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From a Quantum Paradox to Counterportation

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We uncover a new quantum paradox, where a simple question about two identical quantum systems reveals unsettlingly paradoxical answers when weak measurements are considered. Our resolution of the paradox, from within the weak measurement framework, amounts to a demonstration of exchange-free communication for the generalised protocol for sending an unknown qubit without any particles travelling between the communicating parties, i.e. counterfactually. The paradox and its resolution are reproduced from a consistent-histories viewpoint. We go on to propose a novel, experimentally feasible implementation of this counterfactual disembodied transport that we call counterportation, based on cavity quantum electrodynamics, estimating resources for beating the no-cloning fidelity limit—except that unlike teleportation no previously-shared entanglement nor classical communication are required. Our approach is several orders of magnitude more efficient in terms of physical resources than previously proposed techniques, while being remarkably tolerant to device imperfections, paving the way for an experimental demonstration. Surprisingly, while counterfactual communication is intuitively explained in

terms of interaction-free measurement and the Zeno effect, we show based on our proposed scheme that neither is necessary, with implications in support of an underlying physical reality.

Introduction

“It is wonderful that we have met with a paradox. Now we have some hope of making progress,” Neils Bohr once said. Paradoxes are puzzles that highlight some of the stranger aspects of physical theories, pointing to gaps in our present understanding. Their resolution, as Bohr enthused, often marks genuine progress. But never mind the ramifications of a future resolution of Schrodinger’s cat paradox for instance, one just has to think of the advances this imagined feline physics companion has already instigated.

While the new paradox we present here, about where a photon has or has not been within nested interferometers, is interesting in its own right, it also has a direct bearing on the possibility of counterfactual communication, that is sending information without sending particles (*1, 2*)—previously thought impossible. But if particles, as we show, did not carry information in such a scenario, then what did? And what does this say about the reality of the wave-function?—that mathematical construct that has divided scientists as to whether it merely represents a state of knowledge or objective physical reality.

In an optical setting, we make the assumption that communication is explainable by one or more of the following: 1) detectable photons, possibly including entangled ones, traversing the channel between the two communicating parties 2) measurements carried out between an initial state and a final state, and 3) an underlying objective physical state represented by the wave-function. By categorically ruling out the first two in our scheme, we provide evidence that an underlying objective physical state is what has carried information across space.

Given the recent experimental demonstration of sending classical information without ex-

changing particles (3), we show that it is experimentally feasible to demonstrate sending not only classical but also quantum information this way, based on the generalised protocol, Salih14, for counterfactually transporting an unknown qubit (4, 5), what we here call counterportation. The key primitive of the protocol is the counterfactual CNOT gate, which we show here how to implement based on cavity quantum electrodynamics (6), and ideas inspired by electromagnetically induced transparency (7). Our proposed demonstration is remarkably tolerant to device imperfections and noise, offering several-orders-of-magnitude saving of physical resources, and as we show, is feasible with current technology. This realistic CNOT gate along with single-qubit operations allows us to transport an unknown quantum state between two remote parties without the use of previously shared entanglement or classical communication: counterportation.

A Paradox of Weak Measurements

Let's get straight to the heart of the new paradox. We will first discuss the setup shown in Fig. 1 generally, before framing the discussion in the context of counterfactual communication, revealing some of its dramatic consequences. Consider the two outer interferometers, nested within each are two inner interferometers, shown in Fig. 1. All beam-splitters are polarising beam-splitters, transmitting horizontally polarised photons and reflecting vertically polarised photons, and all half-wave plates HWP's rotate polarisation by 45 degrees. The evolution of a photon between times t_0 and t_4 , the first cycle, is identical to its evolution between times t'_0 and t'_4 , the second cycle. Moreover, the photon is in the same state at times t_0 , t_4 , t'_0 , and t'_4 , in terms of both path and polarisation, provided that it is not lost to either of the detectors D_3 . (As we will see shortly, this definition of a cycle is not arbitrary.) We want to know whether a photon detected at detector D_0 at the bottom was in any of the arms labeled C on the right-hand side.

We follow the photon's evolution starting with the photon in arm S at the top at time t_0 ,

H-polarised. The combination of the first half-wave plate HWP1 and polarising beamsplitter puts the photon in an equal superposition of traveling along arm A, H-polarised, and along arm D, V-polarised. The combined action on the V-polarised part by two successive half-wave plates HWP2's, one in front of each inner interferometer, is to rotate V polarisation all the way to H. This part of the superposition proceeds towards the first detector D_3 . If D_3 does not click, then we know that the photon is instead in arm S at time t_4 , H-polarised, having traveled along arm A. Exactly the same happens between times t'_0 and t'_4 , which means that given it is not lost, the photon will be in arm F at t_{final} , H-polarised, causing detector D_0 at the bottom to click.

