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Modified Extended Object Tracker for Lidar Data using Random Matrix Model

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Abstract: Random Matrix (RM) model is a typical extended object modeling method, and has been widely used in extended object tracking. However, the existing RM based filters usually under a assumption that the measurements obey Gaussian distribution, which may lead to the decrease of accuracy when the filter applied to Lidar system. This paper proposed a new observation model and used it to modified a RM smoother by considering the characteristics of Lidar data. Simulation results show that the proposed method achieved better performance compared with the original RM tracker in Lidar system.

Keywords: Extended object, random matrix, RM, target tracking, Bayesian smoothing.

1. INTRODUCTION

Extended object tracking (EOT) is a major topic in data fusion and has been applied to a lot of different scenarios, such as RGB-D sensor [1], imaging sonar [2], marine radar [3], Lidar [4], etc. The extended object many generate more than one measurement per scan, thus a EOT tracker needs to estimate the objects' kinematical and extended state at the same time.

The major EOT methods mainly include the Random Hypersurface models (RHMs) and the RM model. In the RHMs, the kinematical state of a object is updated by the Kalman filter, and the corresponding extended state is described as a given shape model, e.g., ellipsoidal model [5], star-convex model [6], or level-set model [7]. By estimating the parameters of the given shape model, the kinematical and extended state of a object can be obtained respectively. However, as a complex mathematical model, although a RHMs based tracker can accurately estimate the state of the extended object with different shape, this approach lacks simplicity and robustness. Compared with the RHMs, the RM model is an effective and versatile approach with concise mathematical equations. It assumes that the object extended matrix follows the Wishart distribution and the kinematical vector follows the Gaussian distribution, thus the RM model is also known as the Gaussian inverse Wishart (GIW) model.

The RM model was first presented by professor Koch to estimate the state of group targets [8], and soon received widespread attention. A lot of work focuses on improving the RM model, and the modified RM models have been applied to many scenarios, such as multiple extended object tracking [9-12], Bayesian Smoothing [16], etc. However, although some RM based approaches have shown their tracking results of Lidar data, e.g. [9], [11], [12], [17], the tracking accuracy decreases according to the Lidar can only detect one side of the whole object.

This paper proposed a modified RM tracking model

which is more suitable for Lidar scenarios. The detection characteristics of Lidar system in the real world are fully considered, thus the proposed method can better estimate the kinematical and extended state of the object. The main contributions of this paper are as follows:

- Proposed a novel method to estimate the kinematical vector and extended matrix of the object.
- Applied the proposed method to the Bayesian filtering and smoothing method in [16], which is an robust and effective RM tracker.
- The performance between the proposed method and the original method with Lidar data is compared.

The paper is organized as follows. Section 2 introduces the tracking problems. In Second 3, the RM model is improved based on the detection characteristics of Lidar data. Section 4 presents the modified RM tracker for Lidar system. Section 5 shows the simulation results. Section 6 contains the conclusions.

2. PROBLEM STATEMENTS

Let ξ_k denote the extended state at time k . In the RM model, it is usually defined as $\xi_k = \{x_k, X_k\}$, where x_k is the kinematical vector, X_k is a $d \times d$ matrix to describe the object's extent, and d is the dimension of the tracking scenario. Using Bayesian filtering framework, the state prediction is

$$p(x_k, X_k | Z_{k-1}) = \int p(x_k, X_k | x_{k-1}, X_{k-1}) \times p(x_{k-1}, X_{k-1} | Z_{k-1}) dx_{k-1} dX_{k-1}, \quad (1)$$

where $p(x_k, X_k | x_{k-1}, X_{k-1})$ is the transition density. The state update is

$$p(x_k, X_k | Z_k) = \frac{p(Z_k | x_k, X_k) p(x_k, X_k | Z_{k-1})}{\int p(Z_k | x_k, X_k) p(x_k, X_k | Z_{k-1}) dx_k dX_k}. \quad (2)$$

The RM model assumes that x_k follows Gaussian distribution and X_k follows inverse Wishart distribution. Therefore, according to [18], the object state density based on factorised model is

$$p(x_k, X_k | Z_k) = p(x_k | Z_k) p(X_k | Z_k) \\ = N(x_k | m_k, P_k) \times IW(X_k | \nu_k, V_k), \quad (3)$$

where $N(\cdot)$ denotes Gaussian distribution and $IW(\cdot)$ denotes inverse Wishart distribution.

