.35	41.52	83.98	42.46
2.40	42.42	91.71	49.29
2.45	43.32	99.61	56.29
2.50	44.22	107.67	63.45
2.55	45.12	115.89	70.77
2.60	46.02	124.28	78.26
2.65	46.92	132.83	85.91
2.70	47.82	141.54	93.72
2.75	48.72	150.41	101.69
2.80	49.62	159.45	109.83
2.85	50.53	168.65	118.12
2.90	51.43	178.01	126.58
2.95	52.33	187.54	135.21
3.00	53.23	197.22	143.99

499

500



502 **Table 4** Parameters for Bay Area landslide (Chen et al. 2017)

$c'$ (kPa)	1.0
$\varphi'$ (°)	20
$\varphi^b$ (°)	6
$\alpha$ (°)	34
$\gamma_w$ (kN/m <sup>3</sup> )	9.81
$\gamma_d$ (kN/m <sup>3</sup> )	20.00
$\theta_s$	0.50
$\alpha$ (°)	34
$k_s$ (m/s)	$2.59 \times 10^{-8}$
$q_{rain}$ (m/s)	$2.52 \times 10^{-7}$
$h_{ab}$ (m)	0.54
$h_{wb}$ (m)	0.23

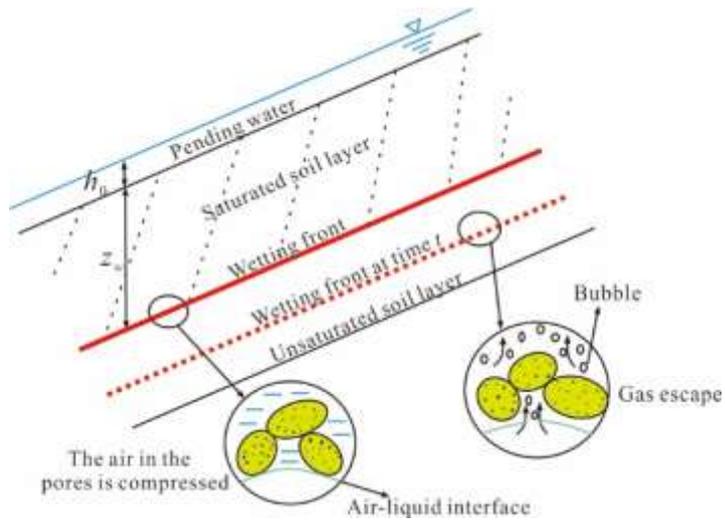
503

504 **Table 5** The FOS of the colluvial soil layer

$z_w$	FOS	
	The proposed method	Chen's method
0.2	2.10	2.34
0.3	1.88	2.11
0.4	1.63	1.87
0.5	1.30	1.53
0.6	1.18	1.42
0.7	1.09	1.35
0.8	1.05	1.33
0.9	1.03	1.31
1.0	1.01	1.28

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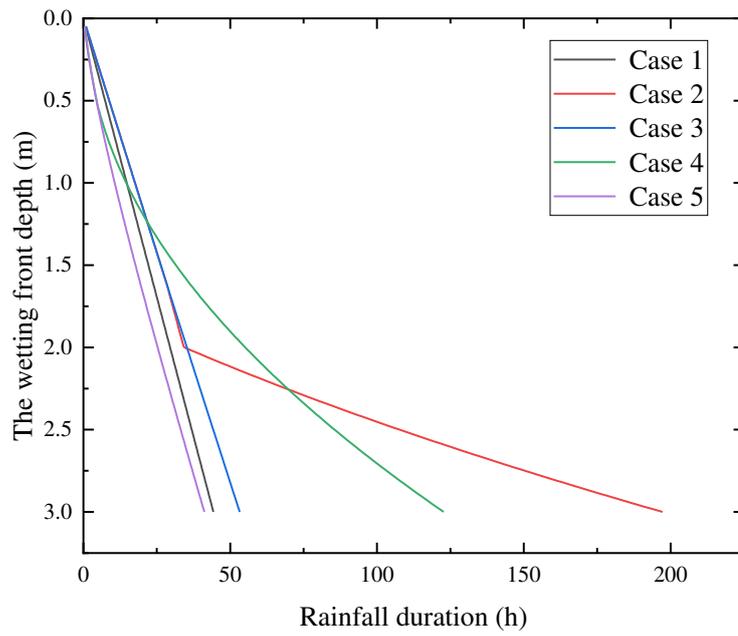