An interesting approach for investigating whether a photon detected at D_0 at the bottom has traveled along any of the arms C, an approach that has proven controversial in this context (8–14), is weak measurement (15, 16). The idea is to perform sufficiently weak measurements such that their effect on individual photons lies within the uncertainty associated with the observable measured, in this case the photon's path. When averaged over a sufficiently large number of photons, however, these measurements acquire definite, predictable values. One could, for instance, cause various mirrors in the setup to vibrate at different frequencies, as in reference (8), then check which of these frequencies show up in the power spectrum of classical light detected by a quad-cell photodetector D_0 .

An intuitive way of predicting the outcomes of such weak measurements—at least to a first order approximation—is the two-state vector formulation, TSVF (8, 15, 16). According to the TSVF, each photon detected at detector D_0 is described by a forward-evolving quantum state created at the photon source at the top, and a backward-evolving quantum state created at detector D_0 at the bottom. Unless these two states overlap at a given point in space, in which case a well defined associated quantity, proportional to the weak measurement, called the weak value is nonzero, any weak measurement performed there will be vanishingly small.

We can thus consider a weak measurement in arm C at time t_2 , carried out on a beam of

light. Starting from the photon source at the top, and following the photon's unitary evolution, the forward-evolving state is present in arm C. And starting from detector D_0 at the bottom, the backward-evolving state is also present in arm C. The weak value is nonzero and consequently a weak measurement in arm C at time t_2 is also nonzero.

We can similarly consider a weak measurement in arm C at time t'_2 . Starting from the photon source at the top, the forward-evolving state is present in arm C. However, starting from detector D_0 at the bottom, the backward-evolving state is *not* present in arm C. The weak value is zero and therefore a weak measurement in arm C at time t'_2 is vanishingly small. However, in the absence of weak measurements, the first outer cycle and the second outer cycle are identical as far as standard quantum mechanics is concerned—the photon undergoes the same transformations in each cycle, starting and finishing each cycle in the same state.

The issue here is that weak measurement disturbs interference in the inner interferometers (beyond that due to device imperfections and the effect of the environment, both of which can be made negligible enough, as in (8)) leading to a flux of V-polarised photons in arm D between times t_3 and t_4 (t'_3 and t'_4). This flux finds its way to detector D_0 at the bottom only in the case of a weak measurement in the first outer cycle, as an artifact caused by the action of HWP1 *after* the end of the first outer cycle. In other words, the result of a weak measurement during the first outer cycle is not only dependent on what happens during that cycle, but also what happens *afterwards*. While it is remarkable that the TSVF correctly predicts the outcome of such weak measurement experiments (most of the time), it manifests the inherent unusualness of the time-symmetric formulation of physics, based on the TSVF, where the present is not only dependent on the past but equally on the future (17). This is neither a criticism of the TSVF nor the time-symmetric formulation.

One runs into trouble, however, when taking a nonzero weak value to necessarily mean *the photon* was there regardless of whether a weak-measurement experiment is actually per-

formed or not, a position advocated by Vaidman (18). The problem comes into sharp focus, in the absence of a weak-measurement experiment, when photon leakage from the inner interferometer through arms D tends to zero, except for the negligible device imperfections and environmental effects mentioned above. In this case the evolution of the photon during the first cycle approaches being identical to its evolution during the second cycle, in which case saying the photon was in arm C for the first cycle but not for the effectively identical second cycle is unsettlingly paradoxical.

Our resolution of the paradox—from within the weak measurement framework—is based on the observation that the strong measurement by detector D_3 at the end of each outer cycle projects the state of the photon onto arm S, where we know it should be H-polarised. This is our post-selected state. Therefore for the first outer cycle, starting with the pre-selected state, where the photon is in S at time t_0 H-polarised, the forward-evolving state is present in arm C at time t_2 (and t_3). Starting with the post-selected state, however, where the photon is in S at time t_4 H-polarised, the backward-evolving state is *not* present in arm C at time t_2 (or t_3). The weak value is thus zero and a weak measurement will not find the photon there. Exactly the same applies for the second outer cycle, starting with the pre-selected state, the photon in S at time t'_0 H-polarised, the forward-evolving state is present in arm C at time t'_2 (and t'_3). Starting with the post-selected state, the photon in S at time t'_4 H-polarised, the backward-evolving state is *not* present in arm C at time t'_2 (or t'_3). The weak value is thus zero and a weak measurement will *not* find the photon there. Due to imperfections, the post-selected state in S at time t'_2 (and t'_3) may have a very small V-polarised component. Nonetheless, because the forward evolving state and a post-selected V-polarised state are orthogonal, a weak measurement in C will be negligibly small, as the experiment in (8) demonstrates for the case of the forward evolving state and the post-selected state not overlapping. We can say that the photon was not in C during the first outer cycle. It was not in C during the second outer cycle. Therefore it was never in C.

This generalises straightforwardly to any larger number of inner and outer interferometers. We reproduce the paradox and its resolution, in the Appendix, from a consistent histories point of view. The fact that the paradox persists from a different point of view, specifically designed to avoid quantum paradoxes, is an indication of the seriousness of the new paradox uncovered.

