SIS-BAM Algorithm for Angular Parameter Estimation of 2-D Incoherently Distributed Sources

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SIS-BAM Algorithm for Angular Parameter Estimation of 2-D Incoherently Distributed Sources

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Abstract For two-dimensional (2-D) incoherently distributed sources, this paper presents an effective angular parameter estimation method based on shift invariant structure (SIS) of the beamspace array manifold (BAM), named as SIS-BAM algorithm. In the proposed method, a shift invariance structure (SIS) of the observed vectors is firstly established utilizing a generalized array manifold of an uniform linear orthogonal array (ULOA). Secondly, based on Fourier basis vectors and the SIS, a beamspace transformation matrix can be performed. It projects received signals into the corresponding beamspace, so as to carry out dimension reduction of observed signals in beamspace domain. Finally, according to the SIS of beamspace observed vectors, the closed form solutions of the nominal azimuth and elevation are derived. Compared with the previous works, the presented SIS-BAM method provides better estimation performance, for example: 1) the computational complexity is reduced due to dealing with low-dimension beamspace signals and avoiding spectral search; 2) it can not only improve the angular parameter estimation accuracy but also have excellent robustness to the change of signal-to-noise ratio (SNR) and snapshot number. The theoretical analysis and simulation results confirm the effectiveness of the proposed method.

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1 Introduction

In millimeter wave (mmWave) massive multiple-input multiple-output (MIMO) systems, accurate estimation of direction of arrival (DOA) can not only improve the energy efficiency, but also enhance the performance of key techniques, such as spatial modulation and NOMA [1]- [2]. Most of DOA estimation algorithms are based on the point source model, while diffusion and multipath propagation cannot be neglected in numerous practical applications. Therefore, the distributed source is more suitable than the point one [3]. In general, the distributed sources can be divided into coherently distributed (CD) sources and incoherently distributed (ID) sources [4]. This paper will focus on the the angular parameter estimation of ID sources.

In recent years, several high-resolution angular parameter estimation methods for ID sources have been proposed. The dispersed signal parameter estimator (DISPARE) is one of the efficient subspace-based approaches, which exploits the orthogonality between the quasi-signal subspace and the quasi-noise subspace [5] to obtain the nominal DOA and the angular spread estiamtion. Unfortunately, DISPARE requires multidimensional spectrum search to estimate the angular parameters, which may result in high computational costs. Therefore, several computationally simpler algorithms are proposed, which estimate the angular parameters via the closed form expression. For example, a low-complexity algorithm based on a novel propagator is given for the nominal DOA estimation, which avoids spectrum search and the eigendecomposition [6]. A generalized shift invariance property and an extended cross-correlation matrix are used to derive the closed form expressions of the nominal DOAs in [7] and [8], respectively. Besides, via beamspace transformation, the matrix dimension can be reduced, which is also able to reduce the computational cost [9].

In large-scale MIMO systems, two-dimensional (2-D) DOAs are necessary for beamforming, wireless location and interference coordination [10]. Some of the existing parameter estimation approaches for one-dimensional (1-D) ID sources can be extended to two-dimensional (2-D) sources [11] [12]. In spite of good performance, the computational costs of the above works are relatively high. To overcome this obstacle, several low-complexity parameter estimation solutions for 2-D sources via the rotational invariance technique are proposed [13]- [16]. A new approach based on parallel uniform linear arrays is proposed in [13], the presented algorithm firstly estimates the nominal elevation by the modified TLS-ESPRIT method, and then the nominal azimuth is estimated by one-dimensional searching. However, the angular spreads are not taken into account in the aforementioned method. An ESPRIT-based approach is presented for the estimation of the nominal DOAs and their angular spreads, which enjoys low computational cost since it avoids spectral search [14].

In consideration of the signal non-circular property, a computational efficient method for DOA estimation is given [15], which can enhance the DOA estimation performance such as higher estimation accuracy, without spectrum searching and pairing, etc. A modified conjugate ESPRIT algorithm is presented, which considers the mixed circular and noncircular signals [16]. However, the computational complexity of the aforementioned algorithms may be high due to high dimensional received signals.

Motivated by this, a low-complexity 2-D DOA estimation method based on the beamspace transformation is presented in this paper. The generalized array manifold (GAM) model is firstly established via the first-order Taylor expansion, in which the nominal DOA is decoupled from the angular spread. Secondly, a beamspace transformation matrix is derived from the shift invariant structure (SIS) of observed signals impinging on the uniform linear orthogonal array (ULOA). Furthermore, via the transformation matrix, the observed signals is projected into the beamspace to constructed the beamspace data, which has lower dimension than the observed signals. Finally, the closed form solution of the nominal azimuth and the nominal elevation are obtained by the SIS of the beamspace signals. The rest of this paper is organized as follows. Sect. 2 briefly introduces the data model of ID sources. The proposed method is formulated in Sect. 3. In Sect. 4, simulation results are presented to verify the performance of the proposed method. Finally, the conclusion is provided in Sect. 5.

