A Novel Multi-Criteria Decision Analysis Technique Considering Various Essential Characteristics

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A Novel Multi-Criteria Decision Analysis Technique Considering Various Essential Characteristics

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Abstract

This paper has proposed a novel Multi-Criteria Decision Analysis (MCDA) technique that considers relationships among the criteria, relationships among the alternatives, relationships among the criteria and the alternatives, the uncertainty or dilemma that the decision makers face in their decision making, the entropy among the criteria. The dilemma of the decision makers has been captured through the use of Hesitant Fuzzy Elements; the information content among the criteria has been captured by applying the concept of entropy through the application of a technique called IDOCRIW. A kind of sensitivity analysis has been performed to verify the effectiveness of the proposed technique. The proposed method has also been compared with four different types of already existing MCDA techniques, AHP, MAUT, MACBETH and MOORA. Both the sensitivity analysis and the comparison with other methods establish the effectiveness of the proposed technique.

Keywords: Novel MCDA Technique; IDOCRIW; Hesitant Fuzzy Elements; Spearman's rank correlation

1 Introduction

The existing literature shows vast variety of Multi-Criteria Decision Analysis (MCDA) techniques – the benchmark techniques (Saaty, 1980; Brans and Mareschal, 2005; Behzadian et al., 2012; Saaty, 2004; Figueira et al., 2010) and their various modifications, other less frequently applied techniques and the hybridization among various MCDA techniques and with the other techniques (Nixon et al., 2013; Oztaysi, 2014). Some of these methods are distance based techniques (such as, TOPSIS), some are pair-wise comparison based techniques (such as, PROMETHEE), and some are utility based techniques (such as, MAUT). Besides, there are techniques for calculating weights of the criteria for a MCDA problem (such as, IDOCRIW). The existing literature shows techniques which endeavored to establish relations among the alternatives and the relationships among the criteria, (such as, AHP, ANP). Therefore, the existing literature highlights some essential requirements for any MCDA techniques. The most important among those are the relationships among the alternatives, relationships among the criteria, relationships between the alternatives and the criteria, the information content among the criteria, uncertainty and dilemma in assigning the weights to the criteria by the decision makers, and unbiased assignment of the weights to the criteria. In search of the better MCDA techniques over the previously proposed techniques, the researchers all over the world are still proposing significant number of techniques.

However, the most important issue regarding the application of these techniques in the practical applications is the difficulty to identify the best technique for a given problem under study. The

difficulty arises because of the fact that the application of different MCDA techniques may lead to different rankings and thus the most appropriate technique for a problem cannot be identified. Although, no universal method to identify the most suitable technique for a particular problem is possible since every practical problem has its own characteristics regarding the decision matrix elements, the type of criteria and the types of alternatives, but the existing literature shows significant number of methods to compare among these techniques. However, some of these techniques are the measures of rank correlations which basically establish the associations among the rankings; some are the sensitivity analysis which basically verifies how robust a particular ranking is; some are based on the differentiation in terms of the methods and the characteristics of these techniques. However, there is very few papers which endeavored to propose methods which can identify a technique as the most suitable technique among several other techniques.

The contribution of this paper is to propose a novel MCDA technique which establishes the relationships among the alternatives, relationships among the criteria, relationships among the alternatives and the criteria, application of a well-recognized technique to determine the weights of the criteria which considers the entropy or information content in the criteria, capturing the uncertainty or dilemma of the decision makers in deciding the elements of the decision matrix. In other words, this paper proposes a MCDA technique which endeavors to incorporate all the required features in the proposed technique. The relationships among the criteria and among the alternatives are captured by calculating covariance matrices; the relationships among the alternatives and the criteria are captured through matrix of variances; the weights of the criteria is determined by partial application of an already existing technique called IDOCRIW which captures the entropy in the criteria; the dilemma of the decision makers is captured through the application of Hesitant Fuzzy Elements.

The following sections are organized in the following way – Section 2 presents some preliminary ideas which will be applied in this paper; section 3 reviews the existing literature on various aspects of this paper; section 4 presents a case study, based on which all the experimentations are done in this paper; section 5 presents the proposed MCDA technique; section 6 applies the proposed technique on the case study; Section 7 analyzes the proposed MCDA technique; section 8 ranks the alternatives by the four already existing techniques; section 9 compares the proposed technique with the other already existing techniques as mentioned in section 8; section 10 concludes this paper.

2 Preliminaries

Before proceeding to the next section, this section depicts some preliminary concepts which will be applied in this paper – the four different MCDA techniques with which the proposed technique has been compared; IDOCRIW technique which has been applied partially in order to determine the weights of the criteria; Spearman's rank correlation which has been used for comparing the proposed technique with the four other already existing techniques; a method as proposed by Bandyopadhyay (2021) which is capable to identify the most suitable technique for a given problem.

2.1 AHP

AHP

1. **Get the decision matrix and deiced the weight of the criteria**
2. **For each criterion *c*, perform pair-wise comparison between each pair of alternatives either as received as input from the decision makers or by following the expression** $r_{ij}^{c} = a_{ij} - a_{ij}$
3. **Calculate the rating values** R_i^c **for each alternative i based on the matrix of comparison as obtained from step 2.**
4. **Calculate weighted average rating for each alternative by expression** $R_i = \sum_c R_i^c \times w_c$ **where** w_j **is the weight of criterion *j***
5. **Rank the alternatives in the descending order of** R_i **values with the alternative having the highest** R_i **value receiving the highest rank.**

Figure 1: Algorithm of AHP

Figure 1 shows the algorithm of AHP (Analytic Hierarchy Process) as proposed by Saaty (1980). AHP is based on pair-wise comparison between each pair of alternatives for particular criteria. Thus, for each criterion, a matrix of pair-wise comparison between each pair of alternatives is obtained. Then, the aggregate rating for each alternative is obtained for each of the matrices of pair-wise comparison. The ranking of the alternatives is done based on the weighted sum of these average ratings in the descending order of values.

2.2 MAUT

MAUT

1. **Get the decision matrix and deiced the weight of the criteria**
2. **Normalize the decision matrix by following the expression** $a_{ij}^{n} = \dfrac{a_{ij} - \min_i a_{ij}}{\max_i a_{ij} - \min_i a_{ij}}$ **for the benefit type of criteria or by following the expression** $a_{ij}^{n} = 1 + \left(\dfrac{\min_i a_{ij} - a_{ij}}{\max_i a_{ij} - \min_i a_{ij}} \right)$ **for the cost type of criteria.**
3. **Calculate the Marginal Utility Score (MUS) by following the expression** $MUS_{ij} = \dfrac{e^{(a_{ij}^{n})^2} - 1}{1.71}$
4. **Find the aggregate** MUS_i **by following the expression** $MUS_i = \sum_j MUS_{ij} \times w_j$ **where** w_j **is the weight of criterion *j***
5. **Rank the alternatives in the descending order of aggregate marginal utility values with the alternative having the highest aggregate marginal utility value receiving the highest rank.**

Figure 2: Algorithm of MAUT

Figure 2 shows the algorithm of MAUT (Multi-Attribute Utility Theory) (Emovon et al., 2016) is based on the calculation of aggregate marginal utility score for each alternative. Marginal utility scores are calculated by certain exponential expression as shown in step 3 of Figure 2, based on the weighted normalized elements of the decision matrix.

2.3 MACBETH

MACBETH

1. **Get the decision matrix and deiced the weight of the criteria**
2. **Identify the maximum values $r_{j,\max}$ for the benefit type of criteria and minimum values $r_{j,\min}$ for the cost type of criteria. These values are known as Reference values.**
3. **Determine Macbeth score M_{ij} by following the expression $M_{ij} = \frac{a_{ij} - r_{j,\min}}{r_{j,\max} - r_{j,\min}} \times (100 - 0)$**
4. **Calculate overall Macbeth score for each alternative by following the expression $OMS_i = M_{ij} \times w_j$ where w_j is the weight of criterion *j***
5. **Rank the alternatives in the descending order of OMS_i with the alternative having the highest OMS_i receiving the highest rank.**

Figure 3: Algorithm of MACBETH

Figure 3 shows the algorithm of MACBETH (Measuring Attractiveness by a Categorical Based Evaluation TecHnique) (Bana e Costa and Chagas, 2004) which ranks the alternatives based on the aggregate MACBETH score for each alternative. MACBETH score for each alternative is calculated by the expression as mentioned in step 3 in Figure 3, for each element of the decision matrix based on a reference value for each criterion.

