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Contact dynamics and stress field of fluid-conveying GRC nanotubes

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Abstract

Fluid-conveying nanotubes play key roles in micro-nano electromechanical systems. The contact dynamic response and stress field of the fluid-conveying graphene reinforced composite (GRC) nanotube under lateral low-velocity impact are studied. The size-dependent models considering slip flow, nonlocal stress and strain gradient effect are established. The governing equations of flow-inducing post buckling and contact vibration are derived based on a refined beam theory, in which the post-buckling equilibrium provides the initial configuration for the impact vibration analysis. A computation mode of two-step perturbation-Galerkin truncation-Runge-Kutta method is developed to study the contact dynamic responses. Through the convergence analysis, the truncation terms required to ensure the accuracy are obtained. The contact force curves and the midspan displacement time history curves are acquired, and the dynamic snap-through instability behaviors of the nanotube in the flow-inducing post-buckling state are revealed. Also, the stress field in the impact process is obtained to provide theoretical results to guide the strength design. Numerical results reveal the dual influence law of nonlocal stress and strain gradient on the contact dynamic response and stress field and provide the flow velocity range sensitive to the nano effects.

Keywords: Slip flow; Nonlocal strain gradient theory; Fluid-conveying nanotube; Low-velocity impact; Stress field

1. Introduction

Fluid-conveying nanotubes are important carriers for fluid storage and transportation in nanoscale, whose static and dynamic mechanical performances have been widely studied [1]. Limited by the experimental conditions and the time cost of molecular dynamics simulation, the establishment of generalized continuum theory to study the mechanical properties of nano materials has become a research topic of concern. Over the years, some non-classical continuum theories have been developed, such as surface energy theory [2,3], nonlocal elastic theory [4-6], strain gradient theory [7,8], nonlocal strain gradient theory [9,10], nonlocal strain gradient theory coupled with surface effect [11]. Based on the above theoretical framework, static and dynamic stability [12], and other nonlinear dynamic problems [13-16] of fluid-conveying nanotubes have been extensively studied to reveal the influence of nano effects.

The chaos in an embedded fluid-conveying single-walled carbon nanotube under transverse harmonic load was studied by the generalized multi-symplectic method [17]. Free vibration and instability analysis of a nanotube conveying nanoflow was investigated based on the nonlocal strain gradient theory [18]. The dynamic behaviors of two mutually interacting single-walled boron nitride nanotubes under a moving nanoparticle was revealed [19]. A nonlinear surface-stress-dependent nanoscale shell model was developed for free vibration analysis of the fluid-conveying nano-shell [20]. The chaotic vibrations mechanism of cantilevered microtubes conveying fluid was revealed using the Galerkin procedure [21,22]. The effect of transverse magnetic field on the vibration of cantilever micro and nanotubes conveying fluid was investigated by considering the small size effects [23]. The effect of the residual surface stress on the dynamic instability of the fluid conveying double-walled nanotubes was explored [24]. The dynamic stiffness method was used to study the free vibration of a fluid-conveying nanotube composed of carbon nanotube and boron nitride nanotube [25]. However, few research is found on the theoretical prediction for contact dynamic response of nanotubes under impact load.

The existing reports on the calculation methods of contact dynamic response are mainly divided into the following categories, finite difference-Newmark method [26], the two-step perturbation method [27,28], differential quadrature

method (DQM) [29], Fourier expansion and Galerkin truncation [30-33], Ritz method [34-36] and finite element method [37,38]. The stochastic sensitivity analysis was dealt with of functionally graded material (FGM) plates subjected to low-velocity impact to identify the most influential parameters in the respective analyses [39]. An experimentally validated analytical model for low-velocity impact between a sphere and a plate was presented [40]. An integrated model was proposed for prediction of the impact dynamic behaviors on hybrid fiber metal laminates embedded with a viscoelastic layer [41]. However, little attention was paid to the stress field in the process of contact, which is important for strength failure prediction. The stress components of the central points were obtained and the damage analysis and dynamic response were conducted of an elasto-plastic laminated composite shallow spherical shell under low velocity impact [42]. The contact stress time history curves at different temperatures were acquired of functionally graded plates [43]. However, these studies did not study the stress fields and did not focus on the change of the position of stress peak in the process of contact. Also, there are few reports on contact and impact of the fluid-conveying pipes. The limited studies are listed below. The impact dynamics of a cantilevered pipe in the presence of two support walls along the pipe axis were explored [44]. The low velocity impact response of prestressed functionally graded hybrid pipes were studied [45]. It was revealed that the existence of a low-frequency vibration bandgap in the locally resonant pipe can effectively suppress the vibration propagation under external impact and fluid impact excitation [46]. The dynamic response of the nanobeam impacted by a highspeed C_{60} was obtained by molecular dynamics simulations and the potential was found to measure the mass of a molecular under a known incident velocity [47].

Through the above literature review, it can be found that the influences of nano-effects on the contact dynamic response of fluid-conveying nanotubes are still unknown. Also, there is few research on the effects of internal flow on the contact dynamic responses of tube under impact. Besides, to obtain the refined stress field in the process of contact, an efficient and high-precision calculation method requires to be developed. As a novel study, the contact dynamic responses of the nanotube in the flow-inducing pre-and post-buckling states under lateral low-velocity impact are studied. In addition, the computation mode of the two-

step perturbation-higher-order Galerkin truncation-Runge-Kutta method is developed.

Here, the nanotube is assumed to be simply supported with immovable ends and to be a three-layer structure, where the inner and outer layers are graphene-reinforced composite materials while the interlayer is an isotropic viscoelastic material. The nonlocal strain gradient theory is employed in establishing the reduced constitutive relationship of nanotubes. The Knudsen number is introduced to characterize the slip flow effect and modify the kinetic energy of nanofluid. The Hertz contact law is employed to model the contact force. The flow-inducing post-buckling equilibrium path is obtained as the initial configuration of the dynamic response of the nanotube under impact by using the two-step perturbation method. The computation method of two-step perturbation-higher-order Galerkin truncation-Runge-Kutta is developed to obtain the contact force, dynamic response, modal and stress field. Through the convergence analysis, the truncation orders with sufficient accuracy are obtained respectively. Through numerical analysis, the influence of nano effects on the impact dynamic response, stress field of the nanotube in flow-inducing pre- and post-buckling states are revealed.

2. Formulations of the fluid-conveying nanotubes

Fig. 1 Structure diagram of fluid-conveying GRC nanotubes

2.1 Displacement field

As is shown in Fig. 1, steady nanofluid flow through a nanotube and the midspan is impacted laterally by an impactor. The boundary conditions of the two ends are assumed simply supported and immovable. Under the action of the internal fluid flow and the impact, both the flow-inducing post-buckling and the impact vibration are assumed to move in the x - z plane, namely, the displacement

in the y direction is zero. To characterize the geometric features of nanotube, a refined beam model is utilized [48,49] that utilizes the Laurent series expansion form for the displacement field.

$$\begin{aligned}\hat{u}(x, y, z, t) &= \bar{u}(x, t) + f(y, z) \frac{\partial \bar{w}(x, t)}{\partial x} + g(y, z) \bar{\mathcal{G}}(x, t) \\ \hat{v}(x, y, z, t) &= 0 \\ \hat{w}(x, y, z, t) &= \bar{w}(x, t)\end{aligned}\quad (1)$$

where $\hat{u}$, $\hat{v}$, $\hat{w}$ denote the displacement of a point of the nanotube in x -, y - and z -directions, and $\bar{u}$, $\bar{w}$, $\bar{\mathcal{G}}$ are the mid-plane displacements in geometry. Specifically, $\partial \bar{w} / \partial x$ is the rotation angle of the mid-plane and $\bar{\mathcal{G}}$ is the angle between the tangent of the warped section at the mid-plane and the positive z -axis. Because the deformation of the nanotube is composed of initial post-buckling deformation and additional impact vibration, the mid-plane displacements also consist of two parts, namely, $\bar{w}(x, t) = w(x, t) + w^*(x)$, $\bar{u}(x, t) = u(x, t) + u^*(x)$, $\bar{\mathcal{G}}(x, t) = \mathcal{G}(x, t) + \mathcal{G}^*(x)$, where $w^*(x)$, $u^*(x)$ and $\mathcal{G}^*(x)$ are static flow-inducing post-buckling displacements and $w(x, t)$, $u(x, t)$ and $\mathcal{G}(x, t)$ are additional dynamic displacements. Besides, f , g are the higher-order shape functions, defined by

$$f = \frac{z}{R_o^2 + R_i^2} (R_o^2 R_i^2 r^{-2} - r^2 / 3), g = f + z \quad (2)$$

where $r = \sqrt{y^2 + z^2}$

By employing the von Kármán's nonlinear strain-displacement relations considering the initial deflection, the geometric relationships can be written as follows.

