Event-triggered Control of Uncertain Nonlinear Discrete-time Systems with Extended State Observer

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Abstract This paper is dedicated to extended state observer-based event-triggered model free iterative learning control (ET-MFILC) for nonlinear systems with disturbances. A modified MFILC scheme is proposed by using the estimated uncertainties for the disturbed system. With the help of the estimated errors, an iterative extended state observer (IESO) is constructed to estimate the unknown uncertainties. New triggering mechanism integrated true tracking errors, the estimated errors and the estimated uncertainties is designed for multiple inputs and multiple outputs (MIMO) systems. Sufficient conditions are proposed to make the resultant ET-MFILC tracking error systems be uniformly ultimately bounded. An illustrative example is presented to demonstrate the effectiveness of the proposed scheme.

Keywords Networked control systems, event-triggered control, extended state observer.

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1 Introduction

Since network control systems are affected by the limited bandwidth, packet losses and time-delay occur frequently in practical applications [1–4]. Considering the negative influence of these phenomena to system performance, event-triggered control (ETC) has been adopted widely as an effective method to save the communication resources [5–11]. Specially, different from the event-triggered mechanism in [5] requiring special hardwares to measure the states of system continuously, [6, 7] put forward a periodic ETC strategy by combining ETC and traditional periodic sampling data control. In [8], a self-triggered control (STC) scheme is presented instead of checking the triggering condition passively. Employing the estimated errors to adjust the triggering condition, this STC is further extended to an adaptive paradigm by [9]. Contrasted to these static triggering rules, a novel dynamic triggering mechanism is supplied in [10] via introducing an internal dynamic variable.

Note that the above mentioned ETC methods require systems with accurate model. However, this requirement is unrealistic because of complex production process. Therefore, how to construct controllers independent of system models is a meaningful topic in control community. Data-driven control (DDC) refers that the design of controller employs system inputs and outputs merely. Based on this concept, there are

some DDC methods called differently, such as model-free adaptive control (MFAC) [11, 12], unfalsified control (UC) [13, 14], virtual reference feedback tuning (VRFT) [15, 16]. Among them, MFAC is discussed in [17–20]. In particular, by introducing pseudo partial derivative (PPD) parameter and dynamic linearization technology, an equivalent data model is proposed in [17]. Motivated by this idea, [18] presents an ETC strategy based on MFAC by using the tracking error. Different from [18], [19] develops an ETC mechanism for model-free iterative learning control (MFILC) input by applying Lyapunov theory to the tracking error on the iteration domain. Further, [20] adopts the output-s received by the controller to improve the ET-MFILC rule for nonlinear systems with fading channels. However, according to the author's knowledge, considering a MFILC scheme for nonlinear systems with disturbances through event-triggered outputs is yet to be investigated.

Motivated by the above discussions, an ET-MFILC method for MIMO nonlinear systems with disturbances is studied in this paper. Utilizing the estimated uncertainties obtained by an IESO, new triggering mechanism for MIMO systems is built in terms of the true tracking errors, the estimated errors and the estimated uncertainties. The uniformly ultimately boundedness of the resultant ET-MFILC tracking error systems is ensured by the the established ET-MFILC scheme with the help of the Lyapunov method. The validity of the proposed scheme is demonstrated by an example.

This paper consists of five parts. Part 2 covers the problem formulation, some assumptions and lemmas. Part 3 shows the main results. The effectiveness of the proposed scheme for MIMO is presented in Part 4. Part 5 conclusions this paper.

2 Problem Formulation

Consider the repetitive disturbed MIMO nonlinear system as below:

$$\begin{aligned} \mathbf{y}(\Xi, \delta + 1) = & \mathbf{f}(\mathbf{y}(\Xi, \delta), \dots, \mathbf{y}(\Xi, \delta - c_{\mathbf{y}}), \\ & \mathbf{u}(\Xi, \delta), \dots, \mathbf{u}(\Xi, \delta - c_{\mathbf{u}})) + \mathbf{d}(\Xi, \delta) \end{aligned} \quad (1)$$

where $\mathbf{y}(\Xi, \delta) \in \mathcal{R}^m$ denotes the system output, $\mathbf{u}(\Xi, \delta) \in \mathcal{R}^m$ denotes the system input and $\mathbf{d}(\Xi, \delta) \in \mathcal{R}^m$ denotes the disturbance. The iteration direction is presented by $\Xi \in \{0, 1, 2, \dots\}$ and $\delta \in \{0, 1, \dots, T\}$ is the time domain index. $c_{\mathbf{y}}$ and $c_{\mathbf{u}}$ are the unknown order. Nonlinear function $\mathbf{f}(\cdot)$ is continuously differentiable.

Some assumptions and lemmas are presented for the following derivation.

Assumption 1 [14] *The partial derivative of $\mathbf{f}(\cdot)$ with respect to $\mathbf{u}(\Xi, \delta)$ is continuous.*

Assumption 2 [14] *Nonlinear system (1) satisfies the generalized Lipschitz condition, i.e. $\|\Delta \mathbf{y}(\Xi, \delta + 1)\| \leq b \|\Delta \mathbf{u}(\Xi, \delta)\|$, where $\Delta \mathbf{y}(\Xi, \delta + 1) = \mathbf{y}(\Xi, \delta + 1) - \mathbf{y}(\Xi - 1, \delta + 1)$, $\Delta \mathbf{u}(\Xi, \delta) = \mathbf{u}(\Xi, \delta) - \mathbf{u}(\Xi - 1, \delta)$, $b > 0$.*

Assumption 3 [18] *The signs of all elements in matrix $\Theta(\Xi, \delta) \in \mathcal{R}^{m \times m}$ are invariant and diagonal ones are dominant. Moreover,*

$$|\Theta_{pj}(\Xi, \delta)| \leq b_1, b_2 < |\Theta_{pp}(\Xi, \delta)| \leq ab_2, p, j \in [1, m], p \neq j, b_2 > b_1(2a + 1)(m - 1), a \geq 1.$$

Lemma 1 [16] *If system (1) meets Assumptions 1 and 2, then it can be transformed as*

$$\Delta \mathbf{y}(\Xi, \delta + 1) = \Theta(\Xi, \delta) \Delta \mathbf{u}(\Xi, \delta) + \boldsymbol{\vartheta}(\Xi, \delta) \quad (2)$$

where $\|\Theta(\Xi, \delta)\| \leq b$ ($b > 0$), $\|\boldsymbol{\vartheta}(\Xi, \delta)\| \leq c_{\boldsymbol{\vartheta}}$ is uncertainties.