2.4 MOORA

MOORA

1. **Get the decision matrix and deiced the weight of the criteria**
2. **Normalize the decision matrix by following the expression, $a_{ij}^n = \frac{a_{ij}}{\sum_i a_{ij}^2}$**
3. **Find the weighted normalized decision matrix by following the expression, $a_{ij}^w = a_{ij}^n \times w_j$ where w_j is the weight of criterion *j***
4. **Find the reference points for each criterion. The reference point for each of the benefit type of criteria is the maximum $R_j = a_{j,\max}$ of all the values for that criterion. Similarly, the reference point for each of the cost type of criteria is the minimum $R_j = a_{j,\min}$ of all the values for that criterion.**
5. **Calculate the assessment values by following the expression, $v_{ij} = \frac{a_{ij} - R_j}{n}$**
6. **Find the maximum value $v_{i,\max}$ of v_{ij} for each alternative**
7. **Rank the alternatives in the descending order of $v_{i,\max}$ with the alternative having the highest $v_{i,\max}$ receiving the highest rank.**

Figure 4: Algorithm of MOORA

Figure 4 shows the algorithm of MOORA (Multi-Objective Optimization Ratio Analysis) (Brauers et al., 2006). MOORA is a very simple method which first identifies a reference point for each criterion based on the weighted normalized decision matrix. Based on this reference point, assessment values are calculated for each element of the decision matrix. The maximum values of these assessment values for each alternative are used to rank the alternatives in the descending order of the maximum assessment values of the alternatives.

2.5 IDOCRIW

IDOCRIW (Integrated Determination of Objective CRIteria Weights) (Alinezhad and Khalili, 2019) is a type of multiple criteria based technique that is used to measure the weights of the criteria. Significant number of researchers, as evident from the existing literature has applied

IDICRIW technique in order to measure weights of criteria. The basic feature of this technique is that this technique captures use of the entropy or the information content among the criteria. This paper has also made partial use of this technique in measuring the weights of the criteria rather than using the random assignment of the weights of the decision makers. The portion of IDOCRIW technique that has been applied in this paper is depicted below.

At first, the normalized value for each element of the decision matrix is calculated by expression (1). Next, the entropy of each of the criteria is captured by the expression (2). Now, the deviation rate of the entropy is calculated by expression (3). The normalized values of these deviations are taken to be the weights of the criteria.

$$\bar{a}_{ij} = \frac{a_{ij}}{\sum_i a_{ij}} \tag{1}$$

$$entropy_j = -\frac{1}{\ln n}\sum_i \bar{a}_{ij} \times \ln \bar{a}_{ij} \tag{2}$$

$$deviation_j = 1 - entropy_j \tag{3}$$

2.6 Spearman's Rank Correlation

This paper has made use of Spearman's rank correlation (Sheskin, 2004) in order to establish the association among various rankings as obtained from various MCDA techniques as considered in this paper. The expression for Spearman's rank correlation is shown in expression (4).

$$\tau = 1 - \frac{6\sum d^2}{n(n^2-1)} \tag{4}$$

Where, d is the difference in rankings between MCDA techniques; n is the number of alternatives. Generally, the value of τ lies +1 and -1. The value $\tau = +1$ indicates perfect positive association between two variables which means if one of those variables increases then the other also increases. The value $\tau = -1$ indicates perfect negative association meaning, if one increases then the other decreases.

2.7 Method as Proposed by Bandyopadhyay (2021)

The use of rank correlations and some other methods to compare among the MCDA techniques only establishes the associations and thus, are unable to identify the most appropriate MCDA technique for the problem under study. Towards this direction, the method of comparison as proposed by Bandyopadhyay (2021) is a performance-based method of comparison, which at first, identifies the highest weighted criterion and then, identifies the MCDA technique as the most appropriate technique for the problem under study based on the cumulative values of the decision elements upto m number of sorted alternatives, where m is the number of best alternatives to be chosen by the decision maker. The method is very simple and identifies the most beneficial MCDA technique for the problem at hand. For the detailed understanding of the method, the work of Bandyopadhyay (2021) may be consulted.

3 Literature Review

The topic of this paper contains several components – Multi-Criteria Decision Analysis Techniques (MCDA), Hesitant Fuzzy Elements (HFE), IDOCRIW technique, and comparison among MCDA techniques. Therefore the following subsections reviews the related existing literature on each of these topics.

3.1 Review on Multi-Criteria Decision Analysis Techniques (MCDA)

Multi-Criteria Decision Analysis (MCDA) techniques are widely practiced among the researchers belonging to Scientific, Technical and Management fields of study. These techniques are so popular since the real world decision making is abundant with complexity. In most cases, decision makers face situations where they are to choose any alternative from among several alternatives, based on certain conditions (criteria). Such decision making happens both in our everyday life and industrial scenarios. Therefore, the researchers all over the world have found significant number of MCDA techniques for several decades (Alinezhad and Khalili, 2019; Ishizaka and Nemery, 2013; Tzeng and Huang, 2011). Each of the MCDA techniques has its own characteristics and applicability. Because of the variety in the applicability of these techniques, it becomes difficult to choose the most appropriate technique for a given problem at hand. However, there are some MCDA techniques that are frequently applied and some are not so frequently applied techniques. The basic features of some of these techniques are enlisted in Table I.

Table I: Basic Features of MCDA Techniques

MCDA Technique	Authors	Feature
AHP (Analytic Hierarchy Process	Saaty, 1980	This technique is based on comparison matrix containing comparison value among each of the pairs of alternatives for each criterion. The final ranking of the alternatives is done based on some kind of aggregation of the values for each alternative.
PROMETHEE (Preference Ranking Organization METHod for Enrichment of Evaluations)	Brans and Mareschal, 2005	This technique is based on pair-wise comparison between each pair of alternatives for each criterion followed by the weighted sum of those comparisons. The final ranking is based on the difference between how each alternative is preferred over other alternatives and how inferior, each alternative is, compared to other alternatives.
TOPSIS (Technique of Order Preference Similarity to the Ideal Solution)	Behzadian et al., 2012	This technique is based on the aggregate Euclidian distance measure between the weighted normalized elements of the decision matrix and the best and the worst values for each criterion. The final ranking is based on the ratio of relative aggregate distance from the worst solutions to the sum of distances from the worst and the best solutions.
ANP (Analytic Network Process)	Saaty, 2004	This technique is based on both the relationship among the alternatives and among the criteria. These relationships form the weighted super matrix which is raised to certain power following Markov process, until stable values are

		obtained. These values are used to rank the alternatives.
ELECTRE (ELimination Et Choix Traduisant la REalite)	Figueira et al., 2010	This technique is based on the calculation of domination matrix, concordance matrix and discordance matrix. This technique has several major versions like ELECTRE I, ELECTRE II, ELECTRE III and some more.
MAUT (Multi-Attribute Utility Theory)	Emovon et al., 2016	This technique calculates marginal utility score for each element of the decision matrix based on the weighted normalized elements. Final ranking of the alternatives is done based on the final utility score of each alternative, which is actually the weighted sum of marginal utility scores for each alternative.
MACBETH (Measuring Attractiveness by a Categorical Based Evaluation TecHnique)	Bana e Costa and Chagas, 2004	This technique calculates MACBETH score for each element of the decision matrix based on reference levels for each criterion. The final ranking of the alternatives is done based on the overall MACBETH score for each alternative, which is actually the weighted sum of MACBETH scores of the elements for each alternative.
SMART (Simple Multi-Attribute Rating Technique)	Edwards and Barron, 1994	Simple technique based on logarithmic calculation and geometric progression.
REGIME	Hinloopen and Nijkamp, 1986	This technique is based on the identifying the superior criteria, separate ranking of alternatives based on individual criteria, forming the REGIME matrix based on pair-wise comparison.
ORESTE	Roubens, 1982	This technique is based on distance measurement. Block distance for each alternative is calculated as a weighted sum based on a position matrix. The weights are termed as succession rate.
VIKOR	Opricovic and Tzeng, 2002	Here, the best and worst values for each criterion are calculated at first. Based on that, weighted sum of relative distances of the elements from the best values are calculated for each of the alternatives. This helps in calculating VIKOR index which is used to rank the alternatives.
EVAMIX (EVAluation of MIXed data)	Voogd, 1983	In this technique, superiority rate for each alternative is calculated, and it helps to determine differential matrix, which in turn, helps to determine total dominance for each alternative. The final ranking is done based on this total dominance.
ARAS (Additive Ratio ASsessment)	Zavadskas et al., 2010	This technique calculates optimality function for each alternative based on the weighted normalized decision matrix. The final ranking of alternatives is done based on utility degree for each alternative which is calculated based on optimality function.
MOORA (Multi-Objective	Brauers et al., 2006	This is a very simple technique in which final ranking of alternatives are done by calculating the assessment value for