2 Data Model

Consider a uniform linear orthogonal array (ULOA), which consists of two uniform linear arrays (ULAs) X and Y as shown in Fig. 1. Arrays X and Y have M_x and M_y sensors with interelement spacing d, respectively. Assume that $K(K \leq M)$ uncorrelated narrowband ID sources impinge on the ULOA. The array output vectors $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ of arrays X and Y can be expressed as [17]

$$\begin{aligned} \boldsymbol{x}(t) &= \sum_{k=1}^{K} s_k(t) \sum_{l=1}^{L_k} \boldsymbol{\beta}_{kl}(t) \boldsymbol{a}_x(\theta_{kl}(t), \varphi_{kl}(t)) + \boldsymbol{n}_x(t) \\ \boldsymbol{y}(t) &= \sum_{k=1}^{K} s_k(t) \sum_{l=1}^{L_k} \boldsymbol{\beta}_{kl}(t) \boldsymbol{a}_y(\theta_{kl}(t), \varphi_{kl}(t)) + \boldsymbol{n}_y(t) \end{aligned} \tag{1}$$

where $s_k(t)$ is the complex-valued signal transmitted by the kth source. L_k denotes the number of rays inside the kth signal. $\boldsymbol{\beta}_{kl}(t)$ stands for the complex-valued ray gain. $\theta_{kl}(t)$ and $\varphi_{kl}(t)$ represent the azimuth angle and the elevation angle of the lth ray from the kth signal, respectively. $\boldsymbol{n}_x(t)$ and $\boldsymbol{n}_y(t)$ stand for the additive Gaussian white noise with zero-mean and equal power σ_n^2 in arrays X and Y, respectively. $\boldsymbol{a}_x(\theta_{kl}(t), \varphi_{kl}(t))$ and $\boldsymbol{a}_y(\theta_{kl}(t), \varphi_{kl}(t))$ are the array manifold vectors of the arrays X and Y, which can be expressed as

$$\boldsymbol{a}_x(\theta_{kl}(t),\varphi_{kl}(t)) = [1, e^{j\frac{2\pi}{\lambda}d\cos(\theta_{kl}(t))\cos(\varphi_{kl}(t))}, \cdots, e^{j\frac{2\pi}{\lambda}(M_x-1)d\cos(\theta_{kl}(t))\cos(\varphi_{kl}(t))}]^{\mathrm{T}}$$

$$\boldsymbol{a}_y(\theta_{kl}(t),\varphi_{kl}(t)) = [1, e^{j\frac{2\pi}{\lambda}d\sin(\theta_{kl}(t))\cos(\varphi_{kl}(t))}, \cdots, e^{j\frac{2\pi}{\lambda}(M_y-1)d\sin(\theta_{kl}(t))\cos(\varphi_{kl}(t))}]^{\mathrm{T}}$$

where λ is the wavelength of the signal. The superscript $(\cdot)^{\mathrm{T}}$ represents the transpose operation.

The azimuth angle and the elevation angle $\theta_{kl}(t)$ and $\varphi_{kl}(t)$ can be expressed as

$$\theta_{kl}(t) = \theta_k + \widetilde{\theta}_{kl}(t)$$

$$\varphi_{kl}(t) = \varphi_k + \widetilde{\varphi}_{kl}(t)$$

where θ_k and φ_k stand for the nominal azimuth DOA and the nominal elevation DOA for the kth signal, which are the means of $\theta_{kl}(t)$ and $\varphi_{kl}(t)$, respectively. $\widetilde{\theta}_{kl}(t)$ and $\widetilde{\varphi}_{kl}(t)$ denote the corresponding angular deviations.

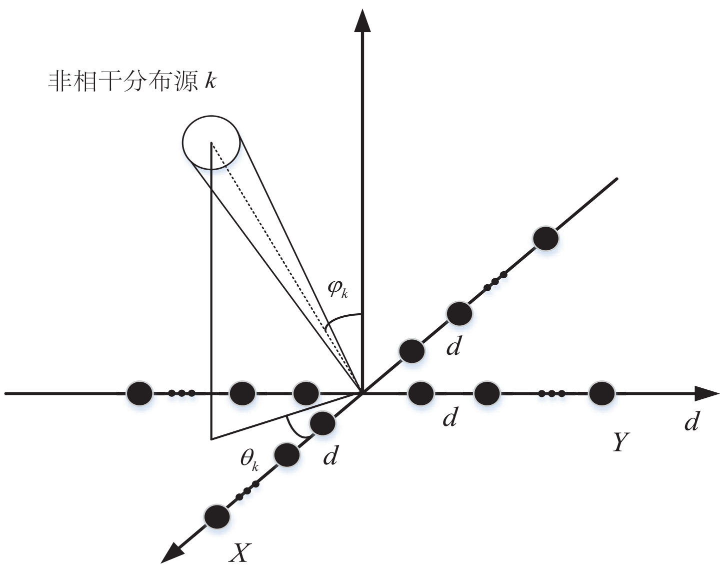


Fig.1. Array geometry of uniform linear orthogonal array

3 Problem Formulation

In this section, the generalized array manifold (GAM) model is firstly established to decouple the nominal DOAs from their angular spreads, so as to estimate the nominal DOAs and the angular spreads separately. Then, via the

SIS, the beamspace transformation matrix is derived to perform dimensionality reduction for the observed vectors. Finally, via the SIS of the beamspace received signals, the closed form solutions are given for the estimation of the nominal azimuth DOA and elevation DOA.

3.1 GAM Model and SIS

Using the first order Taylor series expansion around θ_k and φ_k, the array manifold vectors $\boldsymbol{a}_x(\theta_{kl}(t), \varphi_{kl}(t))$ and $\boldsymbol{a}_y(\theta_{kl}(t), \varphi_{kl}(t))$ can be approximated as

$$\begin{aligned}\boldsymbol{a}_x(\theta_{kl}(t), \varphi_{kl}(t)) &\approx \boldsymbol{a}_x(\theta_k, \varphi_k) + \boldsymbol{a}'_{x\theta}(\theta_k, \varphi_k)\widetilde{\theta}_{kl}(t) + \boldsymbol{a}'_{x\varphi}(\theta_k, \varphi_k)\widetilde{\varphi}_{kl}(t)\\ \boldsymbol{a}_y(\theta_{kl}(t), \varphi_{kl}(t)) &\approx \boldsymbol{a}_y(\theta_k, \varphi_k) + \boldsymbol{a}'_{y\theta}(\theta_k, \varphi_k)\widetilde{\theta}_{kl}(t) + \boldsymbol{a}'_{y\varphi}(\theta_k, \varphi_k)\widetilde{\varphi}_{kl}(t)\end{aligned} \tag{2}$$