Remark 1 (2) *bridges the relationship from inputs and uncertainties to outputs without practical mechanism meaning. As [16], the subsequent derivation depends on $\Theta(\Xi, \delta)$ with slow change in the iterative direction.*

Lemma 2 [19] Let $\mathcal{A} = [c_{pj}] \in \mathcal{R}^{m \times m}$. The Gerschgorin disk is expressed as $\mathcal{D}_p = \{z \mid |z - c_{pj}| \leq \sum_{j \neq p, j=1}^n |c_{pj}|\}$, $p = 1, \dots, m$. The union of $\mathcal{D}_p$ is denoted by the Gerschgorin domain $\mathcal{D}_{\mathcal{A}} = \bigcup_{p=1}^m \mathcal{D}_p$. Gerschgorin domain $\mathcal{D}_{\mathcal{A}}$ contains all the eigenvalues of matrix $\mathcal{A}$.

Referring to the MFILC method proposed in [17], a MFILC scheme for disturbed MIMO systems is modified as below.

$$\hat{\Theta}(\Xi + 1, \delta) = \hat{\Theta}(\Xi, \delta) + \frac{\eta \mathbf{E}(\Xi, \delta) \Delta \mathbf{u}(\Xi, \delta)^\top}{\mu + \|\Delta \mathbf{u}(\Xi, \delta)\|^2}, \quad (3)$$

$$\mathbf{u}(\Xi + 1, \delta) = \mathbf{u}(\Xi, \delta) + \frac{\rho \hat{\Theta}(\Xi + 1, \delta)^\top \mathbf{Z}(\Xi, \delta)}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2} \quad (4)$$

where $\mathbf{E}(\Xi, \delta) = \Delta \mathbf{y}(\Xi, \delta + 1) - \hat{\Theta}(\Xi, \delta) \Delta \mathbf{u}(\Xi, \delta) - \hat{\boldsymbol{\vartheta}}(\Xi, \delta)$, $\mathbf{Z}(\Xi, \delta) = \mathbf{e}(\Xi, \delta + 1) - \hat{\boldsymbol{\vartheta}}(\Xi + 1, \delta)$, $\hat{\boldsymbol{\vartheta}}(\Xi, \delta)$ is the estimation of $\boldsymbol{\vartheta}(\Xi, \delta)$.

Remark 2 As [13] mentioned that $\Theta(\Xi, \delta)$ is reckoned accurately by parameter estimation law (3), therefore, $\Theta(\Xi, \delta)$ can be replaced by $\hat{\Theta}(\Xi, \delta)$ in the subsequent derivation.

Define the event-triggered error $\mathbf{e}_{ET}(\Xi, \delta) = \mathbf{y}(\Xi_l, \delta) - \hat{\mathbf{y}}(\Xi, \delta)$, $\Xi \in [\Xi_l, \Xi_{l+1}]$ and $\{\Xi_l\}$ ($l = 1, 2, \dots$) denotes the triggered iteration instants. As it is known, controllers cannot get the outputs during the inter-event iterations $\Xi \in [\Xi_l, \Xi_{l+1}]$. Therefore, without sacrificing system performance largely, the outputs in this case is estimated by $\hat{\mathbf{y}}(\Xi, \delta) = \hat{\mathbf{y}}(\Xi - 1, \delta) + \hat{\Theta}(\Xi, \delta - 1) \Delta \mathbf{u}(\Xi, \delta - 1) + \hat{\boldsymbol{\vartheta}}(\Xi, \delta - 1)$.

3 Main Results

In this part, the ET-MFILC scheme based on IESO for MIMO systems is proposed firstly. Then the boundedness of $\hat{\Theta}(\Xi, \delta)$ and tracking error $\mathbf{e}(\Xi, \delta)$ under the proposed scheme is established by means of the Lyapunov method.

First, an IESO employing the system errors is utilized to estimate unknown $\boldsymbol{\vartheta}(\Xi, \delta)$ as below.

$$\begin{aligned} \hat{\boldsymbol{\vartheta}}(\Xi, \delta) &= \hat{\boldsymbol{\vartheta}}(\Xi - 1, \delta) + \alpha \hat{\mathbf{e}}(\Xi - 1, \delta + 1) \\ \hat{\vartheta}_{p1}(\Xi, \delta) &= \begin{cases} b_{\hat{\boldsymbol{\vartheta}}} & \text{if } \hat{\vartheta}_{p1}(\Xi, \delta) > b_{\hat{\boldsymbol{\vartheta}}} \\ \hat{\vartheta}_{p1}(\Xi, \delta) & \text{if } |\hat{\vartheta}_{p1}(\Xi, \delta)| \leq b_{\hat{\boldsymbol{\vartheta}}} \\ -b_{\hat{\boldsymbol{\vartheta}}} & \text{if } \hat{\vartheta}_{p1}(\Xi, \delta) < -b_{\hat{\boldsymbol{\vartheta}}} \end{cases} \end{aligned} \quad (5)$$

where α is observer gain and selected by designers, $b_{\hat{\boldsymbol{\vartheta}}} > 0$ is a predetermined saturation constant, $\hat{\vartheta}_{p1}(\Xi, \delta)$ is the element of $\hat{\boldsymbol{\vartheta}}(\Xi, \delta)$ in row p and column 1, $\hat{\mathbf{e}}(\Xi, \delta + 1)$ is the estimation error as follows.