The setup in Fig. 1 in fact represents an instance of Salih et al.’s protocol for counterfactual communication (*I*) with one of the communicating parties, Alice, on the left in control of all the optical elements except mirrors MR_B , which are controlled by the other communicating party, Bob, on the right. (The fact that the setup of Fig. 1 employs a very small number of nested interferometers means that there’s photon loss as well as error, which can be avoided in the limit of a very large number of inner and outer interferometers, or as we will later see, by introducing suitable attenuation in the outer interferometers). Bob communicates bit “1” by blocking (i.e. obstructing) the optical channel, marked in grey, to Alice, causing detector D_1 to sometimes click, indicating “1”, provided the photon is neither lost nor ends up at D_0 erroneously. On the other hand, Bob communicates bit “0” by not blocking the optical channel to Alice, as is the case in Fig. 1, causing detector D_0 to click, provided the photon is not lost. Counterfactuality for the case of Bob not blocking means that given a D_0 click, Alice’s photon has not traveled to Bob on the right—which common sense also tells us is the case, since any photon that enters the inner interferometers would necessarily be lost to detector D_3 through the action of the two $HWP2$ ’s. It is this case of Bob not blocking the channel where counterfactuality has been questioned. Counterfactuality for the case of Bob blocking the channel is not in question, since there is agreement that any photon entering the channel would have been lost to Bob’s blocking device. Our proof above that the photon has not been to Bob for the case of Bob not blocking the channel therefore amounts to a proof of counterfactuality not only of Salih et al.’s protocol for counterfactual communication of a classical bit (*I*), but also for Salih’s generalised protocol for counterfactual communication of an unknown quantum bit (*4, 5*)—counterportation—the

focus of the rest of the paper.

Counterfactual CNOT Gate

First proposed in (4, 5), as a generalisation of (1), and drawing on ideas in (19–26), this is the key primitive for counterfactual disembodied transport, or counterportation. In contrast to reference (4, 5), we use the circular polarisation basis here as it ties better with our experimental proposal for Bob’s qubit in the Methods and Results section below. We provide a more succinct derivation in what follows before incorporating a key improvement based on recent work by Aharonov and Vaidman. Consider a right-circular polarised, R, photon entering the top chained quantum Zeno effect module CQZE1 in Fig. 2C. As mentioned previously, the setup of Fig. 1 is equivalent to two outer cycles here, each containing two inner cycles, except that in the present setup of Fig. 2C Bob implements a quantum superposition of blocking, which represents bit “1”, and not blocking the channel to Alice, bit “0”. It should become clear shortly what is meant by inner and outer cycles.

Note that we start with a separable product state of Alice’s photon and Bob’s atom. The action of the detectors helps to vividly explain counterfactuality, but is not necessary in obtaining the final state. This is because their measurements can be deferred, by the deferred measurement principle (27), to the end as that part of the wave-function stops evolving. The overall process can therefore be taken to be unitary.

Switchable mirror SM1 is first switched off to allow the photon into the outer interferometer, before being switched on again. Switchable polarisation rotator SPR1, whose action is described by $|R\rangle \rightarrow \cos \frac{\pi}{2M} |R\rangle + \sin \frac{\pi}{2M} |L\rangle$, and $|L\rangle \rightarrow \cos \frac{\pi}{2M} |L\rangle - \sin \frac{\pi}{2M} |R\rangle$, rotates the photon’s polarisation from R by a small angle $\frac{\pi}{2M}$. Polarising beam-splitter PBS2 passes the R part towards the bottom mirror while reflecting the small L part towards the inner interferometer. Switchable mirror SM2 is then switched off to allow the L part into the inner interferometer,

before being switched on again. Switchable polarisation rotator SPR2 rotates the L part by a small angle $\frac{\pi}{2N}$. Polarising beamsplitter PBS3 then reflects the L part towards the top mirror while passing the small R part towards Bob, who is implementing a superposition, $\alpha |0\rangle + \beta |1\rangle$, of reflecting back any photon, and blocking the channel, respectively. More precisely, inside the inner interferometer, and given the photon is not lost to Bob's detector D_B ,

$$|L\rangle (\alpha |0\rangle + \beta |1\rangle) \rightarrow \alpha (\cos \frac{\pi}{2N} |L\rangle - \sin \frac{\pi}{2N} |R\rangle) |0\rangle + \beta \cos \frac{\pi}{2N} |L\rangle |1\rangle \quad (1)$$

The R and L components of the photonic superposition are brought back together by PBS3 on the way towards SM2. This represents one inner cycle. After N such cycles we have,

$$|L\rangle (\alpha |0\rangle + \beta |1\rangle) \rightarrow -\alpha |R\rangle |0\rangle + \beta \cos^N \frac{\pi}{2N} |L\rangle |1\rangle \quad (2)$$

Switchable mirror SM2 is then switched off to let the photonic component inside the inner interferometer out. Since for large N, $\cos^N \frac{\pi}{2N}$ approaches 1, we have,

$$|L\rangle (\alpha |0\rangle + \beta |1\rangle) \rightarrow -\alpha |R\rangle |0\rangle + \beta |L\rangle |1\rangle \quad (3)$$

where the R-polarised part proceeds through PBS2 towards detector D_A , and the V part is reflected towards SM1.

