Kinematic Calibration of a 7R 6-DOF Robot With Non-spherical Wrist Using Laser Tracker

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Kinematic calibration of a 7R 6-DOF robot with non-spherical wrist using laser tracker

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Abstract In this paper, a kinematic calibration method is proposed to improve the accuracy of a 7R 6-DOF robot with non-spherical wrist, where the parameters identifiability analysis and model reduction are particularly addressed in the identification procedure. Firstly, the kinematic error model of the robot is constructed based on the modified DH (MDH) model. Secondly, in order to derive a minimal model which is more suitable for practical application, model reduction is implemented considering compensability and constraints of the parameters defined in the controller, and QR decomposition is utilized to determine the redundant parameters, which are then removed in practical parameter identification. Finally, by using a laser tracker as the measuring device, a two-step procedure for error parameter estimation is proposed: (1) estimation of the robot base location with respect to the measurement coordinate system; (2) fine estimation of the identifiable kinematic parameters according to the measurement data. The experiment on a 7R 6-DOF industrial painting robot is implemented, the results show that the proposed method can effectively improve the accuracy of the robot.

Keywords: Robot calibration; Kinematic identification; Parameter identifiability; Mode reduction; 7R 6-DOF robot.

1. Introduction

Repeatability and accuracy are most crucial performance indices of industrial robots. Especially, with the expansion of robots' application area and off-line programming, accuracy is more and more essential. It is well known that if the robot has sufficient repeatability, calibration is an efficient way to improve robot accuracy [1, 2]. In fact, there are many factors could influence the positioning accuracy of industrial robots, e.g. geometric parameter errors, flexible deformation, thermal distortion and expansion and so on. However, as reported by several references [3, 4], the kinematic errors contribute for more than 90% of the total errors. Therefore, this paper mainly focuses on kinematic calibration of a 7R 6-DOF painting robot with non-spherical wrist.

The robot calibration procedure generally involves four steps [5]: modeling, measurement, identification, and compensation. The modeling and identification issues would be particularly addressed in this paper. The modeling step is to construct a mathematical relationship between the kinematic errors and the pose of the end-effector (EE), and it is the premise and foundation of calibration. Schröer [6] pointed out that a perfect model should satisfy three conditions: continuousness, completeness and minimality. To date, the Denavit-Hartenberg (DH) model [7] is universally utilized for robot kinematic calibration [8, 9]. However, DH model is discontinuous when two consecutive joint axes are parallel or near parallel. To solve this problem, Hayati [10] firstly modified this model by adding an extra revolute parameter around the Y axis i.e. the modified DH (MDH) model. Then, authors have presented some other modified models to guarantee continuousness condition, e.g. S model [11], Zero-reference model [12], CPC model [13], and so on. The POE-based model is another competitive choice in the robot calibration [14, 15], because it internally satisfies the continuousness and completeness conditions. However, a potential drawback of these alternate modeling methods is that the compensation cannot be directly implementable in controllers, because DH notation still dominates the kinematic models used in the controller of existing actual industrial manipulators. It should be noted that the kinematic model is generally redundant, in other words, some parameters are not identifiable especially for the modified models with additional parameters to ensure continuousness. This leads to the parameter identifiability analysis and model reduction problem, and there have been several techniques for elimination of these redundant parameters. For example, Khalil and Dombre [16] used QR decomposition of the identification Jacobian for model redundancy analysis, while Gao [17] utilized singular value decomposition (SVD) to solve this problem. Additionally, some of the kinematic parameters cannot be directly changed in controller, because the inverse kinematics may be based on the properties of these parameters e.g. α_i equals $\pm 90^\circ$ or 0° . Therefore, model parameter reduction considering the compensability and constraints defined in the robot controller is still an open issue needs to be further investigated.

As for measurement step, there are mainly two techniques: the closed-loop and open-loop ones. The closed-loop method applies constrains (e.g. point, plane or sphere constraints) on the EE of the robot and collects joint angles by encoders [18, 19]. It does not rely on any expensive measurement device, however the robot must be carefully operated to satisfy particular constrains which requires skilled personnel and

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is time consuming. Therefore, the open-loop methods with external measurement devices e.g. laser tracker [5, 20], measurement arms [15, 21], ball-bar system [22, 23], wire potentiometer [8], photogrammetry system [24, 25], and so on, to measure full or partial pose of the EE have been widely used. In this case, the measured data is described in the measurement coordinate system. So, the estimation of robot base location with respect to the measurement coordinate system is an essential work before kinematic parameter identification, which generally needs additional measurement. Besides, some authors have proposed techniques to improve the parameter identification accuracy by optimal selection of measurement configurations for calibration experiments [26, 27].

The robot researched in this paper is a 7R 6-DOF robot as shown in **Figure 1**, which is not a purely open-loop structure, but rather contains a 4R 3-DOF non-spherical wrist with a coupling joint to increase motion flexibility [28]. This special wrist greatly increases the complexity and difficulty of calibration. To develop a feasible kinematic calibration method more suitable for industrial application, kinematic error model of the robot is constructed based on the modified DH (MDH) model. Then, model reduction and redundancy analysis are implemented considering compensability and constraints of the parameters defined in the controller. Finally, special attention is applied on estimation of the robot base location with respect to the measurement coordinate system before fine identification of the kinematic parameters. Having outlined existing approaches and challenges, this paper is organized in the following manner. In Section 2, the kinematic model of the robot is constructed, and the error parameter estimation method is proposed. Section 3 gives parameters identifiability analysis and model reduction. Section 4 conducts the kinematic calibration experiment based on a 7R 6-DOF painting robot. And the conclusions are drawn in Section 5.

