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Novel optical solitons to the perturbed Gerdjikov-Ivanov equation via collective variables

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Abstract

The objective of this research is to study the collective variable (CV) technique to explore an important form of Schrödinger equation known as the Gerdjikov-Ivanov (GI) equation which expresses the dynamics of solitons for optical fibers in terms of pulse parameters. These parameters are temporal position, amplitude, width, chirp, phase, and frequency known as collective variables (CVs). This is an effective and dynamic mathematical gadget to obtain soliton solutions of non-dimensional as well as perturbed GI equations. Moreover, an established numerical scheme that is the fourth-order Runge-Kutta method is exerted for the numerical simulation of the revealing coupled system of six ordinary differential equations which represent all the CVs included in the pulse ansatz. The CV approach is used to determine the evolution of pulse parameters with the propagation distance and illustrated it graphically. Furthermore, Figures show the compelling periodic oscillations of pulse chirp, width, frequency and amplitude of soliton. For various values of super-Gaussian pulse parameters, the numerical behavior of solitons to illustrate variations in CVs is provided. Other significant aspects with regards to the current investigation are also inferred.

Key words: Runge-Kutta method, Super-Gaussian solitons, Gerdjikov-Ivanov equation, Collective variables.

1 Introduction

In the field of information technology, the transmission mechanism is a crucial component. The significant adoption of optical fiber is in telecommunications [1–5]. This structure has been employed for high-performance and long-distance data transport due to its enhanced bandwidth capabilities. As a result, it is employed to relay signals from communication devices such as the telephone, television and internet between distant areas. Recent research has shown that using dispersion-managed solitons for data transfer will significantly increase the ability of fiber-optic links [2, 3]. Because of its extensive applications in nonlinear optics, solitons theory is gaining a lot of interest. To investigate communication phenomena, a variety of analytical and numerical methods have been presented [4, 5].

Many physical processes such as nonlinear optics, electromagnetism, plasma, shallow water wave propagation, fluid dynamics and many more can be represented using nonlinear partial differential equations [6–9]. There are many efficient and compelling methods that have been made to extract the solution of these equations in recent years [10–14]. The nonlinear Schrödinger equation (NSE) is a well-known soliton problem with numerous applications in mathematics and physics [15–17]. Many scientists have proposed and investigated extended versions of NSE with higher-order nonlinearity [18–20]. In these equations, there are three Chen-Lee-Liu equation, Kaup-Newell equation and GI equation with derivative-type non-linearities, which are known as derivative NSEs [21–23]. Several efficient methods exact as well as

numerical address the soliton solutions for GI and Perturbed GI equation in the existing literature [24–26].

In this paper, we have investigated the optical solitons for both dimensionless as well as generalized perturbed GI equations by applying a CV approach. It is a compelling and latest technique to extract exact solutions of different NSEs [18,27]. This strategy deals with conservative and non-conservative models by showing equations of motion, regardless of dissipative terms and nonlinearities. For mathematical approximation, the traditional fourth-order Runge-Kutta method for integration of the corresponding simultaneous equations is further followed. The assumed solution of the governing equations is divided into two parts using this method, one of them is the soliton part and the other is the residual part. The number of CVs that can be applied is determined by the physical system being considered. A transformation is derived to convert the original field equation in terms of CVs. Furthermore, the parameters such as width, amplitude, chirp, temporal position, phase and frequency have an impact on soliton solutions.

The arrangement of this article is as follows: Sect. 2 provides the governing model, the CV technique is illustrated in Sect.3. The dynamics of the soliton parameter is introduced in sect. 4 and graphical interpretation is given in subsection 4.1.1, 4.2.1 and 4.3.1. Sect. 5 shows its discussion and in Sect. 6 conclusion is given.

2 Governing Equation

Consider the non-dimensional GI equation as:

$$\iota z_t + \sigma_1 z_{xx} + \sigma_2 |z|^4 z + \iota \sigma_3 z^2 z_x^* = 0, \quad (1)$$

where $z^*(x, t)$ denotes the complex conjugation of the complex values wave structure $z(x, t)$ with x as spatial and t is for the temporal variables correspondingly. The first term of Eq.(1) denotes temporal evolution of solitons linearly and the last term of Eq.(1) representing the nonlinear dispersion whereas σ_1 is the group velocity dispersion and σ_2 is the coefficient of the quintic form of nonlinearity.

The famous full nonlinearity structure of the perturbed GI-equation is,

$$\iota z_t + \sigma_1 z_{xx} + \sigma_2 |z|^4 z + \iota \sigma_3 z^2 z_x^* = \iota [\sigma_4 z_t + \sigma_5 (z|z|^{2q})_x + \sigma_6 ((|z|^{2q})_x z)]. \quad (2)$$

Where σ_4 stands for the inter-modal dispersion, σ_5 depicts the self steepening for the short pulses, σ_6 is the higher order dispersion effect and q represents effects of full nonlinearities.

3 Mathematical Methodology

In the innovation for the proposed technique, the solution of GI is supposed to cleave in two parts constituting the residual radiation part and soliton part. The concept is that the solitons depend on CVs that may represent the pulse, amplitude, chirp, frequency, temporal position and width etc. Furthermore, the commencement of CVs continues in a rise in the phase space of the dynamical arrangement of the soliton boundaries. The residuary part of the solution is approximated to zero. The limitations result to a nonlinear dynamical arrangement of the collective parameters that are examined mathematically.