$$\hat{\mathbf{e}}(\Xi, \delta + 1) = \begin{cases} \mathbf{y}^*(\delta + 1) - \mathbf{y}(\Xi, \delta + 1) & \text{if } \Xi = \Xi_l \\ \mathbf{y}^*(\delta + 1) - \hat{\mathbf{y}}(\Xi, \delta + 1) & \text{if } \Xi_l < \Xi < \Xi_{l+1} \end{cases} \quad (6)$$

When there is no triggering, i.e. $\Xi \in [\Xi_l, \Xi_{l+1}]$, $\hat{\Theta}(\Xi, \delta)$ remains unchanged, namely, $\hat{\Theta}(\Xi, \delta) = \hat{\Theta}(\Xi, \delta - 1)$. Therefore, the event-triggered iterative learning parameter estimation law is presented as below:

$$\hat{\Theta}(\Xi + 1, \delta) = \begin{cases} \hat{\Theta}(\Xi, \delta) + \frac{\eta \mathbf{E}(\Xi, \delta) \Delta \mathbf{u}(\Xi, \delta)^\top}{\mu + \|\Delta \mathbf{u}(\Xi, \delta)\|^2}, & \Xi = \Xi_l \\ \hat{\Theta}(\Xi, \delta), & \Xi_l < \Xi < \Xi_{l+1} \end{cases} \quad (7)$$

In the ET-MFILC scheme, iterative learning control law (4) is modified as:

$$\mathbf{u}(\Xi + 1, \delta) = \mathbf{u}(\Xi, \delta) + \frac{\rho \hat{\Theta}(\Xi + 1, \delta)^\top \hat{\mathbf{Z}}(\Xi, \delta)}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2} \quad (8)$$

where $\hat{\mathbf{Z}}(\Xi, \delta) = \hat{\mathbf{e}}(\Xi, \delta + 1) - \hat{\boldsymbol{\vartheta}}(\Xi + 1, \delta)$.

By applying Lyapunov theory to the tracking error on the iterative domain, the event-triggering rule is proposed for MIMO system as below:

$$\chi(\Xi, \delta + 1) = D(\|\mathbf{e}_{ET}(\Xi, \delta + 1)\|^2) - \frac{B(\Xi, \delta + 1)}{2\|\mathbf{v}\|^2} > 0 \quad (9)$$

where $B(\Xi, \delta + 1) = \|\mathbf{e}(\Xi, \delta + 1)\|^2 - 2\|(\mathbf{I} - \mathbf{v})(\mathbf{e}(\Xi, \delta + 1) - \hat{\boldsymbol{\vartheta}}(\Xi + 1, \delta))\|^2$, $\mathbf{v} = \rho \hat{\Theta}(\Xi + 1, \delta) \hat{\Theta}(\Xi + 1, \delta)^\top / (\lambda +$

$\|\hat{\Theta}(\Xi + 1, \delta)\|^2$). $D(\cdot)$ is a conditional function, namely, if $|e(\Xi, \delta + 1)| > H_e$, then $D(e_{ET}^2(\Xi, \delta + 1)) = e_{ET}^2(\Xi, \delta + 1)$; Otherwise, $D(e_{ET}^2(\Xi, \delta + 1)) = 0$ in which H_e is a predetermined positive scalar.

With the above preparations, the ET-MFILC scheme is summarized as below.

$$\Xi = \Xi_l, \quad l = 1, 2, \dots, \quad \text{if } \chi(\Xi, \delta + 1) > 0$$

$$\hat{\Theta}(\Xi + 1, \delta) = \begin{cases} \hat{\Theta}(\Xi, \delta) + \frac{\eta \mathbf{E}(\Xi, \delta) \Delta \mathbf{u}^T(\Xi, \delta)}{\mu + \|\Delta \mathbf{u}(\Xi, \delta)\|^2}, & \Xi = \Xi_l \\ \hat{\Theta}(\Xi, \delta), & \Xi_l < \Xi < \Xi_{l+1} \end{cases} \quad (10)$$

$$\begin{aligned} \hat{\Theta}_{pp}(\Xi, \delta) &= \hat{\Theta}_{pp}(1, 1), \\ \text{if } \text{sign}(\hat{\Theta}_{pp}(\Xi, \delta)) &\neq \text{sign}(\hat{\Theta}_{pp}(1, 1)) \\ \hat{\Theta}_{pj}(\Xi, \delta) &= \hat{\Theta}_{pj}(1, 1), \\ \text{if } |\hat{\Theta}_{pj}(\Xi, \delta)| &> b_1 \\ \text{or } \text{sign}(\hat{\Theta}_{pj}(\Xi, \delta)) &\neq \text{sign}(\hat{\Theta}_{pj}(1, 1)). \end{aligned} \quad (11)$$

$$\mathbf{u}(\Xi + 1, \delta) = \mathbf{u}(\Xi, \delta) + \frac{\rho \hat{\Theta}^T(\Xi + 1, \delta) \hat{\mathbf{Z}}(\Xi, \delta)}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2}, \quad (12)$$