Figure 1 The 7R 6-DOF painting robot with non-spherical wrist (a) Virtual prototype (b) Schematic diagram and coordinate systems

2. Modeling and parameter identification

2.1 Kinematic parameter error modeling

The kinematic error model is a mathematical relationship between the kinematic errors and the pose of EE. As shown in **Figure 1** (a), it is the virtual prototype of a 7R 6-DOF robot with non-spherical wrist which has been proved to be advantageous in painting application due to wider range of motion of the wrist [28]. The 4R 3-DOF wrist of the 7R 6-DOF robot has 4 revolute joints, the second and third of which are coupled with the relation $\theta_6 = -\theta_5$. This paper aims at developing kinematic calibration method that is more suitable for industrial application, then the kinematic model is established based on MDH model considering that the second and third joint axes are parallel. **Figure 1** (b) shows the coordinate systems of the 7R 6-DOF robot, while the corresponding MDH parameters are listed in **Table 1**. For general case, the transformation from coordinate system $\{i-1\}$ to $\{i\}$ is described by four parameters and is expressed as

$${}^{i-1}\mathbf{T} = \text{Rot}(z, \theta_i) \text{Trans}(0, 0, d_i) \text{Rot}(x, \alpha_i) \text{Trans}(a_i, 0, 0)$$

$$= \begin{bmatrix} c_i & -s_i c \alpha_i & s_i s \alpha_i & a_i c_i \\ s_i & c_i c \alpha_i & -c_i s \alpha_i & a_i s_i \\ 0 & s \alpha_i & c \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where $s_i = \sin \theta_i$, $c_i = \cos \theta_i$, $c \alpha_i = \cos \alpha_i$, $s \alpha_i = \sin \alpha_i$.

Table 1 Nominal values of the MDH parameters of the 7R 6-DOF painting robot

i - th	θ_i (deg)	d_i (mm)	a_i (mm)	α_i (deg)	β_i (deg)
1	θ_1	0	0	-90	/
2	θ_2-90	0	1300	0	0
3	θ_3	0	0	-90	/
4	θ_4	1474	0	-35	/
5	θ_5	95	0	70	/
6	θ_6	95	0	-35	/
7	θ_7	85	0	0	0

However, when two consecutive joint axes are parallel, an extra revolute parameter around the Y axis is added, the corresponding transformation matrix can be expressed as

$${}^{i-1}\mathbf{T} = \text{Rot}(z, \theta_i) \text{Trans}(0, 0, d_i) \text{Rot}(x, \alpha_i) \text{Trans}(a_i, 0, 0) \text{Rot}(y, \beta_i)$$

$$= \begin{bmatrix} c_i c \beta_i - s_i c \alpha_i s \beta_i & -s_i c \alpha_i & c_i s \beta_i + s_i s \alpha_i c \beta_i & a_i c_i \\ s_i c \beta_i + c_i s \alpha_i s \beta_i & c_i c \alpha_i & s_i s \beta_i - c_i s \alpha_i c \beta_i & a_i s_i \\ -c \alpha_i s \beta_i & s \alpha_i & c \alpha_i c \beta_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

where $s \beta_i = \sin(\beta_i)$, $c \beta_i = \cos(\beta_i)$. Then, forward kinematics of the robot i.e. the transformation matrix from the world reference frame to its EE frame could be formulated by

$${}_{tool}^{base}\mathbf{T} = \mathbf{T}_{base}^0 \mathbf{T}_1^1 \mathbf{T}_2^2 \mathbf{T}_3^3 \mathbf{T}_4^4 \mathbf{T}_5^5 \mathbf{T}_6^6 \mathbf{T}_7^7 \mathbf{T}_{tool} = \mathbf{T}_{base} \prod_{i=1}^7 {}^{i-1}\mathbf{T}_i \mathbf{T}_{tool} \quad (3)$$

where $\mathbf{T}_{base}$ is the transformation matrix from the world frame to base frame, $\mathbf{T}_{tool}$ is the transformation matrix from frame $\{7\}$ to EE frame. These transformation matrices are described by six parameters and formulated as Eqs. (4) and (5).

$$\begin{aligned}\mathbf{T}_{base} &= \text{Trans}(x_0, y_0, z_0) \text{Rot}(x, \alpha_0) \text{Rot}(y, \beta_0) \text{Rot}(z, \varphi_0) \\ &= \begin{bmatrix} c\beta_0 c\varphi_0 & -c\beta_0 s\varphi_0 & s\beta_0 & x_0 \\ c\alpha_0 s\varphi_0 + s\alpha_0 s\beta_0 c\varphi_0 & c\alpha_0 c\varphi_0 - s\alpha_0 s\beta_0 s\varphi_0 & -s\alpha_0 c\beta_0 & y_0 \\ s\alpha_0 s\varphi_0 - c\alpha_0 s\beta_0 c\varphi_0 & s\alpha_0 c\varphi_0 + c\alpha_0 s\beta_0 s\varphi_0 & c\alpha_0 c\beta_0 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\end{aligned}\quad (4)$$

$$\begin{aligned}\mathbf{T}_{tool} &= \text{Trans}(x_t, y_t, z_t) \text{Rot}(x, \alpha_t) \text{Rot}(y, \beta_t) \text{Rot}(z, \varphi_t) \\ &= \begin{bmatrix} c\beta_t c\varphi_t & -c\beta_t s\varphi_t & s\beta_t & x_t \\ c\alpha_t s\varphi_t + s\alpha_t s\beta_t c\varphi_t & c\alpha_t c\varphi_t - s\alpha_t s\beta_t s\varphi_t & -s\alpha_t c\beta_t & y_t \\ s\alpha_t s\varphi_t - c\alpha_t s\beta_t c\varphi_t & s\alpha_t c\varphi_t + c\alpha_t s\beta_t s\varphi_t & c\alpha_t c\beta_t & z_t \\ 0 & 0 & 0 & 1 \end{bmatrix}\end{aligned}\quad (5)$$

where $[x_0, y_0, z_0]([x_t, y_t, z_t])$ is the translation vector from the world frame to base frame (from frame $\{7\}$ to EE frame), $[\alpha_0, \beta_0, \varphi_0](\alpha_t, \beta_t, \varphi_t)$ is the corresponding Euler rotation angles vector.