Moreover, the soliton field $z(x, t)$ is followed up into $m(x, t)$ representing the soliton part of the solution, and $n(x, t)$ denoting the residual part, here t is the temporal and x stands for the spatial component. Thus,

$$z(x, t) = m(x, t) + n(x, t). \quad (3)$$

In Eq.(3) x is supposed to be $x = x(v_1, v_2, \dots, v_p)$, for the CVs. It is important that the existence of the variables in m produces an expansion in the degree of freedom of the problem. In this manner we modify the Eq.(3) within the sight of these variables as follows.

Applying the following wave transformation

$$z(x, t) = m(v_1(x), v_2(x), \dots, v_p(x), t) + n(x, t), \quad (4)$$

the residual free energy is given as

$$E = \int_{-\infty}^{\infty} |n|^2 dt = \int_{-\infty}^{\infty} |z - m(v_1(x), v_2(x), \dots, v_p(x), t)|^2 dt. \quad (5)$$

Differentiating Eq.(5) partially with respect to v_k , we suppose R_k to be as the accompanying

$$\begin{aligned} R_k &= \frac{\partial E}{\partial v_k} = \frac{\partial}{\partial v_k} \int_{-\infty}^{\infty} |n|^2 dt, \\ &= \frac{\partial}{\partial v_k} \int_{-\infty}^{\infty} n n^* dt, \\ &= \int_{-\infty}^{\infty} \frac{\partial}{\partial v_k} n n^* dt, \\ &= \int_{-\infty}^{\infty} \left(\frac{\partial n}{\partial v_k} n^* + \frac{\partial n^*}{\partial v_k} n \right) dt. \end{aligned} \quad (6)$$

By defining inner product as follows:

$$\langle \beta_1, \beta_2 \rangle = \int_{-\infty}^{\infty} \beta_1(t) \beta_2(t) dt. \quad (7)$$

Therefore, by using above definition, we write Eq.(6) as

$$\begin{aligned} R_k &= \left\langle \frac{\partial n}{\partial v_k}, n^* \right\rangle + \left\langle n, \frac{\partial n^*}{\partial v_k} \right\rangle = \left\langle \frac{\partial n}{\partial v_k}, n^* \right\rangle + \left\langle \frac{\partial n}{\partial v_k}, n^* \right\rangle, \\ &= \left\langle \frac{\partial n^*}{\partial v_k}, n \right\rangle + \left\langle \frac{\partial n^*}{\partial v_k}, n \right\rangle = 2Re \left(\left\langle \frac{\partial n^*}{\partial v_k}, n \right\rangle \right), \\ &= 2Re \left(\left\langle \frac{\partial(z(x, t) - m(v_1, v_2, \dots, v_p, t))^*}{\partial v_k}, n \right\rangle \right), \\ &= 2Re \left(\left\langle \frac{\partial z^*(x, t)}{\partial v_k} - \frac{\partial m^*(v_1, v_2, \dots, v_p, t)}{\partial v_k}, n \right\rangle \right), \\ &= -2Re \left(\left\langle \frac{\partial m^*}{\partial v_k}, n \right\rangle \right) = -2Re \left(\int_{-\infty}^{\infty} \frac{\partial m^*}{\partial v_k} n dt \right). \end{aligned} \quad (8)$$

Here, Re is for the real part and $\frac{\partial z^*(x, t)}{\partial v_k} = 0$. Now, From Eq.(4) we write $n(x, t)$ as

$$n(x, t) = z(x, t) - m(v_1, v_2, \dots, v_p, t), \quad (9)$$

which gives the following relations.

$$\begin{aligned} \frac{\partial n}{\partial v_k} &= -\frac{\partial m}{\partial v_k}, \\ \frac{\partial n^*}{\partial v_k} &= -\frac{\partial m^*}{\partial v_k}. \end{aligned}$$

Using the above relations into Eq.(6) gives

$$\begin{aligned} R_k = \frac{\partial E}{\partial v_k} &= \int_{-\infty}^{\infty} \left(-\frac{\partial m}{\partial v_k} n^* - \frac{\partial m}{\partial v_k} n \right) dt, \\ &= -\int_{-\infty}^{\infty} \left(\frac{\partial m}{\partial v_k} n^* + \frac{\partial m}{\partial v_k} n \right) dt, \end{aligned} \quad (10)$$

or rather

$$R_k = -2Re \int_{-\infty}^{\infty} \frac{\partial m^*}{\partial v_k} n dt. \quad (11)$$

Again, we also get

$$\begin{aligned}\dot{R}_k &= \frac{dR_k}{dx} = -2Re \int_{-\infty}^{\infty} \frac{d}{dx} \left(\frac{\partial m^*}{\partial v_k} n \right) dt, \\ &= -2Re \int_{-\infty}^{\infty} \left(\frac{\partial m^*}{\partial v_k} \frac{\partial n}{\partial x} + n \left(\frac{\partial}{\partial x} \frac{\partial m^*}{\partial v_k} \right) \right) dt,\end{aligned}\quad (12)$$

and

$$\frac{\partial m^*}{\partial x} = \sum_{j=1}^p \frac{\partial m^*}{\partial v_j} \frac{\partial v_j}{\partial x}.\quad (13)$$