507

508 **Fig. 1** Porous air compressed-escaped process

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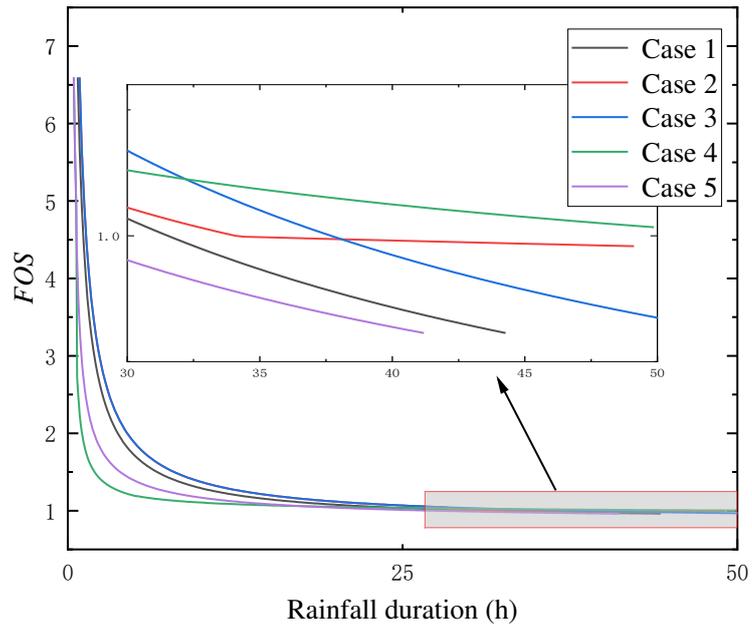


511

512 **Fig. 2** Changes in the depth of the wetting front during rainfall

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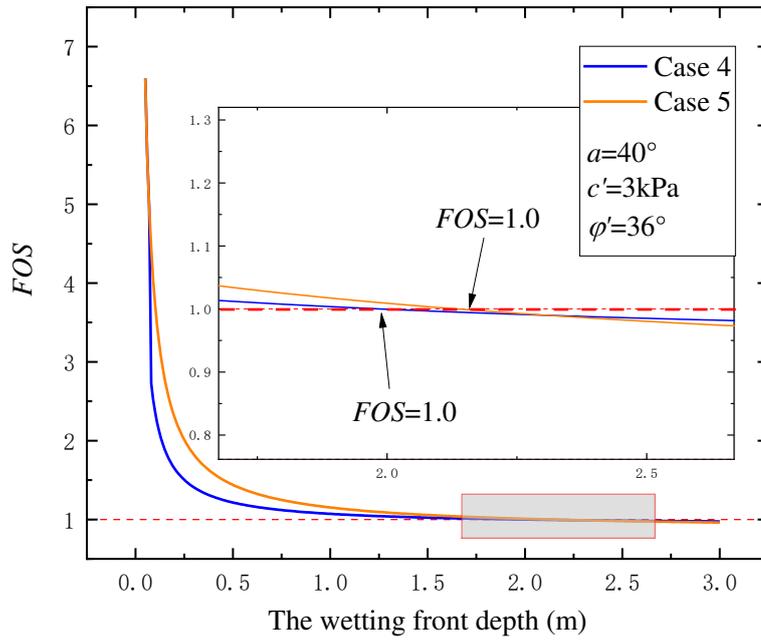


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516 **Fig. 3** Safety factor of 5 cases

