MCGDM based on VIKOR and TOPSIS methods using spherical interval valued fuzzy soft with aggregation operators

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Abstract: Spherical interval valued fuzzy soft set (SIVFS set) has much stronger ability than Pythagorean interval valued fuzzy soft set and interval valued intuitionistic fuzzy soft set. Now, we talk about aggregated operation for aggregating SIVFS decision matrix. TOPSIS and VIKOR methods are strong point of view for multi criteria group decision making (MCGDM), which is a various extensions of interval valued fuzzy soft sets. We talk through a score function based on aggregating TOPSIS and VIKOR method to the SIVFS-positive ideal solution and the SIVFS-negative ideal solution. Also TOPSIS and VIKOR methods are provides the weights of decision makings. To find out the optimal alternative under closeness is introduced.

Keywords: spherical interval valued fuzzy soft set, MCGDM, VIKOR, aggregation operator.

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1 Introduction

Decision making (DM) problem indicates the finding of best optional alternatives. Hwang and Yoon [6] was discussed by multiple criteria decision making (MCDM) methods. The matrix form of MCDM problem as:

$$\mathscr{D}_{n\times m} = \begin{array}{c} \\ \mathscr{A}_1 \\ \mathscr{A}_2 \\ \vdots \\ \mathscr{A}_n \end{array} \begin{array}{c} \begin{array}{cccc} \mathscr{B}_1 & \mathscr{B}_2 & \dots & \mathscr{B}_m \end{array} \\ \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{pmatrix} \end{array}$$

Here $\mathscr{A}_1, \mathscr{A}_2, ..., \mathscr{A}_n$ are called possible alternatives means which decision makers have to choose, $\mathscr{B}_1, \mathscr{B}_2, ..., \mathscr{B}_m$ are called criteria means which alternative effecting are calculated and x_{ij} means estimate of $\mathscr{A}_i$ with respect to $\mathscr{B}_j$.

These two methods (TOPSIS and VIKOR) for DM problems have been studied by Adeel et al. [1], Akram et al. [2], Boran et al. [4], Eraslan et al. [5], Peng et al. [17], Xu et al. [22] and Zhang et al. [27]. In 2021, Zulqarnain et al. discussed the TOPSIS extends to interval valued intuitionistic fuzzy soft sets (shortly IVIFSS). He also discussed a new type of correlation coefficient under IVIFSS's [28]. In TOPSIS method consists of distances to positive ideal solution(PIS) and negative ideal solution(NIS), and calculate a preference order is ranked under relative closeness, and finding a combination of these two distance measures. In VIKOR method focal point on ranking and selecting from a set of alternatives, and compute compromise solutions for a problem with inconsistent criteria, which can help the decision makers to get a final decision [14, 15]. Opricovic et al. [16] discussed VIKOR method using fuzzy logic. Tzeng et al. [19] discussion about comparison of VIKOR with TOPSIS methods using public transportation problem.

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2 Preliminaries

Definition 2.1 *[9] Let* $\mathbb{U}$ *be a non-empty set of the universe, spherical interval valued fuzzy set* X *in* $\mathbb{U}$ *is of the following form :* $\widetilde{X} = \left\{u, (\widetilde{\delta_X}(u), \widetilde{\eta_X}(u), \widetilde{\gamma_X}(u)) | u \in \mathbb{U}\right\}$, *where* $\widetilde{\delta_X}(u) = [\delta_X^L(u), \delta_X^U(u)]$ *and* $\widetilde{\eta_X}(u) = [\eta_X^L(u), \eta_X^U(u)]$ *and* $\widetilde{\gamma_X}(u) = [\gamma_X^L(u), \gamma_X^U(u)]$ *represent the degree of positive, neutral and negative-membership of* X *respectively. Consider the mapping* $\widetilde{\delta_X} : \mathbb{U} \rightarrow D[0,1]$, $\widetilde{\eta_X} : \mathbb{U} \rightarrow D[0,1]$, $\widetilde{\gamma_X} : \mathbb{U} \rightarrow D[0,1]$ *and* $0 \leq (\widetilde{\delta_X}(u))^2 + (\widetilde{\eta_X}(u))^2 + (\widetilde{\gamma_X}(u))^2 \leq 1$ *means* $0 \leq (\delta_X^U(u))^2 + (\eta_X^U(u))^2 + (\gamma_X^U(u))^2 \leq 1$. *The degree of refusal is determined as* $\widetilde{\pi_X}(u) = [\pi_X^L(u), \pi_X^U(u)] = \left[\sqrt{1-(\delta_X^U(u))^2-(\eta_X^U(u))^2-(\gamma_X^U(u))^2}, \sqrt{1-(\delta_X^L(u))^2-(\eta_X^L(u))^2-(\gamma_X^L(u))^2}\,\right]$. *Here* $\widetilde{X} = \left([\delta_X^L, \delta_X^U], [\eta_X^L, \eta_X^U], [\gamma_X^L, \gamma_X^U]\right)$ *is called an interval valued spherical fuzzy number(SIVFN).*

Definition 2.2 *Let* $\mathbb{U}$ *and* E *be the universe and set of parameter respectively. The pair* $\widetilde{(\Upsilon, X)}$ *or* $\widetilde{\Upsilon_X}$ *is called a SIVFS set on* $\mathbb{U}$ *if* $X \sqsubseteq E$ *and* $\Upsilon : X \rightarrow SIVF^{\mathbb{U}}$, *where* $SIVF^{\mathbb{U}}$ *is denote the set of all spherical interval valued fuzzy subsets of* $\mathbb{U}$. *(ie)*

$$\widetilde{\Upsilon_X} = \left\{ \left(e, \left\{\frac{u}{\left(\left[\delta^L_{\Upsilon_X}(u), \delta^U_{\Upsilon_X}(u)\right], \left[\eta^L_{\Upsilon_X}(u), \eta^U_{\Upsilon_X}(u)\right], \left[\gamma^L_{\Upsilon_X}(u), \gamma^U_{\Upsilon_X}(u)\right]\right)}\right\}\right) : e \in X, u \in \mathbb{U} \right\}.$$

Remark 2.3 *Let* $\widetilde{p_{ij}} = \widetilde{\delta_{\Upsilon_X}}(e_j)(u_i)$, $\widetilde{q_{ij}} = \widetilde{\eta_{\Upsilon_X}}(e_j)(u_i)$ *and* $\widetilde{r_{ij}} = \widetilde{\gamma_{\Upsilon_X}}(e_j)(u_i)$, *where* i *various from 1 to m and* j *various from 1 to n. Then the SIVFS set* $\widetilde{\Upsilon_X}$ *defined in matrix form:*

$$\widetilde{\Upsilon_X} = [(\widetilde{p_{ij}}, \widetilde{q_{ij}}, \widetilde{r_{ij}})]_{m\times n} = \begin{bmatrix} (\widetilde{p_{11}}, \widetilde{q_{11}}, \widetilde{r_{11}}) & (\widetilde{p_{12}}, \widetilde{q_{12}}, \widetilde{r_{12}}) & \cdots & (\widetilde{p_{1n}}, \widetilde{q_{1n}}, \widetilde{r_{1n}}) \\ (\widetilde{p_{21}}, \widetilde{q_{21}}, \widetilde{r_{21}}) & (\widetilde{p_{22}}, \widetilde{q_{22}}, \widetilde{r_{22}}) & \cdots & (\widetilde{p_{2n}}, \widetilde{q_{2n}}, \widetilde{r_{2n}}) \\ \vdots & \vdots & \ddots & \vdots \\ (\widetilde{p_{m1}}, \widetilde{q_{m1}}, \widetilde{r_{m1}}) & (\widetilde{p_{m2}}, \widetilde{q_{m2}}, \widetilde{r_{m2}}) & \cdots & (\widetilde{p_{mn}}, \widetilde{q_{mn}}, \widetilde{r_{mn}}) \end{bmatrix}$$

This matrix is called spherical interval valued fuzzy soft matrix (SIVFSM).

Remark 2.4 *Using fundamental operations of arithmetic leads to the following.*
(i) $[u,v] + [w,x] = [u+w, v+x]$
(ii) $[u,v] - [w,x] = [u-x, v-w]$
(iii) $[u,v] \cdot [w,x] = [uw, vx]$, *whenever* $u \geq 0$ *and* $v \geq 0$
(iv) $\frac{1}{[u,v]} = [\frac{1}{v}, \frac{1}{u}]$, *whenever* $0 \notin [u,v]$, $u,v,w,x \in \mathbb{R}$.

3 MCGDM based on SIVFS sets

Definition 3.1 *The cardinal set of the SIVFS set* $\widetilde{\Upsilon_X}$ *over* $\mathbb{U}$ *is a SIVFS set over* E *and is defined as* $c\widetilde{\Upsilon_X} = \left\{\frac{e}{\left(\left[\delta^L_{c\theta_X}(e), \delta^U_{c\theta_X}(e)\right], \left[\eta^L_{c\xi_X}(e), \eta^U_{c\xi_X}(e)\right], \left[\gamma^L_{c\varphi_X}(e), \gamma^L_{c\varphi_X}(e)\right]\right)} : e \in E\right\} = \left\{\frac{e}{\left(\widetilde{\delta}_{c\theta_X}(e), \widetilde{\eta}_{c\xi_X}(e), \widetilde{\gamma}_{c\varphi_X}(e)\right)} : e \in E\right\}$, *where* $\widetilde{\delta}_{c\theta_X}$, $\widetilde{\eta}_{c\xi_X}$ *and* $\widetilde{\gamma}_{c\varphi_X} : E \rightarrow D[0,1]$ *are mapping respectively, where* $\widetilde{\delta}_{c\theta_X}(e) = \frac{|\widetilde{\theta}_X(e)|}{|\mathbb{U}|}$, $\widetilde{\eta}_{c\xi_X}(e) = \frac{|\widetilde{\xi}_X(e)|}{|\mathbb{U}|}$ *and* $\widetilde{\gamma}_{c\varphi_X}(e) = \frac{|\widetilde{\varphi}_X(e)|}{|\mathbb{U}|}$, *where* $|\widetilde{\theta}_X(e)|$, $|\widetilde{\xi}_X(e)|$ *and* $|\widetilde{\varphi}_X(e)|$ *denote the scalar cardinalities of the SIVFS sets* $\widetilde{\theta}_X(e)$, $\widetilde{\xi}_X(e)$ *and* $\widetilde{\varphi}_X(e)$ *respectively, and* $|\mathbb{U}|$ *represents cardinality of the universe* $\mathbb{U}$. *The collection of all cardinal sets of SIVFS sets of* $\mathbb{U}$ *is represented as* $cSIVF^{\mathbb{U}}$. *If* $X \subseteq E = \{e_i : i = 1,2,...,n\}$, *then* $c\widetilde{\Upsilon_X} \in cSIVF^{\mathbb{U}}$ *may be represented in matrix form as* $\left[\left([p^L_{1j}, p^U_{1j}], [q^L_{1j}, q^U_{1j}], [r^L_{1j}, r^U_{1j}]\right)\right]_{1\times n} =$

$\left[\left([p^L_{11}, p^U_{11}], [q^L_{11}, q^U_{11}], [r^L_{11}, r^U_{11}]\right), \left([p^L_{12}, p^U_{12}], [q^L_{12}, q^U_{12}], [r^L_{12}, r^U_{12}]\right), ..., \left([p^L_{1n}, p^U_{1n}], [q^L_{1n}, q^U_{1n}], [r^L_{1n}, r^U_{1n}]\right) \right]$, *where* $\left([p^L_{1j}, p^U_{1j}], [q^L_{1j}, q^U_{1j}], [r^L_{1j}, r^U_{1j}]\right) = \left[\mu^L_{r\Upsilon_X}(e_j), \mu^U_{r\Upsilon_X}(e_j)\right], \forall j = 1, 2, ..., n$. *For our convenience matrix form as* $[(\widetilde{p}_{1j}, \widetilde{q}_{1j}, \widetilde{r}_{1j})]_{1\times n} = [(\widetilde{p}_{11}, \widetilde{q}_{11}, \widetilde{r}_{11}), (\widetilde{p}_{12}, \widetilde{q}_{12}, \widetilde{r}_{12}), ..., (\widetilde{p}_{1n}, \widetilde{q}_{1n}, \widetilde{r}_{1n})]$, *where* $(\widetilde{p}_{1j}, \widetilde{q}_{1j}, \widetilde{r}_{1j}) = \widetilde{\mu}_{c\Upsilon_X}(e_j), \forall j = 1, 2, ..., n$. *This matrix is called as cardinal matrix of* $\widetilde{c\Upsilon_X}$ *of* E.