Now that we know how a photonic component entering the inner interferometers evolves, we can look at the first outer cycle. Starting with the photon at SM1 travelling downwards we have, assuming the photon is not later lost to Alice's detector D_A , or to Bob's D_B ,

$$|R\rangle (\alpha |0\rangle + \beta |1\rangle) \rightarrow \alpha \cos \frac{\pi}{2M} |R\rangle |0\rangle + \beta (\cos \frac{\pi}{2M} |R\rangle + \sin \frac{\pi}{2M} |L\rangle) |1\rangle \quad (4)$$

This represents one outer cycle, containing N inner cycles. The photonic superposition has now been brought back together by PBS2 on the way towards SM1. After M such cycles we have,

$$|R\rangle (\alpha |0\rangle + \beta |1\rangle) \rightarrow \alpha \cos^M \frac{\pi}{2M} |R\rangle |0\rangle + \beta |L\rangle |1\rangle \quad (5)$$

Since for large M , $\cos^M \frac{\pi}{2M}$ approaches 1, we have,

$$|R\rangle (\alpha |0\rangle + \beta |1\rangle) \rightarrow \alpha |R\rangle |0\rangle + \beta |L\rangle |1\rangle \quad (6)$$

Switchable mirror SM1 is now switched off to let the photon out. Crucially, this last equation describes the action of a quantum CNOT gate with Bob's as the control qubit, acting on Alice's R-polarised photon. But we want to allow Alice to input a superposition of R and L, hence the two CQZE modules in Fig. 2. Alice sends her photon into PBS1, which passes the R component towards the CQZE1 module as before, while reflecting any L component towards the bottom CQZE2 module. The function of Pockels cells PC before CQZE2 is to flip the polarisation of any incoming L photon, as well as flipping the polarisation of any photon exiting CQZE2. This means that the two CQZE modules are identical. (In fact it might be more practical for the same CQZE module to double as CQZE1 and CQZE2. This can be achieved by using a PBS1 that splits the R and L photonic components into two parallel modes, which are then fed to the same CQZE module.)

By means of optical circulators OC1 and OC2, the outputs of the two CQZE modules are directed to 50-50 beamsplitter BS, where the two outputs are added together at Port2. At Port1, however, the output from CQZE2 acquires a negative phase, by the action of BS, as it is added to the output from CQZE1. This has an effect, given the photon is found at Port1, equivalent to applying a phase-flip Z-gate to the polarisation of Alice's photon before being initially sent into

PBS1. The Z-gate in Fig. 2B corresponds to finding the photon at Port1. This completes our description of our counterfactual CNOT gate.

We have demonstrated recently that the laws of physics do not prohibit counterfactual communication (28). We were happy to lose most photons during communication, so long as counterfactuality was unequivocally demonstrated, which we were able to achieve by employing a single outer cycle. This result extends straightforwardly to the case of Bob implementing a superposition of reflecting the photon back and blocking it.

For the multiple outer-cycles considered here (where the probability of losing the photon can be made arbitrarily close to zero) we have already shown by means of a new quantum paradox that a nonzero weak measurement does not necessarily mean the photon was at Bob. Moreover, we have shown through our resolution of the paradox that the photon was in fact never at Bob. It would be nice, however, if the result of a weak measurement at Bob is zero, given an initial state before the beginning of the first outer cycle and a final state after the end of the last outer cycle. The recent proposed modification by Aharonov and Vaidman (29) of the illustrative version (which does not use polarisation) of Salih et al.'s counterfactual communication protocol (*1*), while not passing the consistent histories test, does make the result of a weak measurement at Bob zero, as a first order approximation. (We briefly explain in the appendix why Aharonov and Vaidman's modification does not pass the consistent histories criteria for counterfactuality, even for a single cycle.)

Here's how to implement Aharonov and Vaidman's clever modification in our counterfactual CNOT-gate, while passing the consistent histories test through the use of the polarisation degree of freedom, in a similar way to the consistent histories analysis in the Appendix. Take the Nth inner cycle, which was previously the last inner cycle during a given outer cycle. After applying SPR2 inside the inner interferometer for the Nth cycle, Alice now makes a measurement by blocking the entrance to channel leading to Bob. (She may alternatively flip the polarisation

and use a PBS to direct the photonic component away from Bob.) And instead of switching SM2 off, it is kept turned on for a duration corresponding to N more inner cycles, after which SM2 is switched off as before. One has to compensate for the added time by means of optical delays. The idea here is that, for the case of Bob not blocking, any lingering V component inside the inner interferometer after N inner cycles (because of weak measurement or otherwise) will be rotated towards H over the extra N inner cycles. This has the effect that, at least as a first order approximation, any weak measurement in the channel leading to Bob will be vanishingly small. In the TSVF mentioned above, the forward state and the backward state do not overlap anywhere in the channel between Alice and Bob.