Optimization Ratio Analysis)		each alternative, which is calculated based the difference between the weighted sums of benefit and cost attributes (criteria).
COPRAS (COmplex PRoportional ASsessment)	Zavadskas et al., 2007	This technique calculates the sums of weighted normalized elements of decision matrix separately for benefit and cost type of attributes. These values are used to calculate the relative significance values for each of the alternatives and the final ranking is done on this basis.
WASPAS (Weighted Aggregates Sum Product Assessment)	Zavadskas et al., 2013	This technique calculates additive and multiplicative relative importance for each alternative based on weighted normalized elements of decision matrix. The final ranking of alternatives is done based on joint generalized criterions which are calculated based on additive and multiplicative relative importance.
TODIM	Gomes, 2009	This technique calculates dominance degree for each alternative followed by overall dominance degree based on relative weights. Final ranking of alternatives is done based on the overall dominance degree of the alternatives.
EDAS (Evaluation based on Distance from Average Solution)	Keshavarz et al., 2017	This technique first calculates the average solution for each attribute (criterion) and then calculates positive and negative distances for the benefit and cost criteria respectively. Then the weighted sum of these distances is calculated for each alternative followed by the overall appraisal score for each alternative, which is used to rank the alternatives.
MABAC (Multi-Attributive Border Approximation area Comparison)	Bozanic et al., 2016	This technique calculates Border Approximation Area (BAA) for each criterion based normalized decision matrix elements. Then the distances of each element from BAA is calculated and aggregated for each alternative, on the basis of which, the final ranking of the alternatives is done.

Table 1 shows certain patterns or view towards proposing different MCDA techniques. For example, some techniques are based on distance measurements, some are utility function based, some are based on pair-wise comparison, and some are based on the relationships among the alternatives and among the criteria. There are also numerous other methods, their modifications and hybrid techniques available in the existing literature (Liu et al., 2013; Collan et al., 2013; Tavana et al., 2013; Peng and Xiao, 2013; Oztaysi, 2014). This paper therefore considers the most important among these approaches – the relationships. There is no single research study that has considered relationship among alternatives, relationships among criteria, along with relationships among the alternatives and the criteria. This paper fills this gap of research by considering the relationships among the alternatives, among the criteria, between the alternatives and the criteria, the uncertainty in deciding the elements and the criteria on the part of the decision makers and the information content in the criteria.

3.2 Review on Hesitant Fuzzy Elements (HFE)

Multiple Criteria Decision Analysis (MCDA) with HFE is a very demanding topic as evident from the existing literature. In Hesitant Fuzzy Set (HFS), the membership function for an element is defined by multiple values. MCDA with HFE has been applied by many researchers such as, the work by Huchang et al. (2020), Sellak et al. (2018), Gou et al. (2017) and so on. The primary purpose of HFS and HFE is that it can incorporate the confusion of decision makers. Therefore, HFE has wide applications in the fields of Management and Technology. Recently, the existing literature shows some MCDA techniques based on HFE. Some of the those research studies include the research studies of Jibin (2017), Wang et al. (2015), Wang et al. (2014), Chen and Hong (2014). However, each of these research studies has some negativity which will be covered in this paper. Jibin (2017) determined priority degrees between each pair of alternatives based on Hesitant Fuzzy Decision matrix and ranked the alternatives based on the aggregate priority degrees for the alternatives. Wang et al. (2015) proposed an outranking approach similar to ELECTRE based on HFE and Hausdorff distance which in turn, helped to find the dominance relations for the alternatives and the ranks for the alternatives. Wang et al. (2014) also proposed an outranking MCDA technique combining HFS. Chen and Hong (2014) used both HFS and aggregation of normal fuzzy set and combined these in order to propose an MCDA ranking.

However, the above proposed techniques did not consider other factors in addition to treating the decision makers' dilemma with HFS. The other factors include several other required features of MCDA techniques, such as considering the relationships among the alternatives, considering relationships among the criteria, considering the relationships among the alternatives and the criteria, considering the information content in the criteria and a logical reliable method for determining the weights of the criteria instead of receiving random priority values for the criteria for deriving the weights for the criteria. This paper considers all these features in the proposed MCDA technique.

3.3 IDOCRIW MCDA Technique

The existing literature is not as abundant with articles dealing with IDOCRIW technique as for other benchmark techniques like TOPSIS, AHP, ANP, PROMETHEE and alike. However, since IDOCRIW technique is basically a technique to determine the weights of criteria, thus there are some articles which have endeavored to applied IDOCRIW technique effectively. Some of these research studies are being discussed in this subsection.

Eghbali-Zarch et al. (2021) used IDOCRIW technique in order to calculate the weights of criteria and used those weights in the application of WASPAS MCDA technique in the managerial decision making for construction projects. Čereška et al. (2018) combined two different techniques – IDOCRIW and CILOS (Criteria Impact Loss) in order to determine weights of criteria and then used these weights to compare screw joints of different diameters and made of different materials based on four MCDA techniques, namely, EDAS, SAW, TOPSIS, COPRAS. Podvezko et al. (2020) and Zavadskas and Podvezko (2016) gave fuzzy orientation to both CILOS and IDOCRIW techniques before combining these techniques with a purpose to consider entropy in determining criteria weights. The weights of criteria were calculated by the combined techniques. Čereška et al. (2018) applied a part of IDOCRIW technique just like the current paper in order to consider the information content or entropy in the criteria weights. Vavrek and Bečica (2020) mentioned some techniques available in the existing literature for calculating the

weights of the criteria, like, ENTROPY, CRITIC,MW, SD, IDOCRIW, CV, IDP, or SVP. Some of the other research studies applying IDOCRIW include the research studies of Zavadskas et al. (2017), Dayyani et al. (2021). Therefore, the review of the existing literature on IDOCRIW shows the effective application of this technique in different applications. This paper applies similar approach of Čereška et al. (2018) to apply ODOCRIW technique partially in determining the weights of the criteria, rather than determining weights based on some random evaluation of criteria by the decision makers. The partial application considers the entropy of the criteria just like it was considered by Čereška et al. (2018).

3.4 Comparison among MCDA Techniques

The existing literature shows some methods of comparison among MCDA techniques. Some of these research studies are mentioned in this subsection. Triantaphyllou (1989) performed comparison among some MCDA techniques based on the methods adopted to calculate the criteria weights. Ishizaka and Nemery (2013) classified some MCDA techniques based on the applications of the techniques to different kinds of problem. Such comparison was not very effective in comparing the algorithms of the techniques. Ishizaka and Siraj (2018) compared three benchmark MCDA techniques based on the opinions of 146 participants and thus such comparison is a survey based comparison techniques in which there are always chances of biased opinions and thus, such method of comparison cannot be taken as a universal one. However, the existing literature basically applied different methods of rank correlation measurements in order to compare among different MCDA techniques.