Definition 3.2 *Let* $\widetilde{\Upsilon_X} \in SIVF^{\mathbb{U}}$ *and* $\widetilde{c\Upsilon_X} \in cSIVF^{\mathbb{U}}$. *The SIVFS set aggregation operator* $SIVFS_{agg} : cSIVF^{\mathbb{U}} \times SIVF^{\mathbb{U}} \to SIVFS(\mathbb{U}, E)$ *is defined as* $SIVFS_{agg}(\widetilde{c\Upsilon_X}, \widetilde{\Upsilon_X}) = \left\{\frac{u}{\widetilde{\mu}_{\Upsilon^*_X}(u)} : u \in \mathbb{U}\right\} = \left\{\frac{u}{\left(\widetilde{\delta}_{\theta^*_X}(u), \widetilde{\eta}_{\xi^*_X}(u), \widetilde{\gamma}_{\varphi^*_X}(u)\right)} : u \in \mathbb{U}\right\}$. *This collection is called aggregate spherical interval valued fuzzy set of SIVFS set* $\widetilde{\Upsilon_X}$. *The positive membership function* $\widetilde{\delta}_{\theta^*_X}(u) : \mathbb{U} \to D[0,1]$ *by* $\widetilde{\delta}_{\theta^*_X}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\widetilde{\delta}_{c\theta_X}(e), \widetilde{\delta}_{\theta_X}(e)\right)(u)$, *neutral membership function* $\widetilde{\eta}_{\xi^*_X}(u) : \mathbb{U} \to D[0,1]$ *by* $\widetilde{\eta}_{\xi^*_X}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\widetilde{\eta}_{c\xi_X}(e), \widetilde{\eta}_{\xi_X}(e)\right)(u)$ *and negative membership function* $\widetilde{\gamma}_{\varphi^*_X}(u) : \mathbb{U} \to D[0,1]$ *by* $\widetilde{\gamma}_{\varphi^*_X}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\widetilde{\gamma}_{c\varphi_X}(e), \widetilde{\gamma}_{\varphi_X}(e)\right)(u)$. *The set* $SIVFS_{agg}(\widetilde{c\Upsilon_X}, \widetilde{\Upsilon_X})$ *is expressed in matrix form as*

$$\left[\left([p^L_{i1}, p^U_{i1}], [q^L_{i1}, q^U_{i1}], [r^L_{i1}, r^U_{i1}]\right) \right]_{m\times 1} = \begin{bmatrix} \left([p^L_{11}, p^U_{11}], [q^L_{11}, q^U_{11}], [r^L_{11}, r^U_{11}]\right) \\ \left([p^L_{21}, p^U_{21}], [q^L_{21}, q^U_{21}], [r^L_{21}, r^U_{21}]\right) \\ \vdots \\ \left([p^L_{m1}, p^U_{m1}], [q^L_{m1}, q^U_{m1}], [r^L_{m1}, r^U_{m1}]\right) \end{bmatrix}$$

where $\left[\left([p^L_{i1}, p^U_{i1}], [q^L_{i1}, q^U_{i1}], [r^L_{i1}, r^U_{i1}]\right) \right] = \left[\mu^L_{\Upsilon^*_X}(u_i), \mu^U_{\Upsilon^*_X}(u_i)\right], \forall i = 1, 2, ..., m$. *This matrix is called SIVFS aggregate matrix of* $SIVFS_{agg}(\widetilde{c\Upsilon_X}, \widetilde{\Upsilon_X})$ *over* $\mathbb{U}$.

Remark 3.3 *Let* $\widetilde{X} = (\widetilde{\delta}_{ij}, \widetilde{\eta}_{ij}, \widetilde{\gamma}_{ij}) \in SIVFSM_{m\times n}$. *Then*
(i) choice matrix of SIVFSM X *is given by* $\widetilde{\mathcal{C}(X)} = \left[\left(\frac{\sum_{j=1}^{n}(\widetilde{\delta}_{ij})^2}{n}, \frac{\sum_{j=1}^{n}(\widetilde{\eta}_{ij})^2}{n}, \frac{\sum_{j=1}^{n}(\widetilde{\gamma}_{ij})^2}{n}\right)\right]_{m\times 1}, \forall i$ *when weights are equal.*
(ii) weighted choice matrix of SIVFSM X *(* $\widetilde{w_j} > 0$ *are weights means weights are unequal) is given by* $\widetilde{\mathcal{C}_w(X)} = \left[\left(\frac{\sum_{j=1}^{n}\widetilde{w_j}(\widetilde{\delta}_{ij})^2}{\sum\widetilde{w_j}}, \frac{\sum_{j=1}^{n}\widetilde{w_j}(\widetilde{\eta}_{ij})^2}{\sum\widetilde{w_j}}, \frac{\sum_{j=1}^{n}\widetilde{w_j}(\widetilde{\gamma}_{ij})^2}{\sum\widetilde{w_j}}\right)\right]_{m\times 1} \forall i$.

Theorem 3.4 *Let* $\widetilde{\Upsilon_X}$ *be an SIVFS set. Suppose that* $\widetilde{M_{\Upsilon_X}}, \widetilde{M_{c\Upsilon_X}}, \widetilde{M^*_{\Upsilon_X}}$ *are matrices of* $\widetilde{\Upsilon_X}, \widetilde{c\Upsilon_X}, \widetilde{\Upsilon^*_X}$ *respectively, then* $\widetilde{M_{\Upsilon_X}} \times \widetilde{M^T_{c\Upsilon_X}} = \widetilde{M^*_{\Upsilon_X}} \times |E|$, *where* $\widetilde{M_{\Upsilon_X}} \times \widetilde{M^T_{c\Upsilon_X}}$ *is called a SIVFSM-product and* $\widetilde{M^T_{c\Upsilon_X}}$ *is the transpose of* $\widetilde{M_{c\Upsilon_X}}$.

Proof. The proof of Theorem 3.4 by Definition 3.1 and Definition 3.2.

We can make a MCGDM based on SIVFS set by the following algorithms:

Algorithm-I

Step 1: Form SIVFS set $\widetilde{\Upsilon_X}$ over the universal $\mathbb{U}$.
Step 2: Calculate the cardinalities and cardinal set $\widetilde{c\Upsilon_X}$ of $\widetilde{\Upsilon_X}$.
Step 3: Compute aggregate SIVFS set $\widetilde{\Upsilon^*_X}$ of $\widetilde{\Upsilon_X}$.
Step 4: Find the score function $S_c(u) = \frac{(\delta^{2L}_u - \eta^{2U}_u - \gamma^{2U}_u) + (\delta^{2U}_u - \eta^{2L}_u - \gamma^{2L}_u)}{2}$ and $-1 \leq S_c(u) \leq 1$, $u \in \mathbb{U}$.
Step 5: Find the best alternative by $\max_i S_c(u_i)$.

Example 3.5 *An automobile company produces ten different types of scooter $\mathbb{U} = \{S_1, S_2, ..., S_{10}\}$ and five parameters namely $E = \{e_1, e_2, ..., e_5\}$ consists of fuel tank capacity, better style, better price, more mileage, more durable respectively. Suppose that a customer has to establish which scooter to be purchased ? Each scooter is evaluated and which is a subset of parameters. That is $X = \{e_1, e_2, e_3, e_4\} \subseteq E$. We appeal to algorithm-I as follows.*

Step-1: Form SIVFS set $\widetilde{\Upsilon_X}$ of $\mathbb{U}$ is defined below:

$$\widetilde{\Upsilon_X} = \Bigg\{ \Bigg(e_1, \Big\{ \frac{S_1}{[0.35,0.4],[0.2,0.25],[0.35,0.4]}, \frac{S_2}{[0.3,0.5],[0.3,0.45],[0.3,0.35]}, \frac{S_4}{[0.2,0.3],[0.25,0.35],[0.4,0.45]},$$

$$\frac{S_5}{[0.15,0.35],[0.25,0.3],[0.1,0.2]}, \frac{S_7}{[0.25,0.3],[0.3,0.4],[0.4,0.5]}, \frac{S_9}{[0.25,0.4],[0.4,0.5],[0.2,0.25]},$$

$$\frac{S_{10}}{[0.35,0.45],[0.2,0.25],[0.3,0.35]} \Big\} \Bigg),$$

$$\Bigg(e_2, \Big\{ \frac{S_2}{[0.3,0.45],[0.25,0.3],[0.3,0.5]}, \frac{S_3}{[0.4,0.5],[0.15,0.2],[0.3,0.4]}, \frac{S_5}{[0.2,0.35],[0.3,0.45],[0.15,0.45]},$$

$$\frac{S_6}{[0.3,0.35],[0.4,0.6],[0.3,0.35]}, \frac{S_8}{[0.15,0.2],[0.45,0.5],[0.3,0.4]}, \frac{S_{10}}{[0.2,0.25],[0.3,0.4],[0.2,0.25]}, \Big\} \Bigg),$$

$$\Bigg(e_3, \Big\{ \frac{S_3}{[0.45,0.55],[0.25,0.3],[0.35,0.4]}, \frac{S_4}{[0.4,0.45],[0.1,0.25],[0.2,0.3]}, \frac{S_6}{[0.4,0.45],[0.25,0.35],[0.5,0.55]},$$

$$\frac{S_8}{[0.5,0.6],[0.2,0.25],[0.15,0.2]}, \frac{S_9}{[0.3,0.35],[0.35,0.45],[0.5,0.6]} \Big\} \Bigg),$$

$$\Bigg(e_4, \Big\{ \frac{S_1}{[0.5,0.55],[0.2,0.25],[0.3,0.5]}, \frac{S_2}{[0.2,0.25],[0.45,0.5],[0.25,0.45]}, \frac{S_4}{[0.35,0.45],[0.15,0.2],[0.3,0.45]},$$

$$\frac{S_5}{[0.2,0.25],[0.4,0.45],[0.25,0.3]}, \frac{S_7}{[0.3,0.4],[0.2,0.35],[0.25,0.3]}, \frac{S_8}{[0.25,0.35],[0.3,0.6],[0.3,0.35]},$$

$$\frac{S_{10}}{[0.15,0.3],[0.3,0.7],[0.15,0.2]} \Big\} \Bigg) \Bigg\}.$$

Step-2: The cardinal set of $\widetilde{\Upsilon_X}$ as $c\Upsilon_X = \Big\{ \frac{e_1}{([0.185,0.27],[0.19,0.25],[0.205,0.25])},$

$$\frac{e_2}{([0.155,0.21],[0.185,0.245],[0.155,0.235])}, \frac{e_3}{([0.205,0.24],[0.115,0.16],[0.17,0.205])}, \frac{e_4}{([0.195,0.255],[0.2,0.305],[0.18,0.255])} \Big\}.$$