$$\begin{aligned}\varepsilon_{xx} &= \frac{\partial \hat{u}}{\partial x} + \frac{1}{2} \left(\frac{\partial \hat{w}}{\partial x} \right)^2 = \varepsilon_{xx}^{(0)} + f \varepsilon_{xx}^{(1)} + g \varepsilon_{xx}^{(2)} \\ \gamma_{xy} &= \frac{\partial \hat{u}}{\partial y} + \frac{\partial \hat{v}}{\partial x} = \frac{\partial f}{\partial y} \gamma_{xy}^{(0)} + \frac{\partial g}{\partial y} \gamma_{xy}^{(1)} \\ \gamma_{zx} &= \frac{\partial \hat{u}}{\partial z} + \frac{\partial \hat{w}}{\partial x} = \left(1 + \frac{\partial f}{\partial z} \right) \gamma_{zx}^{(0)} + \frac{\partial g}{\partial z} \gamma_{zx}^{(1)}\end{aligned}\quad (3)$$

in which $\varepsilon_{xx}^{(i)}$ ($i=0,1,2$) are the generalized in-plane positive strains and $\gamma_{xy}^{(i)}, \gamma_{zx}^{(i)}$ ($i=0,1$) are the generalized shear strains, which are expressed as

$$\varepsilon_{xx}^{(0)} = \frac{\partial \bar{u}}{\partial x} + \frac{1}{2} \left(\frac{\partial \bar{w}}{\partial x} \right)^2, \quad \varepsilon_{xx}^{(1)} = \frac{\partial^2 \bar{w}}{\partial x^2}, \quad \varepsilon_{xx}^{(2)} = \frac{\partial \bar{g}}{\partial x}, \quad \gamma_{xy}^{(0)} = \gamma_{zx}^{(0)} = \frac{\partial \bar{w}}{\partial x}, \quad \gamma_{xy}^{(1)} = \gamma_{zx}^{(1)} = \bar{g} \quad (4)$$

2.2 Slip flow effect

With the structural scale from macro to nano, the fluid also presents the size effect, that is nanofluid is only a series of fluid molecules flow and can no longer be regarded as continuous flow. To measure the deviation state of fluid from continuum, a dimensionless parameter termed Knudsen number (Kn) is introduced which is defined as the ratio of molecular mean free path to an external characteristics dimension of a fluid-conveying system [50]. According to the fluid boundary theory, the Knudsen number of the nanofluid is in the range of 0.001-0.1 and the flow of fluid is in the range of slip region [51]. In this case, it is reasonable to employ the Navier-stokes equations modified at the surface with appropriate velocity-slip conditions. Consequently, a correction factor is introduced that establishes the relation between slip and non-slip average velocities

$$v_s = \kappa_{cf} v \quad (5)$$

where v_s and v are respectively the average slip flow velocity and non-slip flow velocity. Combined with the Navier-Stokes equation, and Beskok-karniadaki model, the axial fluid velocity can be obtained, and the correction factor can be written as

$$\kappa_{cf} = (1 + \lambda Kn) \left[\left(\frac{2 - \sigma_v}{\sigma_v} \right) \left(\frac{4Kn}{1 - \beta Kn} \right) + 1 \right] \quad (6)$$

in which

$$\lambda = \frac{2\lambda_0}{\pi} \tan^{-1} \left[\alpha_0 (Kn)^{\alpha_1} \right] \quad (7)$$

and

$$\lambda_0 = \lim_{Kn \rightarrow \infty} \lambda = \frac{64\beta}{3\pi(1-\frac{4}{\beta})} = \frac{64}{15\pi} \quad (8)$$

In the above relations $\alpha_0, \alpha_1, \beta$ and are constants, which are respectively determined as $\alpha_0=4, \alpha_1=0.4$ and $\beta=-1$ [51].

2.3 Nonlocal strain gradient theory

To characterize the size dependent effect of nanotubes, the nonlocal strain gradient elasticity theory is employed [10] which can describe not only the stiffness hardening effect, but also stiffness softening effect in the linear elastic stage. The differential constitutive equation can be uniformly expressed in the following form

$$\left[1 - (e_0 a)^2 \nabla^2\right] t_{ij} = (1 - l^2 \nabla^2) \sigma_{ij} \quad (9)$$

in which t_{ij} represents the nonlocal stress which is different from the classical Cauchy stress σ_{ij} . e_0 is a dimensionless parameter corresponding to each material and a and l are the intrinsic length of the material reflecting the nonlocal stress and the strain gradient effect.

2.4 Constructive equations

Utteriorly, the constitutive relations of the GRC nanotube considering both nonlocal stress and strain gradient effect can be established by utilizing the nonlocal strain gradient theory.

$$\begin{aligned} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} - (e_0 a)^2 \tilde{\nabla}^2 \begin{pmatrix} \dot{\epsilon}_{xx} \\ \dot{\epsilon}_{yy} \\ \dot{\epsilon}_{xy} \end{pmatrix} &= (1 - l^2 \tilde{\nabla}^2) \begin{pmatrix} \bar{Q}_{11}^{(k)} \\ \bar{Q}_{22}^{(k)} \\ \bar{Q}_{66}^{(k)} \end{pmatrix} \begin{pmatrix} \dot{\epsilon}_{xx} \\ \dot{\epsilon}_{yy} \\ \dot{\epsilon}_{xy} \end{pmatrix} + \begin{pmatrix} \bar{Q}_{12}^{(k)} \\ \bar{Q}_{26}^{(k)} \\ \bar{Q}_{66}^{(k)} \end{pmatrix} \begin{pmatrix} \dot{\epsilon}_{yz} \\ \dot{\epsilon}_{yz} \\ \dot{\epsilon}_{yz} \end{pmatrix} \end{aligned} \quad (10)$$

$$\begin{aligned} \begin{pmatrix} \sigma_{yz} \\ \sigma_{xz} \end{pmatrix} - (e_0 a)^2 \tilde{\nabla}^2 \begin{pmatrix} \dot{\epsilon}_{yz} \\ \dot{\epsilon}_{xz} \end{pmatrix} &= (1 - l^2 \tilde{\nabla}^2) \begin{pmatrix} \bar{Q}_{44}^{(k)} \\ \bar{Q}_{55}^{(k)} \end{pmatrix} \begin{pmatrix} \dot{\epsilon}_{yz} \\ \dot{\epsilon}_{xz} \end{pmatrix} + \begin{pmatrix} \bar{Q}_{45}^{(k)} \\ \bar{Q}_{55}^{(k)} \end{pmatrix} \begin{pmatrix} \dot{\epsilon}_{yz} \\ \dot{\epsilon}_{xz} \end{pmatrix} \end{aligned} \quad (11)$$

Combined with the displacement field, the following assumption is introduced [52]

$$\sigma_{yy} = 0, \epsilon_{yy} = 0, \tau_{yz} = 0, \gamma_{yz} = 0 \quad (12)$$

Substituting Eq. (12) into Eq. (10) and (11) yields:

$$\begin{aligned}\bar{Q}_{12}^{(k)} \varepsilon_{xx}^{(k)} + \bar{Q}_{26}^{(k)} \gamma_{xy}^{(k)} &= 0 \\ \bar{Q}_{44}^{(k)} \gamma_{yz}^{(k)} + \bar{Q}_{45}^{(k)} \gamma_{xz}^{(k)} &= 0\end{aligned}\quad (13)$$

Considering Eqs. (10), (11), and (13), the reduced constructive equations can be written as

$$\begin{aligned}\frac{d}{dt} - (e_0 a)^2 \tilde{N}^2 \frac{\partial}{\partial x} \sigma_{xx}^{(k)} &= (1 - l^2 \tilde{N}^2) \tilde{Q}_{11}^{(k)} \varepsilon_{xx}^{(k)} \\ \frac{d}{dt} - (e_0 a)^2 \tilde{N}^2 \frac{\partial}{\partial x} \tau_{xy}^{(k)} &= (1 - l^2 \tilde{N}^2) \tilde{Q}_{66}^{(k)} \gamma_{xy}^{(k)} \\ \frac{d}{dt} - (e_0 a)^2 \tilde{N}^2 \frac{\partial}{\partial x} \tau_{xz}^{(k)} &= (1 - l^2 \tilde{N}^2) \tilde{Q}_{55}^{(k)} \gamma_{xz}^{(k)}\end{aligned}\quad (14)$$