where $\boldsymbol{a}'_{x\theta}(\theta_k, \varphi_k)$ and $\boldsymbol{a}'_{y\theta}(\theta_k, \varphi_k)$ stand for the partial derivatives of $\boldsymbol{a}_x(\theta_k, \varphi_k)$ and $\boldsymbol{a}_y(\theta_k, \varphi_k)$ with respect to θ_k, respectively. $\boldsymbol{a}'_{x\varphi}(\theta_k, \varphi_k)$ and $\boldsymbol{a}'_{y\varphi}(\theta_k, \varphi_k)$ represent the partial derivatives of $\boldsymbol{a}_x(\theta_k, \varphi_k)$ and $\boldsymbol{a}_y(\theta_k, \varphi_k)$ with respect to φ_k, respectively.

Substituting (2) into (1), the received signals $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ can be rewritten as

$$\begin{aligned}\boldsymbol{x}(t) &= \sum_{k=1}^{K}(\boldsymbol{a}_x(\theta_k, \varphi_k)h_{k,1}(t) + \boldsymbol{a}'_{x\theta}(\theta_k, \varphi_k)h_{k,2}(t) + \boldsymbol{a}'_{x\varphi}(\theta_k, \varphi_k)h_{k,3}(t)) + \boldsymbol{n}_x(t)\\ \boldsymbol{y}(t) &= \sum_{k=1}^{K}(\boldsymbol{a}_y(\theta_k, \varphi_k)h_{k,1}(t) + \boldsymbol{a}'_{y\theta}(\theta_k, \varphi_k)h_{k,2}(t) + \boldsymbol{a}'_{y\varphi}(\theta_k, \varphi_k)h_{k,3}(t)) + \boldsymbol{n}_y(t)\end{aligned} \tag{3}$$

where $h_{k,1}(t) = s_k(t)\sum_{l=1}^{L_k}\boldsymbol{\beta}_{kl}(t)$, $h_{k,2}(t) = s_k(t)\sum_{l=1}^{L_k}\boldsymbol{\beta}_{kl}(t)\widetilde{\theta}_{kl}(t)$ and $h_{k,3}(t) = s_k(t)\sum_{l=1}^{L_k}\boldsymbol{\beta}_{kl}(t)\widetilde{\varphi}_{kl}(t)$.

It is obvious that $h_{k,1}(t)$, $h_{k,2}(t)$ and $h_{k,3}(t)$ in (3) are irrelevant to the nominal DOAs, hence (3) can be reformulated into GAM model as

$$\begin{aligned}\boldsymbol{x}(t) &= \boldsymbol{A}_x(\theta, \varphi)\boldsymbol{h}(t) + \boldsymbol{n}_x(t)\\ \boldsymbol{y}(t) &= \boldsymbol{A}_y(\theta, \varphi)\boldsymbol{h}(t) + \boldsymbol{n}_y(t)\end{aligned}$$

where $\boldsymbol{A}_x(\theta, \varphi) = [\boldsymbol{a}_x(\theta_1, \varphi_1), \boldsymbol{a}'_{x\theta}(\theta_1, \varphi_1), \boldsymbol{a}'_{x\varphi}(\theta_1, \varphi_1), \cdots, \boldsymbol{a}_x(\theta_K, \varphi_K), \boldsymbol{a}'_{x\theta}(\theta_K, \varphi_K), \boldsymbol{a}'_{x\varphi}(\theta_K, \varphi_K)]$ and $\boldsymbol{A}_y(\theta, \varphi) = [\boldsymbol{a}_y(\theta_1, \varphi_1), \boldsymbol{a}'_{y\theta}(\theta_1, \varphi_1), \boldsymbol{a}'_{x\varphi}(\theta_1, \varphi_1), \cdots, \boldsymbol{a}_y(\theta_K, \varphi_K), \boldsymbol{a}'_{y\theta}(\theta_K, \varphi_K), \boldsymbol{a}'_{y\varphi}(\theta_K, \varphi_K)]$ represent the generalized manifold matrix of the arrays X and Y. $\boldsymbol{h}(t) = [h_{1,1}, h_{1,2}, h_{1,3}, \cdots, h_{K,1}, h_{K,2}, h_{K,3}]$.

It is noteworthy that the generalized manifold matrix $A_x(\theta, \varphi)$ and $A_y(\theta, \varphi)$ depend only on the nominal azimuth DOA and the nominal elevation DOA, while $\boldsymbol{h}(t)$ is independent of the nominal DOAs and relates to the angular spread. Accordingly, the decoupling of the nominal DOAs and the angular spreads is realized via GAM model.

As shown in Fig. 1, it is obvious that the received signal in array X has similar property to that in array Y , hence taking the received signal $x(t)$ as an example to illustrate the proposed method.

Similar to ESPRIT-based approaches, the ULA X with M_x sensors need to be divided into two subarrays X_1 and X_2 with equivalent number of sensors in the proposed method, so as to construct the SIS of the observed vectors. The subarray X_1 consists of the first $M_x - 1$ sensors and the subarray X_2 is composed of the last $M_x - 1$ sensors.