Step-3: The aggregate SIVFS set $\widetilde{\Upsilon_X^*}$ of $\widetilde{\Upsilon_X}$ is $\widetilde{M_{\Upsilon_X^*}} = \frac{\widetilde{M_{\Upsilon_X}} \times \widetilde{M^t_{c\Upsilon_X}}}{|E|}$

$$= \frac{1}{5} \left\{ \begin{bmatrix} [0.35,0.4] & [0,0] & [0,0] & [0.5,0.55] & [0,0] \\ [0.3,0.5] & [0.3,0.45] & [0,0] & [0.2,0.25] & [0,0] \\ [0,0] & [0.4,0.5] & [0.45,0.55] & [0,0] & [0,0] \\ [0.2,0.3] & [0,0] & [0.4,0.45] & [0.35,0.45] & [0,0] \\ [0.15,0.35] & [0.2,0.35] & [0,0] & [0.2,0.25] & [0,0] \\ [0,0] & [0.3,0.35] & [0.4,0.45] & [0,0] & [0,0] \\ [0.25,0.3] & [0,0] & [0,0] & [0.3,0.4] & [0,0] \\ [0,0] & [0.15,0.2] & [0.5,0.6] & [0.25,0.35] & [0,0] \\ [0.25,0.4] & [0,0] & [0.3,0.35] & [0,0] & [0,0] \\ [0.35,0.45] & [0.2,0.25] & [0,0] & [0.15,0.3] & [0,0] \end{bmatrix} \begin{bmatrix} [0.185,0.27] \\ [0.155,0.21] \\ [0.205,0.24] \\ [0.195,0.255] \\ [0,0] \end{bmatrix}, \right.$$

$$\begin{bmatrix} [0.2,0.25] & [0,0] & [0,0] & [0.2,0.25] & [0,0] \\ [0.3,0.45] & [0.25,0.3] & [0,0] & [0.45,0.5] & [0,0] \\ [0,0] & [0.15,0.2] & [0.25,0.3] & [0,0] & [0,0] \\ [0.25,0.35] & [0,0] & [0.1,0.25] & [0.15,0.2] & [0,0] \\ [0.25,0.3] & [0.3,0.45] & [0,0] & [0.4,0.45] & [0,0] \\ [0,0] & [0.4,0.6] & [0.25,0.35] & [0,0] & [0,0] \\ [0.3,0.4] & [0,0] & [0,0] & [0.2,0.35] & [0,0] \\ [0,0] & [0.45,0.5] & [0.2,0.25] & [0.3,0.6] & [0,0] \\ [0.4,0.5] & [0,0] & [0.35,0.45] & [0,0] & [0,0] \\ [0.2,0.25] & [0.3,0.4] & [0,0] & [0.3,0.7] & [0,0] \end{bmatrix} \begin{bmatrix} [0.19,0.25] \\ [0.185,0.245] \\ [0.115,0.16] \\ [0.2,0.305] \\ [0,0] \end{bmatrix},$$

$$\left. \begin{bmatrix} [0.35,0.4] & [0,0] & [0,0] & [0.3,0.5] & [0,0] \\ [0.3,0.35] & [0.3,0.5] & [0,0] & [0.25,0.45] & [0,0] \\ [0,0] & [0.3,0.4] & [0.35,0.4] & [0,0] & [0,0] \\ [0.4,0.45] & [0,0] & [0.2,0.3] & [0.3,0.45] & [0,0] \\ [0.1,0.2] & [0.15,0.45] & [0,0] & [0.25,0.3] & [0,0] \\ [0,0] & [0.3,0.35] & [0.5,0.55] & [0,0] & [0,0] \\ [0.4,0.5] & [0,0] & [0,0] & [0.25,0.3] & [0,0] \\ [0,0] & [0.3,0.4] & [0.15,0.2] & [0.3,0.35] & [0,0] \\ [0.2,0.25] & [0,0] & [0.5,0.6] & [0,0] & [0,0] \\ [0.3,0.35] & [0.2,0.25] & [0,0] & [0.15,0.2] & [0,0] \end{bmatrix} \begin{bmatrix} [0.205,0.25] \\ [0.155,0.235] \\ [0.17,0.205] \\ [0.18,0.255] \\ [0,0] \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} [0.039, 0.054] \\ [0.039, 0.054] \\ [0.041, 0.048] \\ [0.041, 0.054] \\ [0.039, 0.054] \\ [0.041, 0.048] \\ [0.039, 0.054] \\ [0.041, 0.051] \\ [0.041, 0.054] \\ [0.037, 0.054] \end{bmatrix}, \begin{bmatrix} [0.04, 0.05] \\ [0.04, 0.061] \\ [0.03, 0.04] \\ [0.038, 0.05] \\ [0.04, 0.061] \\ [0.037, 0.049] \\ [0.04, 0.061] \\ [0.04, 0.061] \\ [0.038, 0.05] \\ [0.04, 0.061] \end{bmatrix}, \begin{bmatrix} [0.041, 0.051] \\ [0.041, 0.051] \\ [0.034, 0.047] \\ [0.041, 0.051] \\ [0.036, 0.051] \\ [0.034, 0.047] \\ [0.041, 0.051] \\ [0.036, 0.051] \\ [0.04, 0.05] \\ [0.041, 0.05] \end{bmatrix} \right\}$$

Hence, $\widetilde{\Upsilon_X^*} = \Big\{ \frac{S_1}{([0.039,0.054],[0.04,0.05],[0.041,0.051])}, \frac{S_2}{([0.039,0.054],[0.04,0.061],[0.041,0.051])}, \frac{S_3}{([0.041,0.048],[0.03,0.04],[0.034,0.047])}, \frac{S_4}{([0.041,0.054],[0.038,0.05],[0.041,0.051])}, \frac{S_5}{([0.039,0.054],[0.04,0.061],[0.036,0.051])}, \frac{S_6}{([0.041,0.048],[0.037,0.049],[0.034,0.047])}, \frac{S_7}{([0.039,0.054],[0.04,0.061],[0.041,0.051])}, \frac{S_8}{([0.041,0.051],[0.04,0.061],[0.036,0.051])}, \frac{S_9}{([0.041,0.054],[0.038,0.05],[0.04,0.05])}, \frac{S_{10}}{([0.037,0.054],[0.04,0.061],[0.041,0.05])} \Big\}$.

Step-4: The score function $S_c(S_i)$ as follows.

$Scooter$	$S_c(S_i)$
S_1	-0.001973
S_2	-0.002583
S_3	-0.00094
S_4	-0.001815
S_5	-0.002391
S_6	-0.001575
S_7	-0.002583
S_8	-0.002468
S_9	-0.001724
S_{10}	-0.002609

Step 5: Since $\max_i S_c(S_i) = -0.00094$. Hence the customer to be purchased by the scooter S_3 .

Algorithm-II

Step-1: Form spherical interval valued fuzzy soft matrix (SIVFS matrix) on the basis of the parameters.

Step-2: Case-I Obtain the choice matrix for the positive, neutral and negative-membership of SIVFS matrix (weights are equal).

Case-II Find the choice matrix for the positive, neutral and negative-membership of SIVFS matrix (weights are unequal).

Step-3: Compute score value $S_c(u) = \frac{(\delta_u^{2L} - \eta_u^{2U} - \gamma_u^{2U}) + (\delta_u^{2U} - \eta_u^{2L} - \gamma_u^{2L})}{2}$, $-1 \leq S_c(u) \leq 1$, $u \in \mathbb{U}$.

Step-4: Find the best alternative by $\max_i S(u_i)$.

Case-I: By Example 3.5,

$$\widetilde{\mathcal{C}(X)} = \left\{ \begin{bmatrix} [0.0745, 0.0925] \\ [0.044, 0.103] \\ [0.0725, 0.1105] \\ [0.0645, 0.099] \\ [0.0205, 0.0615] \\ [0.05, 0.065] \\ [0.0305, 0.05] \\ [0.067, 0.1045] \\ [0.0305, 0.0565] \\ [0.037, 0.071] \end{bmatrix}, \begin{bmatrix} [0.016, 0.025] \\ [0.071, 0.1085] \\ [0.017, 0.026] \\ [0.019, 0.045] \\ [0.0625, 0.099] \\ [0.0445, 0.0965] \\ [0.026, 0.0565] \\ [0.0665, 0.1345] \\ [0.0565, 0.0905] \\ [0.044, 0.1425] \end{bmatrix}, \begin{bmatrix} [0.0425, 0.082] \\ [0.0485, 0.115] \\ [0.0425, 0.064] \\ [0.058, 0.099] \\ [0.019, 0.0665] \\ [0.068, 0.085] \\ [0.0445, 0.068] \\ [0.0405, 0.0645] \\ [0.058, 0.0845] \\ [0.0305, 0.045] \end{bmatrix} \right\}$$

Score value =

$Scooter$	$S_c(S_i)$
S_1	0.002348
S_2	-0.009923
S_3	0.0053
S_4	-0.000795
S_5	-0.007144
S_6	-0.008208
S_7	-0.003521
c_8	-0.006452
S_9	-0.008882
S_{10}	-0.009394

Case-II: Weights $(\widetilde{w_j}) = \{[0.16, 0.165], [0.14, 0.145], [0.18, 0.19], [0.17, 0.175], [0.15, 0.155]\}$.

By Example 3.5,

$$\widetilde{\mathcal{C}_w(X)} = \left\{ \begin{bmatrix} [0.0748, 0.0992] \\ [0.0407, 0.1019] \\ [0.0709, 0.1172] \\ [0.0675, 0.111] \\ [0.0193, 0.0611] \\ [0.0499, 0.0703] \\ [0.0305, 0.0536] \\ [0.0708, 0.1195] \\ [0.0316, 0.0621] \\ [0.035, 0.0728] \end{bmatrix}, \begin{bmatrix} [0.0159, 0.0266] \\ [0.0694, 0.1128] \\ [0.0173, 0.0286] \\ [0.0188, 0.0489] \\ [0.06, 0.0996] \\ [0.0405, 0.0943] \\ [0.0255, 0.0598] \\ [0.0613, 0.1389] \\ [0.0574, 0.0997] \\ [0.0413, 0.1491] \end{bmatrix}, \begin{bmatrix} [0.042, 0.0877] \\ [0.0453, 0.1149] \\ [0.0417, 0.067] \\ [0.058, 0.1074] \\ [0.0185, 0.0646] \\ [0.0694, 0.094] \\ [0.0436, 0.0713] \\ [0.0385, 0.0653] \\ [0.0619, 0.0984] \\ [0.0287, 0.0453] \end{bmatrix} \right\} \quad \text{Score value} =$$

$Scooter$	$S_c(S_i)$
S_1	0.002509
S_2	−0.010365
S_3	0.0057
S_4	−0.000388
S_5	−0.006962
S_6	−0.008388
S_7	−0.003706
S_8	−0.004744
S_9	−0.010945
S_{10}	−0.010146

Algorithm-III

Step-1: Find the spherical interval valued fuzzy weighted averaging numbers(SIVFWANs) under aggregated operation, $\widetilde{\mathcal{C}(X)} = \left(\sum_{j=1}^{n} \widetilde{w_j} \widetilde{\delta_{ij}}, \sum_{j=1}^{n} \widetilde{w_j} \widetilde{\eta_{ij}}, \sum_{j=1}^{n} \widetilde{w_j} \widetilde{\gamma_{ij}} \right)$.

Step-2: Compute the score function $S_c(u) = \frac{(\delta_u^{2L} - \eta_u^{2U} - \gamma_u^{2U}) + (\delta_u^{2U} - \eta_u^{2L} - \gamma_u^{2L})}{2}$ and $-1 \leq S_c(u) \leq 1$, $u \in \mathbb{U}$.

Step-3: Find the best alternative by $\max_i S_c(u_i)$.

Weights $(\widetilde{w_j}) = \{[0.16, 0.165], [0.14, 0.145], [0.18, 0.19], [0.17, 0.175], [0.15, 0.155]\}$.
By Example 3.5,

$$\widetilde{\mathcal{C}(X)} = \left\{ \begin{bmatrix} [0.141, 0.1623] \\ [0.124, 0.1915] \\ [0.137, 0.177] \\ [0.1635, 0.2138] \\ [0.086, 0.1523] \\ [0.114, 0.1363] \\ [0.091, 0.1195] \\ [0.1535, 0.2043] \\ [0.094, 0.1325] \\ [0.1095, 0.163] \end{bmatrix}, \begin{bmatrix} [0.066, 0.085] \\ [0.1595, 0.2053] \\ [0.066, 0.086] \\ [0.0835, 0.1403] \\ [0.15, 0.1935] \\ [0.101, 0.1535] \\ [0.082, 0.1273] \\ [0.15, 0.225] \\ [0.127, 0.168] \\ [0.125, 0.2218] \end{bmatrix}, \begin{bmatrix} [0.107, 0.1535] \\ [0.1325, 0.209] \\ [0.105, 0.134] \\ [0.151, 0.21] \\ [0.0795, 0.1508] \\ [0.132, 0.1553] \\ [0.1065, 0.135] \\ [0.12, 0.1573] \\ [0.122, 0.1553] \\ [0.1015, 0.129] \end{bmatrix} \right\} \quad \text{Score value} =$$

$Scooter$	$S_c(S_i)$
S_1	−0.000193
S_2	−0.038378
S_3	0.004683
S_4	−0.010561
S_5	−0.029206
S_6	−0.021865
S_7	−0.014961
S_8	−0.023486
S_9	−0.028474
S_{10}	−0.026591

3.1 Comparison Analysis for SIVFS-Methods:

Comparison analysis of final ranking as follows:

$Methods$	$Ranking\ of\ alternatives$	$Optimal\ alternatives$
$Algorithm - I$	$S_{10} \leq S_7 \leq S_2 \leq S_8 \leq S_5 \leq S_1 \leq S_4 \leq S_9 \leq S_6 \leq S_3$	S_3
$Algorithm - II\ Case - (i)$	$S_2 \leq S_{10} \leq S_9 \leq S_6 \leq S_5 \leq S_8 \leq S_7 \leq S_4 \leq S_1 \leq S_3$	S_3
$Algorithm - II\ Case - (ii)$	$S_9 \leq S_2 \leq S_{10} \leq S_6 \leq S_5 \leq S_8 \leq S_7 \leq S_4 \leq S_1 \leq S_3$	S_3
$Algorithm - III$	$S_2 \leq S_5 \leq C_9 \leq S_{10} \leq S_8 \leq S_6 \leq S_7 \leq S_4 \leq S_1 \leq S_3$	S_3

Therefore the customer to be purchased by the scooter S_3.

