

Type Synthesis of 3-Translational Redundant Actuated Parallel Mechanisms With Closed-Loop Branch Chains Based On Graphical Approach

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Title page

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ORIGINAL ARTICLE

Type Synthesis of 3-Translational Redundant Actuated Parallel Mechanisms with Closed-loop Branch Chains Based on Graphical Approach

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Abstract: The 3-translational parallel mechanism is widely used in industrial, medical, and military fields, among others. With the development of the national logistics industry, a pressing need for a kind of 3-translational parallel mechanism emerged. Such mechanisms have high stiffness and high bearing capacity and are used for cargo handling and sorting. A novel method based on the graphical approach was proposed for the synthesis of 3-translational redundancy actuated parallel mechanism with closed-loop branch chains. The new mechanism has four symmetrically arranged branch chains, which eases subsequent kinematics and dynamics analyses while providing good mechanical properties. Based on the graphical approach theory, according to the constraint number contained in the branch chain, two types of redundant driven branch chains with closed-loop structures were constructed. The first type includes rotation constraint in one direction, while the second type includes the rotation constraint in two directions. Based on various combinations of two branch chain types, their allocation schemes can be divided into two types. Moreover, said these two allocation schemes can be integrated into at least 500 and 400 types of 3-translational redundant actuated parallel mechanisms with closed-loop branch chains. Then, the degree of freedom properties of representative mechanisms were tested using the screw theory. A large number of novel mechanisms were

integrated assessed using this method, and branch chains such mechanisms were symmetrically distributed. They have a strong bearing capacity, simple calculation, and control, and can be applied to the handling and sorting of goods, large-scale precision machine tools, and large construction machinery vibration isolation systems, among others.

Keywords: Graphical approach • 3-Translation • Parallel mechanism • Closed-loop • Redundancy • Type synthesis

1 Introduction

The parallel mechanism first emerged during the 1930s. Gwinnett et al. [1-3] applied the parallel mechanism in the industry, starting the preliminary parallel mechanism development, which was developed rapidly in recent decades. Compared to the serial mechanism, its parallel counterpart has high accuracy, compact structure, robust stability, and has no cumulative error. As a subtype of parallel mechanism, the 3-translational parallel mechanism, was widely used in industrial manufacturing, logistics transportation, and medical treatment, among many other industries. In the early 20th century, Ciavel [4] invented the 3-translational Delta mechanism, which was widely used in packaging and sorting. Beltrami [5] et al. developed a micro-robot based on the Delta mechanism while Tasi [6] designed a typical three-translation parallel mechanism that was later patented in the United States. Li et al. [7] invented a 3-translation mechanism and carried out kinematics analysis. Zeng et al. [8] designed a 3-translational Tri-Pyramid parallel mechanism and analyzed its forward and inverse solution characteristics and Jacobian matrix. Hence, the 3-translational parallel mechanism has an important role in recent mechanism research. However, the development of a novel synthesis

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method for the three-translational parallel mechanism remains necessary, along with its design.

Improving the performance of parallel manipulators by adding redundancy is among the key research directions in the field. Zanganeh et al. [9-10] proposed a type of redundant actuated parallel manipulator; the use of redundant actuators increased the mechanism workspace, while also improving its singularity and dexterity. Furthermore, Qin et al. [11] through calculation concluded that introducing redundant branch chains will increase the parallel mechanism stiffness. By applying the condition needed to satisfy the working space, the redundant driving mechanism has shown obvious advantages. Guo et al. [12-13] integrated a series of low-DoF(Degree of Freedom) redundant actuated parallel mechanisms such as 3 translation mechanisms and 3 rotation mechanisms with screw theory. The result was used to carry out kinematics analysis, workspace analysis, and singularity analysis. Moreover, Wu et al. [14-15] proposed a 3-DoF redundant actuated parallel mechanism obtained by adding the redundancy to the traditional parallel mechanism and analyzed it, concluding that the redundant actuation can improve the machine tool performance.

The closed-loop branch chain can further improve the parallel mechanism stiffness and make its branch chain structure more compact. Within the literature [16-20], a sub-closed-loop parallel mechanism was adopted to effectively adjust the posture and isolate the vibration of multiple drive units. Shen et al. [21] introduced a parallel mechanism with two degrees of freedom consisting of a plane and a five-bar or six-bar loop within the branch chain; the kinematics complexity analysis was also carried out. Next, Zhao et al. [22] modified the open-loop branch chain structure of the traditional 3-RRS parallel mechanism into a closed-loop structure, obtaining a new 3-5RS parallel mechanism with a five-bar closed-loop branch chain. The authors then compared the static stiffness of two mechanisms, finding that the latter had significantly increased stiffness. Li et al. [23-25] synthesized a class of 2T1R and 2R1T parallel mechanisms using the graphical approach, including the redundancy to mechanisms aiming to improve their stiffness and stability. Similarly, Wen Yong [26] synthesized a class of 3-translational parallel mechanisms and carried out stiffness analysis of one of the mechanisms, the pose error analysis was also carried out at the end of the mechanism, considering only joint clearance. Zhao et al. [27] integrated a parallel mechanism and a closed-loop sub-chain, after which they carried out both the kinematic and dynamic analysis.

In this paper, a novel synthesis method for

3-translational redundancy actuated parallel mechanism with closed-loop branch chains was presented; the method was based on the graphical approach. By converting the mechanism kinematic pair and associated constraints into the pattern form, parallel mechanism kinematic characteristics could be expressed visually. A large number of equivalent branch chain DoF diagrams were developed using the line equivalence principle. The redundant lines were added to the DoF diagrams to build the closed-loop branch chains, which served as the basis for forming the innovative branch chain configuration. In the second part of the paper, the branch chain selection scheme was divided into two parts; the representative cases were selected as synthesis examples. The screw theory was then used to verify the degree of freedom property of two new mechanisms, verifying the correctness of the proposed method.

2 Underlying theory

2.1 Graphical approach theory

A graphical approach is a tool that can be used in the design of both rigid and complex flexible mechanisms. As such, the approach has a broad prospect of application as it provides a simple and practical way to design the subsequent mechanism.

The graphical approach includes four graph elements, as shown in Table 1. Those are the black line, black line with arrows at both ends, red line, and red line with arrows at both ends. They respectively represent the constraint force, constraint couple, rotational DoF, and translational DoF. The line corresponds to the screw theory line vector, while the line with arrows corresponds to the screw theory dual vector quantity. Finally $\r and $\$$ represent the constraints and kinematic screw, respectively.

Table 1 Graph elements

Graph elements	Physical meaning	Screw type
	Constraint force	Vector $\$^r = (\$, \$_0)$
	Constraint couple	Dual vector $\$^r = (0; \$)$
	Rotational DoF	Vector $\$ = (\$, \$_0)$
	Translational DoF	Dual vector $\$ = (0; \$)$

2.2 The blanding rule

The blanding rule, also known as the reciprocal line rule, states that if there are n non-redundant lines in a pattern, there will be $(6 - n)$ non-redundant lines in a reciprocal line pattern. Also, each line in the pattern will intersect with all the lines in its reciprocal line pattern.

By applying the blanding rule, a unique freedom line pattern can be obtained from the constraint line pattern, or a unique constraint line pattern can be obtained from the freedom line pattern. By considering the blanding rule and the graphical approach together, the transformation of both the constraint and freedom line patterns can be achieved.

2.3 The principle of line equivalence

The principle of planar parallel lines equivalence states

that if a freedom or constraint line pattern contains only one dual vector and one line perpendicular to the dual vector, then the pattern is equivalent to a pattern with planar parallel lines (see Fig. 1).



Figure 1 Planar parallel lines equivalence

By extending the principle of planar parallel line equivalence to spatial coordinates, the principle of spatial parallel line equivalence can be obtained. The derivation process is shown in Figure 2.

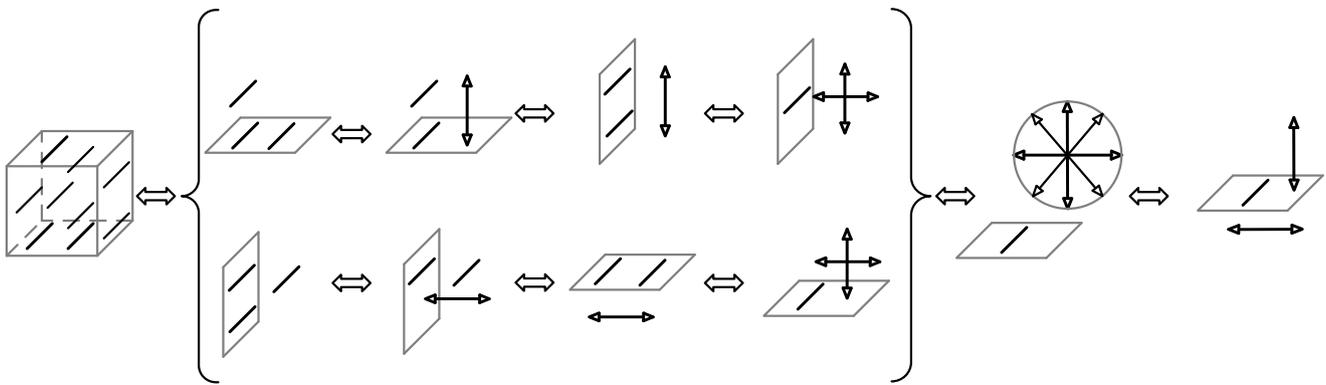


Figure 2 The derivation process of the principle of spatial parallel line equivalence

3 The Process of Mechanism Type Synthesis Based on Graphical Approach

In this section, a type synthesis method of 3-translational redundancy actuated parallel mechanisms with closed-loop branch chains based on the graphical approach was presented. The steps are shown as follows.

- (1) Determine the DoF and freedom properties of the moving platform according to the movement requirements.
- (2) Draw the freedom space of the moving platform.
- (3) Draw the constraint space reciprocal to the freedom space according to the blanding rule.
- (4) Decompose the constraint space to find multiple subspaces.
- (5) Determine the arrangement, quantity, and number of branch chain constraints; in general, increase the mechanism stability, branch chains arrangement requires a certain level of geometric symmetry. The more symmetric the mechanism is, the better its comprehensive performance will be.

- (6) Select the branch chain constraint line patterns according to their constraint properties.
- (7) Draw the branch chain freedom spaces according to the blanding rule.
- (8) Decompose the freedom space according to the branch chain structure characteristics.
- (9) Expand the line pattern according to the principle of line equivalence.

The rotational freedom line pattern can be equivalent to the translation freedom line pattern, and vice versa.

- (10) Create the redundant closed-loop branch chains by utilizing the freedom line pattern.