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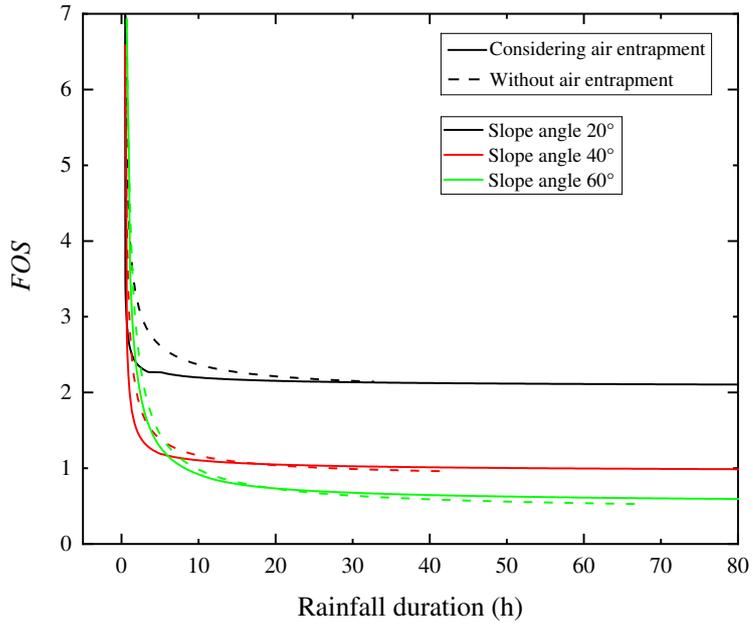


519

520 **Fig. 4** Safety factor and wet front depth change curve of Case 4 and Case 5

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522

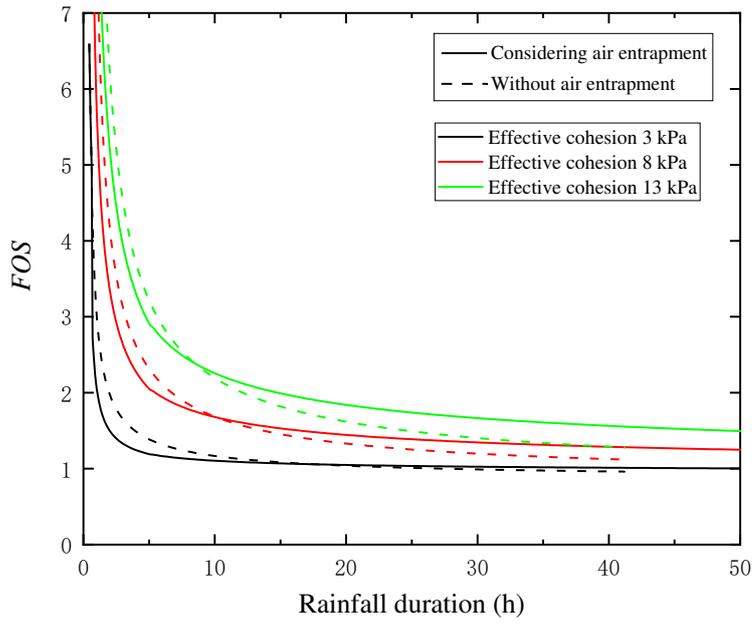


523

524 **Fig. 5** Comparison of three different slope angles

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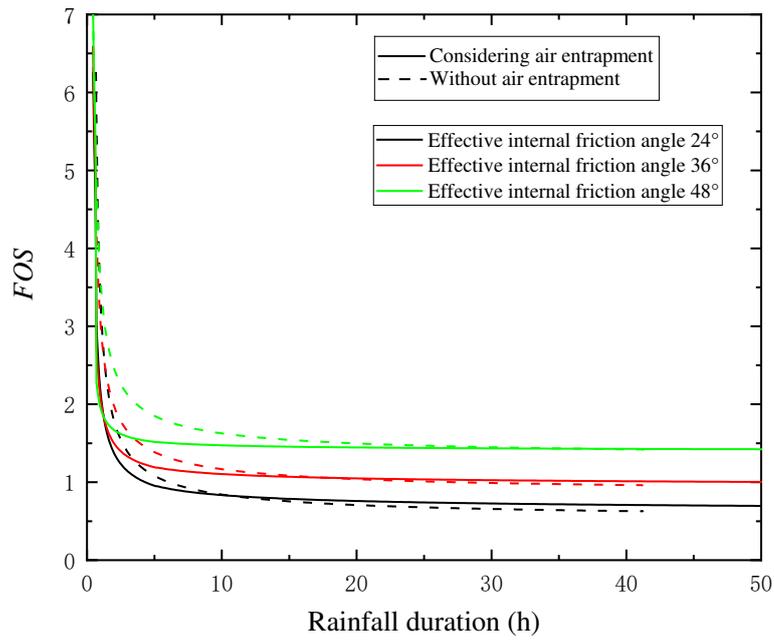