In radar observation model, as shown in Fig. 1 (a), the measurement set $Z_k = \{z_k^{(i)}\}$ is typically assumed that each $z_k^{(i)}$ is generated from the Gaussian distribution independently. Therefore, x_k and X_k are usually estimated by using the mean and covariance of the measurement set Z_k , respectively. However, In Lidar system, as shown in Fig. 1 (b), the measurements are distributed along the contour of the object, and only one side can be detected. The mean and covariance of Z_k can not describe the object's real position and extent accurately. Therefore, the problem of RM model is that: it can not estimate the real state of object when the observation data are from Lidar system.

This paper analyzed the detection process of Lidar system, and modified the RM model according to considering the characteristics of Lidar data.

3. MODIFIED RM MODEL FOR LIDAR

3.1. Measurement model of Lidar

The raw data of Lidar are usually in polar format, e.g. the $z^{(i)} = \{d_L^{(i)}, r_L^{(i)}\}$, where $d_L^{(i)}$ and $r_L^{(i)}$ denote the distance and radian of the i th ranging per revolution, respectively. According to the detection principle of Lidar in real world (as shown in Fig.2), the following assumptions are put forward:

Assumption1: The Lidar detects n_r times per revolution, and the i th observation radian $r_L^{(i)}$ follows the Gaussian distribution $N(r_L^{(i)} | i \cdot 2\pi/n_r, \sigma_r^2)$, where σ_r^2 is the given variance.

Assumption2: The noise of ranging follows zero mean Gaussian distribution with σ_d^2 as variance.

Assumption3: According to interference factors, such as weather or strong light, the object has the probability of p_{D_1} to be detected.

Assumption4: If the object can be detected, according to the difference of object surface material, the detection probability of each ranging is p_{D_2} .

Based on the above assumptions, the measurement model can be described as:

- Let $Z = \{z^{(i)}\}_{i=1}^n$ denotes the measurement set per revolution, according to **Assumption3**, there is $1 - p_{D_1}$ probability that Z is an empty set.
- When Z is not empty, according to **Assumption4**,

there is $1 - p_{D_2}$ probability that $z^{(i)}$ is an empty set.

- Let μ_L and $\hat{z}$ denotes the positions of the Lidar and the object, according to **Assumption1** and

Fig. 1. The difference between Gaussian distribution data and Lidar real data. (a) Data samples with Gaussian distribution; (b) Lidar real data generated by a car.

Assumption2, $d_L^{(i)}$ and $r_L^{(i)}$ follow

$$d_L^{(i)} \sim N(d_L^{(i)} | c(\hat{z}, \mu_L, r_L^{(i)}), \sigma_d^2), \quad (4a)$$

$$r_L^{(i)} \sim N(i \cdot 2\pi/n_r, \sigma_r^2), \quad (4b)$$

where $c(\cdot)$ denotes the detectable contour function of the object. For example, $c(\hat{z}, \mu_L, r_L^{(i)})$ is the ranging result of the object with center coordinate $\hat{z}$. The coordinate of Lidar is μ_L , and the detection radian is $r_L^{(i)}$.

3.2. Proposed method

Given a measurement set $Z_k = \{z_k^{(i)}\}_{i=1}^{n_k}$ in rectangular coordinates at time k (for convenience of calculation, polar coordinates are not used), according to formula (4), state estimation errors will be caused by the use of the mean of Z_k . Supposing that the object extent can be approximated to an ellipse, it's center $\hat{z}_k$ can be approximately estimated as

$$\hat{z}_k = (\mu_1 + \mu_2)/2, \quad (5)$$

where μ_1 and μ_2 are two points on the straight line l_3 passing through the center of the ellipse. As has shown in Fig.3(a), μ_1 and μ_2 can be approximately obtained from the perpendicular of the tangent of the ellipse l_1 and l_2 . The reason for this is that according to the mathematical properties of the ellipse, l_1 and l_2 are the bisectors of the angle formed by the tangent point and focuses of the ellipse. Therefore, the distance from the center to μ_1 and μ_2 is similar, and exactly equal when l_1 and l_2 are parallel.

According to the detection mode of Lidar, $z_k^{(1)}$ and $z_k^{(n_k)}$ can be approximately regarded as the two tangent points. Suppose that $z_k^{(1)} = \{x_1, y_1\}$, $z_k^{(n_k)} = \{x_{n_k}, y_{n_k}\}$ and $\mu_L = \{x_L, y_L\}$. The coordinate equation of straight line l_1 can be obtained by