In this step, the branch chain instantaneities should be considered. Directions of the moving pairs in some branch chains changes during the movement, which may affect their constraints and change their freedom properties. These properties are determined by the branch chain structure; when the branch chain is constructed, if there is a moving pair, the moving pair should be installed

on the stable platform. On the other hand, if the branch chain is constructed with two moving pairs, they can be fixed on both the moving platform and the stable platform. Finally, if the moving pair cannot be fixed, the change direction of the moving pair should be considered during the branch chain movement, especially when the pair moves perpendicularly to the initial position. Similarly, the direction change of the partial rotating pairs will also change during the movement and should be taken into account during the construction.

The branch chain construction is primarily carried out using the following principles: (a) By satisfying the motion condition; when the branch chain contains a moving pair, said pair can be fixed on a stable platform, so its movement direction will not change. (b) When selecting the branch chain with two moving pairs, both branch chain pairs can be fixed to both the stable and moving platforms. (c) If it is not possible to fix the moving pair to a platform, it is necessary to consider changing the freedom property caused

by the moving pair direction change. (d) When the direction of the branch chain rotating pair is perpendicular to the initial position, it is necessary to consider whether the chain constraint direction. The direction will be changed to avoid changing the moving platform freedom property.

After the branch chain is constructed, it should be checked whether the branch chain kinematic pair is reciprocal to the constraints.

- (11) Select branch chains according to the constraint requirements and structural rationality.
- (12) Assemble each branch chains into parallel mechanisms according to requirements.
- (13) Apply the screw theory to verify the freedom property of the new mechanism.
- (14) Verify the continuity of parallel machinery.
- (15) Select the driver pair; in general, either the mobile or rotating pair connected with the frame can be selected as the driving pair.
- (16) Configuration completed.

The process diagram of the above-presented method is shown in Figure 3.

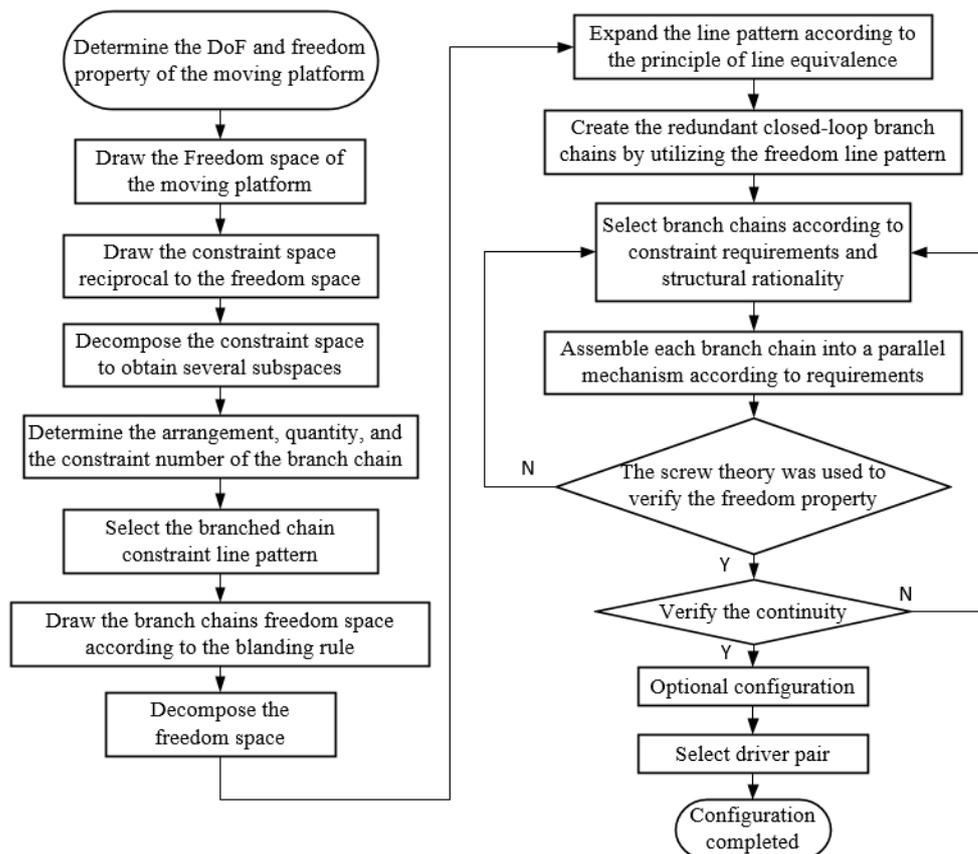


Figure 3 Type synthesis process diagram

4 Mechanism Type Synthesis Based on Graphical Approach

The freedom space of the 3-translational parallel mechanism has three spatial mobility degrees of freedom. According to the graphical approach theory, the 3-translational mechanism is represented by a line pattern, as shown in Fig. 4

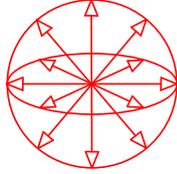


Figure 4 Freedom space of 3-translational parallel mechanism

The constraint space is shown in Fig. 5.

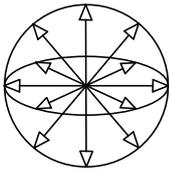


Figure 5 Constraint space of 3-translational parallel mechanism

Table 2 Decomposed constraint line pattern and reciprocal freedom space

Constraint the branch chain contains	Reciprocal freedom space
Constraint X rotation 	
Constraint Y rotation 	
Constraint Z rotation 	

Table 2 (continued)

Constraint the branch chain contains	Reciprocal freedom space
Constraint X, Y rotation 	
Constraints X and Z rotation 	
Constraint Y and Z rotation 	

Considering that the branch chain distribution in the parallel mechanism has a certain geometric symmetry, the mechanism has improved stiffness and kinematic properties. Further, it is beneficial to the subsequent kinematic, dynamic, workspace, and other mechanism calculations. Additionally, in this synthesis method, the number of branch chains was set to four, and the chains were arranged symmetrically.

For the freedom space, whose branch chain constraint is X rotation, the freedom space can be decomposed. According to the line equivalence principle, the translation DoF was transformed into the rotational DoF and the branch chain design could be carried out according to both the translational and rotational DoF (see Table 3).

Table 3 Decomposition and equivalence of the branch chain freedom space with X rotational constraint

Freedom space	Decompose the freedom space	Translational DoF transformed into the rotational DoF

Based on the freedom space and freedom line pattern shown in Table 3, the redundant driving branch chains with closed-loop units were constructed, as shown in Table 4.

Due to the limited space, only 25 branch chain types are shown in Table 4; the actual number is larger.

Table 4 Redundant drive branch chains with closed-loop units of X rotation constraint

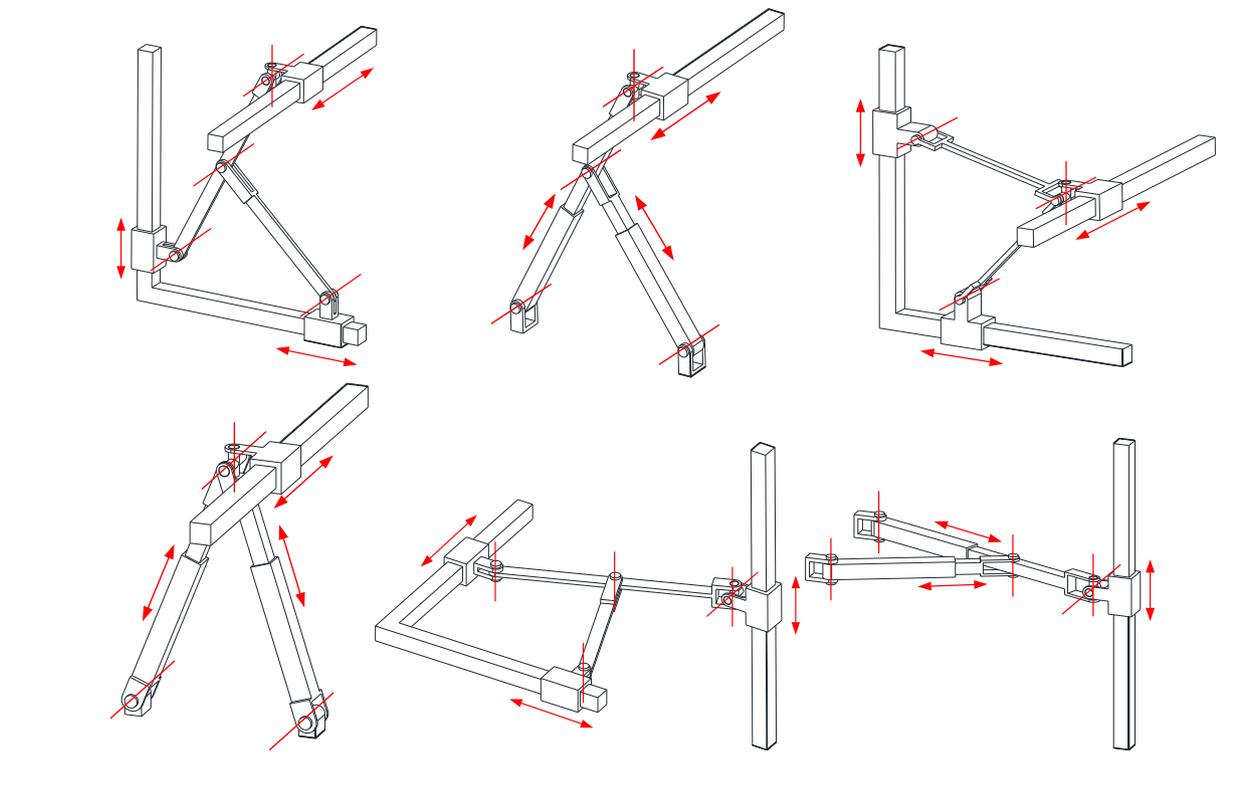


Table 4 (continue)