On the other hand, the Kevin Voigt viscoelastic model is introduced to express the stress terms of the interlayer,

$$\begin{aligned}\frac{d}{dt} - (e_0 a)^2 \tilde{N}^2 \frac{\partial}{\partial x} \sigma_{xx}^{(c)} &= (1 - l^2 \tilde{N}^2) \tilde{E} \varepsilon_{xx}^{(c)} + c \frac{\varepsilon_{xx}^{(c)}}{t} \\ \frac{d}{dt} - (e_0 a)^2 \tilde{N}^2 \frac{\partial}{\partial x} \tau_{xy}^{(c)} &= (1 - l^2 \tilde{N}^2) \tilde{G} \gamma_{xy}^{(c)} + j \frac{\gamma_{xy}^{(c)}}{t} \\ \frac{d}{dt} - (e_0 a)^2 \tilde{N}^2 \frac{\partial}{\partial x} \tau_{xz}^{(c)} &= (1 - l^2 \tilde{N}^2) \tilde{G} \gamma_{xz}^{(c)} + j \frac{\gamma_{xz}^{(c)}}{t}\end{aligned}\quad (15)$$

where the elastic modulus and shear modulus at one point of the nanotube are expressed as follows

$$\begin{aligned}\tilde{Q}_{11}^{(k)} &= \frac{\bar{Q}_{11}^{(k)}}{E} - \frac{(\bar{Q}_{12}^{(k)})^2}{\bar{Q}_{22}^{(k)}} \quad (\text{Face Sheets}) \\ \tilde{Q}_{66}^{(k)} &= \frac{\bar{Q}_{66}^{(k)}}{G} - \frac{\bar{Q}_{16}^{(k)} \bar{Q}_{26}^{(k)}}{\bar{Q}_{12}^{(k)}} \quad (\text{Face Sheets}) \\ \tilde{Q}_{55}^{(k)} &= \frac{\bar{Q}_{55}^{(k)}}{G} - \frac{(\bar{Q}_{45}^{(k)})^2}{\bar{Q}_{44}^{(k)}} \quad (\text{Face Sheets})\end{aligned}\quad (16)$$

in which $\bar{Q}_{ij}^{(k)}$ is the stiffness coefficients of the GRCs, and can be determined by the Halpin-Tsai meso model [53].

2.5 Contact model

The Hertz contact law [54,55] is employed to model the contact force, and is expressed as

$$F = \begin{cases} K_c (w_i - w_{x=x_0})^{1.5} & \text{Loading process} \\ F_{\max} \left(\frac{\alpha - \alpha_0}{\alpha_{\max} - \alpha_0} \right)^{2.5} & \text{Unloading process} \end{cases} \quad (17)$$

where w_i and $w_{x=x_0}$ represent the displacement of the impactor and that of the nanotube at the contact point, respectively. α and $\alpha_{\max}$ denote the indentation and maximum indentation of the nanotube. α_0 represents the permanent plastic indentation. Here, it is assumed that the whole contact process is in the elastic stage and hence $\alpha_0 = 0$. $F_{\max}$ is the maximum contact force in the process of contact. The contact stiffness K_c is defined as

$$K_c = \frac{4}{3} \left(\frac{1 - (\mu_i)^2}{E_i} + \frac{1}{E_{33}} \right)^{-1} \sqrt{r_i} \quad (18)$$

in which E_i , μ_i and r_i are the elastic modulus, Poisson's ratio and radius of the impactor. E_{33} is the surface contact stiffness of the nanotube, which is related to the graphene volume fraction [56].

2.6 Governing equations

2.6.1 Governing equations for the impactor

Consider a rigid impactor and ignore its gravity, and the differential equation of motion in the contact process can be obtained

$$m_i \ddot{w}_i + F_c(t) = 0 \quad (19)$$

The initial conditions of the impactor can be written as

$$\left. \frac{\partial w_i}{\partial t} \right|_{t=0} = v_i, \quad w_i|_{t=0} = 0 \quad (20)$$

2.6.2 Governing equations for the nanotube

The Hamilton energy-variational principle can be expressed as

$$\int_{t_1}^{t_2} (\delta K_T + \delta K_f - \delta U + \delta W) dt = 0 \quad (21)$$

in which δU is the virtual strain energy, δK_T , δK_f are the virtual kinetic energy of the nanotube and the fluid, respectively, and δW is the virtual external force work

$$\begin{aligned} \delta U &= \int_0^L \int_A t_{ij} \delta \varepsilon_{ij} dA dx \\ \delta K_p &= \int_0^L \int_A \rho \dot{u}_i \delta \dot{u}_i dA dx \\ \delta K_f &= \int_0^L \int_{A_f} \rho_f \left[\left(\dot{w} + \kappa_{cf} (v \cdot \nabla) w \right) \delta \left(\dot{w} + \kappa_{cf} (v \cdot \nabla) w \right) \right. \\ &\quad \left. + \left(\dot{u} + \kappa_{cf} v + \kappa_{cf} (v \cdot \nabla) u \right) \delta \left(\dot{u} + \kappa_{cf} v + \kappa_{cf} (v \cdot \nabla) u \right) \right] dA_f dx \\ \delta W &= \int_0^L F_c(t) \delta (x - x_0) \delta w dx \end{aligned} \quad (22)$$

where x_0 represent the contact point which is set as $L/2$ in this study and ρ , ρ_f denote the mass density of the nanotube and the fluid.

Under the assumption that the axial inertia terms and strain gradient are small, and the flow velocity is steady, namely, $\dot{v} = 0$, the governing equations can be derived and simplified through the variational operations.

$$\begin{aligned} \delta u : \frac{\partial N_x}{\partial x} &= 0 \\ \delta w : \frac{\partial Q}{\partial x} + \frac{\partial}{\partial x} \left(N_x \frac{\partial \bar{w}}{\partial x} \right) - \frac{\partial^2 M_x}{\partial x^2} + F_c(t) \delta (x - x_0) - m_f \left[\left(\kappa_{cf} v \right)^2 \frac{\partial^2 \bar{w}}{\partial x^2} + 2 \kappa_{cf} v \frac{\partial \dot{w}}{\partial x} + \ddot{w} \right] \\ &= \rho_0 \ddot{w} - \rho_1 \frac{\partial^2 \ddot{w}}{\partial x^2} - \rho_2 \frac{\partial \ddot{g}}{\partial x} \\ \delta g : \frac{\partial P_x}{\partial x} - Q &= \rho_2 \frac{\partial \ddot{w}}{\partial x} + \rho_3 \ddot{g} \end{aligned} \quad (23)$$

where N_x, M_x, P_x, Q are the generalized internal force, expressed as

$$\begin{aligned}
N_x &= \sum_{k=1}^n \iint_{A_k} \sigma_{xx} dA \\
M_x &= \sum_{k=1}^n \iint_{A_k} f \sigma_{xx} dA \\
P_x &= \sum_{k=1}^n \iint_{A_k} g \sigma_{xx} dA \\
Q &= \sum_{k=1}^n \iint_{A_k} \left(\frac{\partial g}{\partial z} \tau_{xz} + \frac{\partial f}{\partial y} \tau_{xy} \right) dA
\end{aligned} \tag{24}$$

and $m_f, \rho_0, \rho_1, \rho_2, \rho_3$ are the generalized mass densities, defined by

$$\begin{aligned}
m_f &= \int_{A_f} \rho_f dA \\
(\rho_0, \rho_1, \rho_2, \rho_3) &= \sum_{k=1}^n \iint_{A_k} \rho^{(k)} (1, f^2, fg, g^2) dA
\end{aligned} \tag{25}$$

Substituting Eqs. (14) and Eqs. (15) into Eqs. (24), one can obtain

$$\begin{aligned}
\left[1 - (e_0 a)^2 \nabla^2 \right] \begin{bmatrix} N_x \\ M_x \\ P_x \end{bmatrix} &= (1 - l^2 \nabla^2) \begin{bmatrix} \Gamma_0 & \Gamma_1 & \Gamma_2 \\ & \Gamma_3 & \Gamma_4 \\ sym & & \Gamma_5 \end{bmatrix} \begin{bmatrix} \mathcal{E}_{xx}^{(0)} \\ \mathcal{E}_{xx}^{(1)} \\ \mathcal{E}_{xx}^{(2)} \end{bmatrix} \\
\left[1 - (e_0 a)^2 \nabla^2 \right] Q &= (1 - l^2 \nabla^2) \left(\Gamma_6 \gamma_{xy}^{(0)} + \Gamma_7 \gamma_{xy}^{(1)} + \Gamma_8 \gamma_{zx}^{(0)} + \Gamma_9 \gamma_{zx}^{(1)} \right)
\end{aligned} \tag{26}$$

where the reduced stiffness and damping coefficients can be uniformly expressed

as $\Gamma_i = \left(A_i + \bar{A}_i \frac{\partial}{\partial t} \right)$ ($i = 0, 1, \dots, 9$), and

$$\begin{aligned}
(A_0, A_1, A_2, A_3, A_4, A_5) &= \sum_{k=1}^n \iint_{A_k} \tilde{Q}_{11}^{(k)} (1, f, g, f^2, fg, g^2) dA \\
(\bar{A}_0, \bar{A}_1, \bar{A}_2, \bar{A}_3, \bar{A}_4, \bar{A}_5) &= \iint_{A_c} c (1, f, g, f^2, fg, g^2) dA
\end{aligned} \tag{27-a}$$

$$\begin{aligned}
(A_6, A_7) &= \sum_{k=1}^n \iint_{A_k} \tilde{Q}_{66}^{(k)} \left[\left(\frac{\partial f}{\partial y} \right)^2, \left(\frac{\partial f}{\partial y} \frac{\partial g}{\partial y} \right) \right] dA \\
(\bar{A}_6, \bar{A}_7) &= \iint_{A_c} j \left[\left(\frac{\partial f}{\partial y} \right)^2, \left(\frac{\partial f}{\partial y} \frac{\partial g}{\partial y} \right) \right] dA
\end{aligned} \tag{27-b}$$

$$\begin{aligned}
(A_8, A_9) &= \sum_{k=1}^n \iint_{A_k} \tilde{Q}_{55}^{(k)} \left[\frac{\partial g}{\partial z} \left(1 + \frac{\partial f}{\partial z} \right), \left(\frac{\partial g}{\partial z} \right)^2 \right] dA \\
(\bar{A}_8, \bar{A}_9) &= \iint_{A_c} j \left[\frac{\partial g}{\partial z} \left(1 + \frac{\partial f}{\partial z} \right), \left(\frac{\partial g}{\partial z} \right)^2 \right] dA
\end{aligned} \tag{27-c}$$