$$y = A_1 x + B_1, \quad (6a)$$

$$B_1 = y_1 - A_1 x_1, \quad (6b)$$

$$A_1 = -\frac{x_1 - x_L}{y_1 - y_L}, \quad (6c)$$

and the coordinate equation of straight line l_2 is

$$y = A_2 x + B_2, \quad (7a)$$

$$B_2 = y_{n_k} - A_2 x_{n_k}, \quad (7b)$$

$$A_2 = -\frac{x_{n_k} - x_L}{y_{n_k} - y_L}. \quad (7c)$$

The line l_3 passing through the center of the ellipse can be estimated by Z_k . As has shown in Fig.3(b), according to **Assumption1**, the polar coordinate system can be divided into sectors by the radian $2\pi/n_r$, and the number of measurements in each sector may be zero, one or more. By matching the measurements in the corresponding sector, the set $\bar{Z}_k = \{\bar{z}_i\}_{i=1}^{\bar{n}}$ of the midpoints can be obtained. Note that if there are multiple measurements in a sector, the mean value of these measurements is used for calculation. If there are no measurements in a sector, then the measurements in its paired sectors do not participate in the calculation of $\bar{Z}_k$. Supposing that $\bar{z}_i = \{\bar{x}_i, \bar{y}_i\}$, l_3 can be fitted by least square method [19]-[20], i.e.

$$y = A_3 x + B_3, \quad (8a)$$

$$\begin{bmatrix} A_3 \\ B_3 \end{bmatrix} = (C^T C)^{-1} C^T D, \quad (8b)$$

$$C = \begin{bmatrix} 1 & \Lambda & 1 \\ \bar{x}_1 & \Lambda & \bar{x}_{\bar{n}} \end{bmatrix}^T, \quad (8c)$$

$$D = [\bar{y}_1 \quad \Lambda \quad \bar{y}_{\bar{n}}]^T. \quad (8d)$$

The equations of l_1 , l_2 and l_3 have been solved, thus μ_1 and μ_2 can be obtained by

$$\mu_1 = \left[\frac{B_3 - B_1}{A_1 - A_3}, A_1 \frac{B_3 - B_1}{A_1 - A_3} + B_1 \right]^T, \quad (9a)$$

$$\mu_2 = \left[\frac{B_3 - B_2}{A_2 - A_3}, A_2 \frac{B_3 - B_2}{A_2 - A_3} + B_2 \right]^T. \quad (9b)$$

Note that if $\bar{n}$ is too small, the equation of l_3 may be inaccurate, thus a threshold $\mathcal{G}$ needs to be given. When $\bar{n} > \mathcal{G}$, $\hat{z}_k$ will be obtained by the proposed method, while when $\bar{n} \leq \mathcal{G}$, $\hat{z}_k$ will be solved by the weighted mean of Z_k , i.e formula (5) should be rewritten as

$$\hat{z}_k = \begin{cases} (\mu_1 + \mu_2)/2, & \bar{n} > \mathcal{G} \\ \sum_{i=1}^{n_k} \omega^{(i)} z_k^{(i)}, & \bar{n} \leq \mathcal{G}, \end{cases} \quad (10)$$

where $\omega^{(i)}$ is the weight of $z_k^{(i)}$. The reason why formula (10) uses the weighted form is that the inclination of the object contour will affect the measurement density (as shown in Fig. 2). Therefore,

$\omega^{(i)}$ can be calculated by

$$\omega^{(i)} = \tau \frac{(z_k^{(i)} - z_k^{(j)})^T (z_k^{(i)} - z_k^{(j)})}{\hat{d}^2(i, j) (2 - 2 \cos(r_L^{(i)} - r_L^{(j)}))}, \quad (11a)$$

$$\hat{d}(i, j) = \min(d_L^{(i)}, d_L^{(j)}), \quad (11b)$$

$$\tau = \frac{n_k}{\sum_{i=1}^{n_k} \omega^{(i)}}, \quad (11c)$$

where $z_k^{(j)}$ denotes the measurement closest to $z_k^{(i)}$. $\{d_L^{(i)}, r_L^{(i)}\}$ and $\{d_L^{(j)}, r_L^{(j)}\}$ are the distance and radian of the i th and j th measurement, respectively. τ denotes normalization. In fact, the upper part of formula (11a) represents the length of the red line in Fig.2, while the lower part is the length of the blue line. Through the weighting of formula (11), the influence of the object contour inclination on the statistical accuracy can be reduced. Similarly, The covariance matrix $\hat{Y}$ of measurement set Z_k can be obtained by

$$\hat{Z} = \sum_{i=1}^{n_k} \omega^{(i)} (z_k^{(i)} - \hat{z}_k) (z_k^{(i)} - \hat{z}_k)^T. \quad (12)$$