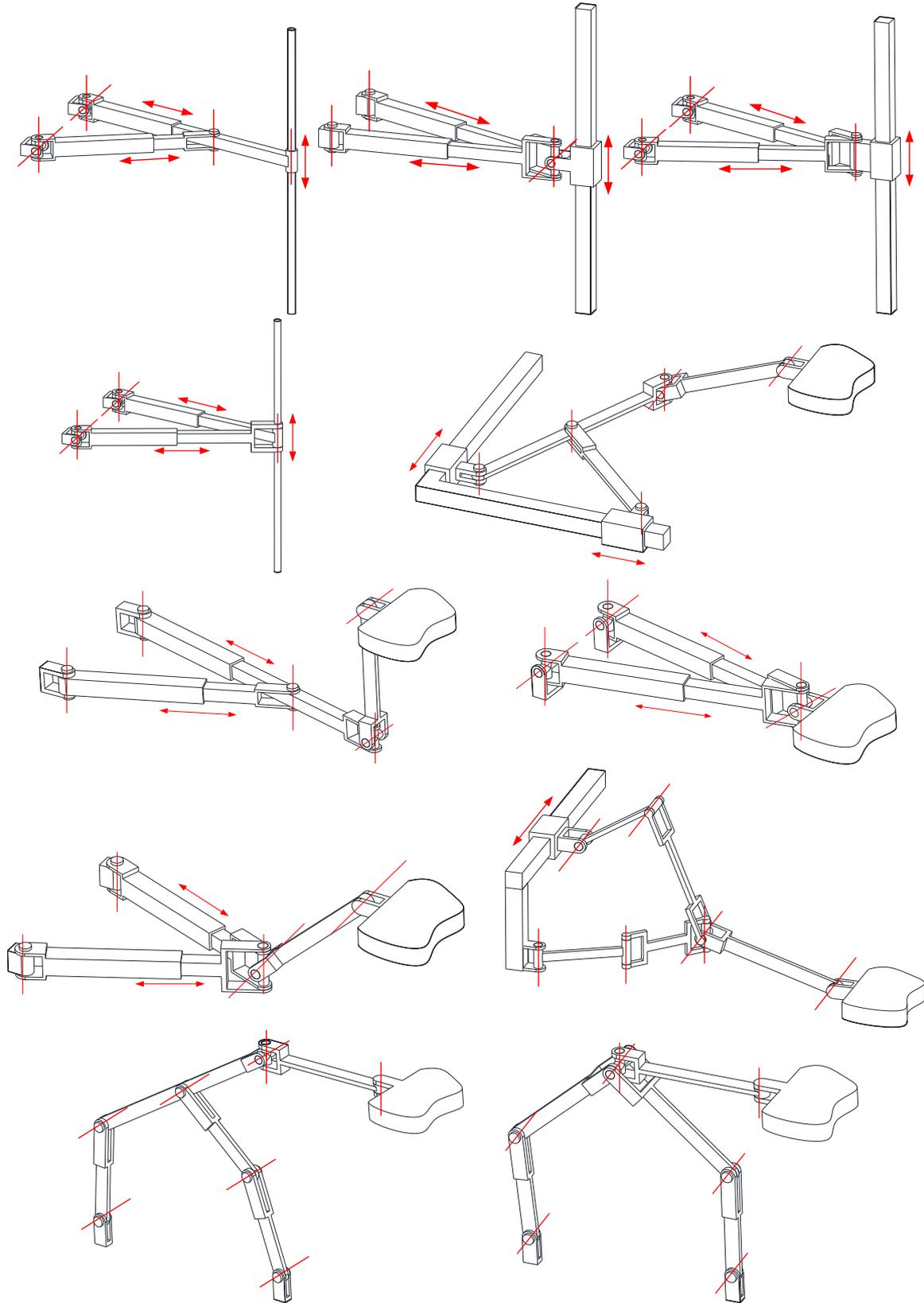
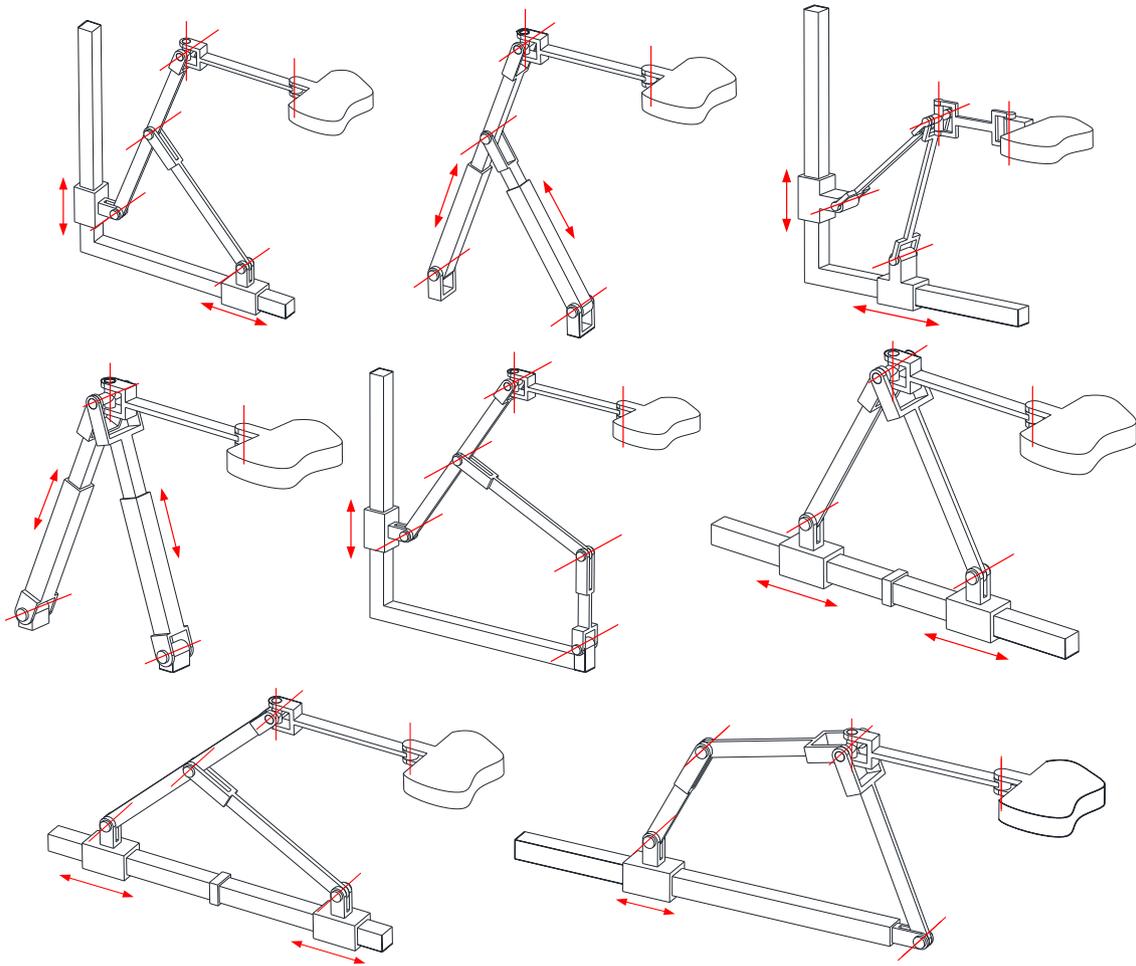


Table 4 (continue)

Similarly to the previous cases, the branch chain freedom space with Y rotational constraint can also be decomposed. According to the line equivalence principle, the translational DoF can be transformed into their rotational counterparts, branch chain design can be carried out according to both the translation and rotational DoF. Additionally, as you see in table 2, the branch chain freedom space with X rotational constraint rotate 90°

around the Z-axis, that is the branch chain freedom space with Y rotational constraint. Thus, The branch chains with X rotational constraint in Table 4 rotate 90° around the Z-axis, that is the branch chains constructed by the freedom space with Y rotational constraint, as shown in Table 5, the number of branch chains is equal to that in Table 4 (a total of 25 types). Due to limited space, only six branch chains were shown (see Table 5).

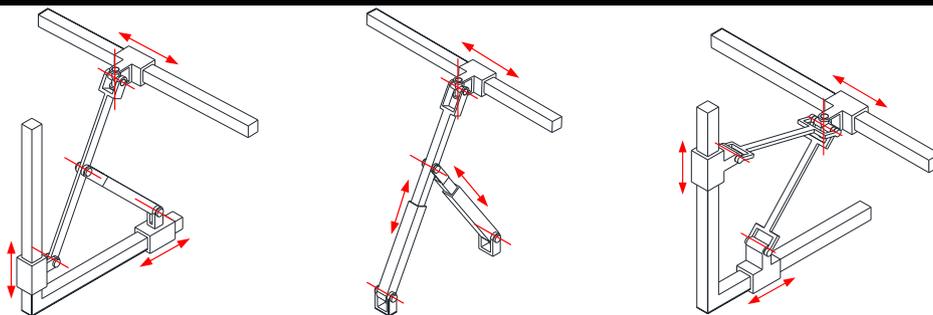
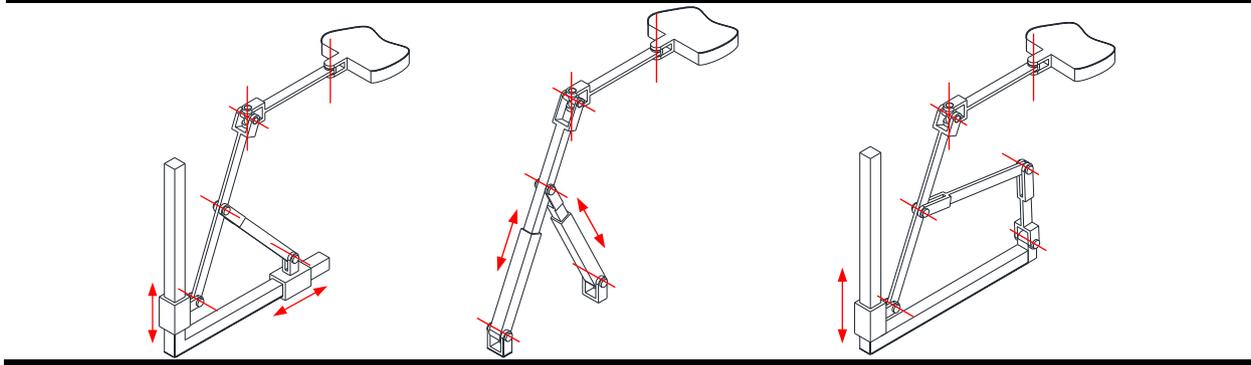
Table 5 Redundant drive branch chains with closed-loop units of rotation constraint Y

