SBL Kalman Filter Based Hybrid Precoder/Combiner for mmWave MIMO Systems: A Frequency Domain Approach

Shoukath Ali k (✉ shoukathlk@gmail.com)

Bannari Amman Institute of Technology <https://orcid.org/0000-0001-9256-373X>

Sampath Palaniswami

Bannari Amman Institute of Technology

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SBL Kalman Filter Based Hybrid Precoder/ Combiner for mmWave MIMO Systems: A Frequency Domain Approach

*Shoukath Ali K and Sampath P

Department of ECE

Bannari Amman Institute of Technology, Sathyamangalam, Tamilnadu, India.

*shoukathlk@gmail.com and sampathpalaniswami@gmail.com

Abstract:

The optimal design of hybrid precoder/ combiner for Millimetre Wave (mmWave) Multiple Input and Multiple Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) system is developed presented. In the frequency domain approach, Sparse Bayesian Learning - Kalman Filter (SBL-KF) algorithm is used to design the optimal hybrid precoder/ combiner in mmWave MIMO OFDM systems. Sparse signal recovery problem for Single Measurement Vector (SMV) is discussed, that is close to the design of ideal digital baseband precoder by maximizing the mutual information from the hybrid precoder. SBL-KF scheme select the minimum number of active Radio Frequency (RF) chains based on the hyper parameter estimator. The minimum number of RF chain is approximate the ideal digital precoder / combiner design. Proposed SBL-KF scheme achieve low power consumption and enhanced spectral efficiency, when compared to the SBL, Orthogonal Matching Pursuit (OMP), Simultaneous OMP (SOMP) and Least Square schemes, which activate a fixed data streams and fixed number of RF chains.

Keywords: Kalman Filter, Sparse Bayesian Learning, Orthogonal Matching Pursuit, Millimetre Wave

1.Introduction

Millimeter wave (mmWave) communication available spectrum range is given from 30 GHz-300 GHz mmwave band. mmWave technology significantly increases the high spectral efficiency for future wireless 5G technologies [1-2]. Due to high path loss, signal communication is highly challenging in the mmWave band, and also increased the hardware complexity. Hybrid precoder/ combiner design requires less number of RF chains, compared to the number of transmit /receive antennas in the mmWave MIMO transceiver system [3-5]. Hybrid precoder /combiner technique play an important role in mmWave MIMO technology, because it attains high performance gain for the mmWave systems. Further, the digital baseband precoding /combining technique achieve high performance gain and multiplexing gain.

Recent researches focused on spatial grid based OMP scheme to overcome the sparse channel vector problem. The suitable selection of baseband and RF precoder matrices is carried out using OMP scheme for narrowband mmWave MIMO system [6-9]. The condition for Angle

of Arrivals (AoAs) and Angle of departures (AoDs) are considered as known, which is not viable in practical situations. The problem of optimal precoder is solved by using the greedy Simultaneous Orthogonal Matching Pursuit (SOMP). SOMP greedy framework leads to convergence errors because the performance of selection dictionary matrix and stopping criterion is more sensitive [10]. The sparse solution is obtained by selecting the global minima from the SBL cost function, which ensure the representation of sparse vector in the digital precoder [11]. Further, the Expectation–Maximization (EM) algorithm is suited for ideal hybrid precoder /combiner and it is convergence that the SBL technique to a fixed point log likelihood functions [12].

In the above works, mmWave MIMO architecture is considered as fixed. The number of RF chain and data streams are fixed and this can be adjusted depends on the Channel State Information (CSI). RF chains contain the ADC/DAC and power amplifier, which significantly increase the power consumption in mmWave MIMO hybrid system. The power budget is effectively reduced by activating the adaptive less RF Chains at the transceiver, but this is not satisfying the multiplexing gain of the system. SBL scheme is used in the hybrid precoder/combiner for frequency selective mmWave MIMO channel, which significantly improve the spectral and power efficiency of the mmWave MIMO systems [13-15].

1.1. Contributions

In the previous work, temporal correlation is not discussed which improve the system performance. Sparse channel estimation scheme is developed for time and wideband mmWave MIMO channel, which overcome the shortcoming of the existing literature [16-20]. SBL-KF based hybrid precoder / combiner design for mmWave MIMO channel is presented in the frequency domain. These exploit the time evolution and time varying spatial multipath component of the sparse mmWave MIMO channel vector. The proposed SBL-KF scheme selects the minimum number of active Radio Frequency (RF) chain based on the hyper parameter estimator. This strategy leads to improve significantly the higher spectral and power efficiency.

The organization of the paper is as follows: Section II, discuss about System Model for millimetre wave MIMO-OFDM system. In section III, design of hybrid precoder/ combiner using SBL-KF is proposed. In section IV, simulation results of the proposed SBL-KF scheme for hybrid precoder/combiner is discussed. Finally, conclusions and future work is presented in section V.

Notation:

Bold lower case **a** letters denotes the column vector, non-bold lower case *a* denotes the scalar values. Bold uppercase letters denotes the **A** matrices, $\bar{\mathbf{A}}$ denotes the conjugate matrix of **A**, $\mathbf{A}^*$ denotes conjugate transpose matrix of **A**, $\mathbf{A}^T$ denotes the transpose matrix of **A** and **I** denote the identity matrix.