4. IMPLEMENTATION

In this section, the proposed method is used to modify an advanced RM tracker presented in [16]. The tracker has three steps: prediction, updating and smoothing. Let function $M(x)$ denote the change mode of the object extent, use the factorised Gaussian inverse Wishart densities [18]. if $M(x)$ is a given $d \times d$ invertible matrix A , the prediction steps are as follows:

$$m_{k|k-1} = f_{k-1}(m_{k-1|k-1}), \quad (13a)$$

$$P_{k|k-1} = \tilde{F}_{k-1} P_{k-1|k-1} \tilde{F}_{k-1}^T + Q_{k-1}, \quad (13b)$$

Fig. 2. Illumination of the key method.

$$v_{k|k-1} = d+1 + \frac{n_{k-1}(v_{k-1|k-1} - d - 1)}{n_{k-1} + v_{k-1|k-1} - 2d - 2}, \quad (13c)$$

$$V_{k|k-1} = \left(1 + \frac{v_{k-1|k-1} - d - 1}{n - d - 1}\right)^{-1} A V_{k-1|k-1} A^T, \quad (13d)$$

where $m_{k+1|k}$ and $P_{k+1|k}$ are the motion state and corresponding covariance, respectively. $V_{k+1|k}$ and $v_{k+1|k}$ are the Wishart scale matrix and degree of freedom. $f_k(\cdot)$ and $\tilde{F}_k$ are the motion model and corresponding state transition matrix. Q_k is the process noise covariance. If $M(x)$ is not a given matrix, formula (13c) and (13d) become

$$v_{k|k-1} = d+1 + \eta^{-1}(v_{k-1|k-1} - d - 1), \quad (14a)$$

$$V_{k|k-1} = \eta^{-1} \left(1 - \frac{d+1}{s}\right) \left(1 - \frac{d+1}{n_{k-1}}\right) C_2, \quad (14b)$$

$$\eta = 1 + (v_{k-1|k-1} - 2d - 2) \left(\frac{1}{s} + \frac{1}{n_{k-1}} - \frac{d+1}{sn_{k-1}}\right), \quad (14c)$$

$$s = \frac{d+1}{d} \text{Tr}\{C_1 C_2 (C_1 C_2 - I_d)^{-1}\}, \quad (14d)$$

$$C_1 = E_{k-1|k-1} \left[\left(M(x) V_{k-1|k-1} M^T(x) \right)^{-1} \right], \quad (14e)$$

$$C_2 = E_{k-1|k-1} \left[M(x) V_{k-1|k-1} M^T(x) \right], \quad (14f)$$

where $\text{Tr}\{\cdot\}$ is the trace of a matrix and $E_{k-1|k-1}[\cdot]$ is expectation.

The state updating steps are as follows:

$$m_{k|k} = m_{k|k-1} + K \varepsilon, \quad (15a)$$

$$P_{k|k} = P_{k|k-1} + K S K^T, \quad (15b)$$

$$v_{k|k} = v_{k|k-1} + n_k, \quad (15c)$$

$$V_{k|k} = V_{k|k-1} + \hat{N} + \hat{Y}, \quad (15d)$$

where,

$$\hat{N} = \hat{X}^{\frac{1}{2}} S^{-\frac{1}{2}} \varepsilon \varepsilon^T \left(S^{-\frac{1}{2}} \right)^T \left(\hat{X}^{\frac{1}{2}} \right)^T, \quad (16a)$$

$$\hat{Y} = \hat{X}^{\frac{1}{2}} Y^{-\frac{1}{2}} \hat{Z} \left(Y^{-\frac{1}{2}} \right)^T \left(\hat{X}^{\frac{1}{2}} \right)^T, \quad (16b)$$

$$Y = \kappa \hat{X} + R, \quad (16a)$$

$$S = \tilde{H} P_{k|k-1} \tilde{H}^T + \frac{Y}{n_k}, \quad (16c)$$

$$K = P_{k|k-1} \tilde{H}^T S^{-1}, \quad (16d)$$

$$\varepsilon = \hat{z} - \tilde{H} m_{k|k-1}, \quad (16e)$$

where κ is a given scale parameter and R is the object observation noise. $\hat{z}$ and $\hat{Z}$ can be calculated by proposed method, i.e. formula (10) and (12). Since this study does not contribute to the smoothing steps, the related steps are the same as the original smoother and can be found in [16, Section V].