According to the aforementioned partition of the array X, the generalized array manifold matrices $\boldsymbol{A}_1(\theta_k, \varphi_k)$ and $\boldsymbol{A}_2(\theta_k, \varphi_k)$ of the subarrays X_1 and X_2 can be respectively expressed as

$$\begin{aligned} \boldsymbol{A}_1(\theta_k, \varphi_k) &= \boldsymbol{J}_1 \boldsymbol{A}_x(\theta_k, \varphi_k) \\ \boldsymbol{A}_2(\theta_k, \varphi_k) &= \boldsymbol{J}_2 \boldsymbol{A}_x(\theta_k, \varphi_k) \end{aligned} \tag{4}$$

where $\boldsymbol{J}_1 = [\boldsymbol{I}_N, \boldsymbol{0}_{(N\times 1)}]$ and $\boldsymbol{J}_2 = [\boldsymbol{0}_{(N\times 1)}, \boldsymbol{I}_N]$ represent selection matrices. $\boldsymbol{A}_1(\theta_k, \varphi_k) = [\boldsymbol{a}_1(\theta_k, \varphi_k), \boldsymbol{a}'_{1\theta}(\theta_k, \varphi_k), \boldsymbol{a}'_{1\varphi}(\theta_k, \varphi_k), \cdots, \boldsymbol{a}_1(\theta_K, \varphi_K), \boldsymbol{a}'_{1\theta}(\theta_K, \varphi_K), \boldsymbol{a}'_{1\varphi}(\theta_K, \varphi_K)]$ and $\boldsymbol{A}_2(\theta_k, \varphi_k) = [\boldsymbol{a}_2(\theta_k, \varphi_k), \boldsymbol{a}'_{2\theta}(\theta_k, \varphi_k), \boldsymbol{a}'_{2\varphi}(\theta_k, \varphi_k) \cdots, \boldsymbol{a}_2(\theta_K, \varphi_K), \boldsymbol{a}'_{2\theta}(\theta_K, \varphi_K), \boldsymbol{a}'_{2\varphi}(\theta_K, \varphi_K)]$ with $\boldsymbol{a}_1(\theta_k, \varphi_k) = \boldsymbol{J}_1 \boldsymbol{a}_x(\theta_k, \varphi_k)$ and $\boldsymbol{a}_2(\theta_k, \varphi_k) = \boldsymbol{J}_2 \boldsymbol{a}_x(\theta_k, \varphi_k)$.

Note that $\boldsymbol{a}_1(\theta_k, \varphi_k)$ and $\boldsymbol{a}_2(\theta_k, \varphi_k)$ have the following relationship

$$\boldsymbol{a}_1(\theta_k, \varphi_k) = \phi(\theta_k, \varphi_k) \boldsymbol{a}_2(\theta_k, \varphi_k) \tag{5}$$

where $\phi(\theta_k, \varphi_k) = \exp(j \frac{2\pi}{\lambda} d \cos\theta_k \cos\varphi_k)$.

The partial derivative of (5) around θ_k and φ_k are respectively given by

$$\begin{aligned} \boldsymbol{a}'_{1\theta}(\theta_k, \varphi_k) &= \phi(\theta_k, \varphi_k) \boldsymbol{a}'_{2\theta}(\theta_k, \varphi_k) + \phi'_\theta(\theta_k, \varphi_k) \boldsymbol{a}_2(\theta_k, \varphi_k) \\ \boldsymbol{a}'_{1\varphi}(\theta_k, \varphi_k) &= \phi(\theta_k, \varphi_k) \boldsymbol{a}'_{2\varphi}(\theta_k, \varphi_k) + \phi'_\varphi(\theta_k, \varphi_k) \boldsymbol{a}_2(\theta_k, \varphi_k) \end{aligned} \tag{6}$$

where $(\cdot)'$ refers to the first-order derivative.

Therefore, referring to (4) - (6), the SIS between the generalized manifold matrixes $\boldsymbol{A}_1(\theta_k, \varphi_k)$ and $\boldsymbol{A}_2(\theta_k, \varphi_k)$ can be given by

$$\boldsymbol{A}_1(\theta_k, \varphi_k) = \boldsymbol{A}_2(\theta_k, \varphi_k) \boldsymbol{\Phi}_x \tag{7}$$

where $\boldsymbol{\Phi}_x = \begin{bmatrix} \boldsymbol{\Lambda}_1 & \boldsymbol{\Lambda}_2 & \boldsymbol{\Lambda}_3 \\ \boldsymbol{0}_{K\times K} & \boldsymbol{\Lambda}_1 & \boldsymbol{0}_{K\times K} \\ \boldsymbol{0}_{K\times K} & \boldsymbol{0}_{K\times K} & \boldsymbol{\Lambda}_1 \end{bmatrix}$ denotes the shift invariant matrix with $\boldsymbol{\Lambda}_1 = \mathrm{diag}\{\phi(\theta_1, \varphi_1), \cdots, \phi(\theta_K, \varphi_K)\}$, $\boldsymbol{\Lambda}_2 = \mathrm{diag}\{\phi'_\theta(\theta_1, \varphi_1), \cdots, \phi'_\theta(\theta_K, \varphi_K)\}$ and $\boldsymbol{\Lambda}_3 = \mathrm{diag}\{\phi'_\varphi(\theta_1, \varphi_1), \cdots, \phi'_\varphi(\theta_K, \varphi_K)\}$.

It is obvious that diagonal elements of $\boldsymbol{\Phi}_x$ are functions of the nominal azimuth DOA and elevation DOA, thereby $\boldsymbol{\Phi}_x$ can be exploited to estimate the nominal azimuth and elevation DOAs.

3.2 Beamspace Transformation Matrix

Note that the implementation of the SIS-based algorithms usually requires eigendecomposition, which may result in high computational cost due to the high dimensional observed signals. Beamspace transformation is one way of reducing observed signal dimension, thereby, an effective beamspace transformation matrix will be given.