$$\begin{aligned} \hat{\boldsymbol{\vartheta}}(\Xi, \delta) &= \hat{\boldsymbol{\vartheta}}(\Xi - 1, \delta) + \alpha \hat{\mathbf{e}}(\Xi - 1, \delta + 1) \\ \hat{\vartheta}_{p1}(\Xi, \delta) &= \begin{cases} b_{\hat{\boldsymbol{\vartheta}}} & \text{if } \hat{\vartheta}_{p1}(\Xi, \delta) > b_{\hat{\boldsymbol{\vartheta}}} \\ \hat{\vartheta}_{p1}(\Xi, \delta) & \text{if } |\hat{\vartheta}_{p1}(\Xi, \delta)| \leq b_{\hat{\boldsymbol{\vartheta}}} \\ -b_{\hat{\boldsymbol{\vartheta}}} & \text{if } \hat{\vartheta}_{p1}(\Xi, \delta) < -b_{\hat{\boldsymbol{\vartheta}}} \end{cases} \end{aligned} \quad (13)$$

where $\hat{\Theta}_{pj}(1, 1)$ is the initial value of $\hat{\Theta}_{pj}(\Xi, \delta)$, $p = 1, 2, \dots, m, j = 1, 2, \dots, m$. Parameters $0 < \eta < 2$, $\mu > 0$, $\rho > 0$, $\lambda > 0$ and they are chosen by designers.

Remark 3 One can see from (13) that $\|\hat{\boldsymbol{\vartheta}}(\Xi, \delta)\|$ is bounded, namely, $\|\hat{\boldsymbol{\vartheta}}(\Xi, \delta)\| \leq c_{\hat{\boldsymbol{\vartheta}}}$, $c_{\hat{\boldsymbol{\vartheta}}}$ is a positive scalar.

Next, the boundness of $\hat{\Theta}(\Xi, \delta)$ is established in Theorem 1.

Theorem 1 If (1) meets Assumptions 1-3, then $\hat{\Theta}(\Xi, \delta)$ estimated by (10) under (9) is ultimate bounded.

Proof: It can be obtained from (10) that the boundness of the triggered $\hat{\Theta}(\Xi, \delta)$ is considered only, namely, $\Xi = \Xi_l$.

$$\begin{aligned} \text{Let } \hat{\Theta}(\Xi, \delta) &= [\hat{\Theta}_1(\Xi, \delta)^T \quad \hat{\Theta}_2(\Xi, \delta)^T \quad \dots \quad \hat{\Theta}_m(\Xi, \delta)^T]^T, \\ \hat{\Theta}_j(\Xi, \delta) &= [\hat{\phi}_{j1}(\Xi, \delta) \quad \hat{\phi}_{j2}(\Xi, \delta) \quad \dots \quad \hat{\phi}_{jm}(\Xi, \delta)] \end{aligned}$$

where $j=1, 2, \dots, m$.

According to (10), one has

$$\hat{\Theta}_j(\Xi + 1, \delta) = \hat{\Theta}_j(\Xi, \delta) + \frac{\mathbf{E}_j(\Xi, \delta) \Delta \mathbf{u}(\Xi, \delta)^T}{\mu + \|\Delta \mathbf{u}(\Xi, \delta)\|^2} \quad (14)$$

where $\mathbf{E}_j(\Xi, \delta) = \Delta \mathbf{y}(\Xi, \delta + 1) - \hat{\Theta}_j(\Xi, \delta) \Delta \mathbf{u}(\Xi, \delta) - \hat{\boldsymbol{\vartheta}}(\Xi, \delta)$.

Subtracting $\Theta_j(\Xi, \delta)$ from both sides of (14), and let $\tilde{\Theta}_j(\Xi, \delta) = \hat{\Theta}_j(\Xi, \delta) - \Theta_j(\Xi, \delta)$, one has

$$\begin{aligned} \tilde{\Theta}_j(\Xi + 1, \delta) &= \Lambda - \frac{\eta \tilde{\boldsymbol{\vartheta}}(\Xi, \delta) \Delta \mathbf{u}^T(\Xi, \delta)}{\mu + \|\Delta \mathbf{u}(\Xi, \delta)\|^2} + \frac{\eta \boldsymbol{\vartheta}(\Xi, \delta) \Delta \mathbf{u}^T(\Xi, \delta)}{\mu + \|\Delta \mathbf{u}(\Xi, \delta)\|^2} \\ &\quad + \Theta_j(\Xi, \delta) - \Theta_j(\Xi + 1, \delta) \end{aligned} \quad (15)$$

$$\text{where } \Lambda = \|\tilde{\Theta}_j(\Xi, \delta) - \frac{\eta \tilde{\Theta}_j(\Xi, \delta) \Delta \mathbf{u}(\Xi, \delta) \Delta \mathbf{u}^T(\Xi, \delta)}{\mu + \|\Delta \mathbf{u}(\Xi, \delta)\|^2}\|.$$

The square of Λ is expressed as

$$\begin{aligned} \Lambda^2 &= \|\tilde{\Theta}_j(\Xi, \delta)\|^2 \\ &\quad + \left(-2 + \frac{\eta \|\Delta \mathbf{u}(\Xi, \delta)\|^2}{\mu + \|\Delta \mathbf{u}(\Xi, \delta)\|^2} \right) \frac{\eta \|\tilde{\Theta}_j(\Xi, \delta) \Delta \mathbf{u}(\Xi, \delta)\|^2}{\mu + \|\Delta \mathbf{u}(\Xi, \delta)\|^2} \end{aligned} \quad (16)$$

By choosing parameters $0 < \eta < 2$ and $\mu > 0$ makes (17) hold

$$\left(-2 + \frac{\eta \|\Delta \mathbf{u}(\Xi, \delta)\|^2}{\mu + \|\Delta \mathbf{u}(\Xi, \delta)\|^2} \right) < 0. \quad (17)$$

Substituting (17) into (16) yields

$$\Lambda \leq g_1 \|\tilde{\Theta}_j(\Xi, \delta)\| \quad (18)$$

where $0 < g_1 < 1$.