For example, Moradian et al. (2019) applied both Graphical method and Spearman's rank correlation to compare the rankings as obtained from applying MOORA, TOPSIS and VIKOR. Zamani-Sabzi et al. (2016) applied both Kendall's tau-b and Spearman's rank correlation to perform comparison among different MCDA techniques, namely, SAW, WPM, CP, TOPSIS, VIKOR and four different versions of AHP. They also compared the results through bar diagrams. Moghassem (2013) compared TOPSIS with VIKOR by performing sensitivity analysis in terms of the changing of ranking order of the alternatives. Javaid et al. (2019) compared some MCDA techniques by performing sensitivity analysis by varying weights of the criteria. Ceballos et al. (2016) applied Spearman's rank correlation to compare among Multi-MOORA, TOPSIS and VIKOR. Özcan et al. (2011) compared AHP, TOPSIS, ELECTRE and Grey Theory by explaining the difference in terms of the characteristics of these techniques. Hodgett (2016) compared among MARE, AHP and ELECTRE III in terms of various characteristics of the techniques along with the time taken in execution. Hajkowicz and Higgins (2008) applied Spearman's rank correlation and Kendall's coefficient of correlation to compare among weighted summation, Compromise Programming, PROMETHEE II and EVAMIX. Selmi et al. (2013) had applied Gini Index, a concept borrowed from Economics, in order to perform comparison among MCDA techniques.

However, the most common methods of comparison among all the methods are the methods of rank correlation. Some other articles applying rank correlations include the research studies of Athawale and Chakraborty (2011); Chitsaz and Banihabib, 2015; Mathew and Sahu, 2018). However, many of these research studies have also acknowledged the fact that such rank correlations cannot identify the most suitable technique for a problem at hand (Athawale and

Chakraborty, 2011; Chitsaz and Banihabib, 2015; Mathew and Sahu, 2018; Sarraf and McGuire, 2020).

However, the thorough review of the existing literature on the comparison among MCDA techniques revealed the fact that the existing literature basically emphasized on comparing the MCDA techniques in terms of characteristics or in term of establishing associations through various rank correlations, or through various types of sensitivity analysis like rank reversal method, varying the weights of the criteria, addition or deleting alternatives and so on. None of these methods is capable to identify the most suitable MCDA technique for a particular problem. However, recently Bandyopadhyay (2021) have proposed a performance based method of comparison in which for a particular problem, decision maker can choose the most beneficial ranking which will maximize the benefit since that is the primary purpose of all the ranking techniques.

Therefore, this paper applies both Spearman's rank correlation which is a traditional method of comparison and the method by Bandyopadhyay (2021) in order to compare the ranking by the proposed MCDA technique with those of the other MCDA techniques as considered in this paper.

4 Case Study

A small company is taking decision on purchasing two cars for official purpose. A total of 11 different cars have been chosen. The determining factors (criteria) for selecting the two cars, as decided by the management are: price (cost criterion), mileage (benefit criterion), fuel tank capacity (benefit criterion), and maximum torque (benefit criterion). The benefit criteria are to be maximized and the cost criterion is to be minimized. The values as collected for the criteria against 11 different types of cars from various car websites are shown in Table II.

Table II: Raw Data Collected from Different Websites

Price (Lakhs)								Mileage (Kmpl)					
Car 1	22.58	22.57	26.97	27.35	22.55	27.35			15.38	18.6	12.95		12
Car 2	96.3	92.35	96.3	96.3	96.3			11.13	10	11.13	11	11.13	14
Car 3	5.35	5.41		6.33	4.66	6.19		21.79	21.79	22.5	21.79	22.5	21.5
Car 4	8.13	7.27	8.62	8.72	7.24	8.72		18.5	18.5	19	18.5	20.4	23
Car 5	32	27.49		30.87				18.6	17.01	15	16.65	16.65	14
Car 6	38.82	40	43.61	43.61					16.5	15.73	17.32		10
Car 7	43.06	38.5	45.7	42.9	37.2	42.9		14.82	10	20.68	14.82	14.82	20
Car 8	13.8	12.42	17.17	17.33	14.23	18.63		15	15.5		15.5	15	15
Car 9	19.81	15	22.35	22.34	17.93			25.35	19.5	16.5	26.8	24	
Car 10	8.6	8.63	12.12	12.29	8.63			20.45	20	18	21.04		
Car 11	12.58	10.99	16.44	16.62				13.83	18.5	14	14.1	14	
	(a)							(b)					

Fuel Tank Capacity							Maximum Torque (nm@rpm)				
Car 1		62	62	62	62		392	400		400	192

Car 2	87		87	87				410			410	410
Car 3	32	32	35	35	32			90	90	113	113	90
Car 4		42	40	40	42			215	119	112	215	120
Car 5	71	60	60	71	60			340	320	340	340	320
Car 6	64	64	64	64				380			380	380
Car 7	51		61	63	51	61		400	400		400	280
Car 8		60	60	60	60	55		319	319	200	319	320
Car 9	50		47	47	47			300			174	174
Car 10	50	50	50	50	50			245		200	200	200
Car 11	63	63	63	50	63			320	320		320	320
	(c)							(d)				

Each of the values of the above tables is normalized by either diving the values by the minimum of that row (for Criterion, Price) or by the maximum of that row (for all other criteria). These normalized values are taken as Hesitant Fuzzy Elements (HFEs). Table III shows the resultant HFEs. Some of the values in this table have been approximated to the nearest fractional values. Wang et al. (2014) approximated the HFEs for each alternative and each criterion by calculating the means. In this paper, median value for each set of HFEs has been calculated. These values have been used as the elements of normalized decision matrix (see Table IV).

Table III: Hesitant Fuzzy Elements

	Price	**Mileage**	**Fuel Tank Capacity**	**Maximum Torque**
Car 1	0.824,0.836,0.999	0.64,0.7,0.83	0.99	0.99
Car 2	0.959	0.71,0.78,0.8	0.99	0.99
Car 3	0.736,0.753,0.861,0.871	0.96,0.97	0.91,0.99	0.71,0.99
Car 4	0.83,0.84,0.891,0.996	0.8,0.83,0.89	0.95,0.99	0.99
Car 5	0.859,0.891	0.75,0.81,0.9,0.91	0.84,0.99	0.71,0.99
Car 6	0.89,0.97	0.58,0.91,0.95	0.99	0.99
Car 7	0.81,0.86,0.87,0.97	0.48,0.72,0.97	0.8,0.97,0.99	0.99
Car 8	0.67,0.72,0.87,0.90	0.99	0.92,0.99	0.7,0.9
Car 9	0.67,0.76,0.84	0.62,0.73,0.9,0.95	0.94,0.99	0.99
Car 10	0.7,0.71,0.997	0.86,0.95,0.97	0.99	0.99
Car 11	0.66,0.67,0.87	0.75,0.76	0.79,0.99	0.71,0.99

Table IV: Median Values Calculated from HFEs

	Price	**Mileage**	**Fuel Tank Capacity**	**Maximum Torque**
A1	0.836	0.7	0.99	0.73
A2	0.959	0.78	0.99	0.99
A3	0.807	0.965	0.95	0.8
A4	0.8655	0.83	0.97	0.55
A5	0.875	0.855	0.915	0.94
A6	0.93	0.91	0.99	0.99

A7	0.865	0.72	0.97	0.7
A8	0.795	0.99	0.955	0.81
A9	0.76	0.815	0.965	0.58
A10	0.71	0.95	0.99	0.82
A11	0.67	0.755	0.89	0.99

5 Proposed Algorithm

The proposed Multi-Criteria Decision Analysis Technique is depicted in Figure 5. Here, the decision matrix is normalized matrix as calculated from HFEs as shown in Table IV. Next, instead of receiving random preference values from the decision makers, this paper has applied IDOCRIW (Integrated Determination of Objective Criteria Weights) partially, in order to calculate the weights of the criteria. The procedure for calculating the weights is shown in Figure 6. Figure 6 shows the method to consider the entropy in calculating the weights of the criteria. Next, the Hesitant Fuzzy Elements (HFEs) are calculated for each criterion for each alternative as depicted in the Case Study section. These HFEs are aggregated by calculating median values following the work of Wang et al. (2014) who had calculated means instead of medians. Now, the weighted normalized matrix $D_{m\times n}$ is calculated by multiplying the calculated weights of the criteria by the aggregate values of the HFEs.