Assume that there is a $P \times P(P<M)$ beamspace transformation matrix $\boldsymbol{W}$, which can be used for reducing the dimension of the received signal $\boldsymbol{x}(t)$ to $P \times N$. The beamspace received signal $\boldsymbol{z}(t)$ can be written as

$$\begin{aligned} \boldsymbol{z}(t) &= \boldsymbol{W}^{\mathrm{H}}\boldsymbol{x}(t) \\ &= \boldsymbol{W}^{\mathrm{H}}\boldsymbol{A}_x(\theta,\varphi)\boldsymbol{h}(t) + \boldsymbol{W}^{\mathrm{H}}\boldsymbol{n}_x(t) \\ &= \boldsymbol{B}(\theta,\varphi)\boldsymbol{h}(t) + \boldsymbol{W}^{\mathrm{H}}\boldsymbol{n}_x(t) \end{aligned} \tag{8}$$

where $\boldsymbol{B} = \boldsymbol{W}^{\mathrm{H}}\boldsymbol{A}_x(\theta,\varphi)$ stands for the beamspace manifold matrix.

Let $\boldsymbol{R}$ denote the covariance matrix of the beamspace received signal $\boldsymbol{z}(t)$. Assume that all the signals are uncorrelated, the covariance matrix $\boldsymbol{R}$ can be written as

$$\begin{aligned} \boldsymbol{R} &= \mathrm{E}\{\boldsymbol{z}(t)\boldsymbol{z}^{\mathrm{H}}(t)\} \\ &= \boldsymbol{E}_s\boldsymbol{\Sigma}_s\boldsymbol{E}_s^H + \boldsymbol{E}_n\boldsymbol{\Sigma}_n\boldsymbol{E}_n^H \end{aligned}$$

where $\mathrm{E}\{\cdot\}$ denotes the mathematical expectation. $\boldsymbol{E}_s$ and $\boldsymbol{E}_n$ are referred to as the signal and noise subspaces, which are formed by the eigenvectors of $\boldsymbol{R}$ corresponding to the $3K$ largest and $(M-3K)$ smallest eigenvalues, respectively. $\boldsymbol{\Sigma}_s$ and $\boldsymbol{\Sigma}_n$ are the diagonal matrices, which are composed of the $3K$ largest and $(M-3K)$ smallest eigenvalues of $\boldsymbol{R}$, respectively.

Referring to the theorem in [19], it can be known that there is a $P \times P$ beamspace transformation matrix $\boldsymbol{W}$ satisfying the following constrains

$$\begin{aligned} \boldsymbol{W}^{\mathrm{H}}\boldsymbol{W} &= \boldsymbol{I}_P \\ \boldsymbol{J}_1\boldsymbol{W} &= \boldsymbol{J}_2\boldsymbol{W}\boldsymbol{F} \end{aligned} \tag{9}$$

where $\boldsymbol{F}$ is a nonsingular matrix. $(\cdot)^{\mathrm{H}}$ stands for the conjugate transpose. $\boldsymbol{I}_P$ represents the $P \times P$ identity matrix.

Furthermore, if there is a matrix $\boldsymbol{Q}$ with $\boldsymbol{Q}\boldsymbol{w}_M = \boldsymbol{0}$ and $\boldsymbol{Q}\boldsymbol{F}^{\mathrm{H}}\boldsymbol{w}_1 = \boldsymbol{0}$, then

$$\boldsymbol{Q}\boldsymbol{B}(\theta,\varphi) = \boldsymbol{Q}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{A}_x(\theta,\varphi) = \boldsymbol{Q}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{J}_1^{\mathrm{H}}\boldsymbol{J}_1\boldsymbol{A}_x(\theta,\varphi) \tag{10}$$

$$\boldsymbol{Q}\boldsymbol{F}^{\mathrm{H}}\boldsymbol{B}(\theta,\varphi) = \boldsymbol{Q}\boldsymbol{F}^{\mathrm{H}}\boldsymbol{W}^{\boldsymbol{H}}\boldsymbol{A}_x(\theta,\varphi) = \boldsymbol{Q}\boldsymbol{F}^{\mathrm{H}}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{J}_2^{\mathrm{H}}\boldsymbol{J}_2\boldsymbol{A}_x(\theta,\varphi)$$

where $\boldsymbol{w}_M$ and $\boldsymbol{w}_1$ denotes the last and first rows of the beamspace transformation matrix $\boldsymbol{W}$, respectively.

Substituting (5) and (8) into (9), it follows that

$$\boldsymbol{Q}\boldsymbol{B}(\theta,\varphi) = \boldsymbol{Q}\boldsymbol{F}^{\mathrm{H}}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{J}_2^{\mathrm{H}}\boldsymbol{J}_2\boldsymbol{A}_x(\theta,\varphi)\boldsymbol{\Phi}$$

Therefore, the following equation can be easily obtained

$$\boldsymbol{Q}\boldsymbol{B}(\theta,\varphi)=\boldsymbol{Q}\boldsymbol{F}^{\mathrm{H}}\boldsymbol{B}(\theta,\varphi)\boldsymbol{\Phi} \tag{11}$$

