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MODELISATION OF THERMALLY INDUCED JITTER IN A SLENDER STRUCTURE

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The thermomechanical interactions of onboard space vehicles is an interesting field of research and study. Since the pioneering paper by Bruno Boley, published in 1954, many authors have given their relevant contribution to the comprehension of phenomena not otherwise investigable if not with a cross-sectoral approach and a multidisciplinary methodology. The anomaly that occurred to the spacecraft Alouette 1, in 1962, marked the beginning of a long series of unexpected events due to unconceivable coupling between the mechanical and thermal behaviour of the system. This work aims to emphasise, by means of a simple model, the basic mechanism responsible for elastic vibrations induced by a thermal shock. This is a widespread event experienced by a spacecraft during the transitions shadow-Sun and vice-versa or when a flexible appendage, previously shadowed by the spacecraft's main body, comes to the light as a consequence of an attitude manoeuvre [Ulysses, 1990]. For the investigation, a very slender structure has been considered in order to make the thermal and mechanical characteristic times comparable and realise the conditions of strong coupling. The accurate thermal analysis provides an equivalent thermal bending moment, depending on time, which appears as a boundary condition in the subsequent modal analysis of the structural element, where it plays the role of a trigger of elastic transverse vibrations.

I. Nomenclature

A_p = projected area, m^2
 Q_{IR} = Earth Radiation, $\frac{W}{m^2}$
 R_E = Earth radius, equitorial 6378 Km
 μ_E = Planetary constant, $398600.4 \frac{km^3}{sec^2}$
 σ_s = Stefan – Boltzmann constant, $5.67 * 10^{-8} \frac{W}{m^2 K^4}$
 ϵ = strain
 A = area, m^2
 C = solar constant, $1368 \frac{W}{m^2}$
 E = Young's Modulus, $\frac{N}{m^2}$
 T = temperature, K or C
 a = Albedo factor, nominally, 0.35
 c = Specific heat, $900 kg m^2 s^{-2} K^{-1}$
 k = Thermal Conductivity, $\frac{W}{mK}$

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$H = \text{Thermal Gradient}, \frac{K}{m}$
 $m = \text{mass}, kg$
 $s/c = \text{Spacecraft}$
 $t = \text{time}, second$
 $\alpha = \text{absorptivity}, 0.7$
 $\delta = \text{sun elevation}, degree$
 $\varepsilon = \text{emissivity}, 0.6$
 $\zeta = \text{true anomaly}, degree \text{ or displacement in modal analysis.}$
 $\rho = \text{density}, \frac{Kg}{m^3}$
 $\sigma = \text{stress}, \frac{N}{m^2}$
 $\omega_E = \text{Earth rotation rate}, 7.292115 * 10^{-5} \frac{rad}{sec}$

II. Introduction

Ever Since the 1960's, thermally induced vibration turns out to be an occurring breakdown in spacecraft. Space beams, for instance, beams of spacecraft booms, solar arrays etc., could well go through thermally induced vibration owing to abrupt temperature changes on night-day and day-night alterations in orbit. Thermal flutters, i.e., uneven thermally induced vibrations, are likely to occur as space beams become flexible and larger. The flexible appendages of spacecraft, such as antennae and solar arrays, are exposed to a sudden solar heat flux during the orbital eclipse transition. Thermally-induced deformations and vibrations will result from this type of loading. Thermal flutter occurs when thermally induced vibration becomes unstable, caused by the coupling effect among structural deformations and thermal loading. The elastic deformation of flexible structures is coupled with rigid-body rotation (attitude motion) of an in-orbit spacecraft with flexible appendages such as solar panels. The influence of thermally induced responses of flexible appendages (the thin-walled boom with tip mass and the solar panels) on the spacecraft attitude motion was investigated by Johnston and Thornton [[3], [6]]. The effects of rigid-flexible coupling nonlinear terms on the thermally induced vibration of a flexible spacecraft with large solar panels were investigated by Sun et al. [7].