Then by putting Eq.(13) into Eq.(12), we obtain

$$\begin{aligned}\dot{R}_k &= -2Re \int_{-\infty}^{\infty} \left(\frac{\partial m^*}{\partial v_k} \frac{\partial n}{\partial x} + n \frac{\partial}{\partial v_k} \left(\sum_{j=1}^p \frac{\partial m^*}{\partial v_j} \frac{\partial v_j}{\partial x} \right) \right) dt, \\ &= -2Re \int_{-\infty}^{\infty} \left(\frac{\partial m^*}{\partial v_k} \frac{\partial n}{\partial x} dt + \left(\sum_{j=1}^p \int_{-\infty}^{\infty} \frac{\partial^2 m^*}{\partial v_k \partial v_j} \frac{\partial v_j}{\partial x} n \right) \right) dt.\end{aligned}\quad (14)$$

Moreover, by Dirac's Postulate, it is known that once all the changes in variables cannot be equal to zero, then the function itself is approximately zero.

So, R_k is certainly minimum if

$$R_k \approx 0,\quad (15)$$

and also

$$\dot{R}_k \approx 0.\quad (16)$$

Therefore, from Eq.(1) and Eq.(2) we obtain

$$\iota\sigma_1(m+n)_{tt} + \iota\sigma_2|(m+n)^4|(m+n) - \sigma_3(m+n)^2(m+n)_x^* = 0,\quad (17)$$

$$\begin{aligned}\iota\sigma_1(m+n)_{tt} + \iota\sigma_2|(m+n)^4|(m+n) - \sigma_3(m+n)^2(m+n)_t^* + [\sigma_4(m+n)_x \\ + \sigma_5((m+n)|(m+n)|^{2q})_t + \sigma_6((|(m+n)|^{2q})_t(m+n))] = 0.\end{aligned}\quad (18)$$

More, from the Eq.(3) we have

$$z_x = \frac{\partial m(v_1(x), v_2(x), \dots, v_p(x))}{\partial x} + \frac{\partial n(x, t)}{\partial x},\quad (19)$$

also by combining Eq.(17) and Eq.(18) with Eq.(19), we get

$$\frac{\partial n}{\partial x} = - \sum_{k=1}^p \frac{\partial m}{\partial v_k} \frac{dv_k}{dx} + \xi_l, \quad l = 1, 2\quad (20)$$

with

$$\xi_1 = \iota\sigma_1(m+n)_{tt} + \iota\sigma_2|(m+n)^4|(m+n) - \sigma_3(m+n)^2(m+n)_x^*,\quad (21)$$

$$\begin{aligned}\xi_2 &= \iota\sigma_1(m+n)_{tt} + \iota\sigma_2|(m+n)^4|(m+n) - \sigma_3(m+n)^2(m+n)_t^* + [\sigma_4(m+n)_x \\ &\quad + \sigma_5((m+n)|(m+n)|^{2q})_t + \sigma_6((|(m+n)|^{2q})_t(m+n))].\end{aligned}\quad (22)$$

Similarly, we know that

$$\dot{R}_k = -2Re \left(\int_{-\infty}^{\infty} \frac{\partial m^*}{\partial v_k} \frac{\partial n}{\partial x} dt + \left(\sum_{j=1}^p \int_{-\infty}^{\infty} \frac{\partial^2 m^*}{\partial v_k \partial v_j} \frac{dv_j}{dx} n \right) \right) dt, \quad (23)$$

$$\dot{R}_k = 2Re \sum_{j=1}^p \int_{-\infty}^{\infty} \left(\frac{\partial m^*}{\partial v_k} \frac{\partial m}{\partial v_j} - \frac{\partial^2 m^*}{\partial v_k \partial v_j} n \right) dt \frac{dv_j}{dx} - 2Re \int_{-\infty}^{\infty} \frac{\partial m^*}{\partial v_k} \xi_l dt. \quad (24)$$

Which in compact form can be written as

$$\dot{R}_k = \frac{\partial R}{\partial v} \dot{V} + \mathbf{U}, \quad (25)$$

with

$$\dot{\mathbf{V}} = - \left[\frac{\partial \mathbf{R}}{\partial \mathbf{V}} \right]^{-1} [\mathbf{U}], \quad (26)$$

where

$$\frac{\partial R}{\partial v} = \begin{bmatrix} \frac{\partial R_1}{\partial v_1} & \frac{\partial R_1}{\partial v_2} & \frac{\partial R_1}{\partial v_3} & \frac{\partial R_1}{\partial v_4} & \frac{\partial R_1}{\partial v_5} & \frac{\partial R_1}{\partial v_6} \\ \frac{\partial R_2}{\partial v_1} & \frac{\partial R_2}{\partial v_2} & \frac{\partial R_2}{\partial v_3} & \frac{\partial R_2}{\partial v_4} & \frac{\partial R_2}{\partial v_5} & \frac{\partial R_2}{\partial v_6} \\ \frac{\partial R_3}{\partial v_1} & \frac{\partial R_3}{\partial v_2} & \frac{\partial R_3}{\partial v_3} & \frac{\partial R_3}{\partial v_4} & \frac{\partial R_3}{\partial v_5} & \frac{\partial R_3}{\partial v_6} \\ \frac{\partial R_4}{\partial v_1} & \frac{\partial R_4}{\partial v_2} & \frac{\partial R_4}{\partial v_3} & \frac{\partial R_4}{\partial v_4} & \frac{\partial R_4}{\partial v_5} & \frac{\partial R_4}{\partial v_6} \\ \frac{\partial R_5}{\partial v_1} & \frac{\partial R_5}{\partial v_2} & \frac{\partial R_5}{\partial v_3} & \frac{\partial R_5}{\partial v_4} & \frac{\partial R_5}{\partial v_5} & \frac{\partial R_5}{\partial v_6} \\ \frac{\partial R_6}{\partial v_1} & \frac{\partial R_6}{\partial v_2} & \frac{\partial R_6}{\partial v_3} & \frac{\partial R_6}{\partial v_4} & \frac{\partial R_6}{\partial v_5} & \frac{\partial R_6}{\partial v_6} \end{bmatrix}. \quad (27)$$