From Lemma 1 and Remark 3, one has $\|\Theta_j(\Xi, \delta) - \Theta_j(\Xi + 1, \delta)\| \leq 2b$. Moreover

$$\left\| \frac{\eta \tilde{\boldsymbol{\vartheta}}(\Xi, \delta) \Delta \mathbf{u}(\Xi, \delta)}{\mu + \|\Delta \mathbf{u}^2(\Xi, \delta)\|} \right\| \leq \frac{\eta c_{\hat{\boldsymbol{\vartheta}}}}{2\sqrt{\mu}} \quad (19)$$

$$\left\| \frac{\eta \boldsymbol{\vartheta}(\Xi, \delta) \Delta \mathbf{u}(\Xi, \delta)}{\mu + \Delta \mathbf{u}^2(\Xi, \delta)} \right\| \leq \frac{\eta c_{\vartheta}}{2\sqrt{\mu}} \quad (20)$$

Combining (15) and (18)-(20), one has

$$\begin{aligned} \|\tilde{\Theta}_j(\Xi + 1, \delta)\| &\leq g_1 \|\tilde{\Theta}_j(\Xi, \delta)\| + \sigma \\ &\leq g_1^2 \|\tilde{\Theta}_j(\Xi - 1, \delta)\| + g_1 \sigma + \sigma \\ &\vdots \\ &\leq g_1^{\Xi} \|\tilde{\Theta}_j(1, \delta)\| + \frac{\sigma(1 - g_1^{\Xi})}{1 - g_1} \end{aligned} \quad (21)$$

where $\sigma = \frac{\eta c_{\hat{\vartheta}}}{2\sqrt{\mu}} + \frac{\eta c_{\vartheta}}{2\sqrt{\mu}} + 2b$.

It can be concluded from (21) that $\tilde{\Theta}_j(\Xi, \delta)$ is ultimate bounded as $\Xi \rightarrow \infty$. Therefore, $\hat{\Theta}_j(\Xi, \delta)$ and $\hat{\Theta}(\Xi, \delta)$ are also bounded.

Based on the boundedness of $\hat{\Theta}(\Xi, \delta)$ established in Theorem 1, the boundedness of tracking errors can be obtained in the following theorem.

Theorem 2 *If system (1) meets Assumptions 1-3, then the boundness of tracking errors under ET-MFILC scheme (10)-(13) is guaranteed by choosing parameters ρ and λ properly.*

Proof: Two cases are discussed, namely, at triggering instants and during two consecutive triggering instants.

Case I: At triggering instants $[\Xi = \Xi_l]$

Combining (2) and (12), it provides

$$\begin{aligned} \mathbf{e}(\Xi + 1, \delta + 1) &= \mathbf{y}^*(\delta + 1) - \mathbf{y}(\Xi + 1, \delta + 1) \\ &= \mathbf{y}^*(\delta + 1) - \mathbf{y}(\Xi, \delta + 1) \\ &\quad - \Theta(\Xi + 1, \delta) \Delta \mathbf{u}(\Xi + 1, \delta) - \boldsymbol{\vartheta}(\Xi + 1, \delta) \\ &= \mathbf{e}(\Xi, \delta + 1) - \boldsymbol{\vartheta}(\Xi + 1, \delta) \\ &\quad - \frac{\rho \Theta(\Xi + 1, \delta) \hat{\Theta}^T(\Xi + 1, \delta) (\hat{\mathbf{e}}(\Xi, \delta + 1) - \hat{\boldsymbol{\vartheta}}(\Xi + 1, \delta))}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2} \\ &= \boldsymbol{\Pi} \mathbf{e}(\Xi, \delta + 1) - \boldsymbol{\vartheta}(\Xi + 1, \delta) \\ &\quad + \frac{\rho \Theta(\Xi + 1, \delta) \hat{\Theta}^T(\Xi + 1, \delta) \hat{\boldsymbol{\vartheta}}(\Xi + 1, \delta)}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2} \end{aligned} \quad (22)$$

where

$$\boldsymbol{\Pi} = \mathbf{I} - \frac{\rho \Theta(\Xi + 1, \delta) \hat{\Theta}^T(\Xi + 1, \delta)}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2}. \quad (23)$$

Applying Lemma 2 to (23) yields

$$\begin{aligned} D_j \left\{ z \left\| z - \left(1 - \frac{\rho \sum_{p=1}^m \Theta_{jp}(\Xi + 1, \delta) \hat{\Theta}_{jp}(\Xi + 1, \delta)}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2} \right) \right\| \right. \\ \left. \leq \sum_{h=1, h \neq j}^m \frac{\rho \left| \sum_{p=1}^m \Theta_{jp}(\Xi + 1, \delta) \hat{\Theta}_{hp}(\Xi + 1, \delta) \right|}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2} \right\} \end{aligned} \quad (24)$$

where $j = 1, 2, \dots, m$ and z is the eigenvalue of $\boldsymbol{\Pi}$.