Thermally-induced space structure vibrations are not a new phenomenon. This failure was observed, for example, on the OGO-IV satellite [2]. Such oscillations were discovered during the 1960s flight of the OGO-IV spacecraft. A 60-foot experiment boom sustained a solar-induced large-amplitude oscillation during that flight, which severely harmed spacecraft performance. Consistent with the observed satellite phenomena, a coupled thermal-structural analysis predicted torsional-flexural oscillations. The Voyager space explorer [3]-Voyager 2 is a NASA space probe that was launched on August 20, 1977, to study the planets beyond our solar system. Low-frequency oscillations of PRA booms during LEO operations were experienced as part of the Voyager programme and were attributed to "thermal flutter."

Disturbance of the Hubble space telescope's [1], [3] pointing system caused by solar arrays- The Hubble Space Telescope (HST) was released into low Earth orbit on April 24, 1990, aboard the Space Shuttle Discovery. The HST was successfully deployed from the Discovery payload bay the next day and started its long-awaited operations. Initial imaging data were acquired quickly, and the now-famous problem of the spacecraft mirror's spherical aberration was discovered.

The Upper Atmosphere Research Satellite (UARS) satellite [4] was NASA's orbital observatory, where its mission would be to analyse the Earth's atmosphere, especially the preventive ozone layer. With 57 degrees orbital inclination, it entered Earth orbit at an operational altitude of 600 kilometres (370 miles). It observed a "thermal snap" at the orbital night/day transition, which was attributed to the sudden heating of the solar array.

To avoid this type of failure, extensive research was conducted to properly design the spacecraft components [3], [5]. Boley [6], [7] predicted the thermally-induced vibration of thin beams for the first time. Boley indicated that this is a problem involving transient heat conduction and structural dynamics. To avoid this type of failure, extensive research was conducted in order to design the spacecraft modules properly.

III. Thermal Environment

Sun, Albedo, Earth Infrared Radiation, Power onboard, and Re-emitted Radiation are the primary sources. Internally dissipated power in electronic components (Joule effect) is another example. Aerothermal flux must also be taken into account during the launch and re-entry phases, which is not depicted in the diagram. Only two heat sources remain during the eclipse: Earth's infrared and internal dissipation and the spacecraft will begin to cool. As a result, the satellite's temperature varies cyclically along its orbit, rising in the sun and falling during the eclipse. The primary source of cold is Deep Space, which should be seen as a black body emitting at 3K. Figure 1 depicts several heat sources on a satellite orbiting Earth. Refer [19] for an in-depth understanding of the various sources.

Fig. 1: Spacecraft Thermal Environment

IV. Thermal Analysis

The primary aim of thermal analysis is to find the temperature at the point of interest of the spacecraft along its orbit. There are two types of thermal analysis. They are

- 1) Steady-state thermal analysis
- 2) Transient thermal analysis

Consider a two-dimensional element (e.g. a solar panel) subjected to a radiative-type heat input (solar radiation), which emits from its surface according to the emissivity values

Fig. 2 Solar panel subjected to radiation.

A. Steady-State Thermal Analysis

The equation governing the transmission of heat within the thickness is the Fourier equation and is given by,

$$\text{div}(k \text{grad} T) + q = \rho c \frac{\partial T}{\partial t} \quad (1)$$

The above Fourier equation is simplified further because of the lightness of the structure, the transient is very small, and the absence of volume heat source, $q=0$. It is, therefore, predominantly in stationary conditions. If k is scalar, then at steady-state condition, the above equation becomes,

$$k \text{div}(\text{grad} T) = 0 \quad (2)$$

Solving the above equations gives,

$$T(x) = \left(\frac{T_2 - T_1}{d} x \right) + T_1 \quad (3)$$

With T_1 and T_2 still unknown. For this, we can use the boundary conditions.

$$\left. \frac{dT}{dx} \right|_{x=0} = \frac{T_2 - T_1}{d} ; \left. \frac{dT}{dx} \right|_{x=d} = \frac{T_2 - T_1}{d} \quad (4)$$

$$(\alpha C) - (\sigma \epsilon_1 T_1^4) = -K \left. \frac{dT}{dx} \right|_{x=0} \quad (5)$$

$$-(\sigma \epsilon_2 T_2^4) = -K \left. \frac{dT}{dx} \right|_{x=d} \quad (6)$$

This is a non-linear algebraic system for which the theory does not allow us to assert anything about the existence and uniqueness of the solution. However, physical reality guarantees the existence of at least one significant solution. For this research, we can proceed with different techniques; the simplest of these is the mapping of the two non-linear functions,