In order to construct the kinematic error model, the errors of kinematic parameters are defined as $\delta \mathbf{q}_i = [\Delta\theta_i, \Delta d_i, \Delta\alpha_i, \Delta a_i, \Delta\beta_i]^T$ ($i = 1, \dots, 7$), $\delta \mathbf{q}_k = [\Delta x_k, \Delta y_k, \Delta z_k, \Delta\alpha_k, \Delta\beta_k, \Delta\varphi_k]^T$ ($k = 0, t$). Then, the actual configuration of EE with respect to the world frame can be rewritten as

$$\mathbf{T}_{tool}^{base} = \mathbf{T}_{base}^r \prod_{i=1}^7 {}^{i-1}\mathbf{T}^r \mathbf{T}_{tool}^r = \mathbf{T}_{base}^r (\delta \mathbf{q}_0) \prod_{i=1}^7 {}^{i-1}\mathbf{T}^r (\theta_i, \delta \mathbf{q}_i) \mathbf{T}_{tool}^r (\delta \mathbf{q}_t) \quad (6)$$

where the superscripts “ r ” represents the actual transformation. From Eqs. (3) and (6), the EE transformation matrix errors result from the errors of link, base and tool transformation matrices. The link transformation matrix error can be expressed as

$$d {}^{i-1}\mathbf{T} = {}^{i-1}\mathbf{T}^r (\theta_i, \delta \mathbf{q}_i) - {}^{i-1}\mathbf{T} \quad (7)$$

The actual link transformation matrix can be treated as the result of the normal transformation after serial differential motions, which can be written as

$${}^{i-1}\mathbf{T}^r (\theta_i, \delta \mathbf{q}_i) = {}^{i-1}\mathbf{T} \text{Trans}(\delta x_i, \delta y_i, \delta z_i) \text{Rot}(x, \delta \alpha_i) \text{Rot}(y, \delta \beta_i) \text{Rot}(z, \delta \varphi_i) \quad (8)$$

where $\mathbf{e}_i = [\delta x_i, \delta y_i, \delta z_i, \delta \alpha_i, \delta \beta_i, \delta \varphi_i]^T$ is the link differential error vector. From Eqs. (7) and (8), following equation can be obtained

$$\begin{aligned}d {}^{i-1}\mathbf{T} &= {}^{i-1}\mathbf{T} (\text{Trans}(\delta x_i, \delta y_i, \delta z_i) \text{Rot}(x, \delta \alpha_i) \text{Rot}(y, \delta \beta_i) \text{Rot}(z, \delta \varphi_i) - \mathbf{I}) \\ &= {}^{i-1}\mathbf{T} \Delta_i\end{aligned}\quad (9)$$

where $\Delta_i = \text{Trans}(\delta x_i, \delta y_i, \delta z_i) \text{Rot}(x, \delta \alpha_i) \text{Rot}(y, \delta \beta_i) \text{Rot}(z, \delta \varphi_i) - \mathbf{I}$ is the differential operator. By ignoring the second and higher order error terms, Δ_i can be expressed as

$$\Delta_i = \begin{bmatrix} 0 & -\delta \varphi_i & \delta \beta_i & \delta x_i \\ \delta \varphi_i & 0 & -\delta \alpha_i & \delta y_i \\ -\delta \beta_i & \delta \alpha_i & 0 & \delta z_i \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (10)$$

From Eqs. (9) and (10), ignoring the second and higher order error terms, $\mathbf{e}_i$ can be written as function of the link kinematic errors.

$$\mathbf{e}_i = \begin{bmatrix} f_x(\Delta\theta_i, \Delta d_i, \Delta\alpha_i, \Delta a_i, \Delta\beta_i) \\ f_y(\Delta\theta_i, \Delta d_i, \Delta\alpha_i, \Delta a_i, \Delta\beta_i) \\ \dots \\ f_\varphi(\Delta\theta_i, \Delta d_i, \Delta\alpha_i, \Delta a_i, \Delta\beta_i) \end{bmatrix} = \mathbf{M}_i \delta \mathbf{q}_i \quad (11)$$

where $\mathbf{M}_i \in R^{6 \times 5}$ is the transform matrix between $\mathbf{e}_i$ and $\delta \mathbf{q}_i$. Similarly, the base (tool) differential error vector can be written as function of corresponding kinematic errors. See **Appendix A** for details of these transformation matrices.

$$\mathbf{e}_0 = \mathbf{M}_0 \delta \mathbf{q}_0, \quad \mathbf{e}_t = \mathbf{M}_t \delta \mathbf{q}_t \quad (12)$$

According to the differential theory and linear superposition principle, the total deflections of EE can be obtained with the general Jacobian matrix.

$$\mathbf{E}^k = \sum_{i=1}^7 {}^t \mathbf{S} \mathbf{e}_i + {}^0 \mathbf{S} \mathbf{e}_0 + {}^t \mathbf{S} \mathbf{e}_t \quad (13)$$

where ${}^t \mathbf{S}$ is the general Jacobian matrix [29], and ${}^t \mathbf{S} = \mathbf{I}$. By substituting Eqs. (11) and (12) into Eq. (13), the kinematic error model can be expressed as

$$\mathbf{E}^k = \sum_{i=1}^7 {}^t \mathbf{S} \mathbf{M}_i \delta \mathbf{q}_i + {}^0 \mathbf{S} \mathbf{M}_0 \delta \mathbf{q}_0 + {}^t \mathbf{S} \mathbf{M}_t \delta \mathbf{q}_t = \mathbf{H}^k \delta \mathbf{q} \quad (14)$$

where $\mathbf{E}^k = [\mathbf{E}_p^k; \mathbf{E}_r^k] \in R^6$ is the differential error vector of EE, while $\mathbf{E}_p^k, \mathbf{E}_r^k$ denote respectively the position error vector and orientation error vector, $\delta \mathbf{q} = [\delta \mathbf{q}_1, \delta \mathbf{q}_2, \dots, \delta \mathbf{q}_7, \delta \mathbf{q}_0, \delta \mathbf{q}_t]^T \in R^c$ is the error vector of the kinematic parameters, $\mathbf{H}^k = [\mathbf{G}_1 \mathbf{M}_1, \mathbf{G}_2 \mathbf{M}_2, \dots, \mathbf{G}_7 \mathbf{M}_7, \mathbf{G}_0 \mathbf{M}_0, \mathbf{G}_t \mathbf{M}_t]$ is the identification Jacobian. It is worthwhile to point out that the effects of the coupled joint of the 4R 3-DOF wrist is not considered in the identification model i.e. Eq. (14). To construct the identification model of the 7R 6-DOF robot, effects of the coupled joint i.e. $\theta_6 = -\theta_5$ should be analyzed. It means the joint angle error of joint 5 $\Delta \theta_5$ could result in an additional error on joint angle of joint 6 i.e. $\Delta \theta_{6,add} = -\Delta \theta_5$. Therefore, the actual differential error of EE results from $\Delta \theta_5$ can be written as

$$\mathbf{E}_{\Delta \theta_5}^k = \mathbf{H}^k(:, \Delta \theta_5) \Delta \theta_5 + \mathbf{H}^k(:, \Delta \theta_6) \Delta \theta_{6,add} = [\mathbf{H}^k(:, \Delta \theta_5) - \mathbf{H}^k(:, \Delta \theta_6)] \Delta \theta_5 \quad (15)$$

where $\mathbf{H}^k(:, \Delta \theta_5), \mathbf{H}^k(:, \Delta \theta_6)$ are respectively columns of the identification Jacobian corresponding to $\Delta \theta_5$ and $\Delta \theta_6$. From Eq. (15), the identification Jacobian considering effects of the coupled joint can be derived by adjusting the column corresponding to $\Delta \theta_5$ as

$$\mathbf{H}^k(:, \Delta \theta_5) = \mathbf{H}^k(:, \Delta \theta_5) - \mathbf{H}^k(:, \Delta \theta_6) \quad (16)$$