Employing the triangle inequality $|\mathbb{A} - \mathbb{B}| \leq |\mathbb{A}| + |\mathbb{B}|$ to (24) gives

$$\begin{aligned} D_j \left\{ z \left| z \right| \right. \\ \leq \left| 1 - \frac{\rho \sum_{p=1}^m \Theta_{jp}(\Xi + 1, \delta) \hat{\Theta}_{jp}(\Xi + 1, \delta)}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2} \right| \\ \left. + \sum_{h=1, h \neq j}^m \frac{\rho \left| \sum_{p=1}^m \Theta_{jp}(\Xi + 1, \delta) \hat{\Theta}_{hp}(\Xi + 1, \delta) \right|}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2} \right\} \end{aligned} \quad (25)$$

Combining Assumption 3 and the resetting algorithm (11) gives the following inequality

$$\begin{aligned} 1 - \frac{\rho \sum_{p=1}^m |\Theta_{jp}(\Xi + 1, \delta)| \|\hat{\Theta}_{jp}(\Xi + 1, \delta)\|}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2} \\ \leq 1 - \frac{\rho |\Theta_{jj}(\Xi + 1, \delta)| \|\hat{\Theta}_{jj}(\Xi + 1, \delta)\|}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2} \\ \leq 1 - \frac{\rho b_2^2}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2} \end{aligned} \quad (26)$$

and

$$\begin{aligned}
& \rho \frac{\sum_{h=1, h \neq j}^m |\ominus_{jp}(\Xi+1, \delta) \hat{\ominus}_{hp}(\Xi+1, \delta)|}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \\
& \leq \rho \frac{\sum_{h=1, h \neq j}^m \sum_{p=1}^m |\ominus_{jp}(\Xi+1, \delta)| |\hat{\phi}_{hp}(\Xi+1, \delta)|}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \\
& = \rho \frac{\sum_{h=1, h \neq j}^m |\ominus_{jj}(\Xi+1, \delta)| |\hat{\phi}_{hj}(\Xi+1, \delta)|}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \\
& + \rho \frac{\sum_{h=1, h \neq j}^m \sum_{p=1, p \neq j}^m |\ominus_{jp}(\Xi+1, \delta)| |\hat{\ominus}_{hp}(\Xi+1, \delta)|}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \\
& = \rho \frac{\sum_{h=1, h \neq j}^m |\ominus_{jj}(\Xi+1, \delta)| |\hat{\ominus}_{hj}(\Xi+1, \delta)|}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \\
& + \rho \frac{\sum_{h=1, h \neq j}^m |\ominus_{jh}(\Xi+1, \delta)| |\hat{\ominus}_{hh}(\Xi+1, \delta)|}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \\
& + \rho \frac{\sum_{h=1, h \neq j}^m \sum_{p=1, p \neq j, h}^m |\ominus_{jp}(\Xi+1, \delta)| |\hat{\ominus}_{hp}(\Xi+1, \delta)|}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \\
& \leq \frac{2ab_1b_2(m-1) + b_1^2(m-1)(m-2)}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} < 1
\end{aligned} \tag{27}$$

Recalling Assumption 3, one has

$$b_2 > b_1(2a+1)(m-1) \tag{28}$$

On the other hand, combining (26) and (27) renders

$$\begin{aligned}
& 1 - \frac{\rho \sum_{p=1}^m |\ominus_{jp}(\Xi+1, \delta)| |\hat{\ominus}_{jp}(\Xi+1, \delta)|}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \\
& + \sum_{h=1, h \neq j}^m \frac{\rho |\sum_{p=1}^m \ominus_{jp}(\Xi+1, \delta) \hat{\ominus}_{hp}(\Xi+1, \delta)|}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \leq \Pi_1
\end{aligned} \tag{29}$$

where

$$\Pi_1 = 1 - \rho \frac{b_2(b_2 - 2ab_1(m-1)) - b_1^2(m-1)(m-2)}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2}.$$

Taking (28) into (29) leads to

$$\begin{aligned}
\Pi_1 & < 1 - \rho \frac{b_2b_1(m-1) - b_1^2(m-1)(m-2)}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \\
& < 1 - \rho \frac{b_2b_1(m-1) - b_1^2(m-1)^2}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \\
& = 1 - \rho \frac{b_1(m-1)(b_2 - b_1(m-1))}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2}
\end{aligned} \tag{30}$$

Utilizing (28) again to (30) produces

$$\Pi_1 \leq 1 - \rho \frac{2ab_1^2(m-1)^2}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \tag{31}$$

From Assumption 3 and the resetting algorithm (11), one has $\ominus_{jp}(\Xi+1, \delta) \hat{\ominus}_{jp}(\Xi+1, \delta) > 0, p = 1, 2, \dots, m, j = 1, 2, \dots, m$. Thus, there exist $\lambda_{min} > 0$ with $\lambda > \lambda_{min}$ such that

$$\begin{aligned}
& \frac{\sum_{p=1}^m \ominus_{jp}(\Xi+1, \delta) \hat{\ominus}_{jp}(\Xi+1, \delta)}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \\
& = \frac{\sum_{p=1}^m |\ominus_{jp}(\Xi+1, \delta)| |\hat{\ominus}_{jp}(\Xi+1, \delta)|}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \\
& \leq \frac{a^2b_2^2(m-1)}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} < \frac{a^2b_2^2(m-1)}{\lambda_{min} + \|\hat{\ominus}(\Xi+1, \delta)\|^2} < 1
\end{aligned} \tag{32}$$

Choosing $0 < \rho < 1$ and $\lambda > \lambda_{min}$ makes (33) hold

$$\begin{aligned}
& \left| 1 - \frac{\rho \sum_{p=1}^m \ominus_{jp}(\Xi+1, \delta) \hat{\ominus}_{jp}(\Xi+1, \delta)}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \right| \\
& = 1 - \frac{\rho \sum_{p=1}^m |\ominus_{jp}(\Xi+1, \delta)| |\hat{\ominus}_{jp}(\Xi+1, \delta)|}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2}
\end{aligned} \tag{33}$$

Then, there exists g_3 such that

$$0 < g_3 \leq \frac{2ab_1^2(m-1)^2}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \tag{34}$$

According to (28), (34) is further rewritten as

$$\begin{aligned}
& \frac{2ab_1^2(m-1)^2}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} < \frac{b_2^2}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \\
& \leq \frac{a^2b_2^2 + b_1^2(m-1)}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2}
\end{aligned} \tag{35}$$