$$f_1 = (\alpha C_p) - (\sigma \epsilon_1 T_1^4) + K \frac{T_2 - T_1}{d} \quad (7)$$

$$f_2 = -(\sigma \epsilon_2 T_2^4) + K \frac{T_2 - T_1}{d} \quad (8)$$

We have two equations, and to solve these equations, consider arbitrary value for T_1 and T_2 . The $f_1 = 0$ and $f_2 = 0$ curves are plotted in the T_1, T_2 plane by assigning a value to T_1 and obtaining T_2 (as regards f_1), the vice versa for f_2 . The intersection between the two curves defines the values of T_1 and T_2 , a solution of the non-linear system. By assigning the values, we get,

Fig. 3 Function f1 and f2.

B. Transient Thermal Analysis

For the transient thermal analysis, the Fourier equation is given by,

$$\text{div}(k \text{grad} T) = \rho c \frac{\partial T}{\partial t} \quad (9)$$

The above equation is linear, which we have to solve in space-time. The analytical solution becomes a much more prolonged and tedious process. Moreover, the Boundary condition is non-homogenous and non-linear. So, in order to solve the transient analysis, we proceed with the lumped mass (Nodal Analysis) approach of the structure.

V. Nodal Analysis

The time historical temperature field will be produced throughout a spacecraft's orbital cycle as it travels around the Earth, as shown in Figure 5. During the day-night and night-day transitions in orbit, significant temperature changes are much more likely to occur.

A. Isothermal Analysis

Thermal analysis of a flat panel suddenly exposed to the Sun after an eclipse period is considered and the analytical and numerical solutions are compared. The thermal balance equation which governs the warming up of an s/c in GEO when it leaves the shadow zone is:

$$mc \frac{dT}{dt} = \alpha C A_p + P - \sigma \epsilon A_r T^4 \quad (10)$$

Fig. 4 An illustration of the solar array's thermal environment in orbit

In our case, indeed, the spacecraft is a flat panel with a Sun pointing attitude and then $A_p = A = \text{constant}$; also, the power P , if present, can be considered as constant. Since $A_r = 2A$, the equation can be written,

$$mc \frac{dT}{dt} = \left(\frac{\alpha CA + P}{2\sigma\epsilon A} - T^4 \right) 2\sigma\epsilon A, \quad T_E^4 = \frac{\alpha CA + P}{2\sigma\epsilon A} \quad (11)$$

By solving the above equation analytically, we get,

$$t = \frac{mc}{2\sigma\epsilon A} \left[\frac{1}{4T_E^3} \ln \left| \frac{T_E + T}{T_E - T} \right| + \frac{1}{2T_E^3} \left(\arctan \frac{T}{T_E} - \arctan \frac{T_0}{T_E} \right) \right] \quad (12)$$

Fig. 5 Numerical Vs Analytical Solution.

To plot the curve analytically, we assign a value to the temperature and compute the time elapsed to reach that value. In this particular case, we can assume $P = 0$ and T_0 coincident with the temperature of the panel at the end of the eclipse.

B. Two Node Analysis

For the flat panel with two nodes, the thermal balance equation can be written as,

$$m_1 c_1 \frac{dT_1}{dt} = (\alpha C A_p) - (\sigma \varepsilon_1 A_r T_1^4) - K (T_1 - T_2) \quad (13)$$

$$m_2 c_2 \frac{dT_2}{dt} = -(\sigma \varepsilon_2 A_r T_2^4) + K (T_1 - T_2) \quad (14)$$

From the equation, it is seen that the top panel only receives solar radiation. At the same time, the bottom panel does not receive radiation. Both equations can be made to apply to a variety of real-world situations with suitable boundary conditions; however, they are nonlinear as well as challenging to resolve into the quadrature. Solving the above Non-Linear equation's numerically using MATLAB ODE45 and the trend plot is given in the following. The temperature variation of T_1 and T_2 overtime is given in fig 7.

Fig. 6 Temperature variation of front and back panel.

It could be noted that there is an overshoot in the plot, which is responsible for increasing the amplitude of the thermal jitter.