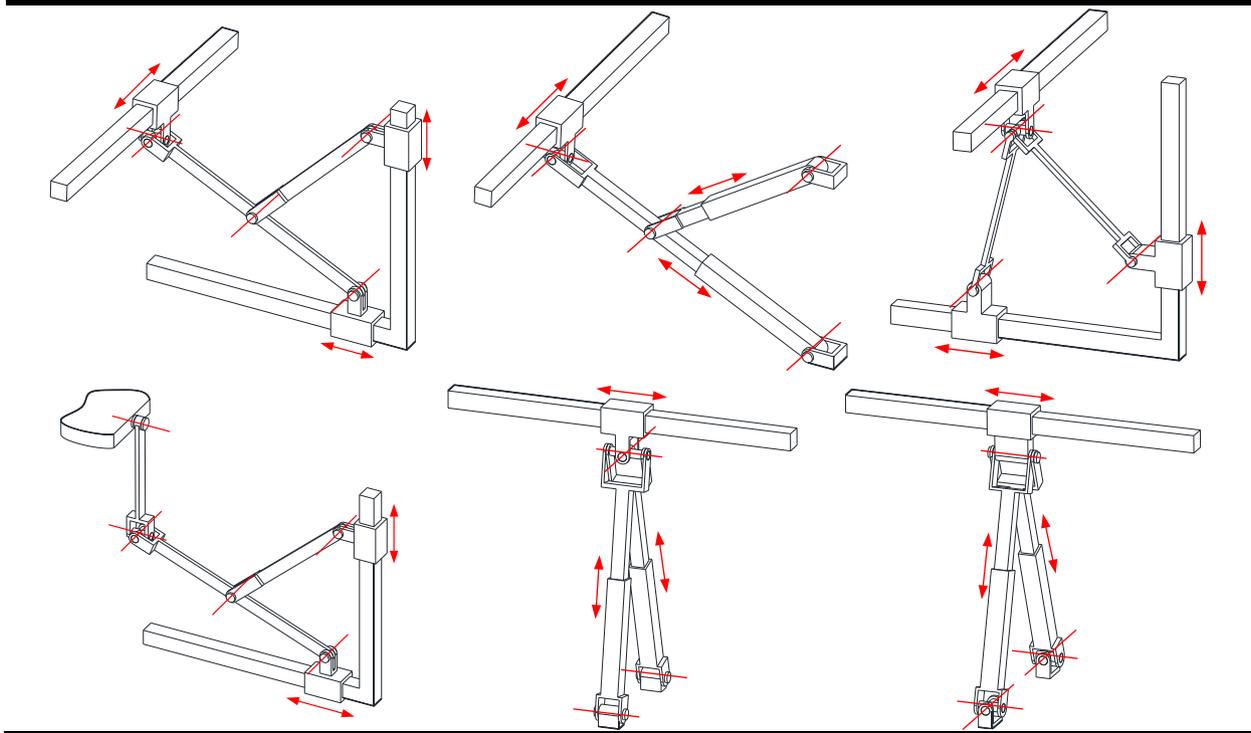
Table 5 (continue)



Similarly to the previous cases, the branch chain freedom space with Z rotational constraint can also be decomposed. According to the line equivalence principle, the translational DoF can be transformed into their rotational counterparts, branch chain design can be carried out according to both the translation and rotational DoF. Additionally, as you see in table 2, the branch chain freedom space with X rotational constraint rotate 90°

around the Y-axis, that is the branch chain freedom space with Z rotational constraint. Thus, The branch chains with X rotational constraint (in Table 4) rotate 90° around the Y-axis, that is the branch chains constructed by the freedom space with Z rotational constraint, as shown in Table 6, the number of branch chains is equal to that in Table 4 (a total of 25 types). Due to limited space, only six branch chains were shown (see Table 6).

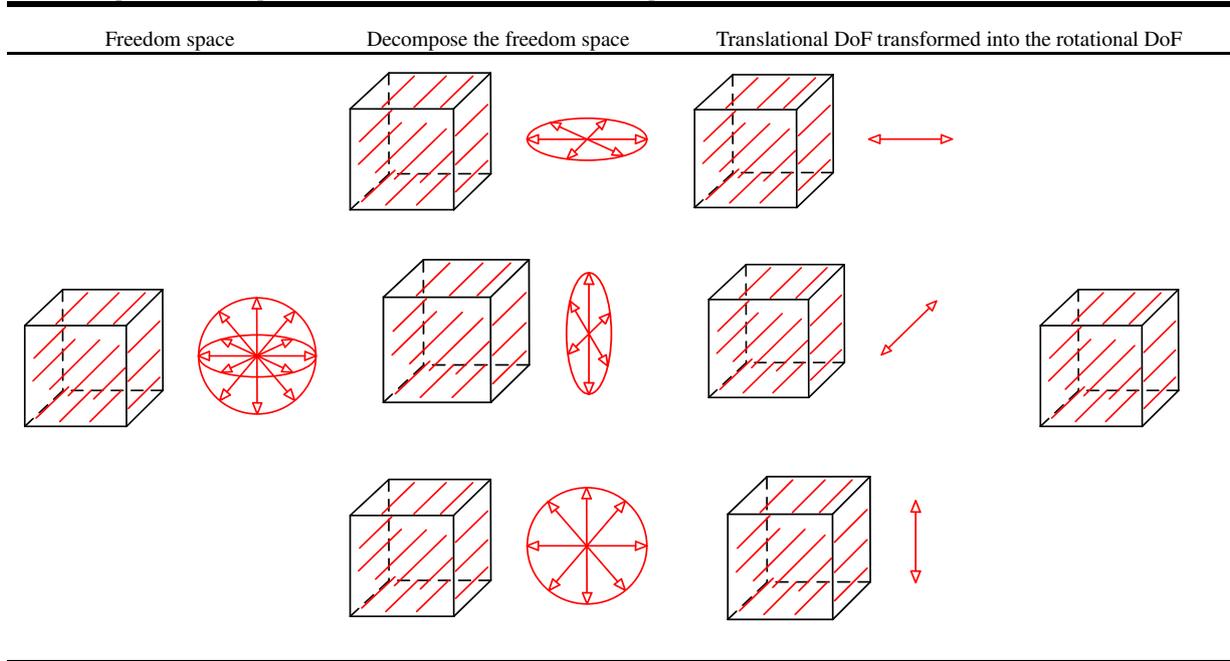
Table 6 Redundant drive branch chains with closed-loop units of rotation constraint Z



For the freedom space with branch chain constraints are the X and Z rotation, the freedom space can be found. According to the line equivalence principle, the translational DoF can be transformed into rotational DoF.

Further, and the branch chain design can be carried out according to both the translation and rotational DoF, as shown in Table 7

Table 7. Decomposition and equivalence of the branch chain freedom space with rotational constraints X and Z



The redundant driving branch chains with closed-loop units are constructed according to the freedom space and freedom line pattern shown in Table 7 and are shown in

Table 8. Due to the limited space, only 20 branch chain types were shown in Table 8 Their actual number is significantly higher.

Table 8. Redundant drive branch chains with closed-loop units of rotation constraints X and Z

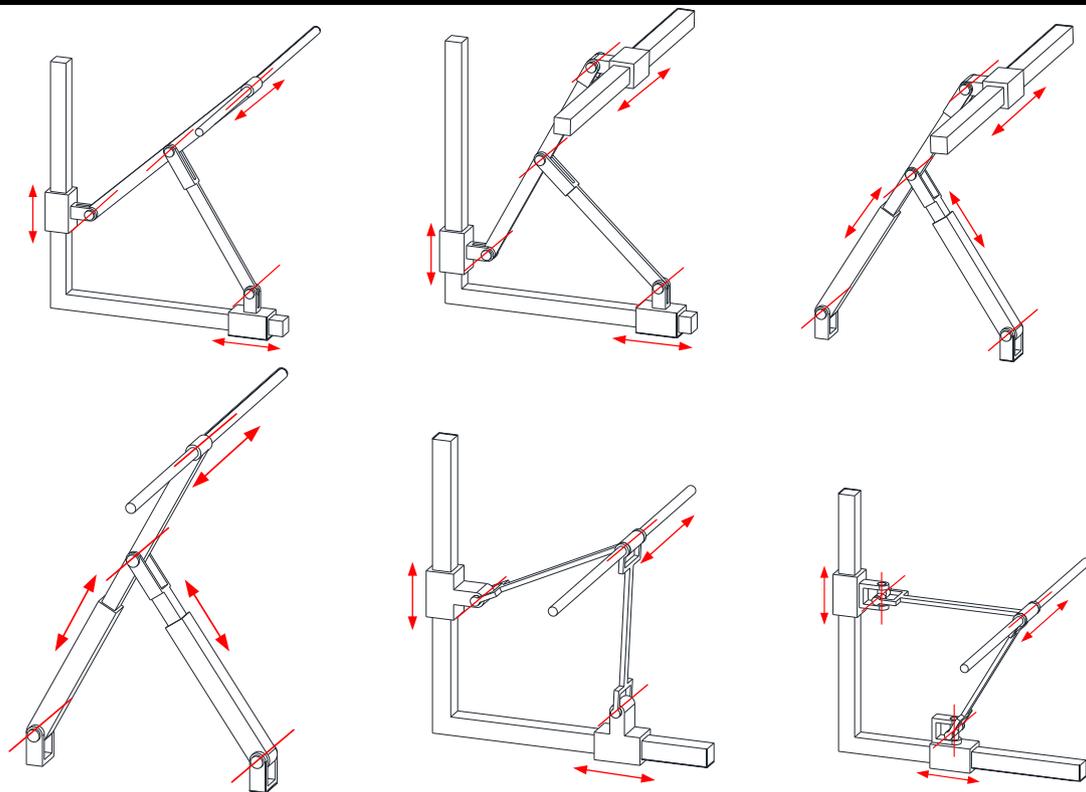
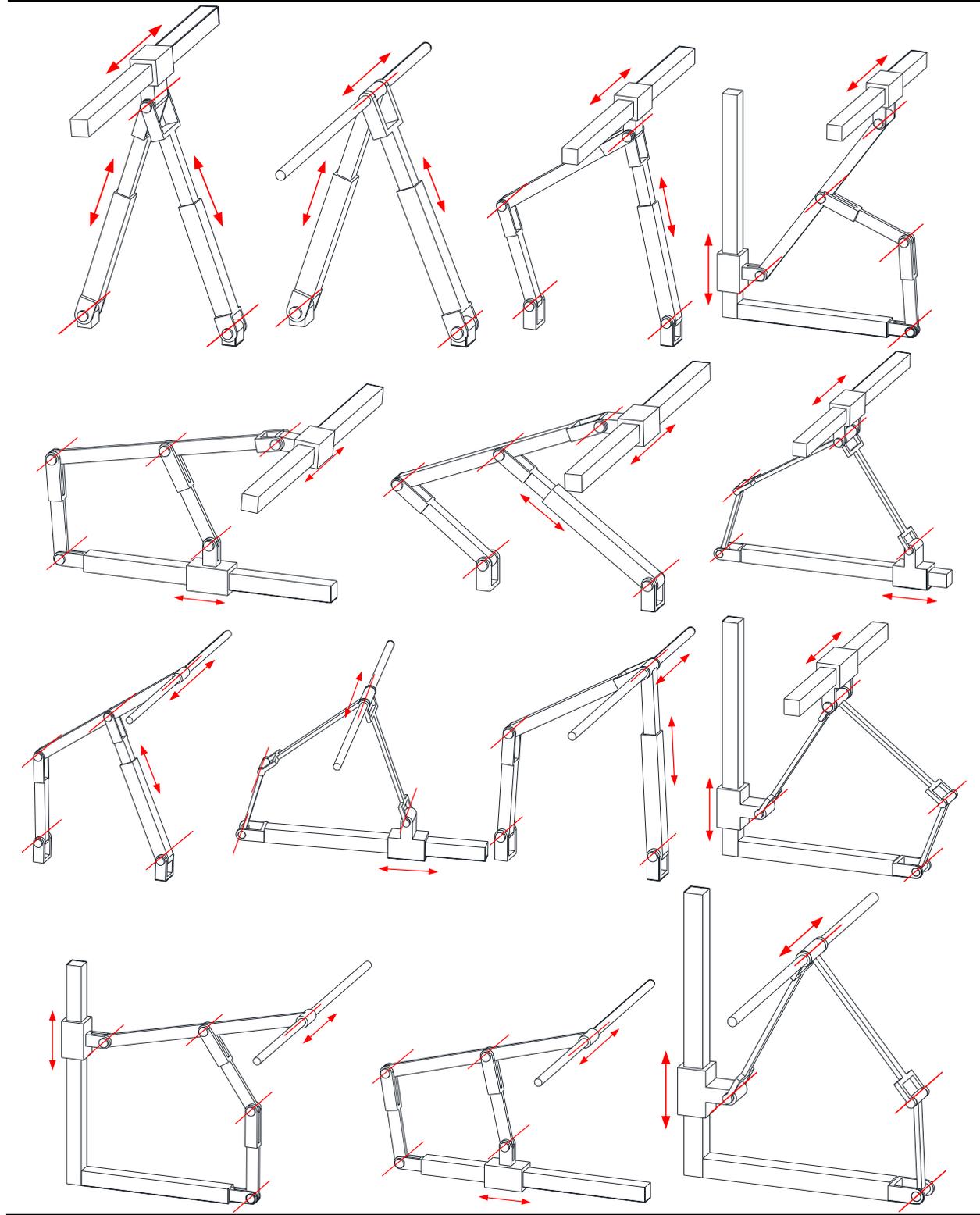