4 MCGDM based on SIVFS-TOPSIS aggregating operator

Algorithm-IV (SIVFS-TOPSIS)

Step-1: Suppose that the finite decision makers namely $\mathscr{D} = \{\mathscr{D}_i : i \in \mathbb{N}\}$ and the finite collection of alternatives namely $\mathscr{C} = \{\ddot{c}_i : i \in \mathbb{N}\}$ and finite family of parameters namely $\mathcal{D} = \{e_i : i \in \mathbb{N}\}$.

Step-2: Form a linguistic variable with weighted parameter matrix

$$\mathcal{P} = \left[\omega_{ij}^L, \omega_{ij}^U\right]_{n\times m} = \begin{bmatrix} \left[\omega_{11}^L, \omega_{11}^U\right] & \left[\omega_{12}^L, \omega_{12}^U\right] & \cdots & \left[\omega_{1m}^L, \omega_{1m}^U\right] \\ \left[\omega_{21}^L, \omega_{21}^U\right] & \left[\omega_{22}^L, \omega_{22}^U\right] & \cdots & \left[\omega_{2m}^L, \omega_{2m}^U\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[\omega_{i1}^L, \omega_{i1}^U\right] & \left[\omega_{i2}^L, \omega_{i2}^U\right] & \cdots & \left[\omega_{im}^L, \omega_{im}^U\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[\omega_{n1}^L, \omega_{n1}^U\right] & \left[\omega_{n2}^L, \omega_{n2}^U\right] & \cdots & \left[\omega_{nm}^L, \omega_{nm}^U\right] \end{bmatrix}$$

Here the weight ω_{ij} means $\mathcal{D}_i$ to $\mathcal{P}_j$ by considering linguistic variables.
Step-3: Obtain weighted normalized decision matrix

$$\widehat{\mathcal{N}} = \left[\widehat{n}_{ij}^L, \widehat{n}_{ij}^U\right]_{n\times m} = \begin{bmatrix} \left[\widehat{n}_{11}^L, \widehat{n}_{11}^U\right] & \left[\widehat{n}_{12}^L, \widehat{n}_{12}^U\right] & \cdots & \left[\widehat{n}_{1m}^L, \widehat{n}_{1m}^U\right] \\ \left[\widehat{n}_{21}^L, \widehat{n}_{21}^U\right] & \left[\widehat{n}_{22}^L, \widehat{n}_{22}^U\right] & \cdots & \left[\widehat{n}_{2m}^L, \widehat{n}_{2m}^U\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[\widehat{n}_{i1}^L, \widehat{n}_{i1}^U\right] & \left[\widehat{n}_{i2}^L, \widehat{n}_{i2}^U\right] & \cdots & \left[\widehat{n}_{im}^L, \widehat{n}_{im}^U\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[\widehat{n}_{n1}^L, \widehat{n}_{n1}^U\right] & \left[\widehat{n}_{n2}^L, \widehat{n}_{n2}^U\right] & \cdots & \left[\widehat{n}_{nm}^L, \widehat{n}_{nm}^U\right] \end{bmatrix}$$

where $\left[\widehat{n}_{ij}^L, \widehat{n}_{ij}^U\right] = \left[\frac{\omega_{ij}^L}{\sqrt{\sum_{i=1}^n \omega_{ij}^{2U}}}, \frac{\omega_{ij}^U}{\sqrt{\sum_{i=1}^n \omega_{ij}^{2L}}}\right]$ is the normalized parameter and weighted vector $\mathcal{W} = \left([m_1^L, m_1^U], [m_2^L, m_2^U] ..., [m_m^L, m_m^U]\right)$, where $[m_i^L, m_i^U] = \left[\frac{\omega_i^L}{\sqrt{\sum_{l=1}^n \omega_{li}^U}}, \frac{\omega_i^U}{\sqrt{\sum_{l=1}^n \omega_{li}^L}}\right]$ is the weight of the j^{th} parameter and $[\omega_j^L, \omega_j^U] = \left[\frac{\sum_{i=1}^n \widehat{n}_{ij}^L}{n}, \frac{\sum_{i=1}^n \widehat{n}_{ij}^U}{n}\right]$.
Step-4: Form SIVFS decision matrix

$$\mathscr{D}_i = \left[c_{jk}^{Li}, c_{jk}^{Ui}\right]_{l\times m} = \begin{bmatrix} \left[c_{11}^{Li}, c_{11}^{Ui}\right] & \left[c_{12}^{Li}, c_{12}^{Ui}\right] & \cdots & \left[c_{1m}^{Li}, c_{1m}^{Ui}\right] \\ \left[c_{21}^{Li}, c_{21}^{Ui}\right] & \left[c_{22}^{Li}, c_{22}^{Ui}\right] & \cdots & \left[c_{2m}^{Li}, c_{2m}^{Ui}\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[c_{j1}^{Li}, c_{j1}^{Ui}\right] & \left[c_{j2}^{Li}, c_{j2}^{Ui}\right] & \cdots & \left[c_{jm}^{Li}, c_{jm}^{Ui}\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[c_{l1}^{Li}, c_{l1}^{Ui}\right] & \left[c_{l2}^{Li}, c_{l2}^{Ui}\right] & \cdots & \left[c_{lm}^{Li}, c_{lm}^{Ui}\right] \end{bmatrix}$$

Here $\left[c_{jk}^{Li}, c_{jk}^{Ui}\right]$ is a SIVFS element for i^{th} decision maker $[\mathscr{D}_i^L, \mathscr{D}_i^U]$ for each i. Find the aggregating matrix by $\left[\mathcal{X}^L, \mathcal{X}^U\right] = \frac{\left[\mathscr{D}_1^L, \mathscr{D}_1^U\right] + \left[\mathscr{D}_2^L, \mathscr{D}_2^U\right] + ... + \left[\mathscr{D}_n^L, \mathscr{D}_n^U\right]}{n} = \left[\dot{c}_{jk}^L, \dot{c}_{jk}^U\right]_{l\times m}$.
Step-5: Find the decision weighted SIVFS matrix

$$[\mathcal{Y}^L, \mathcal{Y}^U] = \left[\ddot{c}_{jk}^L, \ddot{c}_{jk}^U\right]_{l\times m} = \begin{bmatrix} \left[\ddot{c}_{11}^L, \ddot{c}_{11}^U\right] & \left[\ddot{c}_{12}^L, \ddot{c}_{12}^U\right] & \cdots & \left[\ddot{c}_{1m}^L, \ddot{c}_{1m}^U\right] \\ \left[\ddot{c}_{21}^L, \ddot{c}_{21}^U\right] & \left[\ddot{c}_{22}^L, \ddot{c}_{22}^U\right] & \cdots & \left[\ddot{c}_{2m}^L, \ddot{c}_{2m}^U\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[\ddot{c}_{j1}^L, \ddot{c}_{j1}^U\right] & \left[\ddot{c}_{j2}^L, \ddot{c}_{j2}^U\right] & \cdots & \left[\ddot{c}_{jm}^L, \ddot{c}_{jm}^U\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[\ddot{c}_{l1}^L, \ddot{c}_{l1}^U\right] & \left[\ddot{c}_{l2}^L, \ddot{c}_{l2}^U\right] & \cdots & \left[\ddot{c}_{lm}^L, \ddot{c}_{lm}^U\right] \end{bmatrix}$$

Where $\left[\ddot{c}_{jk}^L, \ddot{c}_{jk}^U\right] = \left[m_k^L \times \dot{c}_{jk}^L, m_k^U \times \dot{c}_{jk}^U\right]$.
Step-6: Calculate SIVFSV-PIS and SIVFSV-NIS. Now,
SIVFSV-PIS $= \left(\left[\ddot{c}_1^{L+}, \ddot{c}_1^{U+}\right], \left[\ddot{c}_2^{L+}, \ddot{c}_2^{U+}\right] ..., \left[\ddot{c}_l^{L+}, \ddot{c}_l^{U+}\right]\right)$

$$= \left\{ \left(\vee_k \left[\ddot{c}_{jk}^L, \ddot{c}_{jk}^U\right], \wedge_k \left[\ddot{c}_{jk}^L, \ddot{c}_{jk}^U\right], \wedge_k \left[\ddot{c}_{jk}^L, \ddot{c}_{jk}^U\right]\right) : k = 1, 2, ..., m \right\}$$

and SIVFSV-PIS $= \left(\left[\ddot{c}_1^{L-}, \ddot{c}_1^{U-}\right], \left[\ddot{c}_2^{L-}, \ddot{c}_2^{U-}\right] ..., \left[\ddot{c}_l^{L-}, \ddot{c}_l^{U-}\right]\right)$

$$= \left\{ \left(\wedge_k \left[\ddot{c}^L_{jk}, \ddot{c}^U_{jk}\right], \vee_k \left[\ddot{c}^L_{jk}, \ddot{c}^U_{jk}\right], \vee_k \left[\ddot{c}^L_{jk}, \ddot{c}^U_{jk}\right]\right) : k = 1, 2, ..., m \right\}.$$

Where SIVFS union $\vee$ and SIVFS intersection $\wedge$.

Step-7: Find the SIVFS-Euclidean distances from SIVFSV-PIS and SIVFSV-NIS. Now

$$\left[d_j^{L+}, d_j^{U+}\right] = \Bigg[\sqrt{\sum_{k=1}^m \left\{ \left(\delta^L_{jk} - \delta^{L+}_j\right)^2 + \left(\eta^L_{jk} - \eta^{L+}_j\right)^2 + \left(\gamma^L_{jk} - \gamma^{L+}_j\right)^2 \right\}}, \sqrt{\sum_{k=1}^m \left\{ \left(\delta^U_{jk} - \delta^{U+}_j\right)^2 + \left(\eta^U_{jk} - \eta^{U+}_j\right)^2 + \left(\gamma^U_{jk} - \gamma^{U+}_j\right)^2 \right\}}\Bigg]$$

and

$$\left[d_j^{L-}, d_j^{U-}\right] = \Bigg[\sqrt{\sum_{k=1}^m \left\{ \left(\delta^L_{jk} - \delta^{L-}_j\right)^2 + \left(\eta^L_{jk} - \eta^{L-}_j\right)^2 + \left(\gamma^L_{jk} - \gamma^{L-}_j\right)^2 \right\}}, \sqrt{\sum_{k=1}^m \left\{ \left(\delta^U_{jk} - \delta^{U-}_j\right)^2 + \left(\eta^U_{jk} - \eta^{U-}_j\right)^2 + \left(\gamma^U_{jk} - \gamma^{U-}_j\right)^2 \right\}}\Bigg],$$

where $j = 1, 2, ..., n$.

Step-8: Find the closeness of ideal solution by $\left[C^{L*}(\ddot{c}_j), C^{U*}(\ddot{c}_j)\right] = \left[\frac{d_j^{L-}}{d_j^{U+}+d_j^{U-}}, \frac{d_j^{U-}}{d_j^{L+}+d_j^{L-}}\right]$ hence $C^*(\ddot{c}_j) = \frac{C^{L*}(\ddot{c}_j)+C^{U*}(\ddot{c}_j)}{2} \in [0, 1]$.

