

# Obtaining a relativistic dispersion formula and practice of its application.

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## Research Article

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# **Obtaining a relativistic dispersion formula and practice of its application.**

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## **Abstract.**

This study was based on the hypothesis that the speed of propagation of photons inside a transparent substance depends on the density of electron clouds of atoms of this substance. In the process of research, based on the relativistic formula of Einstein, a new dispersion formula was obtained, which can be applied to the widest range of transparent substances. **The new physical formula was used to calculate 26 refractive indices of light in 5 transparent substances in three states of aggregation.** Comparison of the obtained indicators with laboratory indicators showed the high accuracy of the new dispersion formula, which amounted to  $\pm 10^{-7} - 10^{-5}$  in the calculated wavelength ranges **of more than 100 nm.**

The successful application of the relativistic formula to the processes occurring inside matter at the atomic level allows us to look at the nature of the interaction of light and matter from a new angle.

**Keywords.** Relativistic dispersion formula, new dispersion formula, empirical dispersion formulas, a formula for determining the total energy of a moving body, the energy density index of electron clouds, the calculation of the refractive indices of light.

## **Introduction.**

Currently, there are no physical formulas that can be applied to a wide range of transparent substances. For example, the well-known physical formula of Lorentz-Lorentz, which is based on the dependence of the refractive index of light **on the density of a substance**, is valid only for isotropic media (gases, non-polar

liquids, cubic crystals) and is not applicable for most transparent substances.

Therefore, in practice, to calculate the refractive indices, empirical dispersion formulas (Cauchy, Hartmann, etc.) are used. These formulas are quite accurate, but at the same time they are not physical formulas. One of the main prerequisites for the creation of a new dispersion formula was the external similarity of such processes as the refraction of light in matter and the deflection of light by massive space bodies.

### **Methods.**

Let's start with the well-known formula, where the refractive index of light in a transparent substance is  $n = c/v$ , where  $c$  is the speed of light in vacuum,  $v$  is the speed of light in matter.

In this work, it is hypothesized that the speed of propagation  $v_{\gamma}$  of photons in a transparent substance depends on the energy of electron clouds  $Q_e$  of atoms of the substance: the higher the density of the electron clouds, the lower the speed of the photons and vice versa. In this case, the greater the energy of the photons entering the substance, the more the electron clouds of the atoms of the substance are "condensed" by this energy. As a result of this circumstance, electromagnetic waves with different wavelengths propagate in the same transparent substance at different speeds. Thus, there is an inverse relationship between the energy density  $Q_e$  of electron clouds of atoms of matter and the speed of propagation  $v_{\gamma}$  of photons in matter. To determine this dependence and then obtain the dispersion

formula, we use in this study the relativistic Einstein formula to determine the total energy of a moving body:

$$E_{total} = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{or} \quad E_{total} = \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (1-1)$$

Where  $E_{total}$  is the total energy of a moving body.

$E_0$  – energy of a body at rest.

$v$  is the speed of the body.

Let's transform the formula (1-1) and as a result we get:

$$v = c \sqrt{1 - \frac{E_0^2}{E_{\text{полн.}}^2}} \quad \text{или} \quad v = c \sqrt{1 - Q_b^2} \quad (1-2)$$

Where  $Q_b$  is an indicator of the ratio of the energy of a body at rest to the total energy of a moving body,  $0 < Q_b < 1$ .

Now we apply formula (1-2) to the speed of propagation of photons in a transparent substance:

$$v_\gamma = c \sqrt{1 - Q_e^2} \quad (1-3)$$

Where  $v_\gamma$  is the speed of propagation of photons in the electron clouds of atoms of a transparent substance.

$Q_e$  is a dimensionless indicator of the energy density of electron clouds of a transparent substance,  $0 < Q_e < 1$ .

Let's transform the formula (1-3) and get:

$$\frac{c}{v_\gamma} = \frac{1}{\sqrt{1 - Q_e^2}} \quad \text{or} \quad n = \frac{1}{\sqrt{1 - Q_e^2}} \quad (1-4)$$

Where  $n$  is the refractive index of light in the substance ( $n = c/v_\gamma$ ).

Let's reveal the value of  $Q_e$  in the formula (1-4):

$$n = \frac{1}{\sqrt{1-(Q_0+\Delta Q_\lambda)^2}} \quad (1-5)$$

Where  $Q_0$  is a dimensionless basic indicator of the energy density of electron clouds of a transparent substance.

$\Delta Q_\lambda$  is a dimensionless indicator of the increase in the energy density of electron clouds of a transparent substance.