Integrating (32) and (35), one has

$$\frac{a^2b_2^2 + b_1^2(m-1)}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} < \frac{a^2b_2^2 + b_1^2(m-1)}{\lambda_{min} + \|\hat{\ominus}(\Xi+1, \delta)\|^2} < 1. \tag{36}$$

Combining (31), (33) and (36) gets

$$\begin{aligned}
& \left| 1 - \frac{\rho \sum_{p=1}^m \ominus_{jp}(\Xi+1, \delta) \hat{\ominus}_{jp}(\Xi+1, \delta)}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \right| \\
& + \sum_{h=1, h \neq j}^m \frac{\rho |\sum_{p=1}^m \ominus_{jp}(\Xi+1, \delta) \hat{\ominus}_{hp}(\Xi+1, \delta)|}{\lambda + \|\hat{\ominus}(\Xi+1, \delta)\|^2} \\
& < 1 - \rho g_3 < 1.
\end{aligned} \tag{37}$$

which indicates

$$\text{eig}(\mathbf{II}) \leq 1 - \rho g_3 < 1$$

where $\text{eig}(\mathbf{II})$ denotes the spectral radius of $\mathbf{II}$.

Generally, there exists $\varepsilon > 0$ such that

$$\|\mathbf{II}\|_v < 1 - \rho g_3 + \varepsilon = g_4 < 1 \quad (38)$$

Taking norm operation on (22) with (38) gives

$$\begin{aligned} \|\mathbf{e}(\Xi + 1, \delta)\|_v &\leq \|\mathbf{II}\|_v \|\mathbf{e}(\Xi, \delta)\|_v + \sigma_1 \\ &\leq g_4 \|\mathbf{e}(\Xi, \delta)\|_v + \sigma_1 \\ &\vdots \\ &\leq g_4^{\Xi} \|\mathbf{e}(1, \delta)\|_v + \frac{\sigma_1(1 - g_4^{\Xi})}{1 - g_4} \end{aligned} \quad (39)$$

where σ_1 denotes the bound of

$$\left\| \frac{\rho \Theta(\Xi + 1, \delta) \hat{\Theta}^T(\Xi + 1, \delta) \hat{\boldsymbol{\vartheta}}(\Xi + 1, \delta)}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2} - \boldsymbol{\vartheta}(\Xi + 1, \delta) \right\|_v.$$

Therefore, the tracking error $\mathbf{e}(\Xi, \delta)$ in (39) at triggering instants is ultimate bounded as $\Xi \rightarrow \infty$.

Case II: During two consecutive triggering instants
 $[\Xi_l < \Xi < \Xi_{l+1}]$

Selecting $V(\Xi + 1, \delta + 1) = \|\mathbf{e}(\Xi + 1, \delta + 1)\|^2$, one has

$$\Delta V(\Xi + 1, \delta + 1) = \|\mathbf{e}(\Xi + 1, \delta + 1)\|^2 - \|\mathbf{e}(\Xi, \delta + 1)\|^2 \quad (40)$$

According to (6) and the definition of event-triggered error $\mathbf{e}_{ET}(\Xi, \delta)$, one has $\hat{\mathbf{e}}(\Xi, \delta) = \mathbf{e}(\Xi, \delta) + \mathbf{e}_{ET}(\Xi, \delta)$ for $\Xi_l < \Xi < \Xi_{l+1}$.

Combining (2) and (12) yields

$$\begin{aligned} \mathbf{e}(\Xi + 1, \delta + 1) &= \mathbf{y}^*(\delta + 1) - \mathbf{y}(\Xi + 1, \delta + 1) \\ &= \mathbf{y}^*(\delta + 1) - \mathbf{y}(\Xi, \delta + 1) \\ &\quad - \Theta(\Xi + 1, \delta) \Delta \mathbf{u}(\Xi + 1, \delta) - \boldsymbol{\vartheta}(\Xi + 1, \delta) \\ &= \mathbf{e}(\Xi, \delta + 1) - \boldsymbol{\vartheta}(\Xi + 1, \delta) \\ &\quad - \frac{\rho \Theta(\Xi + 1, \delta) \hat{\Theta}^T(\Xi + 1, \delta) \hat{\mathbf{Z}}(\Xi, \delta)}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2} \\ &= (\mathbf{I} - \mathbf{v}) \mathbf{e}(\Xi, \delta + 1) - \mathbf{v} \mathbf{e}_{ET}(\Xi, \delta + 1) \\ &\quad + \mathbf{v} \hat{\boldsymbol{\vartheta}}(\Xi + 1, \delta) - \boldsymbol{\vartheta}(\Xi + 1, \delta) \end{aligned} \quad (41)$$

where

$$\mathbf{v} = \frac{\rho \Theta(\Xi + 1, \delta) \hat{\Theta}^T(\Xi + 1, \delta)}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2}$$

Considering the slow change feature of $\Theta(\Xi, \delta)$ supplies

$$\mathbf{v} = \frac{\rho \hat{\Theta}(\Xi + 1, \delta) \hat{\Theta}^T(\Xi + 1, \delta)}{\lambda + \|\hat{\Theta}(\Xi + 1, \delta)\|^2} \quad (42)$$

Similarly, $\boldsymbol{\vartheta}(\Xi + 1, \delta)$ is replaced by $\hat{\boldsymbol{\vartheta}}(\Xi + 1, \delta)$, namely, $\hat{\boldsymbol{\vartheta}}(\Xi + 1, \delta) = \boldsymbol{\vartheta}(\Xi + 1, \delta)$. Applying this replacement to (41), one gets

$$\begin{aligned} \mathbf{e}(\Xi + 1, \delta + 1) &= (\mathbf{I} - \mathbf{v})(\mathbf{e}(\Xi, \delta + 1) - \hat{\boldsymbol{\vartheta}}(\Xi + 1, \delta)) - \mathbf{v} \mathbf{e}_{ET}(\Xi, \delta + 1) \end{aligned} \quad (43)$$