5. SIMULATION

This section shows the simulation results of the proposed method and the original method presented in [16]. There are three scenarios: ellipse object scenario, rectangular object scenario and real Lidar data scenario. The motion model is coordinated turn model, i.e. the object motion state is $x_k = [x_k, y_k, v_k^x, v_k^y, \theta_k]^T$, and other parameters are

$$f(x_k) = \begin{bmatrix} 1 & 0 & \frac{\sin(T\theta_k)}{\theta_k} & -\frac{1-\cos(T\theta_k)}{\theta_k} & 0 \\ 0 & 1 & \frac{1-\cos(T\theta_k)}{\theta_k} & \frac{\sin(T\theta_k)}{\theta_k} & 0 \\ 0 & 0 & \cos(T\theta_k) & -\sin(T\theta_k) & 0 \\ 0 & 0 & \sin(T\theta_k) & \cos(T\theta_k) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x_k, \quad (17a)$$

$$Q_k = G \text{diag}([0.1^2, 0.1^2, (\pi/180)^2]) G^T, \quad (17b)$$

$$Q_k = \begin{bmatrix} \frac{T^2}{2} I_2 & O_{2 \times 1} \\ \frac{T^2}{2} I_2 & O_{2 \times 1} \\ O_{1 \times 2} & 1 \end{bmatrix}, \quad (17c)$$

$$M(x) = \begin{bmatrix} \cos(T\theta) & -\sin(T\theta) \\ \sin(T\theta) & \cos(T\theta) \end{bmatrix}, \quad (17d)$$

$$\tilde{H}_k = [I_2 \quad O_{2 \times 3}], \quad (17e)$$

where Lidar scan interval is $T=1$ s. I and O denote identity matrix and zero matrix, respectively.

The Measurements are generated using the model proposed in section 3.1. The detection probability of the object is $p_{D_1}=0.9$, and the detection probability of each ranging is $p_{D_2}=0.9$. The Lidar detects 7200 times per revolution and the observation noise is $\{\sigma_d = 0.1^2, \sigma_r = 0.001^2\}$.

The performance of different methods is judged by the Gaussian Wasserstein Distance (GWD) metric [21]-[22]. It considers both the object motion and extent errors, and is defined as

$$\Delta_{k|l} = \left\| p_k - \hat{p}_{k|l} \right\|^2 + \text{Tr} \left(X_k + \hat{X}_{k|l} - 2 \left(X_k^{\frac{1}{2}} \hat{X}_{k|l} X_k^{\frac{1}{2}} \right)^{\frac{1}{2}} \right), \quad (18)$$

where the first half describes the motion error, and the second half describes the extent error.

5.1. Ellipse object scenario

In this section, the real shape of the object is an ellipse.

Fig.3 shows the filtering and smoothing results of 200 Monte Carlo runs. The GWD errors of the proposed method are lower than that of the original method, which shows that the proposed method improves the tracking accuracy. Fig.4 shows the tracking results of a single run. It can be seen intuitively that, compared with the original method, the object motion and extent states estimated by the proposed method are closer to the real situation,

which is also the reason for the difference of errors in Fig.3.

Fig. 3. Average filtering and smoothing results of ellipse object.

Fig. 4. Ellipse object tracking results of a single run.

5.2. Rectangular Lidar data scenario

In this section, the shape of the target is a rectangle. The shape of the vehicle is often rectangular in real world, thus the rectangular object can well test the performance of the proposed method for tracking non-elliptical objects.

Fig.5 shows the filtering and smoothing results of 200 Monte Carlo runs. The blue and red ellipses are the results of the original and proposed methods, respectively. The green rectangle describes the location of the vehicle. The black square is the location of the Lidar. The proposed method still achieved better performance. Fig.6 shows the results of a single run. It can be seen that although the shape of the object is rectangular, the proposed method can still use ellipse to approximate its shape accurately. Therefore, the proposed method is robust and can be applied to real scenarios such as vehicle tracking.

Fig. 6. Rectangular object tracking results of a single run.

Fig. 5. Average filtering and smoothing results of rectangular object.

5.3. Real Lidar data scenario

In this section, real-world Lidar data are used to test the effectiveness of the proposed method. The object in the scenario is a car, and we used Lidar to collect 60 scans of data. The Lidar detects 1800 times per revolution and the scan interval is $T = 0.1$ s.

Fig.7 shows the tracking results of five scans. The green rectangle is used to describe the vehicle position. It can be seen that the tracking performance of the proposed method is better than that of the original method intuitively. Note that since the real position of the moving vehicle can not be accurately obtained, the green rectangle in the figure is manually marked according to the track and size of the vehicle. Therefore, the detailed GWD errors are not given in this scenario.

Fig. 7. Tracking results of real Lidar data scenario.

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Competing interests

The authors declare that they have no competing interests.

Availability of data and material

The data generated during this study are included in this article.

Authors' contributions

All authors contributed equally. All authors read and approved the final manuscript.

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