Since the material properties are symmetric with respect to the geometric midplane, it can be proved that

$$\Gamma_1 = \Gamma_2 = 0 \tag{28}$$

Introducing the immovable hinged-hinged boundary conditions leads to the relationship between the axial internal force and the mid-plane stretching

$$N_x = \frac{A_0}{2L} \int_0^L \left(\frac{\partial \bar{w}}{\partial x} \right)^2 dx \tag{29}$$

Substitute Eqs. (4), (26) and (29) into Eqs. (23) and the governing equations expressed by the generalized displacements are derived

$$\begin{aligned}
&(1 - l^2 \nabla^2) \left\{ \Gamma_3 \frac{\partial^4 \bar{w}}{\partial x^4} + \Gamma_4 \frac{\partial^3 \bar{g}}{\partial x^3} - (\Gamma_6 + \Gamma_8) \frac{\partial^2 \bar{w}}{\partial x^2} - (\Gamma_7 + \Gamma_9) \frac{\partial \bar{g}}{\partial x} \right\} \\
&+ \left[1 - (e_0 a)^2 \nabla^2 \right] \left\{ \rho_0 \ddot{w} - \frac{\partial}{\partial x} \left(\rho_1 \frac{\partial \ddot{w}}{\partial x} + \rho_2 \ddot{g} \right) - F_c(t) \delta(x - x_0) \right. \\
&\quad \left. + m_f \left[(\kappa_{cf} v)^2 \frac{\partial^2 \bar{w}}{\partial x^2} + 2\kappa_{cf} v \frac{\partial \dot{w}}{\partial x} + \ddot{w} \right] - \left[\frac{A_0}{2L} \int_0^L \left(\frac{\partial \bar{w}}{\partial x} \right)^2 dx \right] \frac{\partial^2 \bar{w}}{\partial x^2} \right\} = 0 \\
&(1 - l^2 \nabla^2) \left[\Gamma_4 \frac{\partial^3 \bar{w}}{\partial x^3} + \Gamma_5 \frac{\partial^2 \bar{g}}{\partial x^2} - (\Gamma_6 + \Gamma_8) \frac{\partial \bar{w}}{\partial x} - (\Gamma_7 + \Gamma_9) \bar{g} \right] \\
&= \left[1 - (e_0 a)^2 \nabla^2 \right] \left(\rho_2 \frac{\partial \ddot{w}}{\partial x} + \rho_3 \ddot{g} \right)
\end{aligned} \tag{30}$$

The physical meaning of Eqs. (30) includes two processes: the initial flow inducing post-buckling, and the impact dynamic response on the post buckling configuration. The post-buckling governing equation can be obtained by removing the dynamic term

$$\begin{aligned}
& (1-l^2\nabla^2)\left\{A_3\frac{\partial^4 w^*}{\partial x^4}+A_4\frac{\partial^3 \mathcal{G}^*}{\partial x^3}-(A_6+A_8)\frac{\partial^2 w^*}{\partial x^2}-(A_7+A_9)\frac{\partial \mathcal{G}^*}{\partial x}\right\} \\
& +\left[1-(e_0 a)^2\nabla^2\right]\left\{-\left[\frac{A_0}{2L}\int_0^L\left(\frac{\partial w^*}{\partial x}\right)^2 dx\right]\frac{\partial^2 w^*}{\partial x^2}+m_f(\kappa_{cf}v)^2\frac{\partial^2 w^*}{\partial x^2}\right\}=0 \quad (31) \\
& (1-l^2\nabla^2)\left[A_4\frac{\partial^3 w^*}{\partial x^3}+A_5\frac{\partial^2 \mathcal{G}^*}{\partial x^2}-(A_6+A_8)\frac{\partial w^*}{\partial x}-(A_7+A_9)\mathcal{G}^*\right]=0
\end{aligned}$$

The governing equations of impact dynamic response of the nanotube is obtained by subtracting Eqs. (30) by Eqs. (31)

$$\begin{aligned}
& (1-l^2\nabla^2)\left\{\Gamma_3\frac{\partial^4 w}{\partial x^4}+\Gamma_4\frac{\partial^3 \mathcal{G}}{\partial x^3}-(\Gamma_6+\Gamma_8)\frac{\partial^2 w}{\partial x^2}-(\Gamma_7+\Gamma_9)\frac{\partial \mathcal{G}}{\partial x}\right\} \\
& +\left[1-(e_0 a)^2\nabla^2\right]\left\{m_f\left[\left(\kappa_{cf}v\right)^2\frac{\partial^2 w}{\partial x^2}+2\kappa_{cf}v\frac{\partial \dot{w}}{\partial x}+\ddot{w}\right]-\frac{A_0}{2L}\frac{\partial^2 w}{\partial x^2}\int_0^L\left(\frac{\partial \bar{w}}{\partial x}\right)^2 dx\right. \\
& \left.-\frac{A_0}{2L}\frac{\partial^2 w^*}{\partial x^2}\int_0^L\left[\left(\frac{\partial w}{\partial x}\right)^2+2\frac{\partial w}{\partial x}\frac{\partial w^*}{\partial x}\right]dx\right. \\
& \left.+\rho_0\ddot{w}-\frac{\partial}{\partial x}\left(\rho_1\frac{\partial \ddot{w}}{\partial x}+\rho_2\ddot{\mathcal{G}}\right)-F_c(t)\delta(x-x_0)\right\}=0 \\
& (1-l^2\nabla^2)\left[\Gamma_4\frac{\partial^3 w}{\partial x^3}+\Gamma_5\frac{\partial^2 \mathcal{G}}{\partial x^2}-(\Gamma_6+\Gamma_8)\frac{\partial w}{\partial x}-(\Gamma_7+\Gamma_9)\mathcal{G}\right] \\
& =\left[1-(e_0 a)^2\nabla^2\right]\left(\rho_2\frac{\partial \ddot{w}}{\partial x}+\rho_3\ddot{\mathcal{G}}\right) \quad (32)
\end{aligned}$$

To facilitate the numerical computation of the governing equations, let $\xi=e_0 a$ and define the following dimensionless parameters

$$\begin{aligned}
X &= \pi \frac{x}{L}, \bar{l} = \pi \frac{l}{L}, \bar{\xi} = \pi \frac{\xi}{L}, (\bar{W}, W^*) = \left(\frac{\bar{w}}{L}, \frac{w^*}{L}\right), (\Theta, \Theta^*) = \left(\frac{\mathcal{G}}{\pi}, \frac{\mathcal{G}^*}{\pi}\right), \\
\lambda &= \frac{\pi \bar{A}_3}{L A_3} \sqrt{\frac{E_m}{\rho_m}}, M_f = \frac{m_f L^2 E_m}{A_3 \rho_m \pi^2}, V_f = v_f \sqrt{\frac{\rho_m}{E_m}}, \tau = \frac{\pi t}{L} \sqrt{\frac{E_m}{\rho_m}}, \\
\lambda_q &= \frac{F_c \delta(x-x_0) L^3}{\pi^4 A_3}, (\psi_0, \psi_1, \psi_2, \psi_3) = \left(\frac{\rho_0 L^2 E_m}{A_3 \rho_m \pi^2}, \frac{\rho_1 E_m}{A_3 \rho_m}, \frac{\rho_2 E_m}{A_3 \rho_m}, \frac{\rho_3 E_m}{A_3 \rho_m}\right), \quad (33) \\
(\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5) &= \left(\frac{A_0 L^2}{A_3 \pi^2}, \frac{A_3}{A_3}, \frac{A_4}{A_3}, \frac{A_5}{A_3}, \frac{(A_6+A_8) L^2}{A_3 \pi^2}, \frac{(A_7+A_9) L^2}{A_3 \pi^2}\right), \\
(\bar{\gamma}_1, \bar{\gamma}_2, \bar{\gamma}_3, \bar{\gamma}_4, \bar{\gamma}_5) &= \left(\frac{\bar{A}_3}{A_3}, \frac{\bar{A}_4}{A_3}, \frac{\bar{A}_5}{A_3}, \frac{(\bar{A}_6+\bar{A}_8) L^2}{\pi^2 \bar{A}_3}, \frac{(\bar{A}_7+\bar{A}_9) L^2}{\pi^2 \bar{A}_3}\right)
\end{aligned}$$

where E_m and ρ_m represent the reference values of the matrix at an ambient temperature of 300K.