In fact one can make the result of a weak measurement at Bob arbitrarily small. The way to do this, we propose, is by repeating the same trick again, employing N more inner cycles—namely blocking the entrance to the channel leading to Bob once more after the $2N$ th application of SPR2, or else directing the photonic component away, then keeping SM2 on for a duration corresponding to N more inner cycles—and if one wishes, this can be repeated again.

Protocol for Counterportation

Starting with the quantum circuits in Fig. 2A and B, from $(4, 5)$, Alice sends her R -polarised photon from the left towards PBS1, as shown in Fig. 2C, where the photon proceeds towards CQZE1 at the top, whose action corresponds to the first CNOT in Fig. 2A. With Bob implementing his qubit as a superposition of reflecting and blocking, $\alpha|0\rangle + \beta|1\rangle$, Alice's photon emerges back maximally entangled with Bob's qubit. Optical circulator OC, and switchable mirror SM0, briefly turned on, then reflect the photon into Port0, where a Hadamard transformation is applied to its polarisation. A Hadamard transformation is also applied to Bob's qubit by means of suitable laser pulses. Alice's photon is then fed again into PBS1 from the left. The R -polarised component incident on PBS1 proceeds towards CQZE1 as before, while the L -

polarised component is reflected by PBS1 towards Pockels cell PC, which flips its polarisation to R, before entering CQZE2. The component that eventually emerges from CQZE2 will have its polarisation flipped again by PC on its way back before being directed by optical circulator OC2 towards beamsplitter BS. This photon component combines with the photon component emerging from CQZE1 and directed by OC1 towards beamsplitter BS. This corresponds to the second CNOT in Fig. 2A and B. A Hadamard transformation is then applied to Bob’s qubit. Similarly, Hadamard transformations are applied to the polarisation of the photon components in ports 1 and 2, before a NOT transformation is applied to the polarisation of the photon component at port 1 in order to compensate for the effective phase-flip introduced by beam-splitter BS at port1, as shown in Fig. 2B. The photonic components at both ports now have identical polarisation. Found in either of Alice’s ports, which happens with unit probability in the ideal limit of an infinite number of cycles and perfect optical components, its polarisation is the desired $\alpha |R\rangle + \beta |L\rangle$, that is Bob’s original qubit.

Methods and Results

Bob needs to implement a superposition of reflecting Alice’s photon, bit “0”, and blocking it, bit “1”. There are various ways to go about this, including cavity optomechanics (30) and quantum dots (31). However, recent breakthroughs in trapped atoms inside optical cavities (6), including the experimental demonstration of light-matter quantum logic gates (32, 33), make trapped atoms an attractive choice.

A single ^{87}Rb atom trapped inside a high-finesse optical resonator by means of a three-dimensional optical lattice constitutes Bob’s qubit (33, 34). Depending on which of its two internal ground states the ^{87}Rb atom is in, a resonant R-polarised photon impinging on the cavity from the left in Fig. 2C will either be reflected as a result of strong coupling, or otherwise enter the cavity on its way towards detector D_B . Unlike references (33, 34), for our purposes

here, the cavity needs to, first, support the two optical modes shown in Fig. 2C (or else support two parallel optical modes impinging on the cavity from the same side, as in (35), which ties in with our earlier suggestion to use a single CQZE module with two optical modes). And second, it needs to have mirror reflectivities such that a photon entering the cavity exists towards detector D_B , similar to (7). By placing the ^{87}Rb in a superposition of its two ground states, by means of Raman transitions applied through a pair of Raman lasers, Bob implements the desired superposition of reflecting Alice's photon back and blocking it. Note that coherence time for such a system is on the order of 0.1 millisecond (34), with longer coherence times possible. Therefore, if the protocol is completed within a timescale on the order of microseconds or tens of microseconds, which is lower-bounded by the switching speed of switchable mirrors and switchable polarisation rotators, on the order of nanoseconds, then decoherence effects can be ignored. (There are experimental tricks for ensuring the correct number of cycles without having to use switchable optical elements, as in, for instance, the recent experimental implementation of Salih et al.'s counterfactual communication by Cao et al. (3).)

We numerically simulate counterfactual disembodied transport by means of recursive relations based on the ones in (1), which track the evolution of Alice's photon from one cycle to the next depending on Bob's bit choice.

Interestingly, by adding suitable attenuation, we can eliminate any dependency for fidelity on the number of inner and outer cycles for the case of perfect components and devices. First, looking at the CQZE1 in Fig.2C, the R component in each outer cycle that goes through the optical delay at the bottom is attenuated, following reference (36), by a factor of $(\cos\frac{\pi}{2N})^N$. Second, we attenuate the exiting L component in the final outer cycle, between SM2 and PBS2, by a factor of $(\cos\frac{\pi}{2M})^M$. What this means is that fidelity approaches 100% as device imperfections approach 0, even for the smallest number of cycles possible. Because loss for the case of Bob blocking is the same for the case of Bob not blocking, fidelity is the same for different

input states, which means the protocol can take mixed states as input. The method, however, is less effective when device imperfections are high, which is the case for current technology, and is therefore not implemented in our simulations below.