Table 8 (continue)



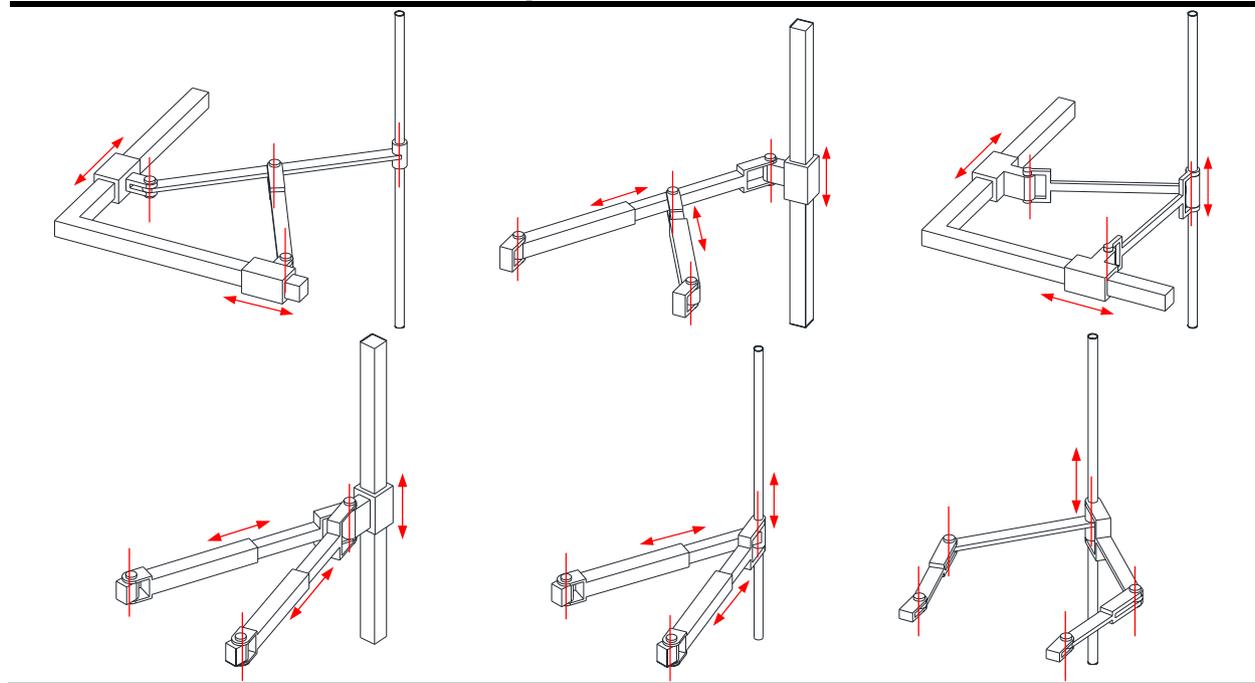
Similarly to the previous cases, the branch chain freedom space with rotational constraint X and Y can also be decomposed, According to the line equivalence principle,

the translational DoF can be transformed into their rotational counterparts, branch chain design can be carried out according to both the translation and rotational DoF.

Additionally, as you see in table 2, the branch chain freedom space with rotational constraint X and Z rotate 90° around the X-axis, that is the branch chain freedom space with rotational constraint X and Y. Thus, The branch chains with rotational constraint X and Z in Table 8 rotate 90° around the X-axis, that is, the branch chains

constructed by the freedom space with rotational constraint X and Y, as shown in Table 9, the number of branch chains is equal to that in Table 8 (a total of 20 types). Due to limited space, only six branch chains were shown (see Table 9).

Table 9 Redundant drive branch chains with closed-loop units of rotation constraints X and Y



Similarly to the previous cases, the branch chain freedom space with rotational constraint Y and Z can also be decomposed, According to the line equivalence principle, the translational DoF can be transformed into their rotational counterparts, branch chain design can be carried out according to both the translation and rotational DoF. Additionally, as you see in table 2, the branch chain freedom space with rotational constraint X and Z rotate 90°

around the Z-axis, that is the branch chain freedom space with rotational constraint Y and Z. Thus, The branch chains with rotational constraint X and Z in Table 8 rotate 90° around the Z-axis, that is, the branch chains constructed by the freedom space with rotational constraint Y and Z, as shown in Table 10, the number of branch chains is equal to that in Table 8 (a total of 20 types). Due to limited space, only six branch chains were shown (see Table 10)

Table 10 Redundant drive branch chains with closed-loop units of rotation constraints Y and Z

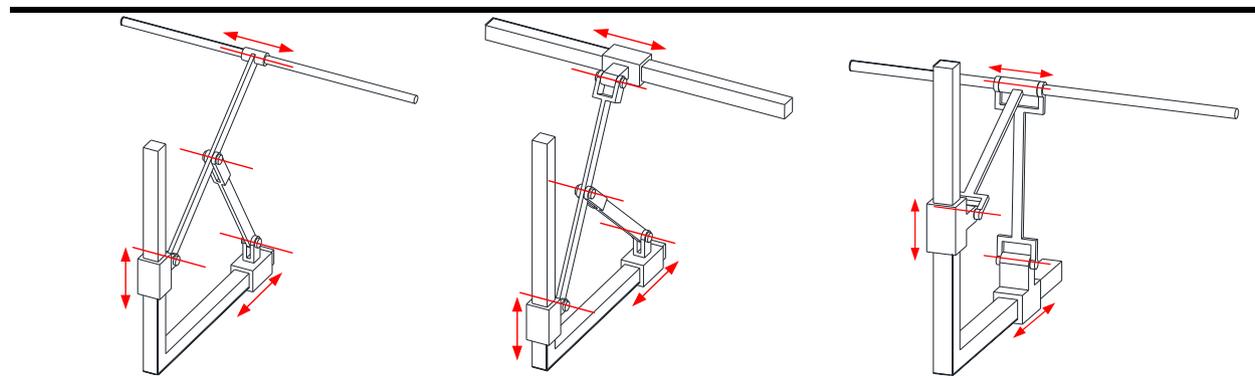
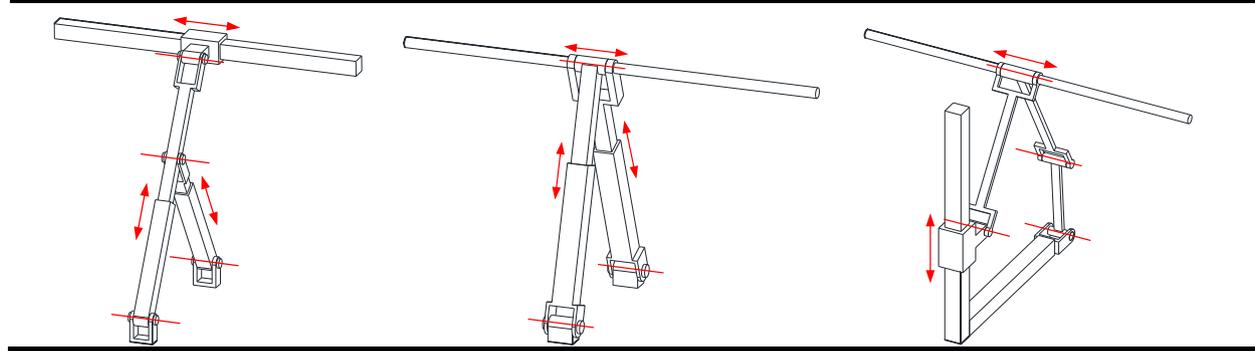


Table 10 (continue)



5. Branch chain selection examples

Considering that the branch chain distribution of the parallel mechanism has a level of geometric symmetry, the mechanism stiffness and kinematic properties were improved. Thus, it is helpful for the subsequent kinematic, dynamic, workspace, and other types of mechanism calculations. In this synthesis method, four symmetrically arranged branch chains were used. Based on the constraints given in Table 2, the number of branch chain constraints is 1 or 2. Combined with both the number and distribution requirements of the mechanism branch chains, this method divides the branch chain selection scheme into two kinds.

In the first scheme, two of the four branch chains have rotational constraints in one direction; which have the same structure and assemble in the opposite direction. Furthermore, the other two branch chains have rotational constraints in the remaining two directions which have the same structure and assemble in the opposite direction. For example, the first two branch chains have rotation constraints in the X direction, while the second two branch chains should include rotation constraints in the Y and Z directions.