2.2 Kinematic parameter identification

In order to identify the actual kinematic parameters, we need to measure poses of EE in different configurations and calculate the differential errors of EE. From Eq. (14), $6 \times N$ scalar equations can be obtained as

$$\begin{bmatrix} \mathbf{E}^1 \\ \mathbf{E}^2 \\ \dots \\ \mathbf{E}^N \end{bmatrix} = \begin{bmatrix} \mathbf{H}^1 \\ \mathbf{H}^2 \\ \dots \\ \mathbf{H}^N \end{bmatrix} \delta \mathbf{q}, \quad \mathbf{E} = \mathbf{H} \delta \mathbf{q} \quad (17)$$

where N is the number of measurement. When $6 \times N > c$, by applying the least-square technique, the error vector of kinematic parameters $\delta \mathbf{q}$ can be obtained as

$$\delta \mathbf{q} = \mathbf{H}^+ \mathbf{E} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{E} \quad (18)$$

where $\mathbf{H}^+ = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T$ is the pseudo-inverse of $\mathbf{H}$. It is notable that the identification Jacobian $\mathbf{H}^k$ in Eq. (14) is function of the kinematic parameters of links, base and tool. In other words, the normal values of these parameters would be used in Eq. (18). For a specific robot, normal values of kinematic parameters of links i.e. MDH parameters are known. In addition, if manufacture errors of the measurement tool are relatively small, the normal values of tool kinematic parameters are also treated as known. Thus, only the base kinematic parameters are unknown. To solve this problem, the existing method is to do a preliminary identification of the base transformation matrix by additional measuring the axes 1 and 2 with external measurement device e.g. laser tracker [1, 30]. In order to simplify the identification process, this paper proposed a two-step procedure for kinematic parameter estimation: (1) estimation of the robot base location with respect to the world frame, (2) fine estimation of the identifiable kinematic parameters. Because no additional measurements are needed, the proposed method is simpler and more suitable for actual industrial application.

In the first step, the kinematic parameters of links and tool transformation are treated as known and take their normal values. From Eq. (6), for each measurement, the configuration of EE with respect to the world frame can be expressed as

$${}_{\text{tool}}^{\text{base}}\mathbf{T}^{r(k)} = \mathbf{T}_{\text{base}} \prod_{i=1}^7 {}_i^{\text{base}}\mathbf{T}^{(k)} \mathbf{T}_{\text{tool}} = \mathbf{T}_{\text{base}} \mathbf{T}_{\text{tool}}^{0(k)} \quad (19)$$

where $\mathbf{T}_{\text{tool}}^{0(k)} = \prod_{i=1}^7 {}_i^{\text{base}}\mathbf{T}^{(k)} \mathbf{T}_{\text{tool}}$ is the configuration of EE with respect to base frame. From Eq. (19), the coarse estimation of the base transformation can be obtained as

$$\tilde{\mathbf{T}}_{\text{base}} = {}_{\text{tool}}^{\text{base}}\mathbf{T}^{r(k)} (\mathbf{T}_{\text{tool}}^{0(k)})^{-1} \quad (20)$$

The normal base transformation is treated as the result of coarse estimation after differential motions.

$$\mathbf{T}_{\text{base}} = \tilde{\mathbf{T}}_{\text{base}} (\mathbf{I} + \Delta_{\text{base}}) \quad (21)$$

where Δ_{base} is the differential operator of base frame and can be written in differential error vector form $\mathbf{e}_{\text{base}} = [\delta\tilde{\mathbf{d}}_{\text{base}}, \delta\tilde{\mathbf{r}}_{\text{base}}]^T = [\delta\tilde{x}_{\text{base}}, \delta\tilde{y}_{\text{base}}, \delta\tilde{z}_{\text{base}}, \delta\tilde{\alpha}_{\text{base}}, \delta\tilde{\beta}_{\text{base}}, \delta\tilde{\phi}_{\text{base}}]^T$. Any homogeneous transformation matrix can be written by rotational submatrix $\mathbf{R}^{3 \times 3}$ and translational vector $\mathbf{P}^{3 \times 1}$ as

$$\mathbf{T} = \begin{bmatrix} \mathbf{R}^{3 \times 3} & \mathbf{P}^{3 \times 1} \\ \mathbf{0}^{1 \times 3} & 1 \end{bmatrix} \quad (22)$$

From Eqs. (19) and (21), for each measurement, the position vector of EE can be expressed as

$$\begin{aligned} {}_{\text{tool}}^{\text{base}}\mathbf{P}^{(k)} &= \tilde{\mathbf{R}}_{\text{base}} (\mathbf{I} + Sp(\delta\tilde{\mathbf{r}}_{\text{base}})) \mathbf{P}_{\text{tool}}^{0(k)} + \tilde{\mathbf{R}}_{\text{base}} \delta\tilde{\mathbf{d}}_{\text{base}} + \tilde{\mathbf{P}}_{\text{base}} \\ &= \tilde{\mathbf{R}}_{\text{base}} \mathbf{P}_{\text{tool}}^{0(k)} - \tilde{\mathbf{R}}_{\text{base}} Sp(\mathbf{P}_{\text{tool}}^{0(k)}) \delta\tilde{\mathbf{r}}_{\text{base}} + \tilde{\mathbf{R}}_{\text{base}} \delta\tilde{\mathbf{d}}_{\text{base}} + \tilde{\mathbf{P}}_{\text{base}} \\ &= \tilde{\mathbf{R}}_{\text{base}} \mathbf{P}_{\text{tool}}^{0(k)} + \tilde{\mathbf{P}}_{\text{base}} + \tilde{\mathbf{R}}_{\text{base}} [\mathbf{I} \quad -Sp(\mathbf{P}_{\text{tool}}^{0(k)})] \begin{bmatrix} \delta\tilde{\mathbf{d}}_{\text{base}} \\ \delta\tilde{\mathbf{r}}_{\text{base}} \end{bmatrix} \end{aligned} \quad (23)$$