It is obvious that when $\boldsymbol{F}$ and $\boldsymbol{Q}$ are determined, $\boldsymbol{\Phi}_x$ can be derived to estimate the nominal azimuth DOA and the nominal elevation DOA. As shown in [19], when standard Fourier basis vectors are used to form $\boldsymbol{W}=[\boldsymbol{c}_1,\cdots,\boldsymbol{c}_P]$, namely, $\boldsymbol{c}_p=[1,e^{2\pi p/M},\cdots,e^{2\pi p(M-1)/M}]$, $\boldsymbol{F}$ and $\boldsymbol{Q}$ can be constructed as

$$\boldsymbol{F}=\mathrm{diag}\{e^{j2\pi/M},\cdots,e^{j2\pi P/M}\} \tag{12}$$

$$\boldsymbol{Q}=\boldsymbol{J}_Q\boldsymbol{\Lambda}_Q$$

where $\boldsymbol{J}_Q=\begin{bmatrix}1 & -1 & \cdots & 0 & 0\\ 0 & 1 & -1 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & 1 & -1\\ 0 & 0 & \cdots & 0 & 0\end{bmatrix}$ and $\boldsymbol{\Lambda}_Q=\mathrm{diag}\{e^{j2\pi(M-1)/M},\cdots,e^{j2\pi P(M-1)/M}\}$.

When the beamspace transformation matrix $\boldsymbol{W}$ is construct based on (12), the dimension of the received signal can be reduced to $P\times N$ ($P<M$). Then, the nominal azimuth DOA and the nominal elevation DOA can be estimated by utilizing the beamspace received signals.

3.3 Estimation of the Nominal Azimuth and Elevation

After constructing the beamspace transformation matrix $\boldsymbol{W}$, the closed form expressions of the nominal azimuth DOA and the nominal elevation DOA can be derived via (11). However, the beamspace manifold matrix $\boldsymbol{B}(\theta,\varphi)$ cannot be obtained directly, hence based on the subspace theory, the signal subspace $\boldsymbol{E}_s$ spans the same column space of $\boldsymbol{B}(\theta,\varphi)$, namely,

$$\boldsymbol{B}(\theta,\varphi)=\boldsymbol{E}_s\boldsymbol{T} \tag{13}$$

where $\boldsymbol{T}$ is a nonsingular matrix.

Substituting (13) into (11), (11) can be rewritten as

$$\boldsymbol{Q}\boldsymbol{E}_s=\boldsymbol{Q}\boldsymbol{F}^{\mathrm{H}}\boldsymbol{E}_s\boldsymbol{\Psi}_x \tag{14}$$

where $\boldsymbol{\Psi}_x=\boldsymbol{T}\boldsymbol{\Phi}_x\boldsymbol{T}^{-1}$ is the eigenvalue transformation matrix. It is obvious that the eigenvalues of $\boldsymbol{\Psi}_x$ are equal to the diagonal elements of $\boldsymbol{\Phi}_x$, which are related to the nominal azimuth DOAs and the nominal elevation DOAs of ID sources. Solving the (14) utilizing the total least squares (TLS) criterion [20], $\boldsymbol{\Psi}_x$ can be found.

Similarly, assume that the eigenvalue transformation matrix of array Y is $\boldsymbol{\Psi}_y$. Then performing eigenvalue decomposition on $\boldsymbol{\Psi}_x$ and $\boldsymbol{\Psi}_y$ yields

$$\begin{aligned}\boldsymbol{\Psi}_x &= \boldsymbol{U}_x \boldsymbol{\Lambda}_x \boldsymbol{U}_x^{\mathrm{H}} \\ \boldsymbol{\Psi}_y &= \boldsymbol{U}_y \boldsymbol{\Lambda}_y \boldsymbol{U}_y^{\mathrm{H}}\end{aligned} \tag{15}$$

where $\boldsymbol{U}_x$ and $\boldsymbol{U}_y$ represent the eigenvectors of $\boldsymbol{\Psi}_x$ and $\boldsymbol{\Psi}_y$, respectively. $\boldsymbol{\Lambda}_x = \mathrm{diag}\{\lambda_{x,1}, \lambda_{x,2}, \cdots, \lambda_{x,3K}\}$ with $\lambda_{x,q}(q = 1, 2, \cdots, 3K)$ being the eigenvalue of $\boldsymbol{\Psi}_x$. $\boldsymbol{\Lambda}_y = \mathrm{diag}\{\lambda_{\mathrm{y},1}, \lambda_{\mathrm{y},2}, \cdots, \lambda_{\mathrm{y,3K}}\}$ with $\lambda_{y,q}(q = 1, 2, \cdots, 3K)$ being the eigenvalue of $\boldsymbol{\Psi}_y$.

Let $\overline{\boldsymbol{\Lambda}}_x = \mathrm{diag}\{\overline{\lambda}_{x,1}, \overline{\lambda}_{x,2}, \cdots, \overline{\lambda}_{x,K}\}$ with $\overline{\lambda}_{x,k} = \frac{1}{3}\sum_{r=1}^{3} \lambda_{x,3(k-1)+r}(k = 1, 2, \cdots, K, r = 1, 2, 3)$ and $\overline{\boldsymbol{\Lambda}}_y = \mathrm{diag}\{\overline{\lambda}_{y,1}, \overline{\lambda}_{y,2}, \cdots, \overline{\lambda}_{y,K}\}$ with $\overline{\lambda}_{y,k} = \frac{1}{3}\sum_{r=1}^{3} \lambda_{y,3(k-1)+r}(k = 1, 2, \cdots, K, r = 1, 2, 3)$. It is noteworthy that the eigenvalues in $\overline{\boldsymbol{\Lambda}}_x$ and $\overline{\boldsymbol{\Lambda}}_y$ are estimated independently based on the received signals of subarrays X and Y. Therefore, they may be mismatched and should be paired using the matching technique in [15].