Step-9: Find the rank of alternatives using closeness coefficients under the order of decreasing (or) increasing.

Step-10: The conclusion of the best alternative.

Example 4.1 *A company plans to invest some cash in stock exchange by purchasing some shares of best five companies. In order to minimize the factor, they establish to invest their cash percentage of* $30, 25, 20, 15$ *and* 10. *Find the top five ranked companies.*

Step-1: A finite set of decision makers namely $[\mathscr{D}^L, \mathscr{D}^U] = \left\{[\mathscr{D}^L_i, \mathscr{D}^U_i] : i = 1, 2, 3, 4, 5\right\}$, the collection of companies/alternatives namely $\mathscr{C} = \{\ddot{c}_i : i = 1, 2, ..., 10\}$ and finite family of parameters namely $\mathcal{D} = \{e_i : i = 1, 2, ..., 5\}$, put $e_1 =$ Momentum, $e_2 =$ Value, $e_3 =$ Growth, $e_4 =$ Volatility, $e_5 =$ Quality.

Step-2: Obtain weighted parameter matrix under the linguistic variables

Linguistic variables	*Interval valued fuzzy weights*
Very Good Crucial(VGC)	$[0.9, 0.95]$
Good Crucial(GC)	$[0.8, 0.9]$
Average Crucial(AC)	$[0.65, 0.8]$
Poor Crucial(PC)	$[0.5, 0.65]$
Very Poor Crucial(VPC)	$[0.35, 0.5]$

Form weighted parameter matrix

$$\begin{aligned} \mathcal{P} &= [\omega^L_{ij}, \omega^U_{ij}]_{5\times 5} \\ &= \begin{bmatrix} PC & VPC & VGC & VPC & GC \\ AC & VPC & PC & VGC & AC \\ VGC & AC & VGC & PC & VPC \\ VPC & VGC & AC & GC & PC \\ VGC & PC & GC & AC & VPC \end{bmatrix} \\ &= \begin{bmatrix} [0.5, 0.65] & [0.35, 0.5] & [0.9, 0.95] & [0.35, 0.5] & [0.8, 0.9] \\ [0.65, 0.8] & [0.35, 0.5] & [0.5, 0.65] & [0.9, 0.95] & [0.65, 0.8] \\ [0.9, 0.95] & [0.65, 0.8] & [0.9, 0.95] & [0.5, 0.65] & [0.35, 0.5] \\ [0.35, 0.5] & [0.9, 0.95] & [0.65, 0.8] & [0.8, 0.9] & [0.5, 0.65] \\ [0.9, 0.95] & [0.5, 0.65] & [0.8, 0.9] & [0.65, 0.8] & [0.35, 0.5] \end{bmatrix} \end{aligned}$$

Here $[\omega^L_{ij}, \omega^U_{ij}]$ means weight of the $\mathcal{D}_i$ to $\mathcal{P}_j$.

Step-3: The weighted normalized decision matrix

$$\widehat{\mathcal{N}} = \left[\widehat{n}_{ij}^{L}, \widehat{n}_{ij}^{U}\right]_{5\times 5}$$

$$= \begin{bmatrix} [0.2832, 0.3681] & [0.2229, 0.3185] & [0.4693, 0.4954] & [0.2012, 0.2875] & [0.5194, 0.5843] \\ [0.3681, 0.4531] & [0.2229, 0.3185] & [0.2607, 0.339] & [0.5175, 0.5462] & [0.422, 0.5194] \\ [0.5097, 0.538] & [0.414, 0.5095] & [0.4693, 0.4954] & [0.2875, 0.3737] & [0.2272, 0.3246] \\ [0.1982, 0.2832] & [0.5732, 0.6051] & [0.339, 0.4172] & [0.46, 0.5175] & [0.3246, 0.422] \\ [0.5097, 0.538] & [0.3185, 0.414] & [0.4172, 0.4693] & [0.3737, 0.46] & [0.2272, 0.3246] \end{bmatrix}.$$

Weighted vector $\mathcal{W}$ as
$\mathcal{W} = ([0.0971, 0.1322], [0.103, 0.1575], [0.092, 0.1182], [0.0968, 0.1366], [0.1027, 0.1641])$.

Step-4: Obtain the aggregated decision matrix $\left[\mathcal{X}^{L}, \mathcal{X}^{U}\right] = \frac{\left[\mathscr{D}_1^L, \mathscr{D}_1^U\right] + \left[\mathscr{D}_2^L, \mathscr{D}_2^U\right] + \ldots + \left[\mathscr{D}_5^L, \mathscr{D}_5^U\right]}{5}$

$$= \left[\begin{array}{ccc}
([0.55, 0.7], [0.38, 0.41], [0.33, 0.35]) & ([0.57, 0.58], [0.45, 0.47], [0.16, 0.26]) & ([0.28, 0.38], [0.36, 0.37], [0.61, 0.71]) \\
([0.45, 0.54], [0.5, 0.62], [0.51, 0.54]) & ([0.45, 0.65], [0.5, 0.53], [0.26, 0.57]) & ([0.35, 0.54], [0.31, 0.46], [0.56, 0.67]) \\
([0.52, 0.55], [0.5, 0.53], [0.52, 0.56]) & ([0.5, 0.56], [0.47, 0.55], [0.35, 0.39]) & ([0.45, 0.55], [0.31, 0.46], [0.57, 0.67]) \\
([0.5, 0.65], [0.55, 0.63], [0.27, 0.28]) & ([0.54, 0.55], [0.55, 0.56], [0.27, 0.47]) & ([0.28, 0.56], [0.44, 0.48], [0.31, 0.49]) \\
([0.4, 0.75], [0.35, 0.45], [0.3, 0.4]) & ([0.23, 0.33], [0.46, 0.58], [0.55, 0.59]) & ([0.41, 0.45], [0.42, 0.46], [0.47, 0.48]) \\
([0.4, 0.68], [0.2, 0.43], [0.27, 0.29]) & ([0.44, 0.46], [0.4, 0.65], [0.5, 0.55]) & ([0.46, 0.5], [0.45, 0.48], [0.54, 0.56]) \\
([0.4, 0.55], [0.56, 0.59], [0.29, 0.3]) & ([0.34, 0.4], [0.29, 0.39], [0.38, 0.48]) & ([0.39, 0.4], [0.34, 0.36], [0.58, 0.6]) \\
([0.41, 0.43], [0.44, 0.52], [0.43, 0.6]) & ([0.37, 0.55], [0.4, 0.63], [0.42, 0.57]) & ([0.28, 0.54], [0.26, 0.46], [0.49, 0.67]) \\
([0.5, 0.57], [0.41, 0.54], [0.29, 0.32]) & ([0.31, 0.5], [0.45, 0.48], [0.36, 0.39]) & ([0.55, 0.67], [0.38, 0.48], [0.31, 0.37]) \\
([0.26, 0.55], [0.5, 0.67], [0.31, 0.46]) & ([0.5, 0.55], [0.4, 0.44], [0.49, 0.65]) & ([0.55, 0.68], [0.54, 0.55], [0.35, 0.38]) \\
 & & \\
([0.26, 0.28], [0.45, 0.46], [0.59, 0.64]) & ([0.48, 0.53], [0.27, 0.49], [0.47, 0.52]) & \\
([0.41, 0.55], [0.23, 0.47], [0.42, 0.46]) & ([0.22, 0.59], [0.29, 0.5], [0.48, 0.55]) & \\
([0.41, 0.55], [0.25, 0.57], [0.42, 0.56]) & ([0.23, 0.59], [0.29, 0.55], [0.49, 0.51]) & \\
([0.38, 0.49], [0.14, 0.34], [0.67, 0.68]) & ([0.52, 0.58], [0.29, 0.41], [0.37, 0.39]) & \\
([0.43, 0.47], [0.44, 0.48], [0.46, 0.49]) & ([0.4, 0.45], [0.61, 0.64], [0.43, 0.47]) & \\
([0.31, 0.43], [0.49, 0.54], [0.38, 0.45]) & ([0.19, 0.29], [0.48, 0.55], [0.63, 0.66]) & \\
([0.49, 0.53], [0.34, 0.48], [0.35, 0.46]) & ([0.42, 0.56], [0.46, 0.58], [0.29, 0.45]) & \\
([0.27, 0.55], [0.43, 0.47], [0.15, 0.46]) & ([0.43, 0.39], [0.35, 0.55], [0.27, 0.5]) & \\
([0.34, 0.47], [0.28, 0.49], [0.54, 0.56]) & ([0.38, 0.43], [0.47, 0.58], [0.33, 0.47]) & \\
([0.49, 0.62], [0.46, 0.48], [0.3, 0.33]) & ([0.26, 0.48], [0.3, 0.47], [0.42, 0.5]) &
\end{array}\right]$$

$$= \left[\dot{c}_{jk}^{L}, \dot{c}_{jk}^{U}\right]_{10\times 5}$$

Step-5: The weighted decision SIVFS matrix $\left[\mathcal{Y}^{L}, \mathcal{Y}^{U}\right] = \left[m_k^L \times \dot{c}_{jk}^{L}, m_k^U \times \dot{c}_{jk}^{U}\right]$

$$= \left[\begin{array}{cc}
([0.0534, 0.0925], [0.0369, 0.0542], [0.032, 0.0463]) & ([0.0587, 0.0913], [0.0464, 0.074], [0.0165, 0.0409]) \\
([0.0437, 0.0714], [0.0485, 0.0819], [0.0495, 0.0714]) & ([0.0464, 0.1024], [0.0515, 0.0835], [0.0268, 0.0898]) \\
([0.0505, 0.0727], [0.0485, 0.07], [0.0505, 0.074]) & ([0.0515, 0.0882], [0.0484, 0.0866], [0.0361, 0.0614]) \\
([0.0485, 0.0859], [0.0534, 0.0833], [0.0262, 0.037]) & ([0.0556, 0.0866], [0.0567, 0.0882], [0.0278, 0.074]) \\
([0.0388, 0.0991], [0.034, 0.0595], [0.0291, 0.0529]) & ([0.0237, 0.052], [0.0474, 0.0913], [0.0567, 0.0929]) \\
([0.0388, 0.0899], [0.0194, 0.0568], [0.0262, 0.0383]) & ([0.0453, 0.0724], [0.0412, 0.1024], [0.0515, 0.0866]) \\
([0.0388, 0.0727], [0.0544, 0.078], [0.0282, 0.0396]) & ([0.035, 0.063], [0.0299, 0.0614], [0.0392, 0.0756]) \\
([0.0398, 0.0568], [0.0427, 0.0687], [0.0417, 0.0793]) & ([0.0381, 0.0866], [0.0412, 0.0992], [0.0433, 0.0898]) \\
([0.0485, 0.0753], [0.0398, 0.0714], [0.0282, 0.0423]) & ([0.0319, 0.0787], [0.0464, 0.0756], [0.0371, 0.0614]) \\
([0.0252, 0.0727], [0.0485, 0.0885], [0.0301, 0.0608]) & ([0.0515, 0.0866], [0.0412, 0.0693], [0.0505, 0.1024]) \\
 & \\
([0.0258, 0.0449], [0.0331, 0.0437], [0.0561, 0.0839]) & ([0.0252, 0.0382], [0.0436, 0.0628], [0.0571, 0.0874]) \\
([0.0322, 0.0638], [0.0285, 0.0544], [0.0515, 0.0792]) & ([0.0397, 0.0751], [0.0223, 0.0642], [0.0407, 0.0628]) \\
([0.0414, 0.065], [0.0285, 0.0544], [0.0525, 0.0792]) & ([0.0397, 0.0751], [0.0242, 0.0778], [0.0407, 0.0765]) \\
([0.0258, 0.0662], [0.0405, 0.0567], [0.0285, 0.0579]) & ([0.0368, 0.0669], [0.0136, 0.0464], [0.0649, 0.0929]) \\
([0.0377, 0.0532], [0.0386, 0.0544], [0.0433, 0.0567]) & ([0.0416, 0.0642], [0.0426, 0.0655], [0.0445, 0.0669]) \\
([0.0423, 0.0591], [0.0414, 0.0567], [0.0497, 0.0662]) & ([0.03, 0.0587], [0.0474, 0.0737], [0.0368, 0.0614]) \\
([0.0359, 0.0473], [0.0313, 0.0426], [0.0534, 0.0709]) & ([0.0474, 0.0724], [0.0329, 0.0655], [0.0339, 0.0628]) \\
([0.0258, 0.0638], [0.0239, 0.0544], [0.0451, 0.0792]) & ([0.0261, 0.0751], [0.0416, 0.0642], [0.0145, 0.0628]) \\
([0.0506, 0.0792], [0.035, 0.0567], [0.0285, 0.0437]) & ([0.0329, 0.0642], [0.0271, 0.0669], [0.0523, 0.0765]) \\
([0.0506, 0.0804], [0.0497, 0.065], [0.0322, 0.0449]) & ([0.0474, 0.0847], [0.0445, 0.0655], [0.0291, 0.0451]) \\
 & \\
([0.0493, 0.087], [0.0277, 0.0804], [0.0483, 0.0854]) & \\
([0.0226, 0.0968], [0.0298, 0.0821], [0.0493, 0.0903]) & \\
([0.0236, 0.0968], [0.02980.0903], [0.0503, 0.0837]) & \\
([0.0534, 0.0952], [0.0298, 0.0673], [0.038, 0.064]) & \\
([0.0411, 0.0739], [0.0627, 0.1051], [0.0442, 0.0771]) & \\
([0.0195, 0.0476], [0.0493, 0.0903], [0.0647, 0.1083]) & \\
([0.0431, 0.0919], [0.0472, 0.0952], [0.0298, 0.0739]) & \\
([0.0442, 0.064], [0.0359, 0.0903], [0.0277, 0.0821]) & \\
([0.039, 0.0706], [0.0483, 0.0952], [0.0339, 0.0771]) & \\
([0.0267, 0.0788], [0.0308, 0.0771], [0.0431, 0.0821]) &
\end{array}\right]$$