We account for two types of imperfections corresponding to Bob reflecting the photon. First, imperfections obstructing the communication channel. Second, Bob’s cavity failing to reflect the photon back, which based on the setup in reference (34) happened with probability 34(2)%. This is caused by scattering or absorption within the cavity. However, dramatically reduced loss is expected for next-generation cavities with increased atom-cavity coupling strength (37). In our simulation we combine these two types of imperfections into one coefficient associated with Bob reflecting the photon. We assume that the probability of imperfections obstructing the channel to lie within the uncertainty for Bob’s object failing to reflect, and can therefore be ignored. For the case of Bob blocking the channel, we account for imperfect optical mode matching, that is imperfect transverse overlap between the free-space mode of the photon and the cavity mode. This according to reference (34) has a probability of 8(3)%, and results in the photon being reflected back when it should not. This is also expected to improve with next-generation cavities.

Fidelity of counterfactual transport is plotted in Fig. 3, averaged over 100 evenly distributed qubits on the Bloch sphere, for a number of outer cycles up to 12, and a number of inner cycles up to 12. Here, an error coefficient of 34% associated with Bob reflecting the photon is assumed, along with an error coefficient of 8% associated with Bob blocking. Fidelity, even for the minimum number of cycles, 4, is above the no-cloning limit of $2/3$ for “classic teleportation” (38). Because of the small number of cycles, efficiency is only 0.1%. But this improves towards unity by increasing the number of cycles and reducing device imperfections. For example, for 20 outer cycles and 75 inner cycles, and error coefficients 4% and 1%, efficiency is above 50%, with fidelity 99%.

Our original proposal of the concept of counterportation (4, 5) in 2014 has lead to a subsequent proposed implementation more than a year later in (39) based on, as Vaidman noted (40), the same basic concept we had already proposed. Importantly, their fidelity falls below the required 2/3 mark for the realistic imperfections described above, where Bob’s device fails to reflect more than 30% of the time, even for thousands or tens of thousands of protocol cycles. In other words, for realistic imperfections, one has to employ an unrealistic hundreds of thousands of cycles in their protocol. By contrast, for the protocol and implementation we give here, for a total number of cycles of just 8, for instance, and Bob’s realistic device failing to reflect 34% of the time and failing to block 8% of the time, fidelity is already 74%. This is promising for a future experimental demonstration of our scheme.

Discussion

Counterfactual communication, and its forerunner counterfactual computation (23), has been inspired by, and so far exclusively explained in terms of interaction-free measurement (19) and the Zeno effect (20). In interaction-free measurement the presence of a measuring device, or the “bomb” in Elitzur and Vaidman’s thought experiment, can sometimes be inferred without any particle triggering it. Whereas in the Zeno effect, used to boost the efficiency of interaction-free measurement, repeated measurement of a quantum state inhibits its evolution, leaving it unchanged (the proverbial watched kettle that does not boil). Let’s consider detectors D_A and D_B in Fig. 2C. By the deferred measurement principle (27), which states that any part of a quantum system that has stopped evolving can be measured straightaway or at a later time, we can imagine detectors D_A and D_B being placed far away such that neither performs any measurement before the photon could exit the protocol. At the end of the protocol, the photon is in Port1 and Port2 with unit probability amplitude in the ideal limit of an infinite number of cycles and perfect implementation, in the desired polarisation state of Bob’s original qubit. No reference

to either interaction-free measurement or the Zeno effect is therefore required, showing that neither is a necessary condition for counterfactuality. Bob communicates information counterfactually by enabling single-photon interference to take place either in the inner interferometers but not the outer, when communicating a “0”, or in the outer interferometers but not the inner, when communicating a “1”, or a linear combination of both scenarios when communicating a quantum bit.

We make the assumption that optical communication is explainable by one or more of the following: 1) detectable photons, possibly including entangled ones, traversing the channel between the two communicating parties 2) measurements carried out in between an initial state and a final state, and 3) an underlying objective physical state represented by the wave-function.

When counterfactual communication of quantum information is cast in terms of interaction-free measurement and the Zeno effect, repeated measurement appears to play a key role in information transfer—with quantum collapse due to measurement, in the words of the authors of the PBR paper (41), a problematic and poorly defined physical process. Since in the limit of infinitely many protocol cycles and perfect implementation the correct transported state in our protocol can be post-selected with unit probability, the updating of the experimenters’ subjective knowledge based on measurement outcomes plays no part in information transfer.