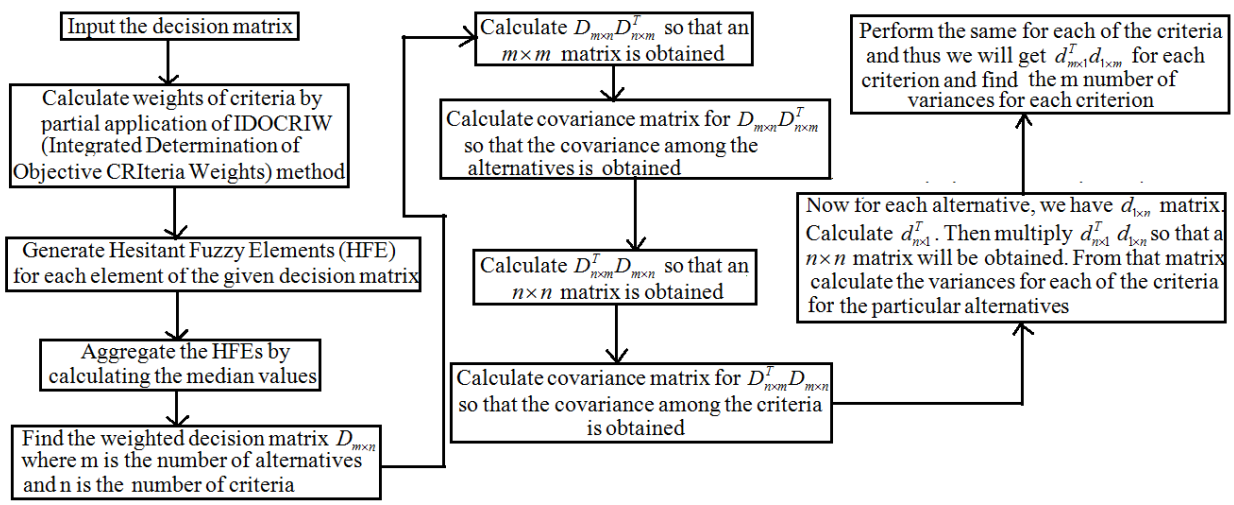

Figure 5: Overview of the Proposed Technique

1. **Normalize each element of the decision matrix by dividing each element under each criterion by the sum of values for that criterion. The normalized element is represented by** X_{ij}
2. **For each element in the decision matrix, calculate** $\bar{X}_{ij} = \ln X_{ij}$
3. **Perform the multiplication** $\bar{X}_{ij}^{e} = X_{ij} * \bar{X}_{ij}$ **for each element of the matrix**
4. **Calculate the sum** $Entropy_j = -\frac{1}{\ln n}\sum_{j=1}^{J}\bar{X}_{ij}^{e}$ **for each of criteria. So, a total of** J **number of weights for** J **number of criteria is calculated. Each of these** J **values I normalized by dividing each value by the total of all the** J **values**
5. **Calculate the deviation rate by the expression** $deviation_j = 1 - Entropy_j$
6. **The deviations are now normalized by dividing each value by the total of all the** J **deviation values**
7. **The resultant normalized** J **number of values is the weights of the criteria.**

Figure 6: Procedure to Calculate Weights of Criteria

Next, following the idea as adopted in Correspondence Analysis (CA) which is frequently used in social research studies, this paper calculates both $D_{m\times n}D_{n\times m}^{T}$ and $D_{n\times m}^{T}D_{m\times n}$ matrices in order to get two square matrices of sizes $m\times m$ and $n\times n$ respectively. Now, covariance matrix is calculated from the $m\times m$ matrix in order to get the covariances among the alternatives. Similarly, covariance matrix is calculated from the $n\times n$ matrix in order to get the covariances among the criteria. For each alternative, we have $d_{1\times n}$ matrix and thus the multiplication $d_{n\times 1}^{T}\, d_{1\times n}$ can be performed, from where variances for each of the criteria for the particular alternatives can be calculated. Similarly, the multiplication $d_{m\times 1}^{T}d_{1\times m}$ provides a matrix from which the variances for each of the alternatives for the particular criteria can be calculated. Next, a bigger matrix of size $(m+n)\times(m+n)$ with the help of covariance matrix for $D_{m\times n}D_{n\times m}^{T}$, covariance matrix for $D_{n\times m}^{T}D_{m\times n}$, variances calculated from $d_{n\times 1}^{T}\, d_{1\times n}$ matrix, variances calculated from $d_{m\times 1}^{T}d_{1\times m}$ matrix, is formed. Such formation of bigger matrix is similar to ANP technique except the fact that in the proposed MCDA technique, the relations among the criteria, relations among the alternatives and the relations among each criterion with the alternatives and the relations among each alternative with the criteria are all being considered. Finally, following Markov process, equilibrium values of this $(m+n)\times(m+n)$ matrix by raising to the powers until stable values are obtained. The first m values are taken for ranking the alternatives in the descending order of the values.

6 Application of Proposed MCDA Technique on the Case Study

In this section, the proposed MCDA technique has been applied on case study, on the HFE based decision matrix as shown in Table IV in section. At first, the weights of the criteria are calculated by the partial application of IDOCRIW (Integrated Determination of Objective Criteria Weights) technique as mentioned in the previous sections. Based on IDOCRIW method, each of elements of the decision matrix in Table IV is divided by the aggregate of the respective criterion (or column) in order to get the normalized decision matrix (See Table V below).

Table V: Normalized HFE Based Decision Matrix

	Price	Mileage	Fuel Tank Capacity	Maximum Torque
A1	0.092147	0.075512	0.093617	0.082022
A2	0.105704	0.084142	0.093617	0.111236
A3	0.08895	0.104099	0.089835	0.089888
A4	0.095398	0.089536	0.091726	0.061798
A5	0.096445	0.092233	0.086525	0.105618
A6	0.102508	0.098166	0.093617	0.111236
A7	0.095343	0.07767	0.091726	0.078652
A8	0.087627	0.106796	0.090307	0.091011
A9	0.08377	0.087918	0.091253	0.065169
A10	0.078258	0.102481	0.093617	0.092135
A11	0.07385	0.081446	0.084161	0.111236

Next, the degree of entropy for each criterion is calculated by expression as depicted in Figure 6 (Alinezhad and Khalili, 2019) and the resultant degree of entropies for the criteria are shown in Table VI. Next the deviation rates are calculated and normalized as depicted in Figure 6, in order to get the weights of the criteria (See Table VII). These weights are now multiplied with the elements of the HFE based decision matrix in order to get weighted decision matrix D of size $m \times n$ as shown in Table VIII. The next steps are to calculate DD^T and $D^T D$ in order to get two matrices of sizes $m \times m$ and $n \times n$ respectively, where m is the number of alternatives and n is the number of criteria. Now, the covariance matrices for these two matrices are calculated so as to get the covariances between each pair of alternatives and between each pair of criteria respectively. The covariance matrices are shown in Table IX and Table X respectively.