$$= \left[\ddot{c}_{jk}^{L}, \ddot{c}_{jk}^{U}\right]_{10\times 5}$$

Step-6: We find SIVFSV-PIS and SIVFSV-NIS can be written as

$[\ddot{c}^{L+}, \ddot{c}^{U+}]$	$SIVFSV-PIS$	$[\ddot{c}^{L-}, \ddot{c}^{U-}]$	$SIVFSV-NIS$
$[\ddot{c}_1^{L+}, \ddot{c}_1^{U+}]$	([0.0587, 0.0925], [0.0277, 0.0437], [0.0165, 0.0409])	$[\ddot{c}_1^{L-}, \ddot{c}_1^{U-}]$	([0.0252, 0.0382], [0.0464, 0.0804], [0.0571, 0.0874])
$[\ddot{c}_2^{L+}, \ddot{c}_2^{U+}]$	([0.0464, 0.1024], [0.0223, 0.0544], [0.0268, 0.0628])	$[\ddot{c}_2^{L-}, \ddot{c}_2^{U-}]$	([0.0226, 0.0638], [0.0515, 0.0835], [0.0515, 0.0903])
$[\ddot{c}_3^{L+}, \ddot{c}_3^{U+}]$	([0.0515, 0.0968], [0.0242, 0.0544], [0.0361, 0.0614])	$[\ddot{c}_3^{L-}, \ddot{c}_3^{U-}]$	([0.0236, 0.0650], [0.0485, 0.0903], [0.0525, 0.0837])
$[\ddot{c}_4^{L+}, \ddot{c}_4^{U+}]$	([0.0556, 0.0952], [0.0136, 0.0464], [0.0262, 0.037])	$[\ddot{c}_4^{L-}, \ddot{c}_4^{U-}]$	([0.0258, 0.0662], [0.0567, 0.0882], [0.0649, 0.0929])
$[\ddot{c}_5^{L+}, \ddot{c}_5^{U+}]$	([0.0416, 0.0991], [0.034, 0.0544], [0.0291, 0.0529])	$[\ddot{c}_5^{L-}, \ddot{c}_5^{U-}]$	([0.0237, 0.0520], [0.0627, 0.1051], [0.0567, 0.0929])
$[\ddot{c}_6^{L+}, \ddot{c}_6^{U+}]$	([0.0453, 0.0899], [0.0194, 0.0567], [0.0262, 0.0383])	$[\ddot{c}_6^{L-}, \ddot{c}_6^{U-}]$	([0.0195, 0.0476], [0.0493, 0.1024], [0.0647, 0.1083])
$[\ddot{c}_7^{L+}, \ddot{c}_7^{U+}]$	([0.0474, 0.0919], [0.0299, 0.0426], [0.0282, 0.0396])	$[\ddot{c}_7^{L-}, \ddot{c}_7^{U-}]$	([0.0350, 0.0473], [0.0544, 0.0952], [0.0534, 0.0756])
$[\ddot{c}_8^{L+}, \ddot{c}_8^{U+}]$	([0.0442, 0.0866], [0.0239, 0.0544], [0.0145, 0.0628])	$[\ddot{c}_8^{L-}, \ddot{c}_8^{U-}]$	([0.0258, 0.0568], [0.0427, 0.0992], [0.0451, 0.0898])
$[\ddot{c}_9^{L+}, \ddot{c}_9^{U+}]$	([0.0506, 0.0792], [0.0271, 0.0567], [0.0282, 0.0423])	$[\ddot{c}_9^{L-}, \ddot{c}_9^{U-}]$	([0.0319, 0.0642], [0.0483, 0.0952], [0.0523, 0.0771])
$[\ddot{c}_{10}^{L+}, \ddot{c}_{10}^{U+}]$	([0.0515, 0.0866], [0.0308, 0.065], [0.0291, 0.0449])	$[\ddot{c}_{10}^{L-}, \ddot{c}_{10}^{U-}]$	([0.0252, 0.0727], [0.0497, 0.0885], [0.0505, 0.1024])

Step-7: We found SIVFS euclidean distances from SIVFSV-PIS and SIVFSV-NIS.

$Alternative\ [(\ddot{c}_i^L), (\ddot{c}_i^U)]$	$[d_i^{L+}, d_i^{U+}]$	$[d_i^{L-}, d_i^{U-}]$
$[(\ddot{c}_1^L), (\ddot{c}_1^U)]$	[0.0867, 0.1183]	[0.0741, 0.1202]
$[(\ddot{c}_2^L), (\ddot{c}_2^U)]$	[0.0655, 0.0867]	[0.0632, 0.0722]
$[(\ddot{c}_3^L), (\ddot{c}_3^U)]$	[0.0543, 0.0804]	[0.0619, 0.0651]
$[(\ddot{c}_4^L), (\ddot{c}_4^U)]$	[0.0859, 0.1054]	[0.1003, 0.1018]
$[(\ddot{c}_5^L), (\ddot{c}_5^U)]$	[0.0535, 0.1126]	[0.0658, 0.1144]
$[(\ddot{c}_6^L), (\ddot{c}_6^U)]$	[0.0798, 0.1267]	[0.0732, 0.1309]
$[(\ddot{c}_7^L), (\ddot{c}_7^U)]$	[0.0457, 0.1117]	[0.0604, 0.1004]
$[(\ddot{c}_8^L), (\ddot{c}_8^U)]$	[0.0671, 0.0854]	[0.0484, 0.0811]
$[(\ddot{c}_9^L), (\ddot{c}_9^U)]$	[0.0504, 0.0722]	[0.0556, 0.0801]
$[(\ddot{c}_{10}^L), (\ddot{c}_{10}^U)]$	[0.0544, 0.0772]	[0.0596, 0.1037]

Step-8: We calculate closeness coefficients from SIVFSV-PIS and SIVFSV-NIS.

$Alternative\ (\ddot{c}_i)$	$[C_i^{L*}, C_i^{U*}]$	C_i^*
$\ddot{c}_1$	[0.3106, 0.7475]	0.5291
$\ddot{c}_2$	[0.3979, 0.5613]	0.4796
$\ddot{c}_3$	[0.4254, 0.5603]	0.4929
$\ddot{c}_4$	[0.4841, 0.5469]	0.5155
$\ddot{c}_5$	[0.2897, 0.9589]	0.6243
$\ddot{c}_6$	[0.2841, 0.8553]	0.5697
$\ddot{c}_7$	[0.2849, 0.9465]	0.6157
$\ddot{c}_8$	[0.2906, 0.7023]	0.4964
$\ddot{c}_9$	[0.3650, 0.7558]	0.5604
$\ddot{c}_{10}$	[0.3295, 0.9095]	0.6195

Step-9: The order of the alternatives for C_i^* is $\ddot{c}_5 \geq \ddot{c}_{10} \geq \ddot{c}_7 \geq \ddot{c}_6 \geq \ddot{c}_9 \geq \ddot{c}_1 \geq \ddot{c}_4 \geq \ddot{c}_8 \geq \ddot{c}_3 \geq \ddot{c}_2$.

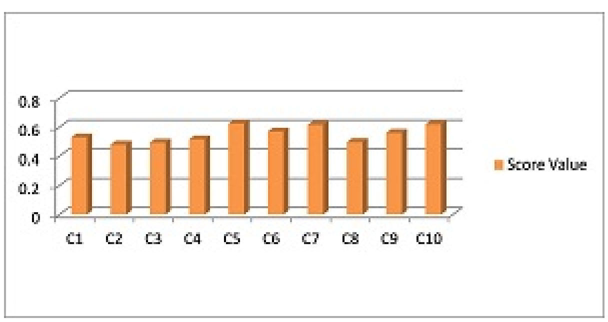

Figure 1 Graphical representation using MCGDM based on TOPSIS.

Step-10: The above ranking, it conclude that the company $\ddot{c}_5$ invest 30% , $\ddot{c}_{10}$ invest 25% , $\ddot{c}_7$ invest 20% , $\ddot{c}_6$ invest 15% and $\ddot{c}_9$ invest 10% .

5 MCGDM based on SIVFS-VIKOR aggregating operator

Algorithm-V (SIVFS-VIKOR)

Step-1: Suppose that the finite decision makers namely $\mathscr{D} = \{\mathscr{D}_i : i \in \mathbb{N}\}$ and the finite collection of alternatives namely $\mathscr{C} = \{\ddot{c}_i : i \in \mathbb{N}\}$ and finite family of parameters namely $\mathcal{D} = \{e_i : i \in \mathbb{N}\}$.
Step-2: Form a linguistic variables with obtain weighted parameter matrix

$$\mathcal{P} = \left[\omega_{ij}^L, \omega_{ij}^U\right]_{n\times m} = \begin{bmatrix} [\omega_{11}^L,\omega_{11}^U] & [\omega_{12}^L,\omega_{12}^U] & \cdots & [\omega_{1m}^L,\omega_{1m}^U] \\ [\omega_{21}^L,\omega_{21}^U] & [\omega_{22}^L,\omega_{22}^U] & \cdots & [\omega_{2m}^L,\omega_{2m}^U] \\ \vdots & \vdots & \ddots & \vdots \\ [\omega_{i1}^L,\omega_{i1}^U] & [\omega_{i2}^L,\omega_{i2}^U] & \cdots & [\omega_{im}^L,\omega_{im}^U] \\ \vdots & \vdots & \ddots & \vdots \\ [\omega_{n1}^L,\omega_{n1}^U] & [\omega_{n2}^L,\omega_{n2}^U] & \cdots & [\omega_{nm}^L,\omega_{nm}^U] \end{bmatrix}$$

Here the weight $\left[\omega_{ij}^L, \omega_{ij}^U\right]$ means that $\mathcal{D}_i$ to $\mathcal{P}_j$.
Step-3: Form weighted normalized decision matrix

$$\widehat{\mathcal{N}} = \left[\widehat{n}_{ij}^L, \widehat{n}_{ij}^U\right]_{n\times m} = \begin{bmatrix} [\widehat{n}_{11}^L,\widehat{n}_{11}^U] & [\widehat{n}_{12}^L,\widehat{n}_{12}^U] & \cdots & [\widehat{n}_{1m}^L,\widehat{n}_{1m}^U] \\ [\widehat{n}_{21}^L,\widehat{n}_{21}^U] & [\widehat{n}_{22}^L,\widehat{n}_{22}^U] & \cdots & [\widehat{n}_{2m}^L,\widehat{n}_{2m}^U] \\ \vdots & \vdots & \ddots & \vdots \\ [\widehat{n}_{i1}^L,\widehat{n}_{i1}^U] & [\widehat{n}_{i2}^L,\widehat{n}_{i2}^U] & \cdots & [\widehat{n}_{im}^L,\widehat{n}_{im}^U] \\ \vdots & \vdots & \ddots & \vdots \\ [\widehat{n}_{n1}^L,\widehat{n}_{n1}^U] & [\widehat{n}_{n2}^L,\widehat{n}_{n2}^U] & \cdots & [\widehat{n}_{nm}^L,\widehat{n}_{nm}^U] \end{bmatrix}$$