The  $Q_0$  index is constant (at constant temperature and pressure). The exponent  $\Delta Q_\lambda$  is a variable. It depends on the energy  $e$  of the electromagnetic wave, where  $e = h\gamma = hc/\lambda$ . From here we get the formula:

$$n = \frac{1}{\sqrt{1-(Q_0+k_\gamma hc/\lambda)^2}} \quad (1-6)$$

Where  $k_\gamma$  is the coefficient of proportionality,  $J^{-1}$ .

Replace  $k_\gamma hc$  with a single coefficient  $k_\lambda$  and obtain a new dispersion formula:

$$n = \frac{1}{\sqrt{1-(Q_0+k_\lambda/\lambda)^2}} \quad (1-7)$$

Where  $k_\lambda$  is the coefficient of proportionality, nm.

$\lambda$  – wavelength, nm.

The coefficient  $k_\lambda$  is individual for each substance and depends on the absorption of electromagnetic waves by atoms. It is relatively stable in the visible range of the electromagnetic spectrum. But in the ultraviolet and infrared ranges, the coefficient  $k_\lambda$  can significantly change its value due to changes in the absorption of electromagnetic waves by matter. For this reason, according to f. (1-

7), the value of the refractive index  $n$  can change sharply up to the adoption of anomalous values. This circumstance introduces a limitation on the use of formula (1-7) in these wave ranges.

Let us check the accuracy of the new dispersion formula. Table 1 presents 39 laboratory indices of refraction of light in the visible range of the spectrum in five transparent substances. 13 conventionally known indices of refraction of light are highlighted in bold, 26 conventionally unknown indices, which must be determined using a new formula, knowing the wavelength, are highlighted in regular font. (These refractive indices are commonly known and readily available on the Internet. The datasets used and analysed during the current study are available from the corresponding author on reasonable request).

To be able to verify with the table data, they will be sent to the editor in a separate file). The first column of the table contains the basic indicators  $Q_0$  (they were determined by solving a system of equations and subsequent selection of the optimal value  $Q_0$ ). The wavelengths are highlighted in bold in the table, where the proportionality coefficients were calculated using the formula  $k_{\lambda 1,2} = \lambda_n$

$\left( \sqrt{\frac{n^2-1}{n^2}} - Q_0 \right)$  (1-8), which will be used for interpolation. As can be seen from the

table, for an inert gas the number of such coefficients was unity for the entire wavelength range, for other substances - 3. (This is due to different amplitudes of fluctuations in the magnitude of the proportionality coefficients in these

substances). Then, using the formula  $k_{\lambda} = \frac{k_{\lambda 1}(\lambda_n - \lambda_2) + k_{\lambda 2}(\lambda_1 - \lambda_n)}{(\lambda_1 - \lambda_2)}$  (1-9), the

coefficients  $k_\lambda$  (they are presented in the table in regular font) and then the refractive indices of light are determined by the formula (1-7).

After that, the calculated indices were rounded off in accordance with the number of digits after the decimal point in laboratory refractive indices. Therefore, for an inert gas, the refractive indices of light were rounded up to 7 decimal places, for water - up to 5 decimal places, for solids - up to 4 decimal places. It should be noted that in those cases when the rounding of the numbers led to a complete coincidence of the refractive indices, then the accuracy was taken to be one order of magnitude greater than that of the other refractive indices. For example, in glass, after rounding, two refractive indices completely coincided for wavelengths of 670, 8, and 643.8 nm. The accuracy here was taken as  $10^{-6}$ . An order of magnitude higher than the rest of the refractive indices in glass, where the accuracy was  $10^{-5}$ . The same method was applied to the rest of the indicators in other substances. The author believes that this approach is the most correct, because the known laboratory parameters, after being obtained experimentally, were also rounded to a certain sign. From this it follows that when comparing the refractive indices, the equality of the commas after zero must be observed, because otherwise, the calculation accuracy indices may increase unreasonably or, conversely, decrease.

After rounding off the calculated indices, they were compared with laboratory refractive indices and the discrepancy between them was determined. The results were tabulated.

**Results.** Table 1 shows the 26 calculated refractive indices of light in 5 transparent substances, which were calculated using the new physical formula.

Comparison of the indicators calculated by the new dispersion formula with laboratory indicators showed the following: in an inert gas the discrepancy was  $10^{-7}$ , in water and solids  $\pm 10^{-6} - 10^{-5}$ . In this case, **the calculated range was more than 100 nm.**

For comparison, the very exact empirical formula of Hartmann has four constants and shows an accuracy of  $\pm 10^{-6} - 10^{-5}$  in the sections of the wavelength ranges **that do not exceed several tens of nm.**

If we compare the new dispersion formula with the physical Lorentz-Lorentz formula, then the advantage of the new formula is obvious. This is a high precision and a much wider range of action among transparent substances, which is equal to the range of known empirical formulas.