527

528 **Fig. 6** Comparison of three effective cohesions

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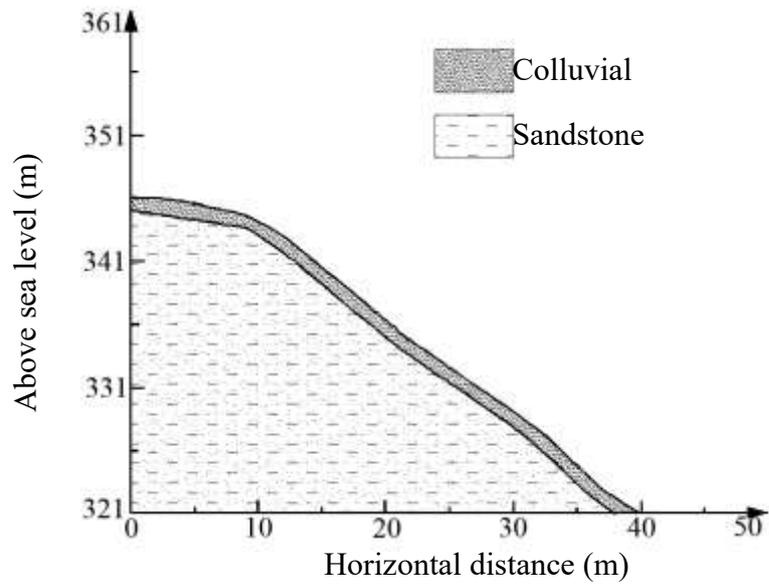


531

532 **Fig. 7** Comparison of three effective internal friction angles

533

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535

536 **Fig. 8** Schematic diagram of the field monitoring hillslope in Alameda County, California ([Chen](#)

537 [et al. 2017](#))