where $Sp(\mathbf{P}_{\text{tool}}^{0(k)})$ is skew symmetric matrix of $\mathbf{P}_{\text{tool}}^{0(k)}$. From Eq. (23), by applying the least-square technique, the differential error vector of base frame can be obtained as

$$\mathbf{e}_{\text{base}} = \begin{bmatrix} \delta\tilde{\mathbf{d}}_{\text{base}} \\ \delta\tilde{\mathbf{r}}_{\text{base}} \end{bmatrix} = (\mathbf{A}_c^T \mathbf{A}_c)^{-1} \mathbf{A}_c^T \Delta \mathbf{P}_c \quad (24)$$

$$\text{where } \mathbf{A}_c = \tilde{\mathbf{R}}_{\text{base}} \begin{bmatrix} \mathbf{I} & -Sp(\mathbf{P}_{\text{tool}}^{0(1)}) \\ \mathbf{I} & -Sp(\mathbf{P}_{\text{tool}}^{0(2)}) \\ \dots & \dots \\ \mathbf{I} & -Sp(\mathbf{P}_{\text{tool}}^{0(N)}) \end{bmatrix}, \quad \Delta \mathbf{P}_c = \begin{bmatrix} {}_{\text{tool}}^{\text{base}}\mathbf{P}^{(1)} - \tilde{\mathbf{R}}_{\text{base}} \mathbf{P}_{\text{tool}}^{0(1)} - \tilde{\mathbf{P}}_{\text{base}} \\ {}_{\text{tool}}^{\text{base}}\mathbf{P}^{(2)} - \tilde{\mathbf{R}}_{\text{base}} \mathbf{P}_{\text{tool}}^{0(2)} - \tilde{\mathbf{P}}_{\text{base}} \\ \dots \\ {}_{\text{tool}}^{\text{base}}\mathbf{P}^{(N)} - \tilde{\mathbf{R}}_{\text{base}} \mathbf{P}_{\text{tool}}^{0(N)} - \tilde{\mathbf{P}}_{\text{base}} \end{bmatrix}. \text{ Finally, from Eqs. (21) and}$$

(24), the normal base transformation matrix can be derived.

In the second step, the normal kinematic parameters of links, base transformation and tool transformation are treated as known. Then from Eq. (18), the error vector of kinematic parameters can be obtained.

3. Model reduction and parameter identifiability analysis

The original MDH model satisfies the continuousness and completeness conditions, however it generally do not satisfy the minimality condition i.e. some kinematic parameters are redundant. Additionally, some kinematic parameters used in controllers cannot be directly changed, because the inverse kinematics may be based on the properties of these parameters e.g. α_i equals $\pm 90^\circ$ or 0° . Therefore, in order to develop a calibration method that is suitable for practical application, model reduction and parameters identifiability analysis should be conducted before parameter identification.

3.1 Model reduction

According to the compensability of the parameters in the robot controller, these parameters can be divided into two groups: G1—parameters that can be compensated by directly changing values in the controller; G2—other parameters. It is notable that the base and tool transformation matrix can be redefined in the controller, therefore the corresponding kinematic parameters belong to G1. Detailed classification of the kinematic parameters of the 7R 6-DOF robot is listed in **Table 2**.

To guarantee the developed calibration method suitable for practical application, parameters in G1 are retained, while those in G2 are removed. By removing columns of $\mathbf{H}^k$ corresponding to the parameters in G2, the reduced model is obtained as

Table 2 Classification of the kinematic parameters of the 7R 6-DOF robot

$i - \text{th}$	θ_i	d_i	a_i	α_i	β_i
1	G1	G1	G1	G2	/
2	G1	G1	G1	G2	G2
3	G1	G2	G1	G2	/
4	G1	G1	G2	G1	/
5	G1	G1	G2	G2	/
6	G2	G1	G2	G2	/
7	G1	G1	G2	G2	G2
x_0	y_0	z_0	α_0	β_0	φ_0
G1	G1	G1	G1	G1	G1
x_t	y_t	z_t	α_t	β_t	φ_t
G1	G1	G1	G1	G1	G1

$$\mathbf{E}^k = \mathbf{H}_r^k \delta \mathbf{q}_r \quad (25)$$

where $\delta \mathbf{q}_r$ is the error vector of kinematic parameters in G1, $\mathbf{H}_r^k$ denotes the corresponding identification Jacobian. It is notable that because of the presence of coupling joint in the 4R 3-DOF wrist, some additional constraints are defined in the controller i.e. $\alpha_5 = -2\alpha_4$, $\alpha_6 = \alpha_4$. In other words, the compensation of α_4 also results changes of α_5 and α_6 . It means the torsion angle error of joint 4 $\Delta\alpha_4$ could result in additional error on torsion angle errors of joint 5 and joint 6 i.e. $\Delta\alpha_{5,add} = -2\Delta\alpha_4$, $\Delta\alpha_{6,add} = \Delta\alpha_4$. Therefore, the actual differential error of EE results from $\Delta\alpha_4$ can be written as

$$\mathbf{E}_{\Delta\alpha_4}^k = [\mathbf{H}^k(:, \Delta\alpha_4) - 2\mathbf{H}^k(:, \Delta\alpha_5) + \mathbf{H}^k(:, \Delta\alpha_6)] \Delta\alpha_4 \quad (26)$$

where $\mathbf{H}^k(:, \Delta\alpha_4)$, $\mathbf{H}^k(:, \Delta\alpha_5)$, $\mathbf{H}^k(:, \Delta\alpha_6)$ are respectively columns of the identification Jacobian corresponding to $\Delta\alpha_4$, $\Delta\alpha_5$ and $\Delta\alpha_6$. Then from Eq. (26), the identification Jacobian considering constrains can be derived by adjusting the column corresponding to $\Delta\alpha_4$ as

$$\mathbf{H}_r^k(:, \Delta\alpha_4) = \mathbf{H}_r^k(:, \Delta\alpha_4) - 2\mathbf{H}^k(:, \Delta\alpha_5) + \mathbf{H}^k(:, \Delta\alpha_6) \quad (27)$$

where $\mathbf{H}_r^k(:, \Delta\alpha_4) = \mathbf{H}^k(:, \Delta\alpha_4)$.