Here $\left[\widehat{n}_{ij}^L, \widehat{n}_{ij}^U\right] = \left[\frac{\omega_{ij}^L}{\sqrt{\sum_{i=1}^n \omega_{ij}^{2U}}}, \frac{\omega_{ij}^U}{\sqrt{\sum_{i=1}^n \omega_{ij}^{2L}}}\right]$ is the normalized parameter and weighted vector $\mathcal{W} = \left([m_1^L, m_1^U], [m_2^L, m_2^U] ..., [m_m^L, m_m^U]\right)$, where $[m_i^L, m_i^U] = \left[\frac{\omega_i^L}{\sqrt{\sum_{l=1}^n \omega_{li}^U}}, \frac{\omega_i^U}{\sqrt{\sum_{l=1}^n \omega_{li}^L}}\right]$ is the weight of the j^{th} parameter and $[\omega_j^L, \omega_j^U] = \left[\frac{\sum_{i=1}^n \widehat{n}_{ij}^L}{n}, \frac{\sum_{i=1}^n \widehat{n}_{ij}^U}{n}\right]$.
Step-4: Form decision SIVFS matrix

$$\mathscr{D}_i = \left[c_{jk}^{Li}, c_{jk}^{Ui}\right]_{l\times m} = \begin{bmatrix} [c_{11}^{Li},c_{11}^{Ui}] & [c_{12}^{Li},c_{12}^{Ui}] & \cdots & [c_{1m}^{Li},c_{1m}^{Ui}] \\ [c_{21}^{Li},c_{21}^{Ui}] & [c_{22}^{Li},c_{22}^{Ui}] & \cdots & [c_{2m}^{Li},c_{2m}^{Ui}] \\ \vdots & \vdots & \ddots & \vdots \\ [c_{j1}^{Li},c_{j1}^{Ui}] & [c_{j2}^{Li},c_{j2}^{Ui}] & \cdots & [c_{jm}^{Li},c_{jm}^{Ui}] \\ \vdots & \vdots & \ddots & \vdots \\ [c_{l1}^{Li},c_{l1}^{Ui}] & [c_{l2}^{Li},c_{l2}^{Ui}] & \cdots & [c_{lm}^{Li},c_{lm}^{Ui}] \end{bmatrix}$$

Here $\left[c_{jk}^{Li}, c_{jk}^{Ui}\right]$ means i^{th} decision maker $[\mathscr{D}_i^L, \mathscr{D}_i^U]$ for each i. Then obtain the aggregating matrix $\left[\mathscr{X}^L, \mathscr{X}^U\right] = \frac{\left[\mathscr{D}_1^L,\mathscr{D}_1^U\right]+\left[\mathscr{D}_2^L,\mathscr{D}_2^U\right]+...+\left[\mathscr{D}_n^L,\mathscr{D}_n^U\right]}{n} = \left[\dot{c}_{jk}^L, \dot{c}_{jk}^U\right]_{l\times m}$.
Step-5: Find the weighted decision SIVFS matrix

$$[\mathcal{Y}^L, \mathcal{Y}^U] = \left[\ddot{c}_{jk}^L, \ddot{c}_{jk}^U\right]_{l\times m} = \begin{bmatrix} [\ddot{c}_{11}^L,\ddot{c}_{11}^U] & [\ddot{c}_{12}^L,\ddot{c}_{12}^U] & \cdots & [\ddot{c}_{1m}^L,\ddot{c}_{1m}^U] \\ [\ddot{c}_{21}^L,\ddot{c}_{21}^U] & [\ddot{c}_{22}^L,\ddot{c}_{22}^U] & \cdots & [\ddot{c}_{2m}^L,\ddot{c}_{2m}^U] \\ \vdots & \vdots & \ddots & \vdots \\ [\ddot{c}_{j1}^L,\ddot{c}_{j1}^U] & [\ddot{c}_{j2}^L,\ddot{c}_{j2}^U] & \cdots & [\ddot{c}_{jm}^L,\ddot{c}_{jm}^U] \\ \vdots & \vdots & \ddots & \vdots \\ [\ddot{c}_{l1}^L,\ddot{c}_{l1}^U] & [\ddot{c}_{l2}^L,\ddot{c}_{l2}^U] & \cdots & [\ddot{c}_{lm}^L,\ddot{c}_{lm}^U] \end{bmatrix}$$

Where $\left[\ddot{c}_{jk}^L, \ddot{c}_{jk}^U\right] = \left[m_k^L \times \dot{c}_{jk}^L, m_k^U \times \dot{c}_{jk}^U\right]$.
Step-6: Calculate SIVFSV-PIS and SIVFSV-NIS. Now,

$$\text{SIVFSV-PIS} = \left(\left[\ddot{c}_1^{L+}, \ddot{c}_1^{U+}\right], \left[\ddot{c}_2^{L+}, \ddot{c}_2^{U+}\right] ..., \left[\ddot{c}_l^{L+}, \ddot{c}_l^{U+}\right]\right)$$
$$= \left\{ \left(\vee_k \left[\ddot{c}_{jk}^{L}, \ddot{c}_{jk}^{U}\right], \wedge_k \left[\ddot{c}_{jk}^{L}, \ddot{c}_{jk}^{U}\right], \wedge_k \left[\ddot{c}_{jk}^{L}, \ddot{c}_{jk}^{U}\right]\right) : j = 1, 2, ..., l \right\}$$

$$\text{and SIVFSV-PIS} = \left(\left[\ddot{c}_1^{L-}, \ddot{c}_1^{U-}\right], \left[\ddot{c}_2^{L-}, \ddot{c}_2^{U-}\right] ..., \left[\ddot{c}_l^{L-}, \ddot{c}_l^{U-}\right]\right)$$
$$= \left\{ \left(\wedge_k \left[\ddot{c}_{jk}^{L}, \ddot{c}_{jk}^{U}\right], \vee_k \left[\ddot{c}_{jk}^{L}, \ddot{c}_{jk}^{U}\right], \vee_k \left[\ddot{c}_{jk}^{L}, \ddot{c}_{jk}^{U}\right]\right) : j = 1, 2, ..., l \right\}.$$

Step-7: Find the values of utility $[\mathcal{S}_i^L, \mathcal{S}_i^U]$, individual regret $[\mathcal{R}_i^L, \mathcal{R}_i^U]$ and compromise $[\mathcal{Q}_i^L, \mathcal{Q}_i^U]$, where $[\mathcal{S}_i^L, \mathcal{S}_i^U] = \left[\sum_{j=1}^{m} m_j^L \cdot \left(\sqrt{\frac{\ddot{c}_{ij}^{2L} - \ddot{c}_j^{2U+}}{\ddot{c}_j^{2U+} - \ddot{c}_j^{2L-}}}\right), \sum_{j=1}^{m} m_j^U \cdot \left(\sqrt{\frac{\ddot{c}_{ij}^{2U} - \ddot{c}_j^{2L+}}{\ddot{c}_j^{2L+} - \ddot{c}_j^{2U-}}}\right)\right]$

and $[\mathcal{R}_i^L, \mathcal{R}_i^U] = \left[\max_{j=1}^{m} m_j^L \cdot \left(\sqrt{\frac{\ddot{c}_{ij}^{2L} - \ddot{c}_j^{2U+}}{\ddot{c}_j^{2U+} - \ddot{c}_j^{2L-}}}\right), \sum_{j=1}^{m} m_j^U \cdot \left(\sqrt{\frac{\ddot{c}_{ij}^{2U} - \ddot{c}_j^{2L+}}{\ddot{c}_j^{2L+} - \ddot{c}_j^{2U-}}}\right)\right]$

and $[\mathcal{Q}_i^L, \mathcal{Q}_i^U] = \left[\kappa\left(\frac{\mathcal{S}_i^L - \mathcal{S}^{U-}}{\mathcal{S}^{U+} - \mathcal{S}^{L-}}\right) + (1-\kappa)\left(\frac{\mathcal{R}_i^L - \mathcal{R}^{U-}}{\mathcal{R}^{U+} - \mathcal{R}^{L-}}\right), \kappa\left(\frac{\mathcal{S}_i^U - \mathcal{S}^{L-}}{\mathcal{S}^{L+} - \mathcal{S}^{U-}}\right) + (1-\kappa)\left(\frac{\mathcal{R}_i^U - \mathcal{R}^{L-}}{\mathcal{R}^{L+} - \mathcal{R}^{U-}}\right)\right]$.

Hence $\mathcal{Q} = \frac{\mathcal{Q}_i^L + \mathcal{Q}_i^U}{2}$, where $[\mathcal{S}_i^{L+}, \mathcal{S}_i^{U+}] = \max_i[\mathcal{S}_i^L, \mathcal{S}_i^U]$, $[\mathcal{S}_i^{L-}, \mathcal{S}_i^{U-}] = \min_i[\mathcal{S}_i^L, \mathcal{S}_i^U]$, $[\mathcal{R}_i^{L+}, \mathcal{R}_i^{U+}] = \max_i[\mathcal{R}_i^L, \mathcal{R}_i^U]$ and $[\mathcal{R}_i^{L-}, \mathcal{R}_i^{U-}] = \min_i[\mathcal{R}_i^L, \mathcal{R}_i^U]$. The real number κ is called a coefficient of decision mechanism. The role of κ is that if majority compromise solution when $\kappa > 0.5$; and consensus compromise solution when $\kappa = 0.5$; and veto compromise solution when $\kappa < 0.5$. Let m_j said to be j^{th} parameter of weight. Let $[m_j^L, m_j^U]$ represents the weight of the j^{th} parameter/criteria.

Step-8: Obtain the rank of choices and derive compromise solution. Arrange $\mathcal{Q}_i$ in increasing order to make ranking list. The alternative $\ddot{c}_\alpha$ will be declared compromise solution if it ranks the best (having least value) in $\mathcal{Q}_i$ and the following two conditions satisfies simultaneously:

$C1$ admissible: If $\ddot{c}_\alpha$ and $\ddot{c}_\beta$ represent top alternatives in $\mathcal{Q}$, then $\mathcal{Q}(\ddot{c}_\beta) - \mathcal{Q}(\ddot{c}_\alpha) \geq \frac{1}{n-1}$, where n is the counting of parameters.

$C2$ admissible: The alternative $\ddot{c}_\alpha$ should be best ranked by $[\mathcal{S}_i^L, \mathcal{S}_i^U] = \frac{\mathcal{S}_i^L + \mathcal{S}_i^U}{2}$ and /or $[\mathcal{R}_i^L, \mathcal{R}_i^U] = \frac{\mathcal{R}_i^L + \mathcal{R}_i^U}{2}$.

If $C1$ and $C2$ are not satisfies simultaneously, then there exist multiple compromise solutions:

(i) If $C1$ is satisfied, then the alternatives $\ddot{c}_\alpha$ and $\ddot{c}_\beta$ are called compromise solutions:

(ii) If $C1$ is not satisfied, then the alternatives $\ddot{c}_\alpha$, $\ddot{c}_\beta$,..., $\ddot{c}_\xi$ are called the compromise solutions, where $\ddot{c}_\xi$ is founded by $\mathcal{Q}(\ddot{c}_\xi) - \mathcal{Q}(\ddot{c}_\alpha) \geq \frac{1}{n-1}$.

Example 5.1 *Now, we can appeal Example 4.1 to Algorithm-V. We conclude that the first five steps are the same process. Hence we solve the problem using VIKOR method entering* **Step 6***.*

Step-6: Find SIVFSV-PIS and SIVFSV-NIS are listed as follows.