while

$$\dot{\mathbf{V}} = \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \\ \dot{v}_4 \\ \dot{v}_5 \\ \dot{v}_6 \end{bmatrix} \quad \text{and} \quad \mathbf{U} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{bmatrix}. \quad (28)$$

Additionally, we finally have U_K as follows

$$\begin{aligned} U_k &= -2Re \int_{-\infty}^{\infty} \frac{\partial m^*}{\partial v_k} \xi_l dt, \\ &= -2Re \int_{-\infty}^{\infty} \iota \sigma_1 \frac{\partial m^*}{\partial v_k} m_{tt} dt - 2Re \int_{-\infty}^{\infty} \iota \sigma_1 \frac{\partial m^*}{\partial v_k} n_{tt} dt \\ &\quad - 2Re \int_{-\infty}^{\infty} i \sigma_2 |m + n|^4 \frac{\partial m^*}{\partial v_k} m dt - 2Re \int_{-\infty}^{\infty} i \sigma_2 |m + n|^4 \frac{\partial m^*}{\partial v_k} n dt \\ &\quad - 2Re \int_{-\infty}^{\infty} -\sigma_3 (m + n)^2 \frac{\partial m^*}{\partial v_k} \frac{\partial m}{\partial t} dt - 2Re \int_{-\infty}^{\infty} -\sigma_3 (m + n)^2 \frac{\partial m^*}{\partial v_k} \frac{\partial n}{\partial t} dt \\ &\quad - 2Re \int_{-\infty}^{\infty} \sigma_4 \frac{\partial m^*}{\partial v_k} \frac{\partial m}{\partial v_k} dt - 2Re \int_{-\infty}^{\infty} \sigma_4 \frac{\partial m^*}{\partial v_k} \frac{\partial n}{\partial v_k} dt \\ &\quad - 2Re \int_{-\infty}^{\infty} \sigma_5 |m + n|_t^{2q} \frac{\partial m^*}{\partial v_k} m dt - 2Re \int_{-\infty}^{\infty} \sigma_5 |m + n|_t^{2q} \frac{\partial m^*}{\partial v_k} n dt \\ &\quad - 2Re \int_{-\infty}^{\infty} \sigma_6 |m + n|_t^{2q} \frac{\partial m^*}{\partial v_k} m dt - 2Re \int_{-\infty}^{\infty} \sigma_6 |m + n|_t^{2q} \frac{\partial m^*}{\partial v_k} n dt \\ &\quad - 2Re \int_{-\infty}^{\infty} \sigma_5 |m + n|^{2q} \frac{\partial m^*}{\partial v_k} m_t dt - 2Re \int_{-\infty}^{\infty} \sigma_5 |m + n|^{2q} \frac{\partial m^*}{\partial v_k} n_t dt. \end{aligned} \quad (29)$$

For which $k \in 1, 2, \dots, p$.

4 Soliton Parameter Dynamics

First of all, by utilizing the constraint equations prior established by formulating the function $m(v_1, v_2, \dots, v_p, t)$ we get the system of equations of motion for the CVs. However, by employing Gaussian ansatz, we assume the soliton function m consists of six CVs in the form:

$$m = v_1 \exp \left(\frac{i}{2} v_4 (t - v_2)^2 - \frac{(t - v_2)^2}{v_3^2} + i v_5 (t - v_2) + i v_6 \right), \quad (30)$$

where $v_i, (i = 1, 2, \dots, 6)$ denoting the centre position, width, amplitude, phase, frequency and chirp. The differential equations for CVs are made by utilizing the basic approximation, alternatively mentioned as the lowest order CV hypothesis. This concept sets the residuary part equals to zero,

$(n(x, t) = 0)$. We find $(\frac{\partial R}{\partial v})$ and U as follows:

$$\left[\frac{\partial R}{\partial v} \right]^{-1} = \begin{bmatrix} \frac{3}{2\sqrt{2\pi}v_3} & 0 & \frac{-1}{\sqrt{2\pi}v_1} & 0 & 0 & 0 \\ 0 & \frac{v_3}{\sqrt{2\pi}v_1^2} & 0 & 0 & \frac{v_3 v_4}{\sqrt{2\pi}v_1^2} & \frac{v_3 v_5}{\sqrt{2\pi}v_1^2} \\ \frac{-1}{\sqrt{2\pi}v_1} & 0 & \frac{\sqrt{\frac{2}{\pi}}v_3}{v_1^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{16\sqrt{\frac{2}{\pi}}}{v_1^2 v_3^5} & 0 & \frac{-2\sqrt{\frac{2}{\pi}}}{v_1^2 v_3^3} \\ 0 & \frac{v_3 v_4}{\sqrt{2\pi}v_1^2} & 0 & 0 & \frac{32\pi^{\frac{5}{2}}v_1^8 \left(\frac{v_4^2 v_3^9}{4} + v_3^5 \right)}{\pi^3 v_1^{10} v_3^8} & \frac{v_3 v_4 v_5}{\sqrt{2\pi}v_1^2} \\ 0 & \frac{v_3 v_5}{\sqrt{2\pi}v_1^2} & 0 & \frac{-2\sqrt{\frac{2}{\pi}}}{v_1^2 v_3^3} & \frac{v_3 v_4 v_5}{\sqrt{2\pi}v_1^2} & \frac{32\pi^{\frac{5}{2}}v_1^8 \left(\frac{v_4^2 v_3^9}{4} + v_3^5 \right)}{\pi^3 v_1^{10} v_3^8} \end{bmatrix}, \quad (31)$$

4.1

For Eq.(1), $U_i \quad i = 1, 2, \dots, 6$ are calculated as:

$$U_1 = 0, \quad (32)$$

$$U_2 = \sqrt{2\pi}v_1^2 \left[-\frac{3\sigma_1 v_3^3 v_4^2 v_5}{4} - \sigma_1 v_3 v_5^3 - \frac{3\sigma_1 v_5}{v_3} + \frac{\sigma_2 v_1^4 v_3 v_5}{\sqrt{3}} + \frac{\sigma_3 v_1^2 v_3^3 v_4^2}{8\sqrt{2}} + \frac{\sigma_3 v_1^2 v_3 v_5^2}{\sqrt{2}} - \frac{\sigma_3 v_1^2}{2\sqrt{2}v_3} \right], \quad (33)$$

$$U_3 = \sqrt{2\pi}v_1^2 [-\sigma_1 v_4], \quad (34)$$

$$U_4 = \sqrt{2\pi}v_1^2 \left[\frac{3\sigma_1 v_3^5 v_4^2}{32} + \frac{\sigma_1 v_3^3 v_5^2}{8} - \frac{\sigma_1 v_3}{8} - \frac{\sqrt{3}\sigma_2 v_1^4 v_3^3}{72} - \frac{\sqrt{2}\sigma_3 v_1^2 v_3^3 v_5}{32} \right], \quad (35)$$

$$U_5 = \sqrt{2\pi}v_1^2 \left[\frac{\sigma_1 v_3^3 v_4 v_5}{2} - \frac{\sigma_3 v_1^2 v_3^3 v_4}{8\sqrt{2}} \right], \quad (36)$$

$$U_6 = \sqrt{2\pi}v_1^2 \left[\frac{\sigma_1 v_3^3 v_4^2}{4} + \sigma_1 v_3 v_5^2 + \frac{\sigma_1}{v_3} - \frac{\sigma_2 v_1^4 v_3}{\sqrt{3}} - \frac{\sigma_3 v_1^2 v_3 v_5}{\sqrt{2}} \right]. \quad (37)$$

Similarly, from Eq.(26) and substituting Eq.(31) and Eq.(32-37), we get

$$\dot{v}_1 = -\sigma_1 v_1 v_4,$$

$$\dot{v}_2 = \left[2\sigma_1 v_5 + \frac{\sqrt{2}\sigma_3 v_1^2}{4} \right],$$

$$\dot{v}_3 = 2\sigma_1 v_3 v_4,$$

$$\dot{v}_4 = -\frac{1}{9v_3^4} \left[18\sigma_1 v_3^4 v_4^2 - 72\sigma_1 + 8\sqrt{3}\sigma_2 v_1^4 v_3^2 + 9\sqrt{2}\sigma_3 v_1^2 v_3^2 v_5 \right],$$

$$\dot{v}_5 = \frac{1}{\sqrt{2}} \left[\sigma_3 v_1^2 v_4 \right],$$

$$\dot{v}_6 = \frac{1}{72v_3^2} \left[72\sigma_1 v_3^2 v_5^2 - 144\sigma_1 + 32\sqrt{3}\sigma_2 v_1^4 v_3^2 + 63\sqrt{2}\sigma_3 v_1^2 v_3^2 v_5 \right].$$

4.1.1 Graphical Interpretation

Chirping Effect

Figure 1: Analysis of collective variables v_i , ($i = 1, 2, \dots, 6$) versus distance of propagation x with initial guess $v_1 = v_2 = v_3 = v_4 = 1, v_5 = v_6 = 0$.

Non-Chirping Effect

Figure 2: Analysis of collective variables $v_i, (i = 1, 2, \dots, 6)$ versus distance of propagation x with initial guess $v_1 = v_2 = v_3 = 1, v_4 = v_5 = v_6 = 0$.