3.2 Parameter identifiability analysis

To avoid potential singularity problem in solving Eq. (17), parameter identifiability analysis is conducted in this section, and the redundant parameters would be removed before practical parameter identification. Here, the QR decomposition technique is utilized to solve this problem. According to QR decomposition, the identification Jacobian $\mathbf{H}$ can be rewritten as

$$\mathbf{H}_{m \times c} = \mathbf{Q}_{m \times m} \begin{bmatrix} \mathbf{R}_{c \times c} \\ \mathbf{0}_{(m-c) \times c} \end{bmatrix} \quad (28)$$

where $\mathbf{Q}_{m \times m}$ is a $m \times m$ orthogonal matrix, $\mathbf{R}_{c \times c}$ is a $c \times c$ upper triangular matrix, and $\mathbf{0}_{(m-c) \times c}$ is a $(m-c) \times c$ null matrix. Then, Eq. (17) can be rewritten as

$$\mathbf{E}_{m \times 1} = \mathbf{Q}_{m \times m} \begin{bmatrix} \mathbf{R}_{c \times c} \\ \mathbf{0}_{(m-c) \times c} \end{bmatrix} \delta \mathbf{q}_{c \times 1} \quad (29)$$

where $m = 6N$ is the number of scalar equations, c is the number of unknown parameters (for original MDH model $c = 42$, for reduced model $c = 28$). From Eq. (29), the unknown parameters whose corresponding elements on the diagonal of $\mathbf{R}_{c \times c}$ are zero can be regarded as redundant. It is notable that the redundant parameters whose corresponding columns of $\mathbf{R}_{c \times c}$ are all zeroes have no effect on differential error of EE, while other redundant parameters are linear correlation with one or more identifiable parameters. And for several linearly dependent parameters, the first k independent

parameters in the vector will be given as the identifiable ones. Therefore, it is worthwhile to rearrange the error vector of kinematic parameters according to complexity of their compensation.

After the redundant parameters are obtained, they are removed to ensure identification Jacobian invertible. If r is the rank of $\mathbf{H}$, there are $c - r$ redundant parameters. For simplicity, the error vector of kinematic parameters after removing the redundant parameters is noted as $\delta \mathbf{q}_a$, and the identification Jacobian after removing corresponding columns is noted as $\mathbf{H}_a$. Then the final identification model is expressed as

$$\mathbf{E}^k = \mathbf{H}_a^k \delta \mathbf{q}_a \quad (30)$$

Following the idea above, for original MDH model, $r = 34$. There are 8 redundant parameters in $\delta \mathbf{q}$: $\delta d_3, \delta z_0, \delta \varphi_0, \delta x_t, \delta z_t, \delta \alpha_t, \delta \beta_t, \delta \varphi_t$. The final identifiable parameter vector is

$$\begin{aligned} \delta \mathbf{q}_a = & [\delta \theta_1, \delta d_1, \delta a_1, \delta \alpha_1, \delta \theta_2, \delta d_2, \delta a_2, \delta \alpha_2, \delta \beta_2, \delta \theta_3, \delta a_3, \delta \alpha_3, \delta \theta_4, \delta d_4, \\ & \delta a_4, \delta \alpha_4, \delta \theta_5, \delta d_5, \delta a_5, \delta \alpha_5, \delta \theta_6, \delta d_6, \delta a_6, \delta \alpha_6, \delta \theta_7, \delta d_7, \delta a_7, \delta \alpha_7, \\ & \delta \beta_7, \delta x_0, \delta y_0, \delta \alpha_0, \delta \beta_0, \delta y_t]^T \end{aligned} \quad (31)$$

While for reduced model, $r = 24$. There are 4 redundant parameters in $\delta \mathbf{q}_r$: $\delta z_0, \delta \varphi_0, \delta z_t, \delta \varphi_t$. The final identifiable parameter vector is

$$\begin{aligned} \delta \mathbf{q}_{r,a} = & [\delta \theta_1, \delta \theta_2, \delta \theta_3, \delta \theta_4, \delta \theta_5, \delta \theta_7, \delta d_1, \delta d_2, \delta d_4, \delta d_5, \delta d_6, \delta d_7, \delta a_1, \delta a_2, \\ & \delta a_3, \delta \alpha_4, \delta x_0, \delta y_0, \delta \alpha_0, \delta \beta_0, \delta x_t, \delta y_t, \delta \alpha_t, \delta \beta_t]^T \end{aligned} \quad (32)$$

4. Experiments and discussion

To verify effectiveness of the proposed method, the kinematic calibration experiment of a 7R 6-DOF painting robot as shown in **Figure 1** has been conducted. In the experiment, the measurement device is a LEICA AT901-LR laser tracker with the maximum observed error of $5\mu\text{m/m}$. It is notable that the proposed method needs measurements of full pose of EE (three translations and three rotations) while the laser tracker with a reflector generally offers the position (three translations) i.e. the center of a sphere reflector. Therefore, a specific end-effector tool has been designed in this paper, as shown in **Figure 2**. The tool is bolted to the end flange of the robot, and it has four reflector holders, the centers of which make a rectangle. Then in order to get full pose of EE, at least three reflectors should be measured at each robot configuration. For simplicity, a tool coordinate system is defined as shown in **Figure 2** (b). The original point is located on the center point of the rectangle i.e. the center of line P_2P_3 , and can be expressed as