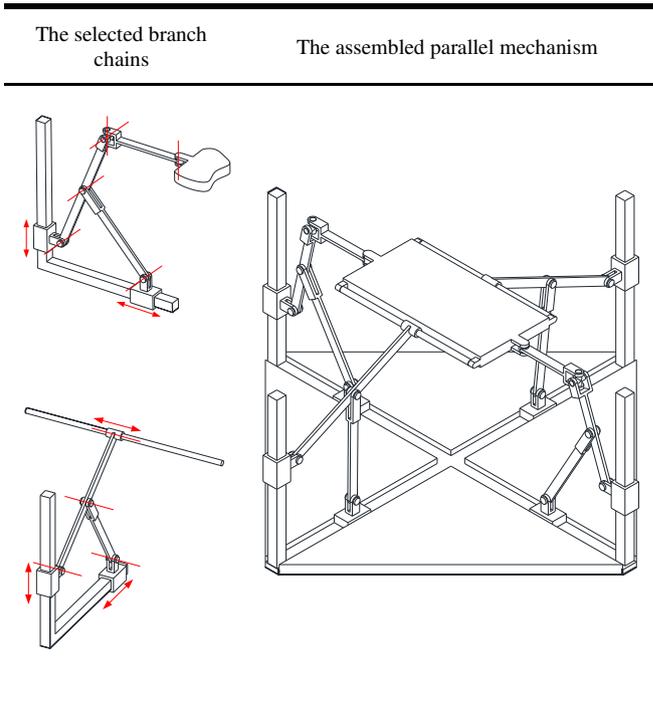
In the second scheme, among the four branch chains, two branch chains have two rotational constraints, which have the same structure and assemble in the opposite direction. The other two branch chains also have two rotational constraints, which have the same structure and assemble in the opposite direction. The rotation constraint directions of the first two branch chains mustn't be equal to those of the second two branch chains. For example, if the first two branch chains have rotation constraints along the X and Z directions, then the second two branch chains will have rotation constraints in the Y direction; therefore, the branch chains with rotation constraints along the X and Y or Y and Z directions can be selected.

According to Tables 4 to 10, there are at least 25 types of single-rotation constraint branch chains and at least 20 types of double-rotation constraint branch chains in the first scheme. Hence, the first scheme can produce at least $25 \times 20 = 500$ types of

mechanisms. In the second scheme, there are at least 20 possible double-rotation constraints branch chains, meaning that it can produce at least $20 \times 20 = 400$ mechanisms.

According to the first branch chain selection scheme, the branch chain with rotation constraints along the X direction were selected in Table 4, while the branch chains with rotation constraints in the Y and Z directions were selected in Table 10. Finally, two branch chains were arranged in opposite directions, as shown in Table 11.

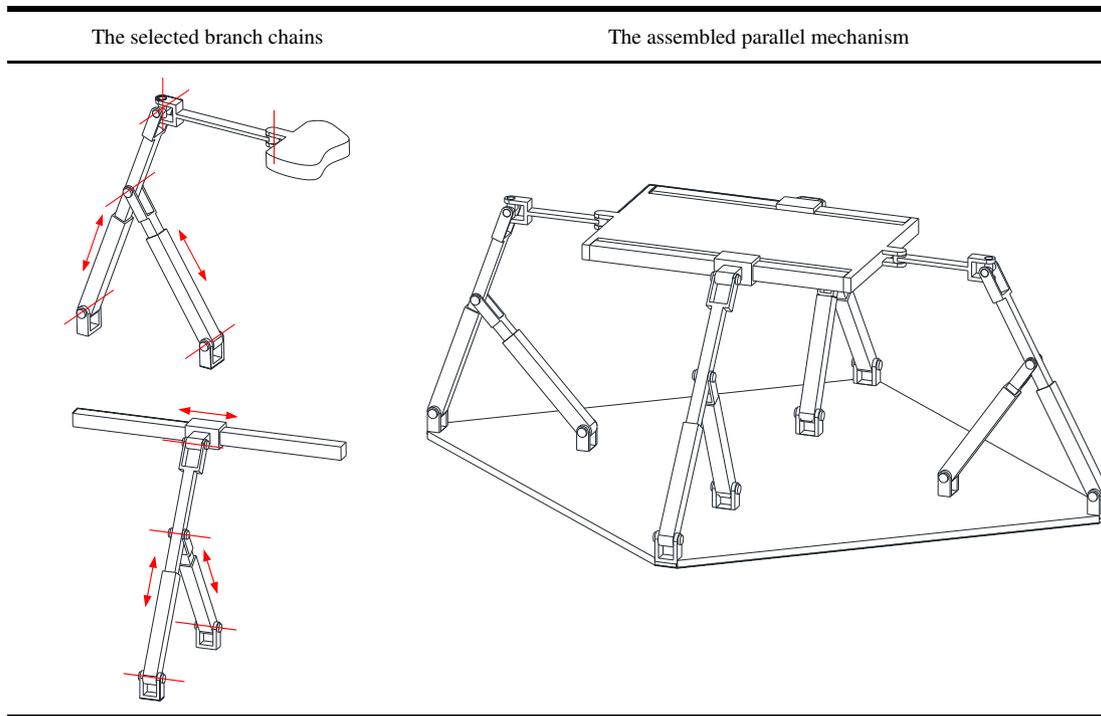
Table 11. Selected branch chains and the assembled parallel mechanism



According to the first branch chain selection scheme, the branch chain with rotation constraints along the X direction were selected in Table 4, while the branch chains with rotation constraints in the Y and Z directions were selected in Table 10.

Finally, two branch chains were arranged in opposite directions, as shown in Table 12.

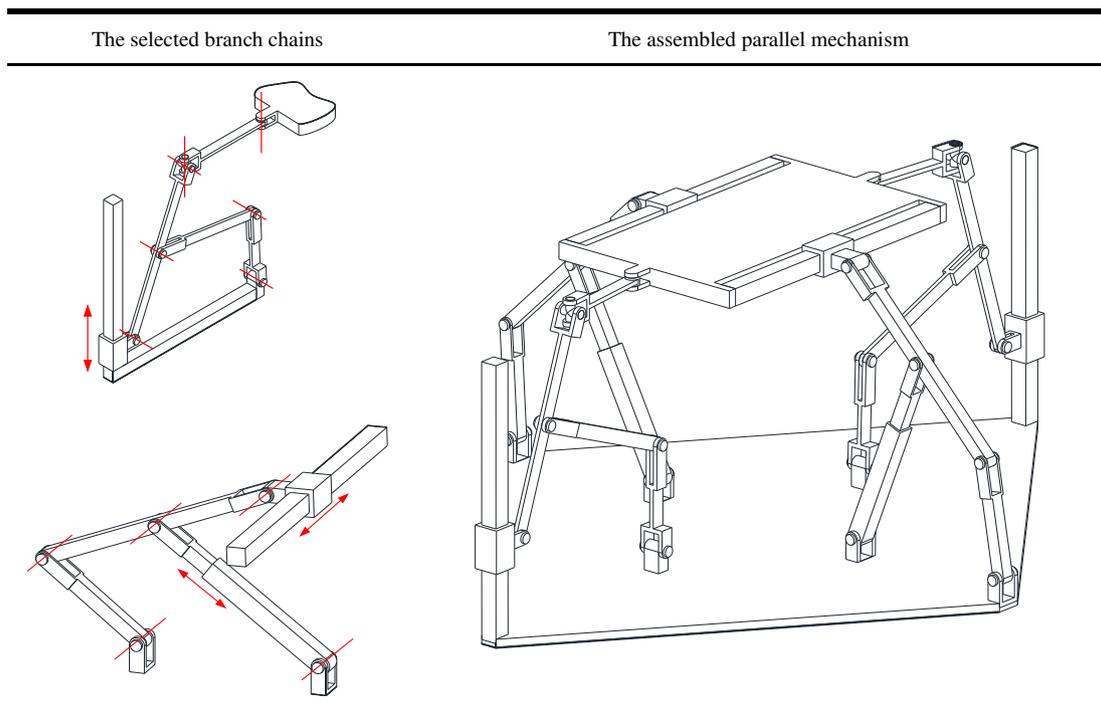
Table 12 Selected branch chains and the assembled parallel mechanism



According to the first branch chain selection scheme, the branch chain with rotation constraints along the Y direction were selected in Table 5, while the branch chains

with rotation constraints in the X and Z directions were selected in Table 8. Finally, two branch chains were arranged in opposite directions, as shown in Table 13.

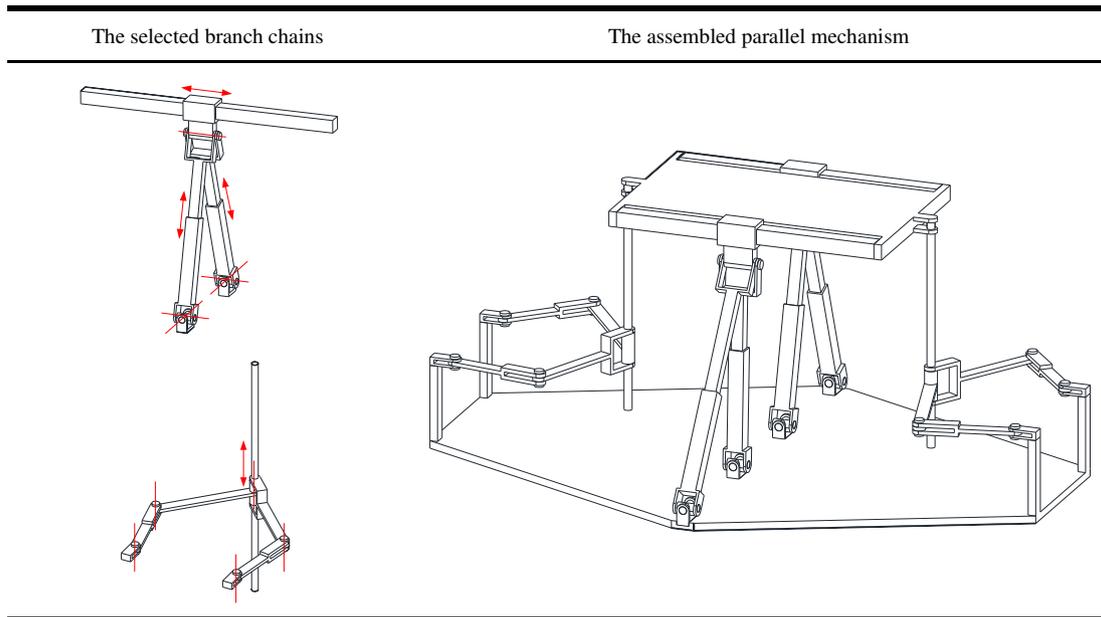
Table 13. Selected branch chains and the assembled parallel mechanism



According to the first branch chain selection scheme, the branch chain with rotation constraints along the Z direction were selected in Table 6, while the branch chains

with rotation constraints in the X and Y directions were selected in Table 9. Finally, two branch chains were arranged in opposite directions, as shown in Table 14.