Table VI: Degree of Entropy for the Criteria

Price	Mileage	Fuel Tank Capacity	Maximum Torque
0.997915	0.997412	0.999878	0.992545

Table VII: Weights of the Criteria

Price	Mileage	Fuel Tank Capacity	Maximum Torque
0.170189	0.211279	0.009927	0.608606

Table VIII: Weighted Decision Matrix

Price	Mileage	Fuel Tank Capacity	Maximum Torque
0.142278	0.147895	0.00982728	0.44428211
0.163211	0.164798	0.00982728	0.60251957
0.137342	0.203884	0.00943022	0.4868845
0.147299	0.175361	0.00962875	0.3347331
0.148915	0.180643	0.00908279	0.57208929
0.158276	0.192264	0.00982728	0.60251957

0.147213	0.152121	0.00962875	0.42602394
0.1353	0.209166	0.00947985	0.49297056
0.129344	0.172192	0.00957912	0.35299127
0.120834	0.200715	0.00982728	0.49905662
0.114027	0.159516	0.00883463	0.60251957

Table IX: Covariance Matrix for the Alternatives

	A1	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**	**A10**	**A11**
A1	0.0018	0.0024	0.0019	0.0013	0.0022	0.0024	0.0017	0.0019	0.0014	0.00197	0.0024
A2	0.0024	0.0032	0.0026	0.0018	0.003	0.0032	0.0023	0.0026	0.0019	0.00266	0.0032
A3	0.0019	0.0026	0.0021	0.0015	0.0025	0.0026	0.0018	0.0021	0.0015	0.00216	0.0026
A4	0.0013	0.0018	0.0015	0.001	0.0017	0.0018	0.0013	0.0015	0.0011	0.0015	0.0018
A5	0.0022	0.003	0.0025	0.0017	0.0029	0.003	0.0022	0.0025	0.0018	0.00253	0.003
A6	0.0024	0.0032	0.0026	0.0018	0.003	0.0032	0.0023	0.0026	0.0019	0.00266	0.0032
A7	0.0017	0.0023	0.0018	0.0013	0.0022	0.0023	0.0016	0.0019	0.0013	0.00189	0.0023
A8	0.0019	0.0026	0.0021	0.0015	0.0025	0.0026	0.0019	0.0022	0.0016	0.00219	0.0026
A9	0.0014	0.0019	0.0015	0.0011	0.0018	0.0019	0.0013	0.0016	0.0011	0.00158	0.0019
A10	0.002	0.0027	0.0022	0.0015	0.0025	0.0027	0.0019	0.0022	0.0016	0.00221	0.0027
A11	0.0024	0.0032	0.0026	0.0018	0.003	0.0032	0.0023	0.0026	0.0019	0.00265	0.0032

Table X: Covariance Matrix for the Criteria

	Price	**Mileage**	**Fuel Tank Capacity**	**Maximum Torque**
Price	0.07527722	0.095526768	0.0050955	0.2738205
Mileage	0.09552677	0.121232147	0.00646638	0.3474754
Fuel Tank Capacity	0.0050955	0.006466376	0.00034492	0.0185348
Maximum Torque	0.27382047	0.347475415	0.01853478	0.9961993

Next, for each alternative, there is a set $d_{1\times n}$ of elements for the criteria. Similarly for each criterion, there is a set $d_{m\times 1}$ of elements for the alternatives. Based on the proposed technique, the matrix multiplications $d_{n\times 1}^{T} d_{1\times n}$ and $d_{m\times 1}^{T} d_{1\times m}$ are performed. For example, for the alternative A1 (Car 1), the relevant vector and the results of respective multiplication are shown in Table XI. The first row in Table XI shows the values of the criteria for alternative A1 (Car 1). Next, the results of matrix multiplication are shown along with the variance of each of the resultant rows. Similarly, for the other alternatives, the same calculations have been performed and the variances are accumulated. The same procedure is applied for each of the criteria and one of the results for criterion, Price along with the respective variances is shown in Table XII.

Table XI: Vector of Elements for A1 and Results of Multiplication $d_{n\times 1}^{T} d_{1\times n}$

	A1	0.142278	0.147895	0.009827	0.444282
	Price	**Mileage**	**Capacity**	**Torque**	**Variance**
Price	0.020243	0.021042	0.001398	0.063212	0.000682
Mileage	0.021042	0.021873	0.001453	0.065707	0.000737
Capacity	0.001398	0.001453	9.66E-05	0.004366	3.2E-06
Torque	0.063212	0.065707	0.004366	0.197387	0.006653

Table XII: Vector of Elements for 'Price' and Results of Multiplication $d_{m\times 1}^{T} d_{1\times m}$

Price	0.142277952	0.1632	0.137	0.1473	0.1489	0.158	0.147	0.135	0.129	0.121	0.114

											Variance
0.0202	0.023	0.0195	0.021	0.021	0.023	0.021	0.019	0.018	0.017	0.016	4.55E-06
0.0232	0.027	0.0224	0.024	0.024	0.026	0.024	0.022	0.021	0.02	0.019	5.98E-06
0.0195	0.022	0.0189	0.0202	0.02	0.022	0.02	0.019	0.018	0.017	0.016	4.24E-06
0.021	0.024	0.0202	0.0217	0.022	0.023	0.022	0.02	0.019	0.018	0.017	4.87E-06
0.0212	0.024	0.0205	0.0219	0.022	0.024	0.022	0.02	0.019	0.018	0.017	4.98E-06
0.0225	0.026	0.0217	0.0233	0.024	0.025	0.023	0.021	0.02	0.019	0.018	5.63E-06
0.0209	0.024	0.0202	0.0217	0.022	0.023	0.022	0.02	0.019	0.018	0.017	4.87E-06
0.0193	0.022	0.0186	0.0199	0.02	0.021	0.02	0.018	0.018	0.016	0.015	4.11E-06
0.0184	0.021	0.0178	0.0191	0.019	0.02	0.019	0.018	0.017	0.016	0.015	3.76E-06
0.0172	0.02	0.0166	0.0178	0.018	0.019	0.018	0.016	0.016	0.015	0.014	3.28E-06
0.0162	0.019	0.0157	0.0168	0.017	0.018	0.017	0.015	0.015	0.014	0.013	2.92E-06

Based on the proposed MCDA technique, the next step is to combine the matrices from Table IX, Table X and the respective variances for each alternative and for each criterion. The combined bigger matrix of size $(m+n)\times(m+n)$ is shown in Table XIII. For convenience in Table XIII, the criteria price, mileage, tank capacity and torque are being represented by C1, C2, C3, C4 respectively. Similarly the alternatives are being represented by A1, A2, …, A11 for CAR 1, CAR 2,…, CAR 11 respectively.

Table XIII: Combined Matrix

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	C1	C2	C3	C4
A1	0.0018	0.0024	0.0019	0.001	0.002	0.0024	0.002	0.002	0.0014	0.002	0.002	5E-06	1E-05	1E-11	0.0019
A2	0.0024	0.0032	0.0026	0.002	0.003	0.0032	0.002	0.003	0.0019	0.003	0.003	6E-06	1E-05	1E-11	0.0034
A3	0.0019	0.0026	0.0021	0.001	0.002	0.0026	0.002	0.002	0.0015	0.002	0.003	4E-06	2E-05	1E-11	0.0022
A4	0.0013	0.0018	0.0015	0.001	0.002	0.0018	0.001	0.001	0.0011	0.002	0.002	5E-06	1E-05	1E-11	0.0011
A5	0.0022	0.003	0.0025	0.002	0.003	0.003	0.002	0.003	0.0018	0.003	0.003	5E-06	1E-05	9E-12	0.0031
A6	0.0024	0.0032	0.0026	0.002	0.003	0.0032	0.002	0.003	0.0019	0.003	0.003	6E-06	2E-05	1E-11	0.0034
A7	0.0017	0.0023	0.0018	0.001	0.002	0.0023	0.002	0.002	0.0013	0.002	0.002	5E-06	1E-05	1E-11	0.0017
A8	0.0019	0.0026	0.0021	0.001	0.003	0.0026	0.002	0.002	0.0016	0.002	0.003	4E-06	2E-05	1E-11	0.0023
A9	0.0014	0.0019	0.0015	0.001	0.002	0.0019	0.001	0.002	0.0011	0.002	0.002	4E-06	1E-05	1E-11	0.0012
A10	0.002	0.0027	0.0022	0.002	0.003	0.0027	0.002	0.002	0.0016	0.002	0.003	3E-06	2E-05	1E-11	0.0023
A11	0.0024	0.0032	0.0026	0.002	0.003	0.0032	0.002	0.003	0.0019	0.003	0.003	3E-06	1E-05	8E-12	0.0034
C1	0.0007	0.0017	0.0008	4E-04	0.001	0.0016	7E-04	8E-04	0.0003	6E-04	9E-04	0.075	0.096	0.005	0.2738
C2	0.0007	0.0018	0.0017	5E-04	0.002	0.0024	7E-04	0.002	0.0006	0.002	0.002	0.096	0.121	0.006	0.3475
C3	3E-06	6E-06	4E-06	2E-06	5E-06	6E-06	3E-06	4E-06	2E-06	4E-06	5E-06	0.005	0.006	3E-04	0.0185
C4	0.0067	0.0237	0.0097	0.002	0.019	0.0234	0.006	0.01	0.0025	0.011	0.025	0.274	0.347	0.019	0.9962

Next, following the Markov Theory, the matrix in Table XIII is raised to the power until the stable values are obtained. The first *m* number of such values for *m* alternatives is taken and is ranked in the descending order of values. Thus the final values along with the respective ranks of the alternatives are shown in Table XIV.