Therefore, the equations for post-buckling (Eqs. (31)) and the impact dynamic response (Eqs. (32)) can be rewritten in the following dimensionless form

$$\begin{aligned}
& (1 - \bar{l}^2 \nabla^2) \left(\gamma_1 \frac{\partial^4 W^*}{\partial X^4} + \gamma_2 \frac{\partial^3 \Theta^*}{\partial X^3} - \gamma_4 \frac{\partial^2 W^*}{\partial X^2} - \gamma_5 \frac{\partial \Theta^*}{\partial X} \right) \\
& - (1 - \bar{\xi}^2 \nabla^2) \left\{ \frac{\pi \gamma_0}{2} \int_0^\pi \left(\frac{\partial W^*}{\partial X} \right)^2 dX \frac{\partial^2 W^*}{\partial X^2} - M_f (\kappa_{cf} V_f)^2 \frac{\partial^2 W^*}{\partial X^2} \right\} = 0 \quad (34) \\
& (1 - \bar{l}^2 \nabla^2) \left(\gamma_2 \frac{\partial^3 W^*}{\partial X^3} + \gamma_3 \frac{\partial^2 \Theta^*}{\partial X^2} - \gamma_4 \frac{\partial W^*}{\partial X} - \gamma_5 \Theta^* \right) = 0
\end{aligned}$$

and

$$\begin{aligned}
& \left(1 - \bar{l}^2 \frac{\partial^2}{\partial X^2} \right) \left\{ \chi_1 \frac{\partial^4 W}{\partial X^4} + \chi_2 \frac{\partial^3 \Theta}{\partial X^3} - \chi_4 \frac{\partial^2 W}{\partial X^2} - \chi_5 \frac{\partial \Theta}{\partial X} \right\} \\
& + \left(1 - \bar{\xi}^2 \frac{\partial^2}{\partial X^2} \right) \left\{ - \left[\frac{\pi \gamma_0}{2} \int_0^\pi \left(\frac{\partial \bar{W}}{\partial X} \right)^2 dX - M_f (\kappa_{cf} V_f)^2 \right] \frac{\partial^2 W}{\partial X^2} \right. \\
& \quad \left. - \lambda_q - \frac{\partial}{\partial X} \left(\Psi_1 \frac{\partial \ddot{W}}{\partial X} + \Psi_2 \ddot{\Theta} \right) + 2 M_f \kappa_{cf} V_f \frac{\partial \dot{W}}{\partial X} \right\} = 0 \quad (35) \\
& \left(1 - \bar{l}^2 \frac{\partial^2}{\partial X^2} \right) \left[\chi_2 \frac{\partial^3 W}{\partial X^3} + \chi_3 \frac{\partial^2 \Theta}{\partial X^2} - \chi_4 \frac{\partial W}{\partial X} - \chi_5 \Theta \right] \\
& = \left(1 - \bar{\xi}^2 \frac{\partial^2}{\partial X^2} \right) \left(\Psi_2 \frac{\partial \ddot{W}}{\partial X} + \Psi_3 \ddot{\Theta} \right)
\end{aligned}$$

in which

$$\chi_i = \left(\gamma_j + \lambda \bar{\gamma}_j \frac{\partial}{\partial \tau} \right) \quad (i = 0, 1, \dots, 9) \quad (36)$$

3. Solutions

3.1 Solution to the post-buckling equilibrium path

The first aim is to obtain the initial flow-inducing post-buckling configuration, namely, the relationship between flow velocity and maximum

deflection. Eqs. (34) can be solved by the two-step perturbation technique [57,58].

In the current case, a perturbation expansion scheme is established

$$\begin{aligned} V^2(X, \varepsilon) &= \sum_{k=0} \varepsilon^k (V^2)^{(k)}(X), \\ W^*(X, \varepsilon) &= \sum_{k=1} \varepsilon^k w_k^*(X), \\ \Theta^*(X, \varepsilon) &= \sum_{k=1} \varepsilon^k \vartheta_k^*(X) \end{aligned} \quad (37)$$

where ε is a dimensionless small perturbation parameter, and k is the number of expansion term.

The first-order post-buckling deflection w_1^* and the corresponding angle ϑ_1^* can be assumed to have the following form.

$$\begin{aligned} w_1^* &= A_1^* \varphi_1(X) \\ \vartheta_1^* &= B_1^* \varphi_1'(X) \end{aligned} \quad (38)$$

For the case of simply supported at both ends, $\varphi_1 = \sin(X)$. By using Eqs. (37), (38) to solve Eqs. (34) step by step, the asymptotic solutions can be obtained

$$W^*(X, \varepsilon) = \sum_{k=1} \varepsilon^k w_k^*(X) = \varepsilon w_1^* + O(\varepsilon^4) = \varepsilon A_1^* \varphi_1(X) \quad (39-a)$$

$$W_m^* = W^* \Big|_{X=\frac{\pi}{2}} = \varepsilon A_1^* \quad (39-b)$$

where W_m^* denote the dimensionless midspan post-buckling deflection and

$$\begin{aligned} V^2 &= (V^2)^{(0)} + \varepsilon (V^2)^{(1)} + \varepsilon^2 (V^2)^{(2)} + O(\varepsilon^3) \\ &= (V^2)^{(0)} + \frac{\gamma_0 \pi^2 (W_m^*)^2}{4 \kappa_{cf}^2 M_f} \end{aligned} \quad (40)$$

in which $V^{(0)}$ denotes the dimensionless critical buckling flow velocity and is written as V_{cr}

$$V_{cr} = V^{(0)} = \frac{1}{\kappa_{cf}} \sqrt{\frac{1 + \bar{l}^2}{1 + \bar{\xi}^2} \left[\frac{\gamma_1 + \gamma_4}{M_f} + \frac{(\gamma_2 + \gamma_5) B_{10}^*}{M_f A_{10}^*} \right]} \quad (41)$$

and

$$\frac{B_{10}^*}{A_{10}^*} = -\frac{\gamma_2 + \gamma_4}{\gamma_3 + \gamma_5} \quad (42)$$

Consequently, the dimensionless post-buckling deflection W^* can be written in the following form providing the initial configuration for impact vibration analysis

$$W^*(X) = W_m^* \varphi_1(X) \quad (43)$$

where W_m^* can be determined by Eq. (40).

3.2 Solution to low-velocity impact response

The two-step perturbation method is used again to discretize the governing equations, and the generalized displacements and load are assumed as

$$\begin{aligned} \lambda_q(X, \varepsilon, \tilde{t}) &= \sum_{m=1}^n \varepsilon^m \lambda_q^{(m)}(X, \tilde{t}), \\ W(X, \varepsilon, \tilde{t}) &= \sum_{m=1}^n \varepsilon^m w_k(X, \tilde{t}), \\ \Theta(X, \varepsilon, \tilde{t}) &= \sum_{m=1}^n \varepsilon^m \vartheta_k(X, \tilde{t}) \end{aligned} \quad (44)$$

where $\tilde{t} = \varepsilon \tau$ is introduced to delay the dynamic terms to higher-order equations.

By substituting Eqs. (44) into the governing equations, a series of perturbation differential equations are derived. The Galerkin truncation is performed on each order equations, so the generalized displacements can be written as follows,

$$\begin{aligned} W(X, \varepsilon, \tilde{t}) &= \varepsilon \sum_{m=1}^n A_1^{(m)}(\tilde{t}) \varphi_m(X) + O(\varepsilon^4) \\ \Theta(X, \varepsilon, \tilde{t}) &= \varepsilon \sum_{m=1}^n B_1^{(m)}(\tilde{t}) \varphi_m'(X) + O(\varepsilon^4) \\ W^*(X) &= A^* \varphi_1(X) \end{aligned} \quad (45)$$

The residual value of load can be written as

$$R(W, X, \tau) = \sum_{k=1}^3 \varepsilon^m \lambda_q^{(m)}(X, \tau) - \lambda_q \quad (46)$$

where $\lambda_q^{(m)}$ can be represented as a function of $A_1^{(m)}$, $\dot{A}_1^{(m)}$, $\ddot{A}_1^{(m)}$, namely,

$$\lambda_q^{(m)}(X, \tau) = \sum_{m=1}^n f^{(m)}(A_1^{(m)}, \dot{A}_1^{(m)}, \ddot{A}_1^{(m)}) \quad (47)$$

The discrete finite order time domain equations can be then obtained by the Galerkin integral

$$\int_0^\pi R(W, X, \tau) \varphi_m(X) = 0 \quad (48)$$

The Runge-Kutta method can be used to solve the coupling equations of impactor and the nanotube, and a series of $\varepsilon A_1^{(m)}(m=1,3,5,\dots)$ can be obtained. Subsequently, by using Eqs. (45), the displacement-time history curve at any position of the nanotube and mode at any moment can be obtained as follows

$$\begin{aligned} W_{X=X_0}(\tau) &= \varepsilon \sum_{m=1}^n A_1^{(m)}(\tau) \varphi_m(X_0) \\ W_{\tau=\tau_0}(X) &= \varepsilon \sum_{m=1}^n A_1^{(m)}(\tau_0) \varphi_m(X) \end{aligned} \quad (49a-b)$$

Besides, by substituting Eq. (49-b) into Eqs. (1), (10) and (11), the stress field at any moment can be acquired.