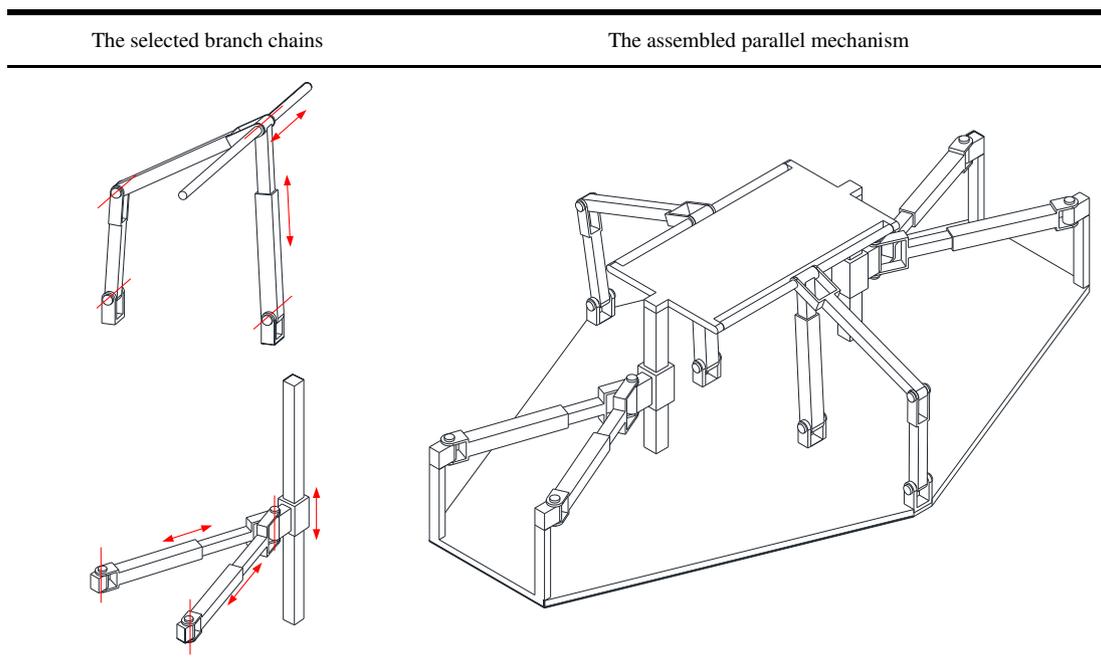
Table 14. Selected branch chains and the assembled parallel mechanism



According to the second branch chain selection scheme, the branch chain with rotation constraints along the X and Z direction were selected in Table 8, while the branch

chains with rotation constraints in the X and Y directions were selected in Table 9. Finally, two branch chains were arranged in opposite directions, as shown in Table 15.

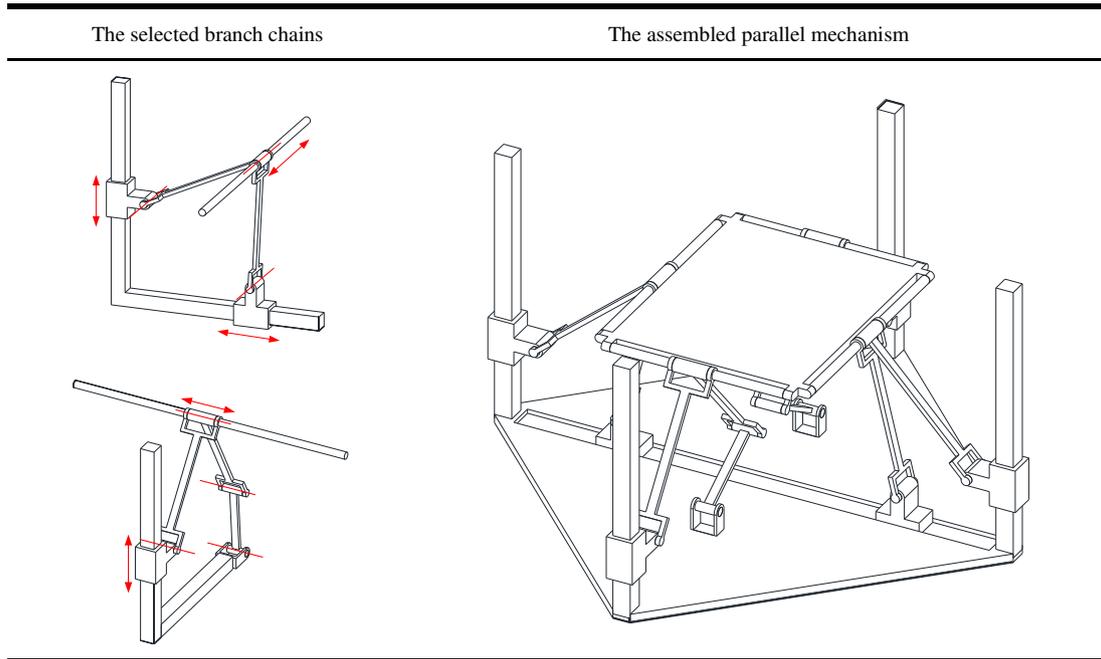
Table 15 Selected branch chains and the assembled parallel mechanism



According to the second branch chain selection scheme, the branch chain with rotation constraints along the X and Z direction were selected in Table 8, while the branch

chains with rotation constraints in the Y and Z directions were selected in Table 10. Finally, two branch chains were arranged in opposite directions, as shown in Table 16.

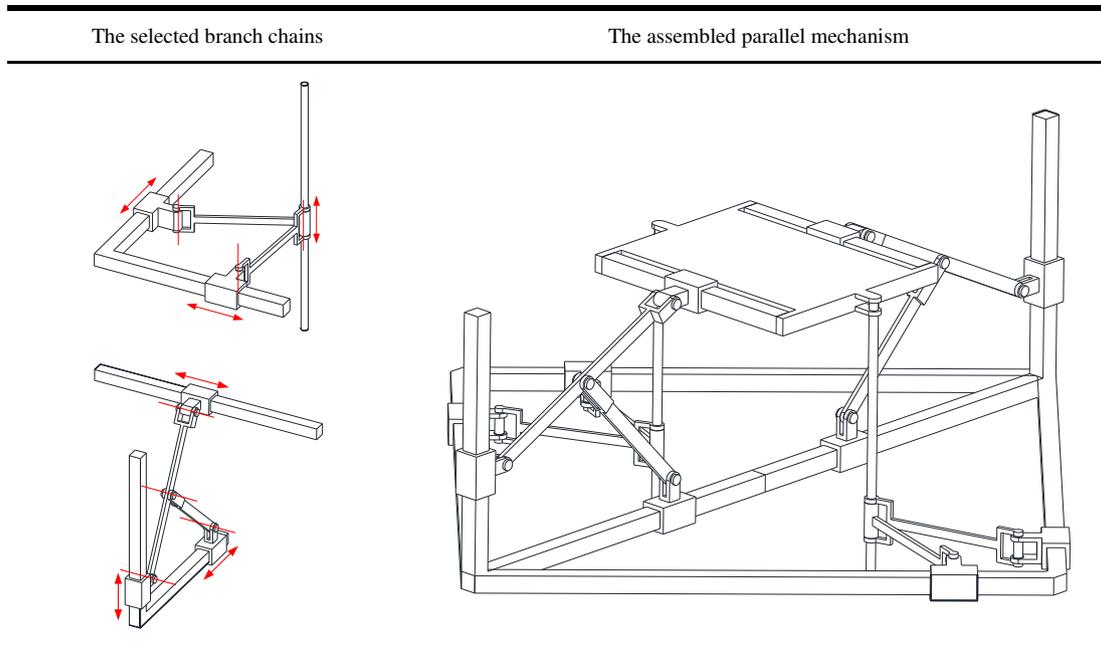
Table 16 Selected branch chains and the assembled parallel mechanism



According to the second branch chain selection scheme, the branch chain with rotation constraints along the X and Y direction were selected in Table 9, while the branch

chains with rotation constraints in the Y and Z directions were selected in Table 10. Finally, two branch chains were arranged in opposite directions, as shown in Table 17.

Table 17 Selected branch chains and the assembled parallel mechanism



6. Analysis of the representative mechanism freedom properties

6.1 Analysis of mechanism 1

A new mechanism from Table 11 was selected for DoF verification, as shown in Figure 6. Firstly, each mechanism branch chain was simplified, along with the closed-loop structure, which is simplified into an open-loop structure. This enabled the authors to assemble the simplified parallel mechanism. Lastly, the mechanism degrees of freedom were verified by the screw theory.

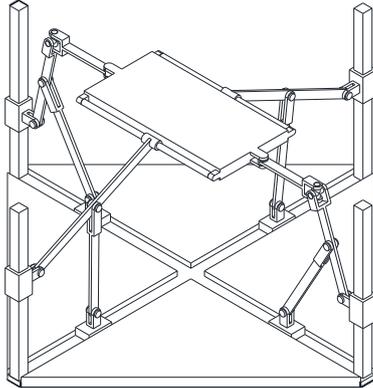


Figure 6 A novel parallel mechanism assembled using Table 11 data

The closed-loop branch chain was simplified to the open-loop branch chain before the calculation, while the DoF property of the simplified branch chain remained unchanged. The results are shown in Table 18.

Table 18. Branch chain simplification

Before the simplification	After the simplification

The simplified branch chains were assembled to obtain a 3-translation parallel mechanism with the open-loop branch chains structure, as shown in Fig. 7.

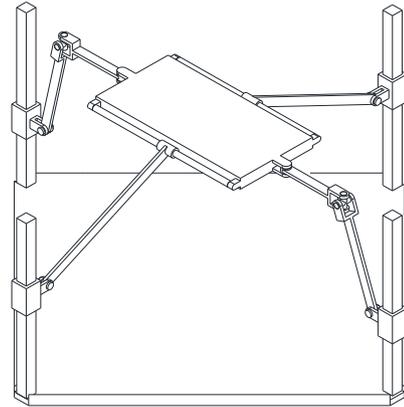


Figure 7 The 3-translation parallel mechanism with open-loop branch chains

The kinematic pairs of some branch chains were represented using symbols to facilitate the subsequent calculations, as shown in Figure 8.