4.2 For q=1

Eq.(2) becomes

$$\begin{aligned} & \iota\sigma_1(m+n)_{tt} + \iota\sigma_2|(m+n)^4|(m+n) - \sigma_3(m+n)^2(m+n)_t^* + [\sigma_4(m+n)_x + \sigma_5((m+n)|(m+n)|^2)_t \\ & + \sigma_6((|(m+n)|^2)_t(m+n))], \end{aligned} \quad (38)$$

while

$$U_1 = -\sqrt{2\pi}\sigma_1v_3, \quad (39)$$

$$\begin{aligned} U_2 = & \sqrt{2\pi}v_1^2 \left[-\frac{3\sigma_1v_3^3v_4^2v_5}{4} - \sigma_1v_3v_5^3 - \frac{3\sigma_1v_5}{v_3} + \frac{\sigma_2v_1^4v_3v_5}{\sqrt{3}} + \frac{\sigma_3v_1^2(v_3^4v_4^2 + 8v_3^2v_5^2 - 4)}{8\sqrt{2}v_3} \right. \\ & \left. - \frac{\sigma_4(v_3^4v_4^2 + 4v_3^2v_5^2 + 4)}{4v_3} + \frac{\sigma_5v_1^2(v_3^4v_4^2 + 8v_3^2v_5^2 + 4)}{8\sqrt{2}v_3} + \frac{(\sigma_5 + \sigma_6)v_1^2}{\sqrt{2}v_3} \right], \end{aligned} \quad (40)$$

$$U_3 = -\sqrt{2\pi}v_1^2 \left[-\sigma_1v_4 + \frac{3\sigma_4}{4v_3} \right], \quad (41)$$

$$U_4 = \sqrt{2\pi}v_1^2 \left[\frac{3\sigma_1v_3^5v_4^2}{32} + \frac{\sigma_1v_3^3v_5^2}{8} - \frac{\sigma_1v_3}{8} - \frac{\sqrt{3}\sigma_2v_1^4v_3^3}{72} - \frac{\sqrt{2}\sigma_3v_1^2v_3^3v_5}{32} - \frac{3\sigma_4v_3^5}{64} + \frac{\sigma_5v_1^2v_3^3v_5}{16\sqrt{2}} \right], \quad (42)$$

$$U_5 = \sqrt{2\pi}v_1^2 \left[\frac{\sigma_1v_3^3v_4v_5}{2} - \frac{\sigma_3v_1^4v_3^3v_4}{8\sqrt{2}} - \frac{\sigma_4v_3^3}{4} - \frac{\sigma_5v_1^2v_3^3v_4}{8\sqrt{2}} \right], \quad (43)$$

$$U_6 = \sqrt{2\pi}v_1^2 \left[\frac{\sigma_1v_3^3v_4^2}{4} + \sigma_1v_3v_5^2 + \frac{\sigma_1}{v_3} - \frac{\sigma_2v_1^4v_3}{\sqrt{3}} - \frac{\sigma_3v_1^2v_3v_5}{\sqrt{2}} - \sigma_4v_3 - \frac{\sigma_5v_1^2v_3v_5}{\sqrt{2}} \right]. \quad (44)$$

Similarly, from Eq.(26) and substituting Eq.(31) and Eq.(39-44), we get

$$\begin{aligned} v_1 &= -\frac{1}{4v_3} \left[4\sigma_1v_1v_3v_4 + 3\sigma_4v_1 - 6\sigma_4v_3 \right], \\ v_2 &= \left[2\sigma_1v_5 + \frac{\sqrt{2}\sigma_3v_1^2}{4} + \sigma_4v_3^2v_5^2 + \frac{\sigma_4v_3^4v_4^2}{4} + \sigma_4v_3^2v_5 + \frac{\sigma_4v_3^4v_4}{4} + \sigma_4 - \frac{3\sqrt{2}\sigma_5v_1^2}{4} - \frac{\sqrt{2}\sigma_6v_1^2}{2} \right], \\ v_3 &= \frac{1}{2v_1} \left[4\sigma_1v_1v_3v_4 + 3\sigma_4v_1 - 2\sigma_4v_3 \right], \\ v_4 &= -\frac{1}{18v_3^4} \left[144\sigma_1 - 36\sigma_1v_3^4v_4^2 - 16\sqrt{3}\sigma_2v_1^4v_3^2 - 18\sqrt{2}\sigma_3v_1^2v_3^2v_5 + 27\sigma_4v_3^4 - 72\sigma_4v_3^2 - 18\sqrt{2}\sigma_5v_1^2v_3^2v_5 \right] \\ v_5 &= \left[\frac{\sqrt{2}\sigma_3v_1^2v_4}{2} + \sigma_4 + \frac{\sigma_4v_3^4v_4^3}{4} + \frac{\sigma_4v_3^4v_4^2}{4} + \sigma_4v_4 + \sigma_4v_3^2v_4v_5^2 + \sigma_4v_3^2v_4v_5 - \frac{\sqrt{2}\sigma_5v_1^2v_4}{2} - \frac{\sqrt{2}\sigma_6v_1^2v_4}{2} \right], \\ v_6 &= \frac{1}{144v_3^2} \left[144\sigma_1v_3^2v_5^2 - 288\sigma_1 + 64\sqrt{3}\sigma_2v_1^4v_3^2 + 126\sqrt{2}\sigma_3v_1^2v_3^2v_5 + 36\sigma_4v_3^6v_4^2v_5 + 36\sigma_4v_3^6v_4v_5 \right. \\ & \quad \left. + 144\sigma_4v_3^4v_5^3 + 144\sigma_4v_3^4v_5^2 - 27\sigma_4v_3^4 + 144\sigma_4v_3^2v_5 + 216\sigma_4v_3^2 - 18\sqrt{2}\sigma_5v_1^2v_3^2v_5 - 72\sqrt{2}\sigma_6v_1^2v_3^2v_5 \right]. \end{aligned}$$

4.2.1 Graphical Interpretation

Chirping Effect

Figure 3: Analysis of collective variables v_i , ($i = 1, 2, \dots, 6$) versus distance of propagation x with initial guess $v_1 = v_2 = v_3 = v_4 = 1, v_5 = v_6 = 0$.