Table XIV: Final Values and Ranks for Proposed MCDA Technique

Alternatives	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11
Final Values	0.184	0.229	0.012	0.618	0.026	0.035	0.029	0.02	0.033	0.035	0.025
Rank	3	2	11	1	8	5	7	10	6	4	9

7 Analysis of the Proposed MCDA Technique

The existing literature shows research studies which have performed sensitivity analysis for proposed MCDA techniques. In general, sensitivity analysis defines how robust a solution is. However, for MCDA techniques, there is “no consensus” about what should be the most appropriate sensitivity analysis for MCDA techniques (Mukhametzyanov and Pamučar, 2018). The reason lies in the facts that any change in criteria or alternatives or decision matrix is definitely going to make influence on the final ranking. Therefore, the concept of robustness is not applicable to the ranking obtained from MCDA techniques. Based on the requirements of the MCDA techniques, the best MCDA ranking for a particular problem is required to be found out. But before the application of such technique to verify the effectiveness of the proposed MCDA technique in later section, this section applies traditional sensitivity analysis in the form of rank reversal in order to establish the validity of the proposed technique.

Among the various sensitivity analysis as proposed in the existing literature, Mukhametzyanov and Pamučar (2018) checked the consistency among ten different MCDA techniques under a

total of eleven criteria. The authors performed statistical analysis of simulation experimentation, in terms of means and variances. Verification of consistency has also been considered by most of the researchers (Triantaphyllou and Mann, 1989; Pamučar et al., 2017). Yu et al. (2012) performed sensitivity analysis for the weightages of the attributes and for the uncertainty for the attributes. The application area for the sensitivity analysis is the selection of onshore environmentally friendly drilling systems. This paper applied a popular method called Rank Reversal. Li et al. (2013) also applied Rank Reversal by varying weights of the attributes for TOPSIS MCDA technique. However, there is significant number of articles on sensitivity analysis among which, the most popular technique is Rank Reversal technique. However, most of the rank reversal techniques have been applied on the already existing MCDA techniques as evident from the review of the existing literature. In this paper, sensitivity analysis for the proposed MCDA technique has been applied – 1) by varying the weights of the criteria, and 2) by varying some of the elements of the decision matrix. The following subsections depict all of these sensitivity analyses.

7.1 Ranks as Obtained by Varying Weights of Criteria

The randomly modified weights for the criteria are shown in Table XV. After applying the same proposed MCDA technique on the same decision matrix formed based on the modified weights of the criteria, the ranks of the alternatives are obtained as shown in the second column of Table XVII.

7.2 Ranks as Obtained by Varying Some Elements of Decision Matrix

In this case, some of the elements of the decision matrix have been modified randomly as shown in Table XVI. The modified values are indicated by bold and italic font. The resultant ranks of the alternatives are shown in the third column of Table XVII.

Table XV: Modified Weights of Criteria

Price	Mileage	Fuel Tank Capacity	Maximum Torque
0.60860563	0.21127889	0.170188937	0.00992655

Table XVI: Modified Decision Matrix

	PRICE	MILEAGE	FUEL TANK CAPACITY	MAXIMUM TORQUE
A1	***0.74***	0.7	0.99	0.73
A2	0.959	***0.79***	0.99	***0.9***
A3	0.807	0.965	***0.81***	0.8
A4	***0.8***	0.83	0.97	0.55
A5	0.875	***0.92***	0.915	0.94
A6	0.93	0.91	0.99	***0.83***
A7	0.865	0.72	0.97	0.7
A8	***0.95***	0.99	***0.91***	0.81
A9	0.76	***0.73***	0.965	0.58
A10	0.71	0.95	0.99	***0.72***

A11	0.67	0.755	***0.92***	0.99

Table XVII: Ranks Obtained from the Sensitivity Analysis

Original Rank	**Sensitivity Analysis 1**	**Sensitivity Analysis 2**
3	6	4
2	1	9
11	7	6
1	4	2
8	3	11
5	2	7
7	5	3
10	8	8
6	9	1
4	10	5
9	11	10

Two different methods of rank correlations have been applied in order to verify the association between the ranking as obtained from the proposed MCDA technique and each of the rankings as obtained by the first and second sensitivity analysis techniques. The rank correlation methods as obtained are Spearman's rank correlation, and Pearson's rank correlation. The results are shown in Table XVIII. Table XVIII shows low but positive association between the ranking from the proposed technique and each of the rankings obtained from the two sensitivity analyses. The positive value of rank correlation value indicates that the ranks of the proposed technique is positively associated with the rankings as obtained from each of the two types of modifications as applied. The low positive association can be justified by the fact that such drastic change of weights of the criteria and the randomly modified value are certainly going to make negative influences on the resultant rankings. However, the validity and guarantee of sensitivity analysis techniques have never been verified for MCDA techniques as commented by several research studies (Mukhametzyanov and Pamučar, 2018).

Table XVIII: Association between the Rankings

	Sensitivity Analysis 1	**Sensitivity Analysis 2**
Spearman's Rank Correlation	0.44	0.39
Pearson's Correlation	0.45	0.39

8 Ranking of the Alternatives by AHP, MAUT, MACBETH and MOORA

In this paper, a total four different types of MCDA techniques (AHP, MAUT, MACBETH and MOORA) have been considered for comparison with the proposed MCDA technique. Among these techniques, AHP is based on the comparison among the alternative for each criterion; MAUT is based on the utility scores of the alternatives; MACBETH is based on the relative distances of the elements of decision matrix from the best and worst values for each criterion; and MOORA is a simple technique which is also based on some kind of distances of the

elements from a reference point. These techniques have been chosen for comparison because of these varying natures of these methods. These methods have been applied on the data as obtained from the case study, after considering the Hesitant Fuzzy Elements.

At first, AHP has been applied and some of the results and the ranks are provided in Table XIX. Next, the relevant calculations based on the algorithms as depicted in the Introduction section of this paper, are provided in Table XX, Table XXI, for the techniques MAUT, MACBETH and MORA respectively. The ranks as obtained from these four MCDA techniques are going to be compared with that for the proposed MCDA technique in the following section.

Table XIX: Relevant Final Calculations and Rank for AHP

Alternatives	**Price**	**Mileage**	**Fuel Tank Capacity**	**Maximum Torque**	**Rank**
CAR 1	0.167903	0.254358	0.009639	0.674544	4
CAR 2	0.146368	0.22827	0.009639	0.497391	10
CAR 3	0.173937	0.184508	0.010045	0.615522	6
CAR 4	0.16218	0.214519	0.009838	0.895304	1
CAR 5	0.16042	0.208246	0.01043	0.523848	9
CAR 6	0.150932	0.19566	0.009639	0.497391	11
CAR 7	0.162274	0.247292	0.009838	0.703453	3
CAR 8	0.176563	0.179849	0.009993	0.607923	7
CAR 9	0.184694	0.218467	0.009889	0.848995	2
CAR 10	0.1977	0.187422	0.009639	0.600509	5
CAR 11	0.209503	0.235828	0.010722	0.497391	8

Table XX: Relevant Final Calculations and Rank for MAUT

Alternatives		**Price**	**Mileage**	**Fuel Tank Capacity**	**Maximum Torque**	**Rank**
CAR 1		6.389453	31.34395	1.004843	6.763721	3
CAR 2		31.34395	10.84425	1.004843	1.004843	5
CAR 3		4.551466	1.318053	3.566858	3.958306	9
CAR 4		9.133986	5.912295	1.883448	31.34395	1
CAR 5		10.27816	4.42426	11.91868	1.436407	8
CAR 6		21.00438	2.393439	1.004843	1.004843	6
CAR 7		9.0778	23.76201	1.883448	8.586599	4
CAR 8		3.967526	1.004843	3.033513	3.674245	10
CAR 9		2.68042	7.064037	2.205107	23.83583	2
CAR 10		1.552369	1.550049	1.004843	3.412072	11
CAR 11		1.004843	14.91544	31.34395	1.004843	7