By presenting a deterministic scheme for counterfactual communication that does not involve such measurements between an initial pre-selected state and a final post-selected state, and in particular no local measurements by the sender, the only thing we are left with as a possible carrier of information—in the absence of particle exchange—is an objective underlying physical state. The authors of the PBR paper, in their seminal investigation of the reality of the quantum wave-function (41) have shown that given a few reasonable assumptions, which others have subsequently managed to relax (42–45), the wave-function is real, in the sense that two wave-functions cannot correspond to the same underlying objective physical state, if such a

state exists. What has been given here is evidence suggesting that such an underlying objective physical state does exist, and is what has carried quantum information between two points in space.

The mystery of communicating quantum bits without sending any particles, the deferred measurement principle tells us, simply comes down to single-particle interference. But as Richard Feynman was quick to point out, such interference “has in it the heart of quantum mechanics”. Notably, the phenomenon of counterportation provides a smoking gun for the existence of an underlying physical reality.

Author Contribution

This is a single-author paper.

Data Availability

The datasets generated and analysed during the current study are available from the corresponding author on reasonable request.

Competing Interests

The author declares that there are no competing interests.

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Appendix

View from Consistent Histories

Another approach to investigating where a photon has or has not been in the setup of Fig. 1 is consistent histories (46), which we have employed in (47) to construct a proof that the photon was never in arm C at any time, analogous to the one just presented. What is meant by a “history” here is a sequence of events between an initial state and a final state—a series of projections at various times during the system’s unitary evolution. Each history has an associated chain-ket, whose inner product with itself gives the probability of that particular history. The idea is to construct a family of histories between the pre-selected state (in this case the photon in S, H-polarised at the start of an outer cycle) and the post-selected state (the photon in S, H-polarised at end of the outer cycle) that, first, has at least one history where the photon is in arm C, and second, the family is consistent, which means all histories are mutually orthogonal. For a consistent history analysis it helps to think of measurements by detectors D_3 to take place after t_{final} , which is permitted by the deferred measurement principle (27). Here is the relevant family of consistent histories for the first outer cycle,

$$S_0 \otimes H_0 \odot \{A_1 \otimes I_1, D_1 \otimes I_1\} \odot \{A_2 \otimes I_2, B_2 \otimes I_2, C_2 \otimes I_2, \} \odot \{A_3 \otimes I_3, B_3 \otimes I_3, C_3 \otimes I_3, \} \odot S_4 \otimes H_4 \quad (7)$$

where S_0 and H_0 are the projectors onto arm S and polarisation H at time t_0 . A_1 and I_1 are the projectors onto arm A and the identity polarisation I at time t_1 , and so on. The curly brackets contain different possible projectors at that particular time. There are 18 possible histories in this family. For example, the history $(S_0 \otimes H_0) \odot (A_1 \otimes I_0) \odot (A_2 \otimes I_2) \odot (A_3 \otimes I_3) \odot (S_4 \otimes H_4)$ has the photon traveling along arm A. Here’s the chain ket associated with this history, $|S_0 \otimes H_0, A_1 \otimes I_1, A_2 \otimes I_2, A_3 \otimes I_3, S_4 \otimes H_4\rangle = (S_4 \otimes H_4)T_{4,3}(A_3 \otimes I_3)T_{3,2}(A_2 \otimes$

$I_2)T_{2,1}(A_1 \otimes I_1)T_{1,0} |S_0 H_0\rangle$, where $T_{1,0}$ is the unitary transformation between times t_0 and t_1 , $T_{2,1}$ is the unitary transformation between times t_1 and t_2 , and so on. By applying these unitary transformations and projections, we see that this chain-ket is equal to, up to a normalisation factor, $|S_4 H_4\rangle$. Other than the history with the photon in arm A, all other 17 histories have probability zero, including the ones where the photon is in arm C. For example the chain-ket $|S_0 \otimes H_0, D_1 \otimes I_1, C_2 \otimes I_2, C_3 \otimes I_3, S_4 \otimes H_4\rangle = S_4 \otimes H_4 |J_4 H_4\rangle$, up to a normalisation factor. Because projectors S and J are orthogonal, as are the projectors H and V, this chain-ket is zero. The photon was not in arm C during the first outer cycle, between times t_0 and t_4 .

Exactly the same goes for the second outer cycle, with the pre-selected state at time t'_0 and the post-selected state at time t'_4 . Here is the relevant family of consistent histories for the second outer cycle,

$$\begin{aligned} & S_{0'} \otimes H_{0'} \odot \{A_{1'} \otimes I_{1'}, D_{1'} \otimes I_{1'}\} \odot \{A_{2'} \otimes I_{2'}, B_{2'} \otimes I_{2'}, C_{2'} \otimes I_{2'}, \} \odot \\ & \{A_{3'} \otimes I_{3'}, B_{3'} \otimes I_{3'}, C_{3'} \otimes I_{3'}, \} \odot S_{4'} \otimes H_{4'} \end{aligned} \quad (8)$$

Since all histories in this consistent family are also zero, except the one where the photon travels down arm A, the photon was not in arm C during the second outer cycle, between times t'_0 and t'_4 . Therefore the photon was not in arm C at any time.