Specifically, for the current case of simply supported nanotubes at both ends, $\varphi_m = \sin(mX)$

The initial conditions are assumed to be

$$W|_{\tilde{t}=0} = \frac{\partial W}{\partial \tilde{t}}|_{\tilde{t}=0} = 0, \quad \Theta|_{\tilde{t}=0} = \frac{\partial \Theta}{\partial \tilde{t}}|_{\tilde{t}=0} = 0 \quad (50)$$

By substituting Eqs. (44) into Eqs. (35), the following 3-order discrete equations can be derived

$$\begin{aligned} \left(1 - \bar{\xi}^2 \frac{\partial^2}{\partial X^2}\right) \lambda_q^{(1)} &= (1 + \bar{\xi}^2) \frac{\pi^2 \gamma_0 A_1^{(1)} (A^*)^2}{2} \sin(X) \\ &+ \sum_{m=1}^n \left[(1 + m^2 \bar{l}^2) \left[(\gamma_1 m^4 + \gamma_4 m^2) + (\gamma_2 m^3 + \gamma_5 m) \frac{B_1^{(m)}}{A_1^{(m)}} \right] A_1^{(m)} \right. \\ &\left. + (1 + m^2 \bar{\xi}^2) \left(\frac{\pi^2 \gamma_0 (A^*)^2}{4} - M_f (k_{cf} V_f)^2 \right) m^2 A_1^{(m)} \right] \sin(mX) \end{aligned} \quad (51-a)$$

$$\begin{aligned}
\left(1 - \bar{\xi}^2 \frac{\partial^2}{\partial X^2}\right) \lambda_q^{(2)} = & \left\{ \sum_{m=1}^n (1 + m^2 \bar{l}^2) \lambda \dot{A}_1^{(m)} \left[(\bar{\gamma}_1 m^4 + \bar{\gamma}_4 m^2) + (\bar{\gamma}_2 m^3 + \bar{\gamma}_3 m) \frac{\dot{B}_1^{(m)}}{\dot{A}_1^{(m)}} \right] \right\} \sin(mX) \\
& + (1 + \bar{\xi}^2) \sum_{m=1}^n \frac{\pi^2 \gamma_0 m^2 A^* (A_1^{(m)})^2}{4} \sin(X) + \left[\sum_{m=1}^n (1 + m^2 \bar{\xi}^2) \left(\frac{\pi^2 \gamma_0 m^2 A^* A_1^{(1)} A_1^{(m)}}{2} \right) \sin(mX) \right. \\
& \left. + 2M_f \kappa_{cf} V_f \sum_{m=1}^n (1 + m^2 \bar{\xi}^2) m A_1^{(m)} \cos(mX) \right]
\end{aligned} \tag{51-b}$$

and

$$\begin{aligned}
\left(1 - \bar{\xi}^2 \frac{\partial^2}{\partial X^2}\right) \lambda_q^{(3)} = & \left[\sum_{m=1}^n (1 + m^2 \bar{\xi}^2) \ddot{A}_1^{(m)} \left(M_f + \psi_0 + \psi_1 m^2 + \psi_2 m \frac{\ddot{B}_1^{(m)}}{\ddot{A}_1^{(m)}} \right) \sin(mX) \right] \\
& + \frac{\pi^2 \gamma_0}{4} \sum_{m=1}^n \left(m^2 (A_1^{(m)})^2 \right) \sum_{m=1}^n (1 + m^2 \bar{\xi}^2) (m^2 A_1^{(m)}) \sin(mX)
\end{aligned} \tag{51-c}$$

where

$$\begin{aligned}
\frac{B_1^{(m)}}{A_1^{(m)}} &= -\frac{m^3 \gamma_2 + m \gamma_4}{m^2 \gamma_3 + \gamma_5} \\
\frac{\dot{B}_1^{(m)}}{\dot{A}_1^{(m)}} &= -\frac{m^3 \bar{\gamma}_2 + m \bar{\gamma}_4}{m^2 \bar{\gamma}_3 + \bar{\gamma}_5} \\
\frac{\ddot{B}_1^{(m)}}{\ddot{A}_1^{(m)}} &= -m \frac{\psi_2}{\psi_3}
\end{aligned} \tag{52}$$

Here, the impact point is at the midpoint of the nanotube, and hence, the anti-symmetric modes does not exist, namely, there are only odd order modes.

$$\int_0^\pi R(W, X, \tau) \sin(mX) dX = 0 \quad (m=1, 3, 5, \dots) \tag{53}$$

Applying Eqs. (53) to Eqs.(51) and replacing $\tilde{t}$ by τ yields the Duffing type higher-order asymptotic expression of load

$$\begin{aligned}
& g_0^{(m)} \varepsilon \ddot{A}_1^{(m)}(\tau) + g_1^{(m)} \varepsilon \dot{A}_1^{(m)}(\tau) + g_2^{(m)} \varepsilon A_1^{(m)}(\tau) \\
& = (1 + m^2 \bar{\xi}^2) \left\{ \begin{aligned} & g_f \left(W_i - W_{X=\frac{\pi}{2}} \right)^{\frac{3}{2}} \quad \text{Loading} \\ & \left[g_f \left(W_i - W_{X=\frac{\pi}{2}} \right)^{\frac{3}{2}} \right]_{\max} \left(\frac{\alpha - \alpha_0}{\alpha_{\max} - \alpha_0} \right)^{\frac{5}{2}} \quad \text{Unloading} \end{aligned} \right. \quad (m=1, 3, 5, \dots) \tag{54}
\end{aligned}$$

where $g_f = \frac{2 \sin(\frac{m\pi}{2}) K_c L^{\frac{7}{2}}}{A_3 \pi^4}$, and according to Eq. 49(a), $W_{X=\frac{\pi}{2}}$ is written as

$$W_{X=\frac{\pi}{2}}(\tau) = \varepsilon \sum_{m=1}^n A_1^{(m)}(\tau) \sin\left(\frac{m\pi}{2}\right) \quad (55)$$

and the coefficients $g_i^{(m)} (i = 0, 1, 2, 3)$ are expressed as

$$\begin{aligned} g_0^{(m)} &= (1+m^2 \bar{\xi}^2) \left(M_f + \psi_0 + \psi_1 m^2 + \psi_2 m \frac{\ddot{B}_1^{(m)}}{\ddot{A}_1^{(m)}} \right) \\ g_1^{(m)} &= \lambda (1+m^2 \bar{l}^2) \left[(\bar{\gamma}_1 m^4 + \bar{\gamma}_4 m^2) + (\bar{\gamma}_2 m^3 + \bar{\gamma}_5 m) \frac{\dot{B}_1^{(m)}}{\dot{A}_1^{(m)}} \right] \\ g_2^{(m)} &= \left\{ \left[\begin{aligned} &(1+m^2 \bar{l}^2) \left[(\gamma_1 m^4 + \gamma_4 m^2) + (\gamma_2 m^3 + \gamma_5 m) \frac{B_1^{(m)}}{A_1^{(m)}} \right] \\ &+ (1+\bar{\xi}^2) \left[\frac{\pi^2 \gamma_0 (A^*)^2}{2} + \frac{\pi^2 \gamma_0 m^2 A^* (\varepsilon A_1^{(m)})}{4} \right] \\ &+ m^2 (1+m^2 \bar{\xi}^2) \left[\left(\frac{\pi^2 \gamma_0 (A^*)^2}{4} - M_f (k_{cf} V_f)^2 \right) \right. \\ &\quad \left. + \left(\frac{\pi^2 \gamma_0 A^* \varepsilon A_1^{(1)}}{2} \right) + \frac{\pi^2 \gamma_0}{4} \sum_{m=1}^n \left(m^2 (\varepsilon A_1^{(m)})^2 \right) \right] \end{aligned} \right\} (m=1) \\ &\quad \left\{ \left[\begin{aligned} &(1+m^2 \bar{l}^2) \left[(\gamma_1 m^4 + \gamma_4 m^2) + (\gamma_2 m^3 + \gamma_5 m) \frac{B_1^{(m)}}{A_1^{(m)}} \right] \\ &+ m^2 (1+m^2 \bar{\xi}^2) \left[\left(\frac{\pi^2 \gamma_0 (A^*)^2}{4} - M_f (k_{cf} V_f)^2 \right) \right. \\ &\quad \left. + \left(\frac{\pi^2 \gamma_0 A^* \varepsilon A_1^{(1)}}{2} \right) + \frac{\pi^2 \gamma_0}{4} \sum_{m=1}^n \left(m^2 (\varepsilon A_1^{(m)})^2 \right) \right] \end{aligned} \right\} (m \geq 2) \end{aligned} \quad (56)$$

It can be seen from Eqs. (54) that there are two kinds of nonlinearities for the impact problem of fluid-conveying nanotubes, one is load nonlinearity, the other is geometric nonlinearity. The square term, cubic term and cross term of Eqs. (54) are reflected in the coefficients $g_2^{(m)}$ in Eq. (56).