Fig. 7 Non-Linear ΔT .

On linearising the equation, the trend plot is shown below. The overshoot, which is responsible for the amplitude of the mechanical vibration, is lost.

Fig. 8 Linear ΔT .

Through the approximations, the parameters that contribute to the Thermal Gradient can be obtained. The existence of a formula that correlates temperature to controllable parameters (such as geometry and thermophysical properties) is extremely useful as a concise explanation of a problem and its solution. The approximation pursues from the linearization of T^4 (now considered a function of T) by substituting it with the first two terms of the Taylor expansion about a mean temperature T_m ; that is, T^4 is replaced by $T_4(T_m) + 4T^3(T_m)[T - T_m]$, which is written as,

$$T^4 \cong 4T_m^3 T - 3T_m^4 \quad (15)$$

From algebra,

$$T^4 - 4T_m^3 T - 3T_m^4 > 0 \text{ where, } T_m = \frac{T_1 + T_2}{2} \quad (16)$$

For arbitrary T_m , the radiative ability of the linearised system is a reduced amount of the exact one that establishes the approximate temperatures always greater than actual temperatures. Since the actual temperature is irrelevant to the selection of T_m . When we linearise the equation, we lose valuable information which is responsible for the thermal excitation. In order to get the overshoot, a term T_m is introduced, which is the average value of the two temperatures, and it has updated periodically. The trend plot is given in the following; based on the above approximation, we got the valuable information that contributes to the overshoot.

Fig. 9 Updated Linear ΔT .

When we linearise the equation, we lose the valuable information which is responsible for the amplitude of the mechanical vibration. In order to get the overshoot, a term T_m is introduced which is the average value of the two temperature and it is updated periodically. The trend plot is given in fig.11; based on the above approximation, we got the valuable information that contributes to the overshoot. From the comparison figure, it is observed that the non-linear and Updated Linear perfectly overlap. Whereas on linearisation, the trend plot becomes monotonic in nature.

Fig. 10 Comparison of Linear, Non-Linear, Updated Linear ΔT .

ANALYTICAL EQUATION

It is better to have the analytical equation representing the above result to proceed further. The analytical expression is given by,

$$\Delta T = A((Bt^2 - 1) * e^{-Ct} + 1)) \quad (17)$$

Where,
A, B, C- Constants; t- time.

Fig. 11 Analytical ΔT.

VI. Thermomechanical Analysis

The thermomechanical analysis of the strip is carried out by considering the strip as Free-Free Beam. The temperature trend across the thickness can be a combination of a constant distribution and a gradient (butterfly pattern).

Fig. 12 Strip with the heat load.

We refer to the relation that links the displacement field to the variations of the thermal field; in the one-dimensional case, we have: Change in length is given by,

$$\Delta L = L_0 \alpha \Delta T \quad (18)$$

We know that stress and strain is given by,

$$\epsilon = \frac{L - L_0}{L_0} \quad (19)$$

$$\sigma = \epsilon E \quad (20)$$

$$\sigma = \alpha E \Delta T \quad (21)$$

If we apply stress, we get the same, but when we apply a thermal load (gradient), we get, in this case, only displacement, not stress. In this computation, the symbols ϵ and σ indicate the classical magnitudes of the science of the constructions, that is, deformation and normal tension, not to be confused therefore with emissivity and constant of Stefan Boltzmann.

$$\sigma = \alpha E \left(\frac{T_1 - T_2}{d} \right) x \quad (22)$$

Fig. 13 Normal thermal tension.

We can evaluate the thermal M_T Moment due to the temperature gradient H , which is given by the difference in temperature over the thickness. This moment is equivalent to a mechanical moment that causes the exact curvature of the structural element without generating a stress state associated with the displacement field. Thermal Bending Moment,

$$M_T = \int_{-\frac{d}{2}}^{\frac{d}{2}} \sigma x dx \quad (23)$$

$$M_T = \int_{-\frac{d}{2}}^{\frac{d}{2}} \alpha E \left(\frac{T_1 - T_2}{d} \right) x^2 dx \quad (24)$$

$$M_T = \alpha E \left(\frac{T_1 - T_2}{d} \right) \left(\frac{d^3}{12} \right) \quad (25)$$