$\left[\ddot{c}^{L+}, \ddot{c}^{U+}\right]$	$SIVFSV-PIS$	$\left[\ddot{c}^{L-}, \ddot{c}^{U-}\right]$	$SIVFSV-NIS$
$\left[\ddot{c}_1^{L+}, \ddot{c}_1^{U+}\right]$	([0.0534, 0.0991], [0.0194, 0.0542], [0.0262, 0.037])	$\left[\ddot{c}_1^{L-}, \ddot{c}_1^{U-}\right]$	([0.0252, 0.0568], [0.0544, 0.0885], [0.0505, 0.0793])
$\left[\ddot{c}_2^{L+}, \ddot{c}_2^{U+}\right]$	([0.0587, 0.1024], [0.0299, 0.0614], [0.0165, 0.0409])	$\left[\ddot{c}_2^{L-}, \ddot{c}_2^{U-}\right]$	([0.0237, 0.052], [0.0567, 0.1024], [0.0567, 0.1024])
$\left[\ddot{c}_3^{L+}, \ddot{c}_3^{U+}\right]$	([0.0506, 0.0804], [0.0239, 0.0426], [0.0285, 0.0437])	$\left[\ddot{c}_3^{L-}, \ddot{c}_3^{U-}\right]$	([0.0258, 0.0449], [0.0497, 0.065], [0.0561, 0.0839])
$\left[\ddot{c}_4^{L+}, \ddot{c}_4^{U+}\right]$	([0.0474, 0.0847], [0.0136, 0.0464], [0.0145, 0.0451])	$\left[\ddot{c}_4^{L-}, \ddot{c}_4^{U-}\right]$	([0.0252, 0.0382], [0.0474, 0.0778], [0.0649, 0.0929])
$\left[\ddot{c}_5^{L+}, \ddot{c}_5^{U+}\right]$	([0.0534, 0.0968], [0.0277, 0.0673], [0.0277, 0.064])	$\left[\ddot{c}_5^{L-}, \ddot{c}_5^{U-}\right]$	([0.0195, 0.0476], [0.0627, 0.1051], [0.0647, 0.1083])

Step-7: Taking $\kappa = 0.5$, we found that the values of utility $[\mathcal{S}_i^L, \mathcal{S}_i^U]$, individual regret $[\mathcal{R}_i^L, \mathcal{R}_i^U]$ and compromise $\mathcal{Q}_i$ for each alternative $[\ddot{c}_i^L, \ddot{c}_i^U]$.

Alternative ($\ddot{c}$)	$[\mathcal{S}_i^L, \mathcal{S}_i^U]$	$[\mathcal{R}_i^L, \mathcal{R}_i^U]$	$[\mathcal{Q}_i^L, \mathcal{Q}_i^U]$	$\mathcal{Q}_i$
$\ddot{c}_1 = [\ddot{c}_1^L, \ddot{c}_1^U]$	[0.4015, 0.5133]	[0.0938, 0.1248]	[0.1197, 0.134]	0.1268
$\ddot{c}_2 = [\ddot{c}_2^L, \ddot{c}_2^U]$	[0.4228, 0.6026]	[0.1120, 0.1409]	[0.7487, 0.8433]	0.7960
$\ddot{c}_3 = [\ddot{c}_3^L, \ddot{c}_3^U]$	[0.3905, 0.5868]	[0.1106, 0.1387]	[0.4635, 0.7292]	0.5963
$\ddot{c}_4 = [\ddot{c}_4^L, \ddot{c}_4^U]$	[0.4001, 0.5274]	[0.0947, 0.1211]	[0.1323, 0.1379]	0.1351
$\ddot{c}_5 = [\ddot{c}_5^L, \ddot{c}_5^U]$	[0.4025, 0.5486]	[0.1044, 0.1380]	[0.3977, 0.5327]	0.4652
$\ddot{c}_6 = [\ddot{c}_6^L, \ddot{c}_6^U]$	[0.4232, 0.5637]	[0.1053, 0.1500]	[0.5794, 0.8126]	0.6960
$\ddot{c}_7 = [\ddot{c}_7^L, \ddot{c}_7^U]$	[0.4026, 0.5023]	[0.0954, 0.1326]	[0.1671, 0.2167]	0.1919
$\ddot{c}_8 = [\ddot{c}_8^L, \ddot{c}_8^U]$	[0.4555, 0.5833]	[0.1022, 0.1464]	[0.7490, 0.8457]	0.7974
$\ddot{c}_9 = [\ddot{c}_9^L, \ddot{c}_9^U]$	[0.3933, 0.4988]	[0.0925, 0.1253]	[0.0213, 0.0734]	0.0474
$\ddot{c}_{10} = [\ddot{c}_{10}^L, \ddot{c}_{10}^U]$	[0.3918, 0.5587]	[0.1084, 0.1378]	[0.4177, 0.5786]	0.4981

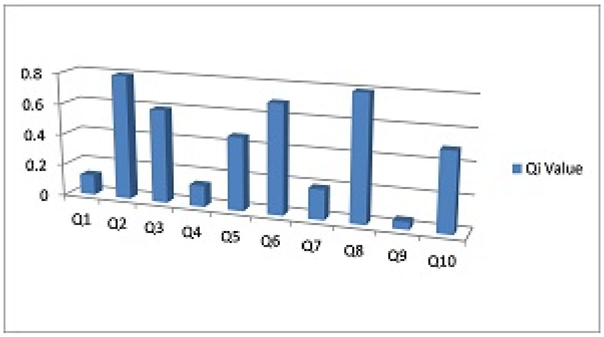

Figure 2 Graphical representation using MCGDM based on VIKOR.

Step-8: The rank of alternatives for $\mathcal{Q}_i$: $\ddot{c}_9 \leq \ddot{c}_1 \leq \ddot{c}_4 \leq \ddot{c}_7 \leq \ddot{c}_5 \leq \ddot{c}_{10} \leq \ddot{c}_3 \leq \ddot{c}_6 \leq \ddot{c}_2 \leq \ddot{c}_8$. Now, $\mathcal{Q}(\ddot{c}_1) - \mathcal{Q}(\ddot{c}_9) = 0.0794 \not\geq \frac{1}{4}$. Thus C1 is false, further more $\mathcal{Q}(\ddot{c}_5) - \mathcal{Q}(\ddot{c}_9) = 0.4178 \geq \frac{1}{4}$. Therefore, we establish $\ddot{c}_9, \ddot{c}_1, \ddot{c}_4, \ddot{c}_7, \ddot{c}_5$ are multiple compromise solutions. Hence the company should invest 30% on $\ddot{c}_9$, 25% on $\ddot{c}_1$, 20% on $\ddot{c}_4$, 15% on $\ddot{c}_7$ and 10% on $\ddot{c}_5$.

6 Comparison and discussion

These two methods are assume a scalar component for each criterion and these two methods are different from normalization approach. In TOPSIS utilize to vector normalization approach and VIKOR utilize to linear normalization approach. The major difference between two methods looks in the aggregation function. We can finding ranking of values using an aggregating function. The best ranked alternative by VIKOR is closest to the ideal solution. However, the best ranked alternative by TOPSIS is the best using ranking index, but doesn't closest to the ideal solution. Hence advantage of VIKOR gives to be compromise solution.

7 Conclusion:

In this present communication, the first three algorithms follow by MCGDM under SIVFS and last two algorithms follow by SIVFS linguistic TOPSIS and VIKOR approaches under aggregation operator. Again we interact SIVFS aggregation operator and score function values based on some technique. Also we have inserted various sorts of statistical charts to image the rankings of alternatives under consideration.

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] A. Adeel, M. Akram and A.N.A. Koam, Group decision-making based on m-polar fuzzy linguistic TOPSIS method, Symmetry 11(735) (2019), 1-20.

[2] M. Akram and M. Arshad, A novel trapezoidal bipolar fuzzy TOPSIS method for group decision making, Group Decision and Negotiation (2018).

[3] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and Systems, 20(1), (1986), 87-96.

[4] F.E. Boran, S. Genc, M. Kurt and D. Akay, A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method, Expert Systems with Applications 36(8), (2009), 11363-11368.

[5] S. Eraslan and F. Karaaslan, A group decision making method based on TOPSIS under fuzzy soft environment, Journal of New Theory, 3, (2015), 30-40.

[6] C.L. Hwang, K. Yoon, Multiple Attributes Decision Making Methods and Applications, Springer, Berlin Heidelberg, 1981.

[7] C. Jana, T. Senapati and M. Pal, Pythagorean fuzzy Dombi aggregation operators and its applications in multiple attribute decision making, International Journal of Intelligent Systems, 34(9), (2019), 2019-2038.

[8] C. Jana and M. Pal, Application of bipolar intuitionistic fuzzy soft sets in decision making problem, International Journal of Fuzzy System Applications, 7(3),(2018) 32-55.

[9] Fatma Kutlu Gndoodu and Cengiz Kahraman, A novel fuzzy TOPSIS method using emerging interval-valued spherical fuzzy sets, Engineering Applications of Artificial Intelligence, 85, (2019), 307323.

[10] T. Mahmood, U. Kifayat, Q. Khan and J. Naeem, An approach toward decision making and medical diagnosis problems using the concept of spherical fuzzy sets, Neural Computing and Applications (2018).

[11] P.K. Maji, R. Biswas and A.R. Roy, Fuzzy Soft Set, Journal of Fuzzy Mathematics, 9(3), (2001), 589-602.

[12] P.K. Maji, R. Biswas and A.R. Roy, On intuitionistic Fuzzy Soft Set, Journal of Fuzzy Mathematics, 9(3), (2001), 677-692.

[13] D. Molodtsov, Soft set theory First results, Computers and Mathematics with Applications, 37, (1999), 19-31.

[14] S. Opricovic, G.H. Tzeng, Extended VIKOR method in comparison with outranking methods, Eur. J. Oper. Res. 178, (2007) 514-529.

[15] S. Opricovic, Fuzzy VIKOR with an application to water resources planning, Expert Syst. Appl. 38, (2011), 12983-12990.

[16] S. Opricovic, G.H. Tzeng, Fuzzy multicriteria model for post-earthquake land-use planning, Natural Hazards Review, 4, (2003), 59-64.

[17] X.D. Peng and J. Dai, Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function, Neural Computing and Applications, 29(10), (2018), 939-954.

[18] X.D. Peng, Y. Yang, J.P. Song, Pythagorean fuzzy soft set and its application, Computer Engineering, 41(7), (2015), 224-229.

[19] G.H. Tzeng, C.W. Lin, S. Opricovic, Multi-criteria analysis of alternative-fuel buses for public transportation, Energy Policy, 33, (2005) 1373-1383.

[20] K. Ullah, T. Mahmood, Z. Ali and N. Jan, On some distance measures of complex Pythagorean fuzzy sets and their applications in pattern recognition, Complex and Intelligent Systems, 1-13, (2019).

[21] Z. Xiao, W.J. Chen, and L.L. Li, A method based on interval-valued fuzzy soft set for multi attribute group decision making problems under uncertain environment, Knowledge and Information Systems, 34, (2013), 653-669.

[22] Z. Xu and X. Zhang, Hesitant fuzzy multi-attribute decision-making based on TOPSIS with incomplete weight information, Knowledge-Based Systems, 52, (2013), 53-64.

[23] R.R. Yager, A.M Abbasov, Pythagorean membership grades, complex numbers, and decision making, International Journal of Intelligent Systems, 28, (2014), 436-452.

[24] R.R. Yager, Pythagorean membership grades in multi criteria decision making, IEEE Trans. Fuzzy Systems, 22, (2014), 958-965.

[25] Yong Yang, Chencheng Liang, Shiwei Ji and Tingting Liu, Adjustable soft discernibility matrix based on picture fuzzy soft sets and its applications in decision making, Journal of Intelligent and Fuzzy Systems, 29, (2015), 1711-1722.

[26] L. A. Zadeh, Fuzzy sets, Information and control 8(3),(1965), 338-353.

[27] X. Zhang and Z. Xu, Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets, International Journal of Intelligent Systems, 29, (2014), 1061-1078.

[28] R.M. Zulqarnain , Xiao Long Xin , Muhammad Saqlain and Waseem Asghar Khan, TOPSIS Method Based on the Correlation Coefficient of Interval-Valued Intuitionistic Fuzzy Soft Sets and Aggregation Operators with Their Application in Decision-Making, Journal of Mathematics, 2021, 1-16.