On the other hand, the dimensionless motion differential equation of impactor is derived

$$\frac{d^2 W_i}{d\tau^2} - \hat{g} = \begin{cases} \bar{K}_c \left(W_i - W_{x=\frac{\pi}{2}} \right)^{\frac{3}{2}} & \text{Loading} \\ \left[\bar{K}_c \left(W_i - W_{x=\frac{\pi}{2}} \right)^{\frac{3}{2}} \right]_{\max} \left(\frac{\alpha - \alpha_0}{\alpha_{\max} - \alpha_0} \right)^{\frac{5}{2}} & \text{Unloading} \end{cases} \quad (57)$$

in which

$$\begin{aligned} \bar{K}_c &= -\frac{L^2 K_c \rho_m}{m \pi^2 E_m} \\ \hat{g} &= \frac{gL \rho_m}{\pi^2 E_m} \end{aligned} \quad (58)$$

The coupling governing equations of the nanotube and the impactor including loading and unloading process can be solved by using the Runge-Kutta method. Subsequently, the free vibration equations of the nanotube after unloading can be obtained by removing the load terms of Eqs. (53). The initial conditions are set as the displacement and velocity at the time of complete unloading. The free vibration equations can also be solved by the fourth-order Runge Kutta method. By substituting a series of solutions of $\varepsilon A_1^{(m)}$ into Eqs. (49), the vibration response of a certain point on the nanotube and the mode at a certain time can be obtained. Ulteriorly, by using Eqs. (1), (10) and (11), the stress field can also be acquired.

4. Numerical results and discussion

This section presents the contact force time history curves, midspan time history curves, modes and flexural normal stress distributions of the fluid-conveying GRC nanotube in the process of contact. The material properties of the damping sandwich material are set as $E = 150 \text{ GPa}$, $\mu = 0.3$, $\rho = 1.9 \times 10^{-6} \text{ kg/mm}^3$, $c = 0.04 \text{ Ns/m}^2$ in Figs. 4-5 and $c = 2.5 \text{ Ns/m}^2$ in Figs. 6-8, $j = c/2(1+\mu)$. Polymethyl methacrylate (PMMA) is selected as the matrix of the composite material. The properties of graphene and the matrix material and the modified parameters in the extended Halpin-Tsai meso model can refer to [53,59]. The

diameter and material of the impactor is set as: $R_i=R_w$, $E_i = 207 \text{ GPa}$, $\mu_i = 0.3$, $\rho_i = 7960 \text{ kg/m}^3$. The piecewise functionally gradient distribution mode is adopted in the composite layers, that is, the volume fraction distribution of graphene in the outer layer composites is V type, namely, $[0.11/0.09/0.07/0.05/0.03]$, and the volume fraction distribution of graphene in the inner layers is Λ type, namely, $[0.03/0.05/0.07/0.09/0.11]$. The laying angle of each layer of graphene is 0° . The main direction of graphene in each composite layer is 0° . The geometric parameters are set as: $R_o=9\text{nm}$, $R_w=10\text{nm}$, $L=30R_w$. Define the thickness of tube wall, inner layer, interlayer layer, and outer layer as d , d_1 , d_2 and d_3 , and set $d_1=d_3= d_2/2= d/4=(R_w-R_o)/4$. The non-dimensional parameters characterizing slip flow effect are set to $K_n = 0.05$, $\kappa_{cf} = 1.4052$ [15]. If not specified, the nanoscale parameters are $e_0a=4R_w$, $l=e_0a$. Besides, v_{cr} is defined as the critical buckling velocity considering the slip flow effects and small-scale effects (*Corresponding to the above nano parameters*)

4.1 Computation accuracy

To validate the theoretical model and computation method and ensure the accuracy, there presents the convergence analysis and the comparison between the current results and those in the literatures. As can be seen from Fig. 2, when neglecting the Poisson effect, the frequency-flow velocity curves are in good agreement with that in [60], which suggests the correctness of the dynamic model of the fluid-conveying tube. However, it can be also found that neglecting the Poisson effect greatly underestimates the curves and hence, this effect is considered in all the following examples. Fig. 3 compares the contact force of first order Galerkin truncation of the GRC beams under low-velocity impact. It can be seen that except for some deviation at the unloading terminal, the curve is completely consistent with [56], and thus the correctness of the computation method can be illustrated.

Fig. 4 presents the convergence analysis of contact force, midspan displacement time history curve, mode and flexural normal stress of upper surface at peak contact force. The flow velocity is $v=0.2v_{cr}$ and the impact velocity is $v_i=4 \text{ m/s}$. It can be found that there only need 9 and 7 terms of truncation for the contact force and displacement response curves to converge, respectively.

However, 55 and 163 terms of truncation are required for the mode and stress distribution to convergence. To give consideration to both the calculation accuracy and efficiency, in the following examples, the 9-order truncation are used for calculating the contact force and displacement time history curves, while the 163-order truncation are used for the mode and stress field.

Fig. 2 Validations of natural frequency-flow velocity relationships

Fig. 3 Validation of the contact force of first-order Galerkin truncation solution

Fig. 4 Convergence analysis, (a) contact force (b) midspan displacement time history curve (c) mode at maximum contact force (d) flexural normal stress distribution of upper surface at maximum contact force

4.2 Contact dynamic responses in flow-inducing pre-and post-bucking states

Figs. 4-5 present the dynamic responses of the fluid-conveying nanotubes under low-velocity impact in the pre-buckling states. It can be further obtained from Fig. 4(b) that the transient vibration waveform is zig-zag type because the higher order modes are excited in the process of impact, but with the continuous free attenuation vibration, the waveform tends to be regular. Fig.5 shows the mode and flexural normal stress of upper surface at the moment of maximum contact force and complete unloading (corresponding to P_1 and P_2 in Fig. 4 (a)). It can be observed from Fig. 5(a) that when the contact force is maximum, the peak value of the mode is at the contact point, and it is a “cusp” requiring higher-order truncation to approach. However, at the moment of complete unloading, the amplitude of the contact point is a local minimum. The stress field of impact process is the most concerned for the problem of strength failure. According to the common sense, the flexural normal stress is the dominant stress and its distribution is plotted in Fig 5(b). At the moment of maximum contact force, the maximum stress is the compressive stress at the impact center, while the maximum stress at the deviation from the midpoint is the tensile stress. As the

unloading process reaches the point where the contact force is 0, the maximum stress decreases greatly and is no longer in the center of impact. This phenomenon gives an important enlightenment to the impact strength design, that is, it is not enough to only consider the strength failure at the impact point, because stress peaks may also occur at non-collision points. It is worth pointing out that the phenomenon that the stress peak shifts from the collision center to the non-collision points cannot be explained as the stress wave propagation. In this paper, the vibration solution rather than the wave solution is presented, which can be understood as the standing stress wave (mode of vibration) formed by the reflection and superposition of the stress wave back and forth through the boundary.

Fig. 5 Mode and flexural normal stress distribution in the process of contact in the pre-buckling state ($v_f=0.2v_{cr}$), (a)mode (b) flexural normal stress distribution of upper surface.