Assume $(\xi_{x,k}, \xi_{y,k})(k = 1, 2, \cdots, K)$ are the pair-matched eigenvalues, the estimated nominal azimuth DOA and the estimated nominal elevation DOA $(\hat{\theta}_k, \hat{\varphi}_k)$ can be expressed as

$$\hat{\theta}_k = \tan^{-1}\left(\frac{\ln(\xi_{y,k})}{\ln(\xi_{x,k})}\right) \tag{16}$$

$$\hat{\varphi}_k = \cos^{-1}\left(\frac{\lambda}{2\pi d}\sqrt{\ln(\xi_{y,k})^2 + \ln(\xi_{x,k})^2}\right) \tag{17}$$

where $k = 1, 2, \cdots, K$.

Based on the above discussion, Table I provides the proposed angular parameter estimation method for ID source. Apparently, the closed form solutions of the nominal azimuth DOA and the nominal elevation DOA are derived in Step.5.

3.4 Computational Complexity

In this section, the complexity of the proposed algorithm is analyzed and compared with two prevalent algorithms, namely, ESPRIT-2D [14] and BS-2D [18]. The computational complexity of the proposed algorithm mainly includes: 1) the computation of the beamspace received signal $\boldsymbol{z}(t)$ in Step (1), of order $\mathcal{O}(PMN)$; 2) the eigendecomposition of an $P \times P$ covariance matrix, of order $\mathcal{O}(P^3)$; 3) the calculation of the eigenvalue transformation matrixes $\boldsymbol{\Psi}_x$ and $\boldsymbol{\Psi}_y$, of order $\mathcal{O}(6K^3)$. 4) the pair matching of eigenvalues, of order $\mathcal{O}(18MK^4)$. Consequently, the overall complexity of the proposed estimation algorithm is of order $\mathcal{O}(PMN)$.

Table 2 compares the computational costs of the proposed algorithm with ESPRIT-2D and BS-2D. Evidently, the complexity of the proposed algorithm is much lower than those of ESPRIT-2D and BS-2D.

Table 1: Proposed algorithm

Algorithm: SIS-BAM
(1) Construct the beamspace transformation matrix $\boldsymbol{W}$ via (12) and obtain the beamspace received signal $\boldsymbol{z}(t)$ using (8);
(2) Calculate the covariance matrix $\boldsymbol{R}$ of the beamspace received signal $\boldsymbol{z}(t)$ and perform the eigendecomposition of $\boldsymbol{R}$ to obtain the signal subspace $\boldsymbol{E}_s$;
(3) Solve (14) utilizing the TLS criterion to get $\boldsymbol{\Psi}_x$. Similarly, $\boldsymbol{\Psi}_y$ can be denoted in the same way;
(4) Perform the eigendecomposition on $\boldsymbol{\Psi}_x$ and $\boldsymbol{\Psi}_y$ to obtain $\overline{\boldsymbol{\Lambda}}_x$ and $\overline{\boldsymbol{\Lambda}}_y$, which is matched using the matching technique in [17];
(5) Estimate the nominal azimuth DOA and elevation DOA $\hat{\theta}_k$ and $\hat{\varphi}_k$ via (16) and (17).

Table 2: Complexities of proposed algorithm and previous algorithms

Algorithm	Computational complexity
The proposed algorithm	$\mathcal{O}(PMN)$
ESPRIT-2D	$\mathcal{O}(M^4N)$
BS-2D	$\mathcal{O}((PM)^2N)$

4 Simulation Results

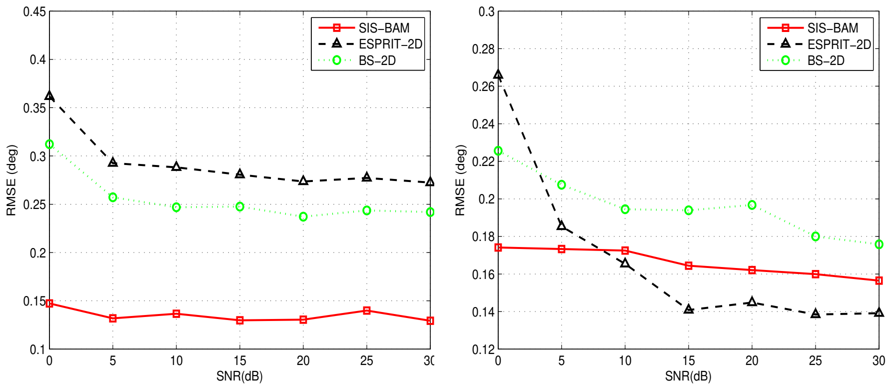
In this section, several simulation results are presented to evaluate the performance of the proposed SIS-BAM algorithm, in which the ULOA is composed of two ULAs and equipped with 36 isotropic sensors, namely, $M_x = 18, M_y = 18$. Meanwhile, this paper compares the performance of the proposed algorithm with two related algorithms, namely, ESPRIT-2D and BS-2D. For ESPRIT-2D and BS-2D, consider an uniform rectangular array with $M_x = M_y = 6$, namely, the total number of sensors is 36. Unless stated otherwise, the signal-to-noise ratio (SNR) and the number of snapshots are set to 20 dB and 512. Two equipowered, uncorrelated ID sources with angular parameters $(\theta_1, \sigma_{\theta 1}, \varphi_1, \sigma_{\varphi 1}) = (10°, 1°, 30°, 1°)$ and $(\theta_2, \sigma_{\theta 2}, \varphi_2, \sigma_{\varphi 2}) = (50°, 1°, 40°, 1°)$ are considered. The number of rays inside the ID sources is set to 200. The root-mean-square error (RMSE) is adopted to measure the estimation performance, which is defined as

$$r_\theta = \sqrt{\frac{1}{RK}\sum_{r=1}^{R}\sum_{k=1}^{K}(\overline{\theta}_k(r) - \theta_k)^2}$$

$$r_\varphi = \sqrt{\frac{1}{RK}\sum_{r=1}^{R}\sum_{k=1}^{K}(\overline{\varphi}_k(r) - \varphi_k)^2}$$

where r_θ and r_φ are the RMSEs of the nominal azimuth and the nominal elevation estimation, respectively. R denotes the number of Monte Carlo simulations, which is set to $R = 200$ in this paper. $\overline{\theta}_k(r)$ and $\overline{\varphi}_k(r)$ stand for the estimation of θ_k and φ_k for the rth Monte Carlo trial ($r = 1, 2, \cdots, R$), and K is the number of signals.