Figure 2 End-effector tool for kinematic calibration (a) 3D (b) Tool coordinate system

$$\mathbf{P}_t = (\mathbf{P}_2 + \mathbf{P}_3) / 2 \quad (33)$$

where $\mathbf{P}_t$ is the position vector of tool frame. The Z axis is defined as the normal vector of plane $P_1P_2P_3$, Y axis is aligned on line P_1P_2 , while X axis is derived according to the right-hand rule. The measured tool transformation matrix can be written as

$${}^{base}_{tool} \mathbf{T}^r = \begin{bmatrix} \mathbf{R}_t & \mathbf{P}_t \\ 0 & 1 \end{bmatrix} \quad (34)$$

where $\mathbf{R}_i = [\mathbf{r}_x, \mathbf{r}_y, \mathbf{r}_z]$ is the orientation matrix of tool frame. And the elements are defined as

$$\mathbf{r}_y = (\mathbf{P}_2 - \mathbf{P}_1) / \|\mathbf{P}_2 - \mathbf{P}_1\| \quad (35)$$

$$\mathbf{r}_z = [(\mathbf{P}_3 - \mathbf{P}_1) / \|\mathbf{P}_3 - \mathbf{P}_1\|] \times \mathbf{r}_y \quad (36)$$

$$\mathbf{r}_x = \mathbf{r}_y \times \mathbf{r}_z \quad (37)$$

The experimental set-up for kinematic calibration of the 7R 6-DOF painting robot is illustrated in **Figure 3**. In order to clearly describe the experiment procedure, the steps are expressed as follows:

1) Select N robot configurations $\boldsymbol{\theta} = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_7]$ for pose measurement, they should be randomly distributed in the robot's usable workspace and ensure at least three reflectors are visible by the laser tracker.

2) The robot is commanded to undergo the measurement configurations successively, the position data of three reflectors are captured by the laser tracker. Then differential error vectors $\mathbf{E}^k$ of EE are derived. The measurement at each configuration is repeated three times and the mean value is retained.

3) From Eqs. (20), (21) and (24), the normal base transformation matrix $\mathbf{T}_{base}$ is estimated.

4) From Eq. (18), the error vector of kinematic parameters $\delta \mathbf{q}$ can be obtained, it is notable that the original identification matrix $\mathbf{H}_a$ and reduced one $\mathbf{H}_{r,a}$ are respectively utilized for comparison.

5) According to the identified kinematic parameters, error compensation is conducted and analyzed. It is worthwhile to point out that the errors of reduced model can be compensated by directly modification of parameters in the controller, while that of the original model should be compensated by adjusting the controller input i.e. target configurations. For error compensation of the original model, the target configurations can be adjusted by

$${}^{base}_{tool} \mathbf{T}_a = {}^{base}_{tool} \mathbf{T} (\mathbf{I} + \Delta_{original}) \quad (38)$$

where ${}^{base}_{tool} \mathbf{T}$, ${}^{base}_{tool} \mathbf{T}_a$ are respectively the original and adjusted target configurations, $\Delta_{original}$ is the differential operator of EE calculated by the original model.

Figure 3 Experiment set-up for kinematic calibration of the 7R 6-DOF painting robot

According to analysis in Section 3.2, there are 34 and 24 kinematic parameters respectively for the original model and the reduced model need to be identified. Then at least 6 measurement configurations are needed i.e. $N \geq 6$. In order to reduce effects of measurement errors on parameter identification accuracy, 50 different measurement configurations are selected in the practical experiment.

From Eqs. (20), (21) and (24), the normal base transformation matrix $\mathbf{T}_{base}$ is firstly estimated by using the measured data. For comparison, the base transformation matrix is also identification by additional measuring the axes 1 and 2 [30], as shown in **Figure 4**. Additional two reflectors are respectively fixed on the waist and upperarm of the robot, by respectively rotating joint 1 and joint2 (with the others being locked) and capturing positions of the reflectors by the laser tracker, the axes along joint 1 and joint 2 can be derived. Then, the base coordinate system is defined as shown in **Figure 4** (b). For simplicity, the

obtained base kinematic parameters by the two methods (the proposed method and additional measurement method) are summarized in **Table 3**.

Figure 4 Measurement of the base frame (a) experiment set-up (b) defining the base coordinate system

Table 3 Base kinematic parameters of the robot

/	x_0 (mm)	y_0 (mm)	z_0 (mm)	α_0 (deg)	β_0 (deg)	φ_0 (deg)
Proposed method	-462.9100	4097.4547	-197.7500	0.6486	0.3299	-59.4643
Additional measurement method	-463.1357	4095.966	-195.6047	0.4100	0.3372	-59.6511

From **Table 3**, errors between the base kinematic parameters by the two methods are really small with the maximum translational and orientational errors are respectively 2.15mm and 0.24° . These errors can be identified and compensated in the following steps of the calibration. In other words, the proposed method for base frame estimation is feasible, and it is more suitable for industrial application because no additional measurements are needed, which is an advantage of the proposed calibration method.

After estimation of the base frame, from Eq. (18), the kinematic parameter errors can be identified. For comparison, the original MDH model and reduced model are both utilized, and the results are shown in **Table 4**.

Table 4 Results of kinematic parameter identification of the robot

/	Normal value $\mathbf{q}$ (mm, deg)	Original model $\mathbf{q}_a$ (mm, deg)	Reduced model $\mathbf{q}_{r,a}$ (mm, deg)
θ_1	0	0.3573	0.1626
d_1	0	4.2324	4.6001
a_1	0	0.1888	0.9890
α_1	-90	-90.0304	-90
θ_2	0	-0.2339	-0.2440
d_2	0	-4.9113	-2.8339
a_2	1300	1302.1207	1302.0667
α_2	0	-0.0150	0
β_2	0	0.1747	0
θ_3	0	-0.0137	-0.0358
d_3	0	0	0
a_3	0	-4.1197	-5.0991
α_3	-90	-90.1729	-90
θ_4	0	-0.2249	0.1229
d_4	1474	1471.5227	1471.9281

a_4	0	2.3213	0
α_4	-35	-34.9859	-34.8464
θ_5	0	2.1370	1.9169
d_5	95	99.7417	98.1055
a_5	0	-1.6359	0
α_5	70	69.8252	70
θ_6	0	-1.6310	0
d_6	95	102.3560	100.7109
a_6	0	-1.0774	0
α_6	-35	-34.5065	-35
θ_7	0	-0.0775	0.0958
d_7	85	81.5858	82.8581
a_7	0	-1.0380	0
α_7	0	-0.3838	0
β_7	0	2.1484	0
x_0	-462.9100	-464.5854	-464.8290
y_0	4097.4547	4101.0744	4101.5041
z_0	-197.7500	-197.7500	-197.7500
α_0	0.6486	0.9123	0.9307
β_0	0.3299	0.4198	0.4376
φ_0	-59.4643	-59.4643	-59.4643
x_i	0	0	-1.1824
y_i	25.6200	24.1504	24.8826
z_i	0	0	0
α_i	0	0	-0.4935
β_i	0	0	2.0693
φ_i	0	0	0

Finally, according to the identified kinematic parameters, error compensation is conducted and analyzed. For the reduced model, errors can be compensated by directly modifying parameters in the controller, while the errors of the original model should be compensated by adjusting the target configurations i.e. Eq. (38). The pose errors of the 50 measurement configurations before and after calibration are shown in **Figure 5**. For simplicity, the position and orientation errors of the robot before and after calibration are also respectively summarized in **Table 5** and **Table 6**.