Fig. 6 demonstrates the low-velocity impact responses of the fluid-conveying nanotube in the post-buckling state. As is shown in Fig. 6(a), the concave surface of the nanotube is assumed to be hit by the impactor. In this example, the flow velocity is $v=1.05v_{cr}$, the viscoelastic coefficient is $c=2.5 \text{ Ns/m}^2$ and two critical impact velocities are $v_i=11.243 \text{ m/s}$ and $v_i=6.329 \text{ m/s}$ is adopted. Obviously, as shown in Fig. 6(b), the greater the impact velocity, the greater the contact force and the shorter the contact time. Subsequently, under the impact of different velocities, different vibration responses occur in the nanotube, which are shown in Fig.6(c-d). It can be seen that there are three equilibrium points at the midpoint of the nanotube, which are two stable focal points F_1 , F_2 and an unstable saddle point S . When the impact velocity is small, the midpoint of the nanotube moves around only one focal point F_2 and $v_i=6.329 \text{ m/s}$ is a critical velocity. When the impact velocity exceeds it, the motion approaches the saddle point S and an instability phenomenon called snap-through will occur [61]. Then the motion changes to an

attenuated vibration around F_1 . When the velocity reaches $v_i=11.243$ m/s, the motion develops into the motion around three equilibrium points, and finally jumps again through saddle point S and changes into attenuation motion around F_1 . The mode and stress field at the moment of maximum contact force and complete unloading (corresponding to P_1 and P_2 in Fig.6(b)) are investigated in Fig.6(e-f). It can be seen that unlike the pre-buckled nanotube, the peak values of mode and stress are always at the contact center. Since the impact is on the post-buckling large-deformation configuration, the nonlinear flexural normal stress at the boundary is no longer zero in the linear solution. Although the deviation between the linear solution and the nonlinear solution is large near the boundary, the results of the stress peak are consistent.

Fig. 6 Contact dynamic responses of the nanotube in the flow-inducing post-buckling state ($v_i=1.05v_{cr}$), (a) impact diagram (b) contact force (c) midspan displacement time history curve (d) phase portrait (e) mode (f) flexural normal stress distribution of upper surface.

4.3 Nano effects

Fig. 7 studies the influence of nonlocal stress and strain gradient on the contact force, midspan displacement, mode and flexural normal stress at the moment of maximum contact force. The viscoelastic coefficient is $c=2.5 \text{ Ns/ m}^2$ and the critical impact velocity is $v_i=11 \text{ m/s}$. When $v = 0.1v_{cr}$ (corresponding to the value of $l = e_0a$), the nanotube is in pre-buckling state in three cases ($l = e_0a$, $l = 1.5e_0a$, $l=0.5 e_0a$). It can be found that when $l > e_0a$, the stiffness hardening effect appears, which is embodied in the higher contact force, lower vibration amplitude, lower modal and stress peak. Similarly, $l < e_0a$ will lead to the opposite result. When $v = 0.95v_{cr}$ and $l =0.5 e_0a$, the stiffness softening effect makes the nanotube in the post-buckling state, and accordingly, the dynamic snap-throughs between the bi-stable states occur, as is shown in Fig. 7(f) and (g). In this case, the stress at the boundary is no longer zero due to initial post-buckling large deformation. However, the state of the nanotube (pre buckling or post buckling) has no additional influence on the contact force curve, as is seen from Fig. 7(e). When $l =1.5 e_0a$, the influence on the result can be analyzed similarly.

In the case of $v = 1.05v_{cr}$ and $l =1.5 e_0a$, the stiffness-hardening effect increases the critical buckling flow velocity and the nanotube is dramatically in the pre-buckling state. On the other hand, when $l < e_0a$, the stiffness softening effect reduces the critical buckling velocity, which leads to the larger post-buckling deflection and nonlinear hardening stiffness of nanotubes at a certain flow rate. Therefore, it can be understood why the vibration snap-through of the curved nanotube occurs when $l = e_0a$, but it does not occur when $l < e_0a$, as is shown in Fig.7(j). *Besides, an interesting conclusion is shown, that is, although $l > e_0a$ has the effect of stiffness hardening, the free vibration amplitude is greater than the case of $l < e_0a$.* These phenomena imply the dual influence mechanism of the small-scale effect. Since the impact vibration of the nanotube is based on the initial flow-inducing pre- and post-buckling configuration, the small-scale effect has both direct and indirect influence on contact dynamic responses. On the one hand, it shows that the nonlocal stress and strain gradient affect the stiffness of the nanotube itself. On the other hand, the critical buckling velocity and post buckling equilibrium path of the nanotube are changed, which determines the tube configuration and nonlinear stiffness. Therefore, the nano-effect affects both the

inherent stiffness of the material and the initial nonlinear stiffness of the nanotube due to large deformation, both of which have an effect on the contact dynamic response at the same time. It can be seen from the Fig. (f) and (j) that the two are often in a competitive relationship. In the pre buckling state, the stiffness of the material itself plays a leading role, while in the post-buckling state, the conclusion changes. Therefore, the phenomenon in Fig. 7(j) can be explained. The stiffness softening effect weakens the stiffness of the nanotube itself, but increases the post-buckling deflection and nonlinear stiffness at a certain flow rate. At this time, the non-linear stiffness dominates. Therefore, the jumping instability does not occur and the amplitude is lower than the case of when $l > e_0a$.

Further, it is found that the sensitivity of the contact force to the nano effect is almost the same at different flow rates. However, the vibration response, mode and stress field are particularly sensitive to the nano effect when the flow velocity approaches the critical buckling velocity. The fundamental reason is that the nano effect makes it easy for the nanotube to change the state between pre-buckling and post-buckling when flow velocity approaches critical buckling value.

Fig. 7 Effects of nonlocal stress and strain gradient on the contact force, midspan displacement response, mode and flexural normal stress of upper surface at maximum contact force at different flow velocities.

Fig. 8 investigates the influence of slip flow on the contact force, midspan displacement, mode and flexural normal stress at the moment of maximum

contact force. It can be known from the existing work that the slip flow effect can reduce the critical buckling velocity. Therefore, at a certain flow rate, for example $v=1.05v_{cr}$, the nanotube is in the post-buckling state, but when the slip flow effect is neglected, the nanotube may be in the pre-buckling state. In this case, the vibration response, mode and stress field are all sensitive to the slip flow effect, but the contact force is almost unaffected, as is shown in Fig. 8(i-l). When $v < v_{cr}$, the nanotube is in the state of pre buckling, regardless of considering or neglecting the slip flow. It can be found that the slip flow effect has little influence on the contact force, mode and stress field. However, the vibration response still follows the rule that the closer the flow velocity is to the critical buckling velocity, the more significant the influence of the slip flow effect is, as is shown in Fig.8(b) and (f). It is worth noting that the slip flow effect does not have a dual effect, because it does not affect the stiffness of the material itself.

Fig. 8 Effects of slip flow on the contact force, midspan displacement response, mode and flexural normal stress of upper surface at maximum contact force at different flow velocities.

5. Conclusion

A nano-model comprehensively considering slip flow and small-scale effects is established and a computation mode of two-step perturbation-Galerkin truncation-Runge-Kutta method is developed to study the contact dynamic response and stress field of the fluid-conveying nanotube. Through numerical analysis, the

contact force, motion law, mode and stress distribution of nanotubes under lateral low-velocity impact are obtained. The sensitivity of motion law and stress field to different flow-inducing pre-buckling and post-buckling initial configurations is revealed. The dual influences of the nano effect are discovered and the flow velocity range where the response sensitive to the nano effect is obtained. In the future, the current theoretical models and methods can be extended to study the influence of nano effect on structural damage and strength failure by introducing the damage criterion and strength criterion. The main conclusions of this paper are as follows.

1. The convergence truncation order of contact force, displacement time history curve, mode and stress field are different. Among them, the stress field requires the highest truncation order.
2. When the nanotube is in the post-buckling stage, repeated dynamic snap-through buckling may occur between bi-stable states after transverse impact.
3. In the pre-buckling state, there is a high compressive stress peak at the contact point at the moment of maximum contact force. In the unloading process, the stress peak drops sharply and may be transferred to the non-contact point. In the post-buckling state, the stress peak is always at the contact point.
4. Influence law of nano effects
 - 1) When the flow velocity is close to the critical buckling value, the contact dynamic response and stress field are most sensitive to the nano effects.
 - 2) The influences of small-scale effect on the contact dynamic responses of the fluid-conveying nanotube are dual. It not only affects the stiffness of the nanotube itself, but also affects the nonlinear stiffness due to initial flow-inducing post-buckling large deformation.

Declarations

All the following sections are applicable.

Abstract

1. Introduction
2. Formulations of the fluid-conveying nanotubes
 - 2.1 Displacement field
 - 2.2 Slip flow effect
 - 2.3 Nonlocal strain gradient theory
 - 2.4 Constructive equations
 - 2.5 Contact model
 - 2.6 Governing equations
 - 2.6.1 Governing equations for the impactor
 - 2.6.2 Governing equations for the nanotube
3. Solutions
 - 3.1 Solution to the post-buckling equilibrium path
 - 3.2 Solution to low-velocity impact response
4. Numerical results and discussion
 - 4.1 Computation accuracy
 - 4.2 Contact dynamic responses in flow-inducing pre-and post-bucking states
 - 4.2 Nano effects
5. Conclusion

Availability of data

Date will be made available on reasonable request.

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Conflicts of interest

The authors declare that they have no conflict of interest.

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