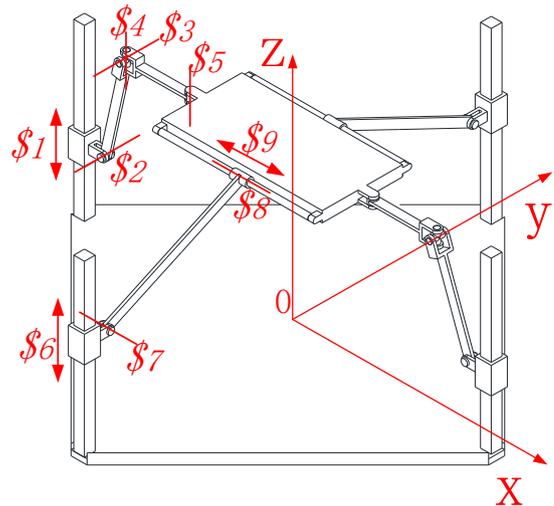


Figure 8 Parallel mechanism and associated symbols and coordinates

The screw theory was used to verify the freedom property:

The basic motion screw system of branch chain 1 is as follows:

$$S_1 = (0, 0, 0; 0, 0, 1);$$

$$S_2 = (0, 1, 0; -b, 0, -a);$$

$$S_3 = (0, 1, 0; -d, 0, -c);$$

$$S_4 = (0, 0, 1; 0, c, 0);$$

$$S_5 = (0, 0, 1; 0, e, 0);$$

Further, the constraint screw of branch chain 1 is:

$$S_1^r = (0, 0, 0; 1, 0, 0)$$

The basic motion screw system of branch chain 2 is:

$$S_6 = (0, 0, 0; 0, 0, 1);$$

$$S_7 = (1, 0, 0; 0, g, f);$$

$$S_8 = (1, 0, 0; 0, d, h);$$

$$S_9 = (0, 0, 0; 1, 0, 0);$$

The constraint screw of branch chain 2 is:

$$S_2^r = (0, 0, 0; 0, 1, 0);$$

$$S_3^r = (0, 0, 0; 0, 0, 1);$$

Since branch chains 3 and 4 are the symmetric forms of branch chains 1 and 2, the branch chains 3 and 4 constraint screws are the same as their branch chain 1 and 2 counterparts respectively. Hence, the constraint screw system of the moving platform is defined as follows:

$$S_1^r = (0, 0, 0; 1, 0, 0);$$

$$S_2^r = (0, 0, 0; 0, 1, 0);$$

$$S_3^r = (0, 0, 0; 0, 0, 1);$$

$$S_4^r = (0, 0, 0; 1, 0, 0);$$

$$S_5^r = (0, 0, 0; 0, 1, 0);$$

$$S_6^r = (0, 0, 0; 0, 0, 1);$$

The three constraint screw expressed above represent the couple constraints in X, Y, and Z directions; therefore, the mechanism loses three rotational DoF.

According to reciprocity relation of the screw, its motion can be written as:

$$S_1 = (0, 0, 0; 1, 0, 0);$$

$$S_2 = (0, 0, 0; 0, 1, 0);$$

$$S_3 = (0, 0, 0; 0, 0, 1);$$

The screw motion shown above represents the freedom of movement in X, Y, and Z directions, meaning that it is a 3-translational mechanism.

6.2 Analysis of mechanism 2

A new mechanism from Table 16 was selected for DOF verification, as shown in Figure 9.

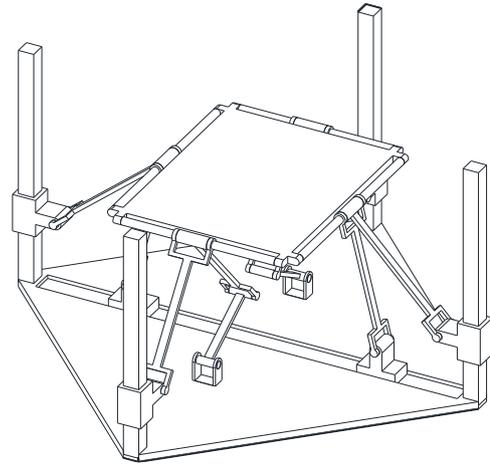


Figure 9 A novel parallel mechanism assembled using Table 16 data

The closed-loop branch chain was simplified to the open-loop branch chain before the calculation, while the DoF property of the simplified branch chain remained unchanged. The results are shown in Table 19.

Table 19. Branch chain simplification

Before the simplification	After the simplification

The simplified branch chains were assembled to obtain a 3-translation parallel mechanism with the open-loop branch chains structure, as shown in Fig. 10.

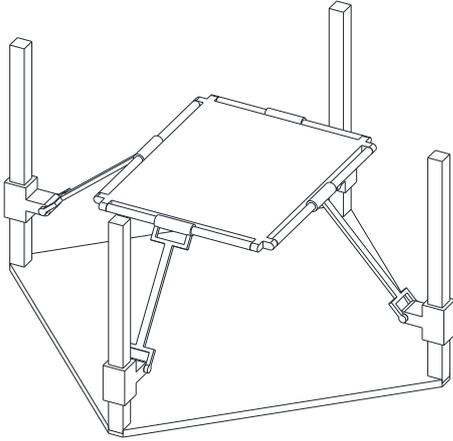


Figure 10 The 3-translation parallel mechanism with open-loop branch chains

The kinematic pairs of some branch chains were represented using symbols to facilitate the subsequent calculations, as shown in Figure 11.

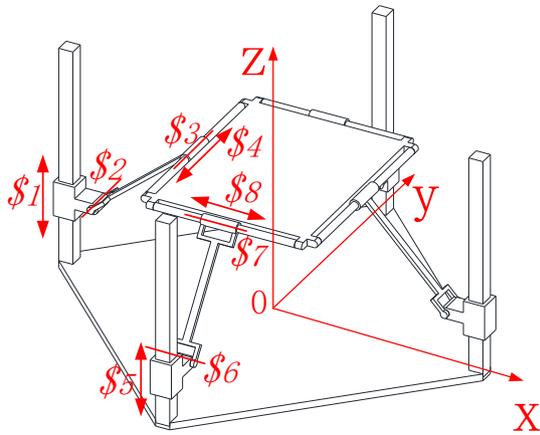


Figure 11 Parallel mechanism and associated symbols and coordinates

The screw theory was used to verify the freedom property:

The basic motion screw system of branch chain 1 is as follows:

$$\begin{aligned} \$_1 &= (0, 0, 0; 0, 0, 1); \\ \$_2 &= (0, 1, 0; -b, 0, -a); \\ \$_3 &= (0, 1, 0; -d, 0, -c); \\ \$_4 &= (0, 0, 0; 0, 1, 0); \end{aligned}$$

Further, the constraint screw of branch chain 1 is:

$$\begin{aligned} \$_1^r &= (0, 0, 0; 1, 0, 0); \\ \$_2^r &= (0, 0, 0; 0, 0, 1); \end{aligned}$$

The basic motion screw system of branch chain 2 is:

$$\begin{aligned} \$_5 &= (0, 0, 0; 0, 0, 1); \\ \$_6 &= (1, 0, 0; 0, f, e); \\ \$_7 &= (1, 0, 0; 0, d, g); \\ \$_8 &= (0, 0, 0; 1, 0, 0); \end{aligned}$$

The constraint screw of branch chain 2 is:

$$\begin{aligned} \$_3^r &= (0, 0, 0; 0, 1, 0); \\ \$_4^r &= (0, 0, 0; 0, 0, 1); \end{aligned}$$

Since branch chains 3 and 4 are the symmetric forms of branch chains 1 and 2, the branch chains 3 and 4 constraint screws are the same as their branch chain 1 and 2 counterparts respectively. Hence, the constraint screw system of the moving platform is defined as follows:

$$\begin{aligned} \$_1^r &= (0, 0, 0; 1, 0, 0); \\ \$_2^r &= (0, 0, 0; 0, 0, 1); \\ \$_3^r &= (0, 0, 0; 0, 1, 0); \\ \$_4^r &= (0, 0, 0; 0, 0, 1); \\ \$_5^r &= (0, 0, 0; 1, 0, 0); \\ \$_6^r &= (0, 0, 0; 0, 0, 1); \\ \$_7^r &= (0, 0, 0; 0, 1, 0); \\ \$_8^r &= (0, 0, 0; 0, 0, 1); \end{aligned}$$

The three constraint screw expressed above represent the couple constraints in X, Y, and Z directions; therefore, the mechanism loses three rotational DoF.

According to reciprocity relation of the screw, its motion can be written as:

$$\begin{aligned} \$_1 &= (0, 0, 0; 1, 0, 0); \\ \$_2 &= (0, 0, 0; 0, 1, 0); \\ \$_3 &= (0, 0, 0; 0, 0, 1); \end{aligned}$$

The screw motion shown above represents the freedom of movement in X, Y, and Z directions, meaning that it is a 3-translational mechanism.

7. Conclusion

(1) A novel type synthesis method for 3-translational redundancy actuated parallel mechanisms with closed-loop branch chains was proposed. The new mechanism has four symmetrically arranged branch chains, which is favorable for subsequent kinematic and dynamic analyses. Finally, the new mechanism has good mechanical properties.

(2) Based on the number of constraints included in

the branch chain, the redundant driven branch chain was divided into two categories. The first category includes rotational constraints in one direction, while the other category contains rotational constraints in both directions. By observing various combinations of the two branch chain types, branch chain distribution schemes can also be divided into two types. Furthermore, at least 500 and 400 3-translation redundant actuated parallel mechanisms with closed-loop branch chains can be summarized using two allocation schemes. The degrees of freedom of the representative mechanisms were tested using the screw theory.

(3) The proposed method combines a large number of new 3-translation parallel mechanisms, whose branch chains contain a closed-loop structure and are symmetrically distributed. The mechanisms are driven and controlled by several driving pairs. With high stiffness and strong bearing capacity, a high-impact and high-load can be applied, such as handling and sorting of goods, large-scale parts precision machine tools, and vibration isolation systems of large-scale engineering machinery.

8 Declaration

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Availability of data and materials

The datasets supporting the conclusions of this article are included within the article.

Authors' contributions

The author's contributions are as follows: YL provided theoretical guidance; HSJ conceived the basic idea, and carried out research, analysis and writing of the manuscript; TZ was in charge of drawing and typesetting; KX was in charge of three-dimensional modeling; HJ was in charge of calculation and verification; LZ contributed the revise of this paper. All authors read and approved the final manuscript.

Competing interests

The authors declare no competing financial interests.

Consent for publication

Not applicable

Ethics approval and consent to participate

Not applicable

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Appendix

Appendix and supplement both mean material added at the end of a book. An appendix gives useful additional information, but even without it the rest of the book is complete: In the appendix are forty detailed charts. A supplement, bound in the book or published separately, is given for comparison, as an enhancement, to provide corrections, to present later information, and the like: A yearly supplement is issue.