538

# Figures

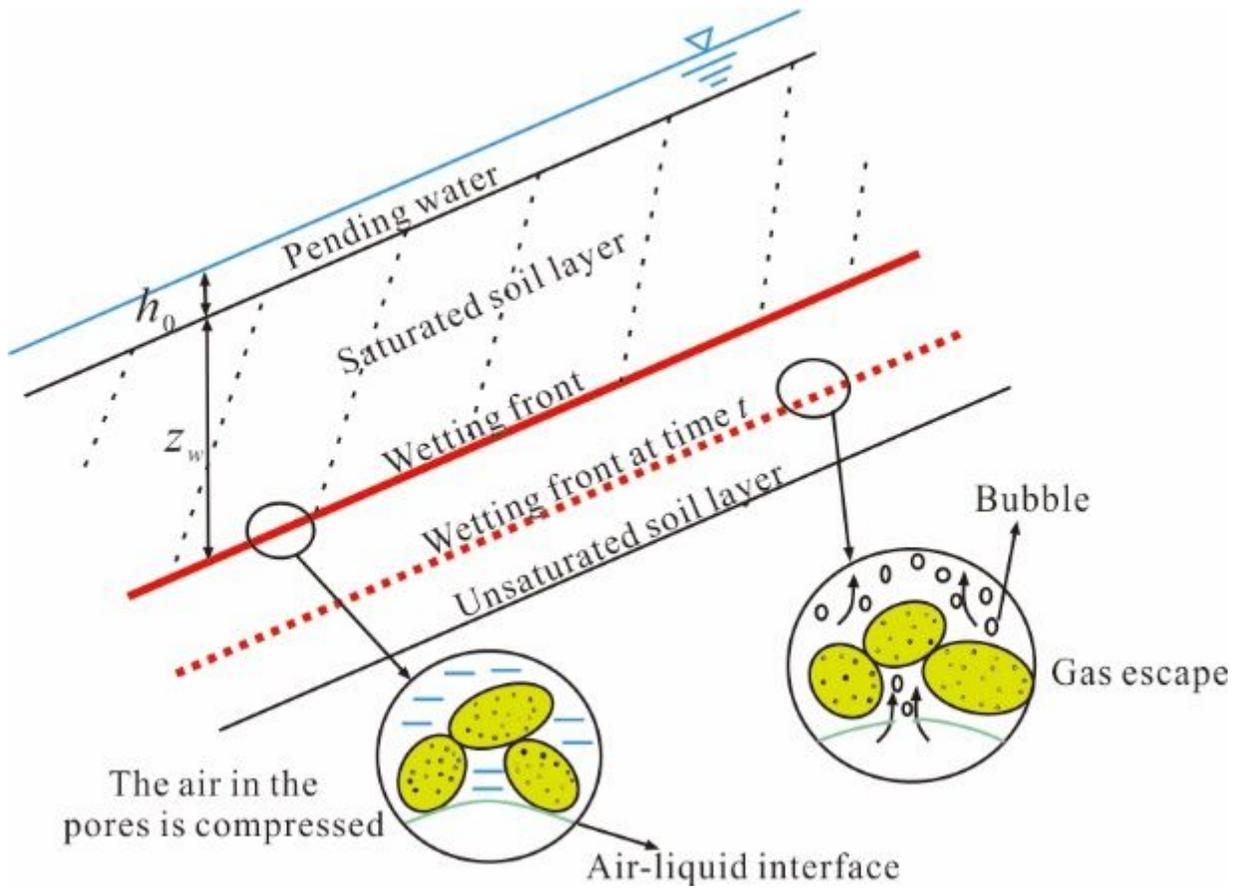
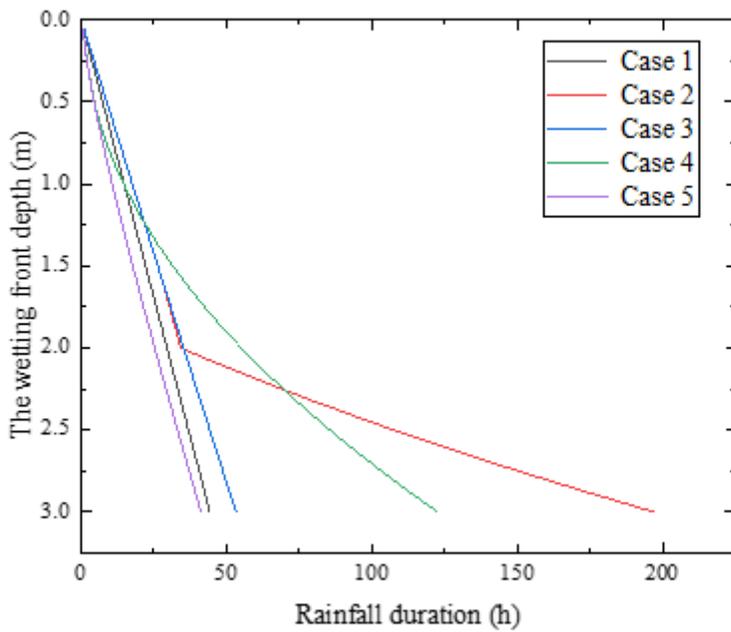