In the first example, Fig. 2 gives the RMSE curves of the nominal azimuth and the nominal elevation estimates for the proposed method, ESPRIT-2D and BS-2D under the hypothesis that the SNR ranges from 0 dB to 30 dB. From Fig. 2, it clearly indicates that the proposed algorithm shows better the nominal azimuth estimation and the nominal elevation estimation than ESPRIT-2D. This is because only one sensor is not employed in the subarray of the proposed method, while for ESPRIT-2D algorithm, there are M_x+M_y-1 sensors are unused. Compared with BS-2D algorithm, the proposed algorithm is able to provide better nominal azimuth estimation performance, but inferior nominal elevation estimation performance in the high SNR region (SNR >10 dB) . Besides, the proposed method is insensitive to the variation of the SNR, which means that SIS-BAM is robust to the noise.

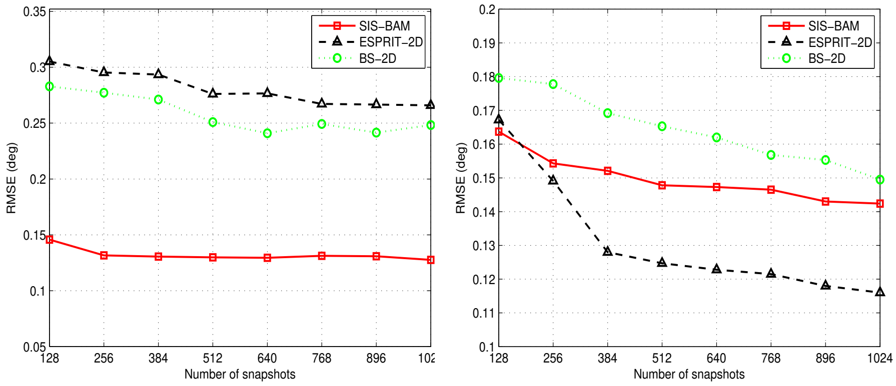


(a) Nominal azimuth estimation (b) Nominal elevation estimation

Fig. 2. RMSE curves of the angular parameter estimates versus the SNR

The impact of the snapshot number on the proposed method is investigated in Fig. 3 when the number of snapshots varies from 128 to 1024. It can be seen that for both nominal azimuth and nominal elevation estimation, the performances of the proposed and related algorithms become better gradually as the number of snapshots increases. In addition, the proposed algorithm offers a steady performance over the whole range of the snapshots number.

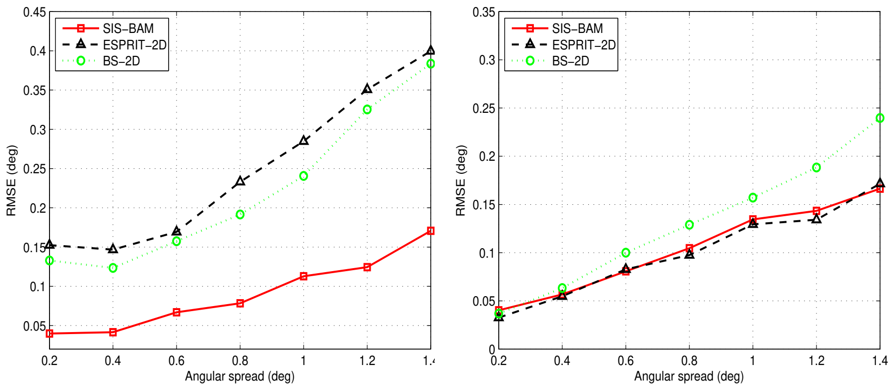
In the last example, the influence of the angular spread is shown in Fig. 4, which plots the RMSEs of the nominal azimuth and the nominal elevation



(a) Nominal azimuth estimation (b) Nominal elevation estimation

Fig. 3. RMSE curves of the angular parameter estimates versus the number of snapshots

estimates. It can be seen that the estimation of the nominal azimuth and the nominal elevation by our method is more accurate than that by ESPRIT-2D method as the angular spread increases. Meanwhile, the RMSEs of the aforementioned algorithms degrade when the angular spread increases. This is very reasonable as the approximation of the array manifold is under the assumption of small angular spread.



(a) Nominal azimuth estimation (b) Nominal elevation estimation

Fig. 4. RMSE curves of the angular parameter estimates versus the angular spread

5 Conclusions

This paper presents an effective SIS-BAM method for 2-D ID source angular parameter estimation. In the proposed method, the first-order Taylor expansion is firstly performed on the steering vector to decouple the nominal DOA and the angular spread. Then, by the beamspace transformation matrix, the low-dimension beamspace data is obtained, whose SIS can be established utilizing generalized array manifold . Finally, the closed-form expressions of the nominal azimuth and the nominal elevation are given through the SIS in the beamspace. The theory analysis shows that the complexity of the proposed algorithm is lower than that of ESPRIT-2D and BS-2D. Simulation results indicate that the proposed algorithm is higher than other related algorithms in terms of accuracy and is robust to SNR and the number of snapshots.

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Declarations

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Data Availability Statement

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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