From **Figure 5**, **Table 5** and **Table 6**, both of the calibration methods based on the original MDH model and reduced model can reduce the pose errors of EE of the 7R 6-DOF painting robot. After kinematic calibration based on the original MDH model and reduced model, the maximum position errors are reduced from 7.2824mm respectively to 1.8561mm and 1.9436mm, the average position errors are reduced from 4.6786mm respectively to 0.7693mm and 0.8313mm, the maximum orientation errors are reduced from 1.6496° respectively to 0.3925° and 0.4001°, the average orientation errors are reduced from 1.0707° respectively to 0.1458° and 0.1493°. Generally speaking, the method based on the original model could get a better result than that based on the reduced model. However, the difference is not so obvious and the compensation of the later is easier. In other words, the proposed kinematic calibration method based on the reduced MDH model is more suitable for practical industrial application, which also validates the rationality of model reduction proposed in this paper.

Figure 5 Pose errors of the robot before and after calibration (a) position error (b) orientation error

Table 5 The position errors of the robot before and after calibration

/	Maximum (mm)	Average (mm)
Before calibration	7.2824	4.6786
Original model	1.8561	0.7693
Reduced model	1.9436	0.8313

Table 6 The orientation errors of the robot before and after calibration

/	Maximum (deg)	Average (deg)
Before calibration	1.6496	1.0707
Original model	0.3925	0.1458
Reduced model	0.4001	0.1493

5. Conclusions

Kinematic calibration problem of a 7R 6-DOF robot with non-spherical wrist is studied in this paper. The main contributions are the parameters identifiability analysis and model reduction in the identification procedure. Firstly, the kinematic error model is constructed based on the modified DH (MDH) model. Secondly, in order to derive a minimal model which is more suitable for practical industrial application, model reduction is implemented considering the compensability and constraints of the parameters defined in the controller, and QR decomposition is utilized to deal with the parameters identifiability analysis. Finally, to simplify the process of calibration with laser tracker, a two-step procedure for error parameter estimation is proposed: (1) estimation of the robot base location with respect to the measurement coordinate system; (2) fine estimation of the identifiable kinematic parameters according to the measurement data.

In order to validate the effectiveness of the proposed method, the calibration experiment on a 7R 6-DOF industrial painting robot is implemented. The results show that the average position errors and orientation errors are reduced from 4.6786mm and 1.0707° respectively to 0.8313mm and 0.1493°, which shows that the proposed method can effectively improve the accuracy of the robot. Additionally, because parameter errors can be compensated by directly modification of the model parameters in the controller, the proposed kinematic calibration method based on the reduced MDH model is more suitable for practical industrial application, which is the main advantage of the proposed method. To further improve accuracy of the robot, our further work will be carried out to investigate the non-kinematic calibration method of the 7R 6-DOF robot.

Declarations

Availability of data and materials

Not applicable.

Acknowledgements

Not applicable.

Authors' contributions

WX was in charge of the whole analyses. ZD and ZC wrote the manuscript. GZ, WM, MX and ML assisted with sampling and laboratory analyses. All authors read and approved the final manuscript.

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Competing interests

The authors declare that they have no competing interests.

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Appendix A. The transform matrix between e_i and δq_i

$$\mathbf{M}_i = \begin{bmatrix} 0 & 0 & 1 & 0 \\ a_i c \alpha_i & s \alpha_i & 0 & 0 \\ -a_i s \alpha_i & c \alpha_i & 0 & 0 \\ 0 & 0 & 0 & 1 \\ s \alpha_i & 0 & 0 & 0 \\ c \alpha_i & 0 & 0 & 0 \end{bmatrix}, \quad i = 1, 3, 4, 5, 6$$

$$\mathbf{M}_i = \begin{bmatrix} a_i s \alpha_i s \beta_i & -c \alpha_i s \beta_i & c \beta_i & 0 & 0 \\ a_i c \alpha_i & s \alpha_i & 0 & 0 & 0 \\ -a_i s \alpha_i c \beta_i & c \alpha_i c \beta_i & s \beta_i & 0 & 0 \\ -c \alpha_i s \beta_i & 0 & 0 & c \beta_i & 0 \\ s \alpha_i & 0 & 0 & 0 & 1 \\ c \alpha_i c \beta_i & 0 & 0 & s \beta_i & 0 \end{bmatrix}, \quad i = 2, 7$$

$$\mathbf{M}_i = \begin{bmatrix} c \beta_i c \varphi_i & c \alpha_i s \varphi_i + c \varphi_i s \alpha_i s \beta_i & s \alpha_i s \varphi_i - c \varphi_i c \alpha_i s \beta_i & 0 & 0 & 0 \\ -c \beta_i s \varphi_i & c \alpha_i c \varphi_i - s \varphi_i s \alpha_i s \beta_i & s \alpha_i c \varphi_i + s \varphi_i c \alpha_i s \beta_i & 0 & 0 & 0 \\ s \beta_i & -c \beta_i s \alpha_i & c \beta_i c \alpha_i & 0 & 0 & 0 \\ 0 & 0 & 0 & c \beta_i c \varphi_i & s \varphi_i & 0 \\ 0 & 0 & 0 & -c \beta_i s \varphi_i & c \varphi_i & 0 \\ 0 & 0 & 0 & s \beta_i & 0 & 1 \end{bmatrix}, \quad i = 0, t$$

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Figures

Figure 1

The 7R 6-DOF painting robot with non-spherical wrist (a) Virtual prototype (b) Schematic diagram and coordinate systems

Figure 2

End-effector tool for kinematic calibration (a) 3D (b) Tool coordinate system

Figure 3

Experiment set-up for kinematic calibration of the 7R 6-DOF painting robot

(a)

(b)

Figure 4

Measurement of the base frame (a) experiment set-up (b) defining the base coordinate system

(a)

(b)

Figure 5

Pose errors of the robot before and after calibration (a) position error (b) orientation error