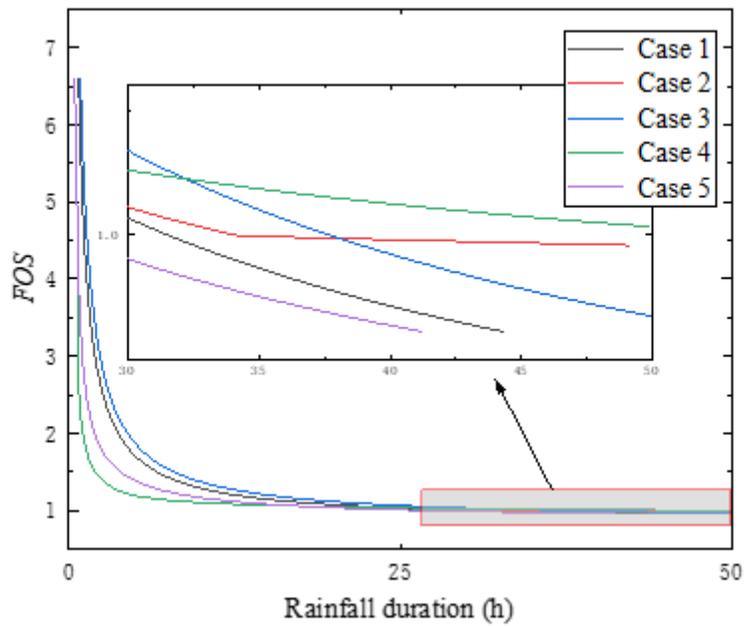
Figure 1

Porous air compressed-escaped process



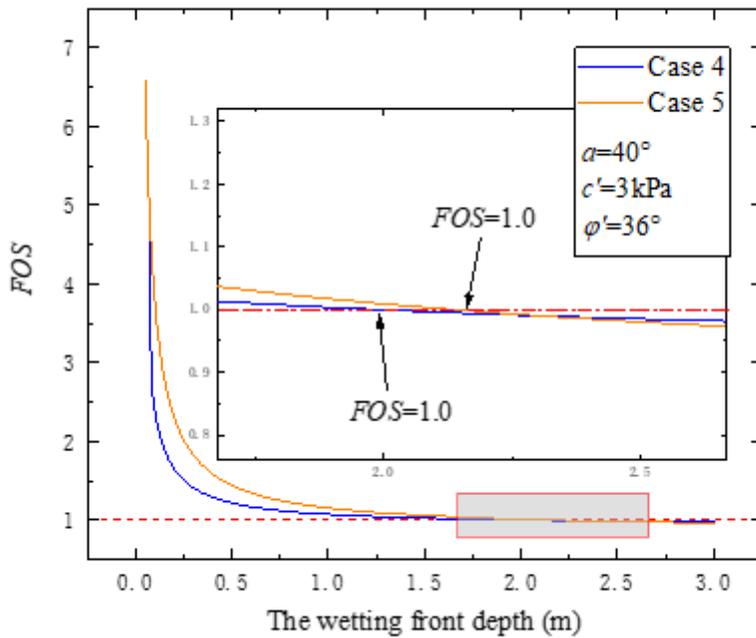
**Figure 2**

Changes in the depth of the wetting front during rainfall



**Figure 3**

Safety factor of 5 cases



**Figure 4**

## Safety factor and wet front depth change curve of Case 4 and Case 5

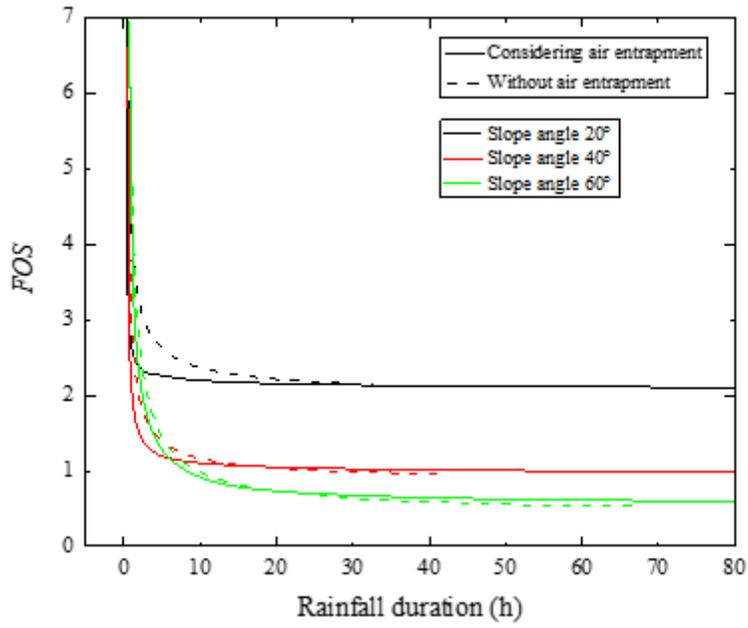


Figure 5