**Conclusions.** In this study, on the basis of Einstein's relativistic formula, a new dispersion formula was obtained with an accuracy of  $\pm 10^{-7} - 10^{-5}$  within the wavelength range of 100 nm. Due to its high accuracy and versatility, the new physical formula can be used to determine the refractive indices of light for almost all transparent substances, which was previously performed only through empirical formulas.

The very fact of the possibility of applying the relativistic formula to the physical processes occurring inside matter at the atomic level allows us to look at the nature of the interaction of light and matter from a new angle.

**Table 1**

Substance	$\lambda$ nm	$k_\lambda$ nm	Calculated refractive index	Laboratory refractive index	Divergence
<b>Krypton</b>  $Q_0 = 0,0228741$	450,4		1,0002750	1,0002752	<b><math>10^{-7}</math></b>
	<b>556,4</b>	<b>0,2573906</b>	-	<b>1,0002724</b>	
	565,1		1,0002723	1,0002722	
	587,3		1,0002719	1,0002719	
	605,8		1,0002715	1,0002716	
	645,8		1,0002709	1,0002711	
<b>Water</b>  $Q_0 = 0,648752$  $t = 20\text{ }^\circ\text{C}$	447,1	7,3594178	1,33931	1,33942	<b><math>10^{-6} - 10^{-5}</math></b>
	<b>471,3</b>	<b>7,3504891</b>	-	<b>1,33793</b>	
	486,1	7,3450358	1,33716	1,33712	
	501,6	7,3393246	1,33640	1,33635	
	<b>546,1</b>	<b>7,322928</b>	-	<b>1,33447</b>	
	577,0	7,3469439	1,33342	1,33338	
	587,6	7,3551824	1,33308	1,33304	
	<b>656,3</b>	<b>7,4085769</b>	-	<b>1,33115</b>	
	670,8	7,4198465	1,33080	1,33080	
706,5	7,447593	1,32999	1,33002		
<b>Sylvin</b>  $Q_0 = 0,727035$  $t = 18\text{ }^\circ\text{C}$	480,0	8,5850021	1,4989	1,4990	<b><math>10^{-6} - 10^{-5}</math></b>
	<b>486,1</b>	<b>8,5766026</b>	-	<b>1,4983</b>	
	508,6	8,5456209	1,4962	1,4961	
	<b>546,1</b>	<b>8,4939848</b>	-	<b>1,4931</b>	
	589,3	8,5471312	1,4905	1,4904	
	643,8	8,6141794	1,4876	1,4877	
	<b>656,3</b>	<b>8,6295574</b>	-	<b>1,4872</b>	
<b>Light crown glass</b>  $Q_0 = 0,741579$  $t = 15\text{ }^\circ\text{C}$	480,0	6,1563048	1,5234	1,5235	<b><math>10^{-6} - 10^{-5}</math></b>
	<b>486,1</b>	<b>6,1545121</b>	-	<b>1,5230</b>	
	546,1	6,1372274	1,5192	1,5191	
	<b>589,3</b>	<b>6,1241824</b>	-	<b>1,5170</b>	
	643,8	6,1806235	1,5149	1,5149	
	<b>656,3</b>	<b>6,1935687</b>	-	<b>1,5145</b>	
<b>Rock salt</b>  $Q_0 = 0,747572$  $t = 18\text{ }^\circ\text{C}$	480,0	8,5940181	1,5541	1,5541	<b><math>10^{-6} - 10^{-5}</math></b>
	<b>486,1</b>	<b>8,5867621</b>	-	<b>1,5534</b>	
	508,6	8,5599976	1,5510	1,5509	
	<b>546,1</b>	<b>8,5153919</b>	-	<b>1,5475</b>	
	589,3	8,5653435	1,5445	1,5443	
	643,8	8,6283611	1,5413	1,5412	
	<b>656,3</b>	<b>8,6428147</b>	-	<b>1,5407</b>	
670,8	8,6360523	1,5399	1,5400		

## Declarations

1. **Availability of data and materials.**

All data obtained and analyzed in the course of this study is included in this article.

2. **Competing interests.** Not applicable (there are no competing interests).

3. **Funding.** Not applicable.

4. **Authors' contributions.** Not applicable.

5. **Acknowledgements.** Not applicable.