The expression of the thermal moment is,

$$M_T = \alpha E H I \quad (26)$$

From the equation of an elastic line of the beam,

$$\frac{d^2 v}{dy^2} = -\frac{M_T}{EI} \quad (27)$$

The double integration provides:

$$v = -\frac{1}{2} \alpha H y^2 + C_1 y + C_2 \quad (28)$$

Fig. 14 Free-Free Beam.

$$C_1 = 0$$

$$y_c = C_2 = \frac{\int_{-\frac{L}{2}}^{\frac{L}{2}} \left(-\frac{1}{2} ax^2 \sqrt{1 + (ax^2)} \right) dx}{\left[L - \int_{-\frac{L}{2}}^{\frac{L}{2}} \sqrt{1 + (ax^2)} dx \right]} \quad (29)$$

$$v = -\frac{1}{2} \alpha H y^2 + \frac{\int_{-\frac{L}{2}}^{\frac{L}{2}} \left(-\frac{1}{2} ax^2 \sqrt{1 + (ax^2)} \right) dx}{\left[L - \int_{-\frac{L}{2}}^{\frac{L}{2}} \sqrt{1 + (ax^2)} dx \right]} \quad (30)$$

$$v = -\frac{1}{2} \alpha H y^2 + \frac{-\frac{1}{2} a \left[\frac{x(1 + ax^2)^{\frac{3}{2}}}{4} - \frac{x(1 + ax^2)^{\frac{1}{2}}}{8} - \frac{\log(x + (1 + ax^2))}{8} \right]_{-\frac{L}{2}}^{\frac{L}{2}}}{\left[L - \left[\frac{x(1 + ax^2)}{2} + \frac{\log(x + (1 + ax^2))}{2} \right]_{-\frac{L}{2}}^{\frac{L}{2}} \right]} \quad (31)$$

where $a = -\frac{1}{2} \alpha H y(t)$. From the above equation, it is clear that the v varies with time, it is because of the fact that the thermal gradient H is a function of time. Solving the above equation gives the following deformation,

Fig. 16 Deflection of Free-Free Beam.

Fig. 17 The Distance between Centroid and midspan changes with time.

VII.Modal Analysis

The transverse vibration of the beam's differential equation of motion is given by,

$$EI \frac{\partial^4 w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} = f(x, t) \quad (32)$$

For free vibration, the external excitation is approximated to zero:

$$f(x, t) = 0 \quad (33)$$

$$c^2 \frac{\partial^4 w}{\partial x^4}(x, t) + \frac{\partial^2 w}{\partial t^2}(x, t) = 0 \quad (34)$$

The shear force and bending moment are both zero at a free end. As a result, the beam's boundary conditions are as follows:

$$Y''(0) = M_T; Y'''(0) = 0 \quad (35)$$

$$Y''(l) = M_T; Y'''(l) = 0 \quad (36)$$

The initial condition is given as,

$$w(x, 0) = w_o(x) \quad (37)$$

$$\left. \frac{\partial w}{\partial x}(x, 0) \right|_{t=0} = \dot{w}_o(x) \quad (38)$$

By proceeding with the homogenous boundary condition, the beam's n^{th} mode shape can be expressed as,

$$w(x, t) = \sum_{n=1}^{\infty} w_n(x, t) = \sum_{n=1}^{\infty} W_n T_n \quad (39)$$

$$W_n(x) = (\cos\beta_n x + \cosh\beta_n x) - \frac{\cos\beta_n l - \cosh\beta_n l}{\sin\beta_n l - \sinh\beta_n l} (\sin\beta_n x + \sinh\beta_n x) \quad (40)$$

The frequency equation is given by,

$$\cos\beta l \cosh\beta l - 1 = 0 \quad (41)$$

$$\omega_n = (\beta_n l)^2 \left(\frac{EI}{\rho A l^4} \right)^{\frac{1}{2}} \quad (42)$$

The value of $\beta_0 l = 0$ precedes a rigid-body mode; When it comes to a free-free beam, there are rotational and translation modes.

Fig. 18 Symmetric Mode Shapes.