Table XXI: Relevant Final Calculations and Rank for MACBETH and MOORA

Alternatives	**Aggregate MACBETH Score**	**Rank**
CAR 1	64.33428402	4

Final Values for MOORA	**Rank**
0.001407	6

CAR 2	15.29950557	10	0.002649	1
CAR 3	37.45024288	6	0.001139	7
CAR 4	78.2219728	2	0.001689	4
CAR 5	22.44252852	9	0.001782	3
CAR 6	7.536161335	11	0.002341	2
CAR 7	65.51752475	3	0.001684	5
CAR 8	34.90271269	8	0.00103	8
CAR 9	81.42762305	1	0.000724	9
CAR 10	41.09183749	5	0.000311	10
CAR 11	35.13242358	7	0	11

9 Comparison between Proposed MCDA Technique and Other Techniques

The proposed MCDA technique has been compared by different methods of comparison with other four MCDA techniques, AHP, MAUT, MACBETH and MOORA. The existing literature has shown various methods of comparison. All of these methods are basically based on establishing the association between different rankings as depicted in Literature Review section of this paper. Some of these techniques are Spearman's rank correlation, Kendall's tau, Kendall's coefficient of correlation (Sheskin, 2004). However, since the application of these techniques provide similar results, thus, this paper has only applied Spearman's rank correlation technique, instead of applying all these techniques. Table XXII shows the rankings as obtained by applying the proposed MCDA techniques and that for AHP, MAUT, MACBETH and MOORA.

Table XXII: Rankings as Obtained

Original Rank	**AHP**	**MAUT**	**MACBETH**	**MOORA**
3	4	3	4	6
2	10	5	10	1
11	6	9	6	7
1	1	1	2	4
8	9	8	9	3
5	11	6	11	2
7	3	4	3	5
10	7	10	8	8
6	2	2	1	9
4	5	11	5	10
9	8	7	7	11

Table XXIII: Application of Spearman's Rank Correlation

	Original Rank	*AHP*
Original Rank	1	
AHP	0.24	1

	Original Rank	*MAUT*
Original Rank	1	
MAUT	0.58	1

(a)

	Original Rank	MACBETH
Original Rank	1	
MACBETH	0.19	1

(b)

	Original Rank	MOORA
Original Rank		
MOORA	0.43	

(c) (d)

Next, Spearman's rank correlations have been calculated between the proposed MCDA technique and each of the other four MCDA techniques as considered in this paper. Table XXIII shows the results of the application of Spearman's rank correlation. Table XXIII shows positive associations between the proposed MCDA technique and all the other four techniques through Table XXIIIa, XXIIIb, XXIIIc and XXIIId for AHP, MAUT, MACBETH and MOORA respectively. The highest positive association is observed for MAUT followed by MOORA, AHP and MACBETH. Except for MAUT, all the other associations show low values of correlation although all the positive associations. Such positive associations indicate that there is some similarity (although, low) between the proposed MCDA technique and the other four MCDA techniques. However, the low values for the associations can be explained by the application of a completely different method in the proposed MCDA technique. In general, positive rank correlation between two variables indicates that the increase in the value of one variable leads to the increase in the value of the other one.

However, the authors applying various rank correlations to compare among various MCDA techniques also acknowledged the fact that such methods for establishing associations among the various MCDA techniques are unable to identify the most appropriate technique for a problem under study as indicated by the review of existing literature as presented in the Literature Review section. Therefore, Bandyopadhyay (2021) has proposed a method of comparison in order to identify the most appropriate technique among several techniques for the problem under study. Bandyopadhyay (2021) has simply identified the most profitable technique for a problem at hand. The method as proposed by Bandyopadhyay (2021) has compared the best cumulative value for the highest weighted criterion for each of the MCDA techniques. For example, for the case study presented in this paper, the highest weighted criterion is "Maximum Torque". Table XXIV, Table XXV and Table XXVI show the cumulative values for different MCDA techniques considered in this paper, for the criterion "Maximum Torque" as considered and proposed in this paper. Based on the current case study, the best two cars will have to be selected. Thus, the second cumulative values for the highest rated criterion (Maximum Torque), for each of the MCDA techniques are to be considered. Table XXIV, XXV and XXVI show that the cumulative values for the proposed MCDA technique, AHP, MAUT, MACBETH and MOORA are 1.79, 1.37, 1.74, 1.37 and 1.72 respectively. Thus the highest value among these cumulative values is 1.79 for the proposed MCDA technique. This indicates that the most appropriate MCDA technique for the current case study is the proposed MCDA technique as it outperforms all the other four techniques. Therefore, the comparison of the proposed MCDA technique establishes its superiority over the other MCDA techniques as considered in this paper. Therefore, such results indicate that the proposed MCDA technique is capable to compete with the other existing MCDA techniques.

Table XXIV: Cumulative Maximum Torque for Proposed Technique and AHP

Proposed Technique	Maximum Torque	Cumulative Torque
3	0.8	0.8
2	0.99	1.79
11	0.99	2.78
1	0.73	3.51
8	0.81	4.32
5	0.94	5.26
7	0.7	5.96
10	0.82	6.78
6	0.99	7.77
4	0.55	8.32
9	0.58	8.9

AHP	Maximum Torque	Cumulative Torque
4	0.55	0.55
10	0.82	1.37
6	0.99	2.36
1	0.73	3.09
9	0.58	3.67
11	0.99	4.66
3	0.8	5.46
7	0.7	6.16
2	0.99	7.15
5	0.94	8.09
8	0.81	8.9

Table XXV: Cumulative Maximum Torque for MAUT and MACBETH

MAUT	Maximum Torque	Cumulative Torque
3	0.8	0.8
5	0.94	1.74
9	0.58	2.32
1	0.73	3.05
8	0.81	3.86
6	0.99	4.85
4	0.55	5.4
10	0.82	6.22
2	0.99	7.21
11	0.99	8.2
7	0.7	8.9

MACBETH	Maximum Torque	Cumulative Torque
4	0.55	0.55
10	0.82	1.37
6	0.99	2.36
2	0.99	3.35
9	0.58	3.93
11	0.99	4.92
3	0.8	5.72
8	0.81	6.53
1	0.73	7.26
5	0.94	8.2
7	0.7	8.9

Table XXVI: Cumulative Maximum Torque for MOORA

MOORA	Maximum Torque	Cumulative Torque
6	0.99	0.99
1	0.73	1.72
7	0.7	2.42
4	0.55	2.97
3	0.8	3.77
2	0.99	4.76
5	0.94	5.7
8	0.81	6.51
9	0.58	7.09
10	0.82	7.91
11	0.99	8.9

10 Conclusion

This paper proposes a novel Multi-Criteria Decision Analysis (MCDA) technique which considers various characteristics of MCDA techniques which have been considered till now, as evident from the existing literature. Such characteristics include considering relationships among the alternatives, relationships among the criteria, relationships among the criteria and the alternatives; the uncertainty or dilemma among the decision makers, the consideration of entropy of the criteria while calculating the weights of the criteria. For calculating the weights of the criteria, this paper has applied IDOCRIW method which also considers the entropy in the criteria while calculating the weights; the uncertainty of the decision makers has been dealt with the Hesitant Fuzzy Elements. Both a kind of sensitivity analysis in the form of rank reversal and the comparison with four other different types of MCDA techniques have been performed in order to establish the effectiveness of the proposed MCDA technique.

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Authorship contributions

The author contributes the following through this paper:

- A novel Multi-Criteria Decision Analysis (MCDA) technique has been proposed.
- The proposed MCDA technique has considered relationships among the alternatives, relationships among the criteria, relationships between the criteria and the alternatives, the dilemma in decision making for the decision makers, consideration of information content in the criteria.
- The proposed MCDA technique has been analyzed by sensitivity analysis.
- The proposed technique has also been compared with other four different MCDA techniques in order to establish its effectiveness and validity.

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