Interestingly, the paradox presented above can be reproduced using consistent histories. Given an initial pre-selected state with the photon at the source at the top of Fig. 1, H-polarised, and a final post-selected state of the photon in arm F on its way to detector D_0 at the bottom, there exists a family of consistent histories that includes histories where the photon is in arm C during the second outer cycle, namely,

$$\begin{aligned}
& S_0 \otimes H_0 \odot \{A_{1'} \otimes I_{1'}, D_{1'} \otimes I_{1'}\} \odot \{A_{2'} \otimes I_{2'}, B_{2'} \otimes I_{2'}, C_{2'} \otimes I_{2'}\} \odot \\
& \{A_{3'} \otimes I_{3'}, B_{3'} \otimes I_{3'}, C_{3'} \otimes I_{3'}\} \odot F_{final} \otimes H_{final}
\end{aligned} \tag{9}$$

It is straightforward to check that this family is consistent, as each chain-ket is zero except the one associated with the history that has the photon in arm A. The photon was not in arm C during the second outer cycle. Now the analogous family that would allow us to ask of the whereabouts of the photon during the first outer cycle is,

$$\begin{aligned}
& S_0 \otimes H_0 \odot \{A_1 \otimes I_1, D_1 \otimes I_1\} \odot \{A_2 \otimes I_2, B_2 \otimes I_2, C_2 \otimes I_2\} \odot \\
& \{A_3 \otimes I_3, B_3 \otimes I_3, C_3 \otimes I_3\} \odot F_{final} \otimes H_{final}
\end{aligned} \tag{10}$$

This family however is not consistent, as its histories are not all mutually orthogonal. Besides the nonzero chain-ket associated with the history that has the photon in arm A, the chain-ket $|S_0 \otimes H_0, D_1 \otimes I_1, C_2 \otimes I_2, B_3 \otimes I_3, F_{final} \otimes H_{final}\rangle$ is also nonzero, rendering the question of whether the photon was in arm C during the first outer cycle meaningless within this framework. We therefore seem to have one conclusion based on consistent histories for the first outer cycle, but a different one for the exactly identical—as far as standard quantum mechanics is concerned—second outer cycle, which once more is unsettlingly paradoxical.

The paradox is resolved by considering each outer cycle *separately*, that is with the pre-selected state at the beginning of the outer cycle and the post-selected state at the end of the outer cycle, as explained earlier. This does not violate the single framework rule, which states that different consistent histories families (or frameworks) cannot be applied during the same time interval (48), which is not the case here. Further, consider the approach given in Ch. 16 of reference (48) (and reiterated in (49) more recently) for combining conclusions drawn based

on two, even incompatible frameworks, “The conceptual difficulty goes away if one supposes that the two incompatible frameworks are being used to describe... the same system during two different runs of an experiment.” Since in the setup we are analysing, each outer cycle is identical, we are effectively looking at the same system during different runs of the experiment. The photon was not in arm C during the first outer cycle. It was not in C during the second outer cycle. Therefore it was never in C.

Note that the question of the counterfactuality of Aharonov and Vaidman’s above mentioned modification (29) can be considered in light of one outer cycle of the setup of Fig. 1, where non-polarising beamsplitters are used instead of polarising beamsplitters, with no polarisation rotators (HWP’s), and with the last beamsplitter in the outer cycle leading straight to two detectors D_0 and D_1 . Importantly, in the absence of the polarisation degree of freedom, there are multiple chain-kets associated with histories that go through the channel that are clearly nonzero. This means that Aharonov and Vaidman’s original modification cannot be counterfactual from a consistent histories point of view.

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Figure 2: A) shows our network for transporting Bob's qubit, $\alpha|0\rangle + \beta|1\rangle$, to Alice by means of two CNOT gates and local operations. The purpose of the Hadamard gates is to keep the control qubit of the second CNOT on the same side as the first. B) a similar network, except for the phase-flip Z-gate acting on Alice's target qubit before the second CNOT, which corresponds to finding the photon in Port1 in Fig. 2C, after the second application of the counterfactual CNOT gate. C) Our general-input counterfactual CNOT gate. A single atom trapped inside an optical resonator constitutes Bob's qubit. Depending on which of two ground states the trapped atom is in, a resonant R-polarised photon impinging on the cavity from the left will either be reflected as a result of strong coupling, or else enter the cavity on its way towards detector D_B . CQZE refer to the chained quantum Zeno effect modules. See text for details of how our proposed CNOT gate works.

Figure 3: Fidelity of counterfactual qubit transport, counterportation, for a number of inner cycles N up to 10, and a number of outer cycles M up to 10, and the realistic imperfections explained in the text, namely Bob's trapped atom failing to reflect 34% of the the time when it should reflect, and failing to not reflect 8% of the time when it should not reflect. Note that the classical fidelity limit for such disembodied transport, from the no-cloning theorem, is $2/3$. Fidelity for each choice of M and N is averaged over 100 evenly distributed qubits, and exceeds the $2/3$ limit even for the minimum number of cycles, 2 outer and 2 inner.