In the second case, boundary conditions are non-homogenous. We take,

$$w = \zeta(x, t) + \sum_{i=1}^4 f_i(t) g_i(x) \quad (43)$$

From Mindlin et al. [22], the non-homogenous boundary conditions are converted to homogenous condition and given by,

$$D_i[\zeta(0, t)] = f_i(t) - \sum_{j=1}^4 D_i[g_j(0)]f_j(t), \quad i = 1, 2 \quad (44)$$

$$D_i[\zeta(l, t)] = f_i(t) - \sum_{j=1}^4 D_i[g_j(l)]f_j(t), \quad i = 3, 4 \quad (45)$$

Finally, the initial conditions are given by,

$$w(x, 0) = w_o - \sum_{i=1}^4 f_i(0)g_i(x) \quad (46)$$

$$\left. \frac{\partial w}{\partial x}(x, 0) \right|_{t=0} = \dot{w}_o - \sum_{i=1}^4 \dot{f}_i(0)g_i(x) \quad (47)$$

The free vibration solution and the n^{th} normal mode are provided by,

$$\zeta(x, t) = W_n(x)T_n(t) \quad (48)$$

$$\zeta(x, t) = \sum_{n=1}^{\infty} w_n(x, t) = \sum_{n=1}^{\infty} W_n T_n \quad (49)$$

$$W_n(x) = (\cos\beta_n x + \cosh\beta_n x) - \frac{\cos\beta_n l - \cosh\beta_n l}{\sin\beta_n l - \sinh\beta_n l} (\sin\beta_n x + \sinh\beta_n x) \quad (50)$$

$$T_n = A_n \cos\omega_n t + B_n \sin\omega_n t + \frac{1}{\omega_n} \int_0^t P_n(\tau) \sin\omega_n(t - \tau) d\tau \quad (51)$$

Where, P_n is given by,

$$P_n(\tau) = Q_n \frac{p(\tau)}{\rho A} - \sum_{i=1}^4 [G_{in} \ddot{f}_i(\tau) + a^2 G_{in}^* f_i(\tau)] \quad (52)$$

Here, there is no external load acting on the beam and Q_n is equal to zero, the equation is reduced to,

$$P_n(\tau) = - \sum_{i=1}^4 [G_{in} \ddot{f}_i(\tau) + a^2 G_{in}^* f_i(\tau)] \quad (53)$$

The functions $g_i(x)$ are chosen to reduce the RHS of the equation to zero. To ensure that the following condition is satisfied,

$$D_i[g_i(0)] = \delta_{ij}, \quad j = 1, 2; \quad i = 1, 2, 3, 4 \quad (54)$$

$$D_i[g_i(l)] = \delta_{ij}, \quad j = 1, 2; \quad i = 1, 2, 3, 4 \quad (55)$$

Where, $\delta_{ij} = 0$ for $i \neq j$ and $\delta_{ij} = 1$ for $i = j$, the polynomial g_i is chosen like below,

$$g_i = c_i x^2 + d_i x^3 + e_i x^4 + f_i x^5, \quad i = 1, 2, 3, 4 \quad (56)$$

Here,

$$f_1 = f_1(t); \quad f_2 = 0; \quad (57)$$

$$f_3 = f_3(t); \quad f_4 = 0; \quad (58)$$

Therefore, $g_2 = g_4 = 0$ and the other values of g is computed and found as,

$$g_1(x) = \frac{1}{2}x^2 - \frac{1}{4L^2}x^4 + \frac{1}{10L^3}x^5 \quad (59)$$

$$g_3(x) = \frac{1}{4L^2}x^4 - \frac{1}{10L^3}x^5 \quad (60)$$

VIII. Conclusion

The research work provides a novel design guideline to analyse the thermal jitter of a slender spacecraft structure that is occurring in space engineering. A step by step procedure is given to study and analyse the phenomenon of thermal jitter in a simplified modelisation. The proposed method is solved analytically using Matlab. The results show that the recommended novel method provides a cost-effective and way to study and analyse the “Thermal Jitter” phenomenon. By understanding the principles behind this and proper mathematical knowledge, numerous applications can be developed based on this occurrence. The research work aims to evaluate the role of different mechanical and thermo-optical properties of the structure in dynamical behaviour.

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