Non-Chirping Effect

Figure 4: Analysis of collective variables $v_i, (i = 1, 2, \dots, 6)$ versus distance of propagation x with initial guess $v_1 = v_2 = v_3 = 1, v_4 = v_5 = v_6 = 0$.

4.3 For q=2

Eq.(2) becomes

$$\begin{aligned} & \iota\sigma_1(m+n)_{tt} + \iota\sigma_2|(m+n)^4|(m+n) - \sigma_3(m+n)^2(m+n)_t^* + [\sigma_4(m+n)_x + \sigma_5((m+n)|(m+n)|^4)_t \\ & + \sigma_6((|(m+n)|^4)_t(m+n))], \end{aligned} \quad (45)$$

while

$$U_1 = -\sqrt{2\pi}\sigma_4v_3, \quad (46)$$

$$\begin{aligned} U_2 = & \sqrt{2\pi}v_1^2 \left[-\frac{3\sigma_1v_3^3v_4^2v_5}{4} - \sigma_1v_3v_5^3 - \frac{3\sigma_1v_5}{v_3} + \frac{\sigma_2v_1^4v_3v_5}{\sqrt{3}} + \frac{\sigma_3v_1^2(v_3^4v_4^2 + 8v_3^2v_5^2 - 4)}{8\sqrt{2}v_3} \right. \\ & \left. - \frac{\sigma_4(v_3^4v_4^2 + 4v_3^2v_5^2 + 4)}{4v_3} + \frac{\sqrt{3}\sigma_5v_1^4(v_3^4v_4^2 + 12v_3^2v_5^2 + 4)}{36v_3} + \frac{4\sqrt{3}(\sigma_5 + \sigma_6)v_1^4}{9v_3} \right], \end{aligned} \quad (47)$$

$$U_3 = \sqrt{2\pi}v_1^2 \left[-\sigma_1v_4 - \frac{3\sigma_4}{4v_3} \right], \quad (48)$$

$$U_4 = \sqrt{2\pi}v_1^2 \left[\frac{3\sigma_1v_3^3v_4^2}{32} + \frac{\sigma_1v_3^3v_5^2}{8} - \frac{\sigma_1v_3}{8} - \frac{\sqrt{3}\sigma_2v_1^4v_3^3}{72} - \frac{\sqrt{2}\sigma_3v_1^2v_3^3v_5}{32} - \frac{3\sigma_4v_3^5}{64} + \frac{\sqrt{3}\sigma_5v_1^4v_3^3v_5}{72} \right], \quad (49)$$

$$U_5 = \sqrt{2\pi}v_1^2 \left[\frac{\sigma_1v_3^3v_4v_5}{2} - \frac{\sigma_3v_1^2v_3^3v_4}{8\sqrt{2}} - \frac{\sigma_4v_3^3}{4} - \frac{\sqrt{3}\sigma_5v_1^4v_3^3v_4}{36} \right], \quad (50)$$

$$U_6 = \sqrt{2\pi}v_1^2 \left[\frac{\sigma_1v_3^3v_4^2}{4} + \sigma_1v_3v_5^2 + \frac{\sigma_1}{v_3} - \frac{\sigma_2v_1^4v_3}{\sqrt{3}} - \frac{\sigma_3v_1^2v_3v_5}{\sqrt{2}} - \sigma_4v_3 - \frac{\sigma_5v_1^4v_3v_5}{\sqrt{3}} \right]. \quad (51)$$

Similarly, from Eq.(26) and substituting Eq.(31) and Eq.(46-51), we get

$$\begin{aligned} v_1 &= -\frac{1}{4v_3} \left[4\sigma_1v_1v_3v_4 + 3\sigma_4v_1 - 6\sigma_4v_3 \right], \\ v_2 &= \left[2\sigma_1v_5 + \frac{\sqrt{2}\sigma_3v_1^2}{4} + \sigma_4v_3^2v_5^2 + \frac{\sigma_4v_3^4v_4^2}{4} + \sigma_4v_3^2v_5 + \frac{\sigma_4v_3^4v_4}{4} + \sigma_4 - \frac{5\sqrt{3}\sigma_5v_1^4}{9} - \frac{4\sqrt{3}\sigma_6v_1^4}{9} \right], \\ v_3 &= \frac{1}{2v_1} \left[4\sigma_1v_1v_3v_4 + 3\sigma_4v_1 - 2\sigma_4v_3 \right], \\ v_4 &= \frac{1}{18v_3^4} \left[144\sigma_1 - 36\sigma_1v_3^4v_4^2 - 16\sqrt{3}\sigma_2v_1^4v_3^2 - 18\sqrt{2}\sigma_3v_1^2v_3^2v_5 + 27\sigma_4v_3^4 - 72\sigma_4v_3^2 - 16\sqrt{3}\sigma_5v_1^4v_3^2v_5 \right], \\ v_5 &= \left[\frac{\sqrt{2}\sigma_3v_1^2v_4}{2} + \sigma_4 + \frac{\sigma_4v_3^4v_4^3}{4} + \frac{\sigma_4v_3^4v_4^2}{4} + \sigma_4v_4 + \sigma_4v_3^2v_4v_5^2 + \sigma_4v_3^2v_4v_5 - \frac{4\sqrt{3}\sigma_5v_1^4v_4}{9} - \frac{4\sqrt{3}\sigma_6v_1^4v_4}{9} \right], \\ v_6 &= \frac{1}{144v_3^2} \left[144\sigma_1v_3^2v_5^2 - 288\sigma_1 + 64\sqrt{3}\sigma_2v_1^4v_3^2 + 126\sqrt{2}\sigma_3v_1^2v_3^2v_5 + 36\sigma_4v_3^6v_4^2v_5 + 36\sigma_4v_3^6v_4v_5 \right. \\ & \quad \left. + 144\sigma_4v_3^4v_5^3 + 144\sigma_4v_3^4v_5^2 - 27\sigma_4v_3^4 + 144\sigma_4v_3^2v_5 + 216\sigma_4v_3^2 - 16\sqrt{3}\sigma_5v_1^4v_3^2v_5 - 64\sqrt{3}\sigma_6v_1^4v_3^2v_5 \right]. \end{aligned}$$