Comparison of three different slope angles

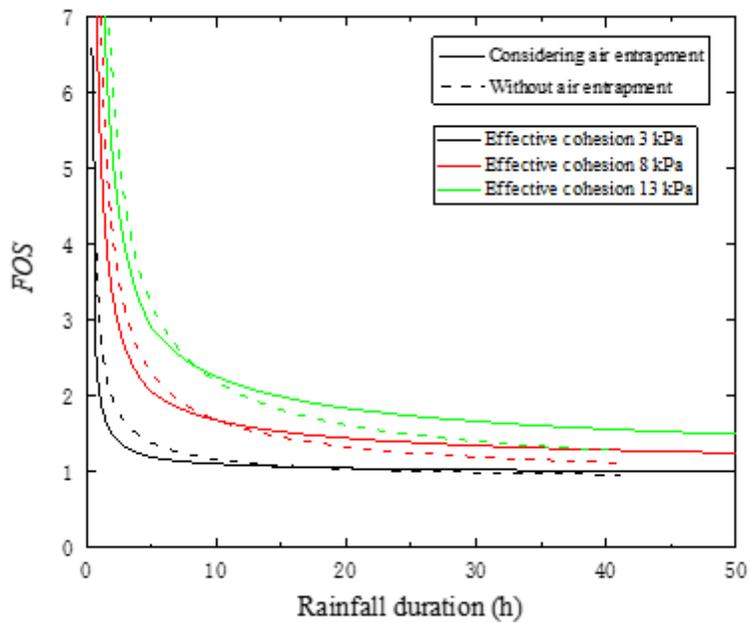


Figure 6

Comparison of three effective cohesions

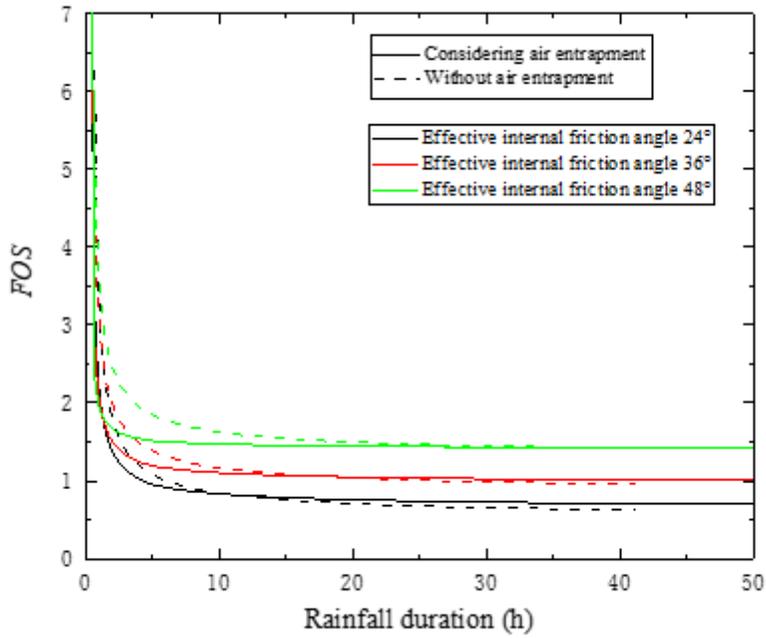


Figure 7

Comparison of three effective internal friction angles

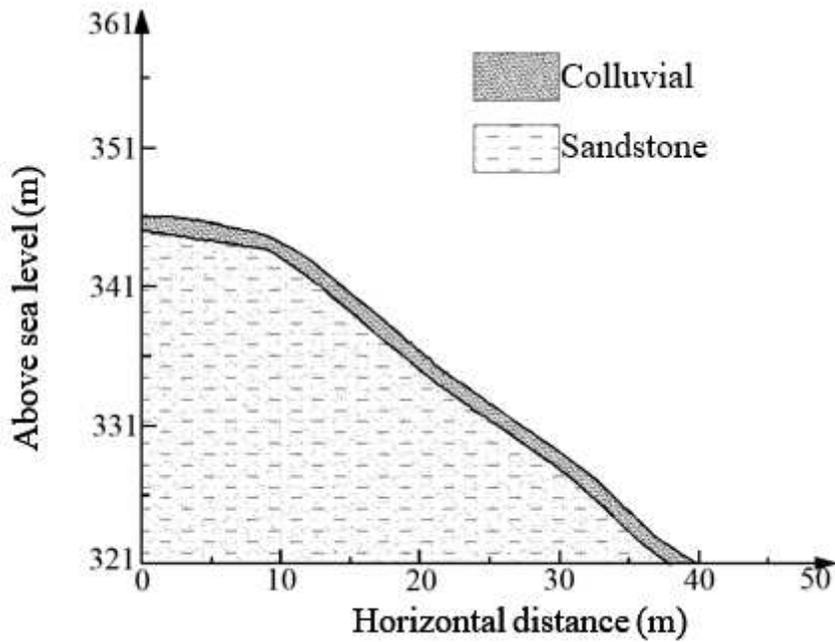


Figure 8

Scematic diagram of the field monitoring hillslope in Alameda County, California (Chen et al. 2017)