Taking (43) into (40) provides

$$\begin{aligned} \Delta V(\Xi + 1, \delta + 1) &= \|(\mathbf{I} - \mathbf{v})(\mathbf{e}(\Xi, \delta + 1) - \hat{\boldsymbol{\vartheta}}(\Xi + 1, \delta)) \\ &\quad - \mathbf{v} \mathbf{e}_{ET}(\Xi, \delta + 1)\|^2 - \|\mathbf{e}(\Xi, \delta + 1)\|^2 \\ &\leq 2\|(\mathbf{I} - \mathbf{v})(\mathbf{e}(\Xi, \delta + 1) - \hat{\boldsymbol{\vartheta}}(\Xi + 1, \delta))\|^2 \\ &\quad + 2\|\mathbf{v}\|^2 \|\mathbf{e}_{ET}(\Xi, \delta + 1)\|^2 - \|\mathbf{e}(\Xi, \delta + 1)\|^2 \end{aligned} \quad (44)$$

From the event-triggering condition (9) one has $\Delta V(\Xi + 1, \delta + 1) \leq 0$.

Combining *Case I* and *Case II*, it means that the tracking error $\mathbf{e}(\Xi, \delta)$ under (10)-(13) is ultimate bounded.

4 Numerical example

Consider the following MIMO nonlinear system in [14]:

$$\begin{cases} y_1(\delta + 1) = \frac{2.5y_1(\delta)}{1 + y_1(\delta)^2 + y_2(\delta)^2} \\ \quad + 1.2u_1(\delta) + 0.5u_2(\delta) + d_1(\delta) \\ y_2(\delta + 1) = \frac{5y_2(\delta)}{1 + y_1(\delta)^2 + y_2(\delta)^2} + 0.5u_1(\delta) \\ \quad + u_2(\delta) + d_2(\delta) \end{cases}$$

where $d_1(\delta) = 0.5 \sin(\delta + \Xi/200)$ and $d_2(\delta) = 0.5 \cos(\delta + \Xi/200)$.

The expected trajectory is

$$\begin{cases} y_1^* = \sin(\delta/50) + 2\cos(\delta/20) \\ y_2^* = \sin(\delta/50) + 5\cos(\delta/20) \end{cases}$$

Some initial parameters are supplied as $\mathbf{y}(\Xi, 1) = \mathbf{y}(\Xi, 2) = [0; 0]$, $\mathbf{u}(\Xi, 1) = \mathbf{u}(\Xi, 2) = [0; 0]$, $\hat{\vartheta}(1, \delta) = [0; 0]$, $\hat{\Phi}(1, 1) = \hat{\Phi}(1, 2) = [2, 1; 1, 2]$, $\rho = 1$, $\mu = 2$, $\eta = 1.8$, $\lambda = 0.33$, $H_e = 0.06$, $b_{\hat{\theta}} = 1$, $\alpha = 0.005$.

Applying the above parameters to Theorem 2, some simulation curves are given in Figs. 1-11 for different iteration times. To be specific, Fig. 1 shows the sum of triggered instants after 50 iterations. Figs. 2-7 present the comparisons of $y_1(\delta)$ and $y_2(\delta)$ at 10th, 20th, 50th iterations with ET-MFILC and MFILC schemes. Figs. 8-9 show the comparison evolution of these outputs under different iterations. The estimated uncertainties of output $d_1(\delta)$ and $d_2(\delta)$ are given in Figs. 10-11.

It is seen from Fig. 1 that only 56% of the instants are triggered during 50 iterations. Additionally, Figs. 2-7 address that the outputs can track desired trajectories effectively after 50th iterations. Figs. 8-9 indicate the fact that the more the iteration times is, the better tracking performance they are. Figs. 10-11 show that the uncertainties can be estimated accurately as well.

Fig. 1: The sum of triggered instants for 50 iterations

Fig. 2: $y_1(\delta)$ under ET-MFAC and MFAC schemes for 10th iterations

Fig. 3: $y_1(\delta)$ under ET-MFAC and MFAC schemes for 20th iterations

Fig. 4: $y_1(\delta)$ under ET-MFAC and MFAC schemes for 50th iterations

Fig. 5: $y_2(\delta)$ under ET-MFAC and MFAC schemes for 10th iterations

Fig. 8: $y_1(\delta)$ under ET-MFAC schemes for different iterations

Fig. 6: $y_2(\delta)$ under ET-MFAC and MFAC schemes for 20th iterations

Fig. 9: $y_2(\delta)$ under ET-MFAC schemes for different iterations

Fig. 7: $y_2(\delta)$ under ET-MFAC and MFAC schemes for 50th iterations

Fig. 10: The estimated $d_1(\delta)$ for different iterations

Fig. 11: The estimated $d_2(\delta)$ for different iterations

5 Conclusions

This paper is dedicated to extended state observer-based ET-MFILC for disturbed nonlinear systems. With the help of the estimated uncertainties, a modified MFILC scheme is proposed. An IESO is constructed to estimate the unknown uncertainties by the estimation errors. New triggering mechanism integrated true tracking errors, the estimated errors and the estimated uncertainties is designed for MIMO systems. Consequently, Sufficient conditions are proposed to make the resultant ET-MFILC tracking error systems be uniformly ultimately bounded. Illustrative example is supplied to confirm the effectiveness of the proposed scheme.

6 Declarations

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Conflict of interest

The authors declare that there is no conflict of interest to this work.

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