4.3.1 Graphical Interpretation

Chirping Effect

Figure 5: Analysis of collective variables v_i , ($i = 1, 2, \dots, 6$) versus distance of propagation x with initial guess $v_1 = v_2 = v_3 = v_4 = 1, v_5 = v_6 = 0$.

Non-Chirping Effect

Figure 6: Analysis of collective variables $v_i, (i = 1, 2, \dots, 6)$ versus distance of propagation x with initial guess $v_1 = v_2 = v_3 = 1, v_4 = v_5 = v_6 = 0$.

5 Results and Discussion

In this section, we discussed the numerical evaluations of the dynamics of pulse propagation variables, which are often seen geometrically to illustrate the achieved performance of the CV strategy. In addition, the Runge-Kutta system of order four is used to solve the ODEs that result from our method. For the following parameter values: $\sigma_1 = \frac{1}{100}$, $\sigma_2 = \frac{1}{5}$ and $\sigma_3 = 1$ for Eq.(1) the dynamics of the attained system is displayed in Fig.1 and Fig.2. Then for the Eq.(2) dynamics of the equation is described for these parametric values: $\sigma_1 = \frac{1}{100}$, $\sigma_2 = \frac{1}{16}$, $\sigma_3 = 1$, $\sigma_4 = 0$, $\sigma_5 = -\frac{1}{2}$ and $\sigma_6 = \frac{1}{50}$ in Figs.3-6. We have presented a graphical interpretation of given model with chirping and non-chirping effect for $q = 1, 2$.

It is worth mentioning here that the pulse propagates, v_1 , v_3 , v_4 and v_5 that is the soliton's amplitude, width, chirp, and frequency oscillates periodically with uniform wavelengths but temporal position v_2 is increasing with propagation distance and phase v_6 is periodic with small amplitude and behaving decreasingly in Figs.1 and 2. However, Figs.3, 4, 5 and 6 illustrate that v_1 , v_3 , v_4 and v_5 are the soliton's amplitude, width, chirp and frequency oscillate periodically with uniform wavelengths but temporal position v_2 is increasing and the phase of pulse v_6 is also periodic with a very small amplitude which is almost linear and showing increasing behavior in Figs. 5 and 6. As a result of this research, it is determined that the energy relies on the amplitude v_1 , width v_3 , chirp v_4 and frequency v_5 . So, the CV technique makes a straightforward investigation of the acquired equations and defines the effect of various parameters in the GI equation.

6 Conclusion

This piece of research introduces the CV approach and the soliton solutions of the GI equation is successfully explored in this work. For pulse propagation in optical fibers, the CV technique regulates a system of ordinary differential equations with higher-order nonlinearities. This investigation exposes solitons in the form of computational effects and provokes an enormous and distinctive class of pulse behavior that occurs for several values of parameters. The basic concept of the CV approach relies on the implementation of constraint, $R_k \approx 0$, on the CVs $v_k(k = 1, \dots, p)$, which gives them adequate physical significance by configuring the Gaussian ansatz function. The main benefit of this technique is to form the derivation of the equations of motion for CVs comparatively. The graphs of acquired solutions provide us admirable thoughts about the dynamics of the GI equation. These consequences may be applicable in optical fibers and play a significant role in optical communications. This method gradually becomes a consistent procedure that can be used to solve a variety of NSEs. Mathematicians, engineers, and physicists may find the investigated soliton solutions useful in recognizing the physical significance of this model.

Author Statement

Zara Hassan: Conceptualization, Methodology, Writing original draft preparation; Nauman Raza: Methodology, Data curation, Writing-Original draft preparation; J.F. Gómez-Aguilar: Conceptualization, Methodology, Writing- Original draft preparation, Software, Supervision.

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Conflicts of Interest

The authors declare no conflict of interest.

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Figures

Figure 1

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Non-Chirping Effect

Figure 2

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Chirping Effect

Figure 3

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Non-Chirping Effect

Figure 4

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Chirping Effect

Figure 5

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Non-Chirping Effect

Figure 6

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