2. System Model For Millimeter Wave MIMO-OFDM System

mmWave MIMO-OFDM hybrid architecture with N_T , N_R and N_{RF} denotes the number of antennas at the transmitter, number of antennas at the receiver and number of RF chains at the transceiver, respectively as shown in Fig.1. The RF Chain falls under the condition of $N_{RF} \leq \min(N_T, N_R)$ and data stream falls under the condition of $N_s \leq N_{RF}$ for K subcarriers. At the transmitter $F_k = F_{RF} F_{BB,k}$ denotes the digital baseband for the $k=0, \dots, K-1$ subcarrier where $F_{BB,k} \in \mathbb{C}^{N_{RF} \times N_s}$ denote the digital baseband for the $k=0, \dots, K-1$ subcarrier and $F_{RF} \in \mathbb{C}^{N_T \times N_{RF}}$ denote analog precoder. At the receiver $W_k = W_{RF} W_{BB,k}$, where $W_{RF} \in \mathbb{C}^{N_R \times N_{RF}}$ denote the analog combiners and $W_{BB,k} \in \mathbb{C}^{N_{RF} \times N_s}$ denote baseband combiner for the $k=0, \dots, K-1$ subcarrier.

Fig 1. Hybrid architecture of mmWave MIMO-OFDM system model for frequency domain channel estimation

In the transmitter side, $s_k \in \mathbb{C}^{N_s \times 1}$ denotes the symbol vector and this should satisfy the condition of $E[s_k s_k^*] = \frac{1}{N_s} I$. The transmitted signal vector is expressed as $x_k = F_{RF} F_{BB,k} s_k \in \mathbb{C}^{N_T \times 1}$, where $s_k \in \mathbb{C}^{N_s \times 1}$ denotes the symbol vector for $k=0, \dots, K-1$ with transmit covariance $E[s_k s_k^H] = \frac{1}{N_s} I$. In order to limit the total transmit power to unity, that is $E[x_k x_k^H] = 1$, one can utilize the equivalent constraint as $x_k = \|F_{RF} F_{BB,k}\|_F^2 = N_s$. $H_k \in \mathbb{C}^{N_R \times N_T}$ that denotes the frequency selective mmWave MIMO channel matrix with delay tap represented as $d=0, 1, \dots, L-1$ for the $k=0, \dots, K-1$.

Finally, the received signal $Y_k \in \mathbb{C}^{N_s \times 1}$ for k^{th} the subcarriers is written as

$$Y_k = \sqrt{P} W_{BB,k}^H W_{RF}^H H_k F_{RF} F_{BB,k} s_k + V_k \quad (1)$$

The wideband block fading mmWave channel model for d^{th} delay tap of the channel $d=0, 1, \dots, N_c-1$ is written as

$$H_d = \sqrt{\frac{N_T N_R}{N_{\text{ray}}}} \sum_{j=1}^{N_{\text{ray}}} \alpha_j P(dT_s - \tau_j) a_R(\theta_j^r) a_T^H(\theta_j^t) \quad (2)$$

Where α_j and $\tau_j \in \mathbb{R}$ denote the complex channel gain and the delay for j^{th} ray respectively. $P(dT_s - \tau_j)$ denotes the pulse shaping filter response evaluated at τ_j . $\theta_j^r \in [0, 2\pi)$ denotes the angle of arrival (AoA) and $\theta_j^t \in [0, 2\pi)$ denotes the angles of departure (AoD) corresponding to the j^{th} ray. $\mathbf{a}_R(\theta_j^r) \in \mathbb{C}^{N_R \times 1}$ and $\mathbf{a}_T^H(\theta_j^t) \in \mathbb{C}^{N_T \times 1}$ denote the antenna array response vectors corresponding to the j^{th} ray of the receiver and transmitter, respectively.

$$\mathbf{a}_R(\theta_j^r) = \frac{1}{\sqrt{N_R}} \left[1, e^{-j\frac{2\pi}{\lambda}d_R \cos \theta_j^r}, \dots, e^{-j\frac{2\pi}{\lambda}(N_R-1)d_R \cos \theta_j^r} \right]^T \quad (3)$$

$$\mathbf{a}_T(\theta_j^t) = \frac{1}{\sqrt{N_T}} \left[1, e^{-j\frac{2\pi}{\lambda}d_T \cos \theta_j^t}, \dots, e^{-j\frac{2\pi}{\lambda}(N_T-1)d_T \cos \theta_j^t} \right]^T \quad (4)$$

The channel model expressed in equation (2) is compactly written as $\mathbf{H}_d = \mathbf{A}_R \Delta_d \mathbf{A}_T^*$.

Where $\Delta_d \in \mathbb{C}^{N_{\text{ray}} \times N_{\text{ray}}}$ denotes the diagonal matrix with non-zero entries of $\alpha_j p_{rc}(dT_s - \tau_j)$. $\mathbf{A}_R \in \mathbb{C}^{N_R \times N_{\text{ray}}}$ and $\mathbf{A}_T \in \mathbb{C}^{N_T \times N_{\text{ray}}}$ denotes the columns of receive array response vectors $\mathbf{a}_R(\theta_j)$ and columns of transmit array response vectors $\mathbf{a}_T(\theta_j)$, respectively.

The complex channel model for the k^{th} subcarrier is expressed in the frequency domain and is given as

$$\mathbf{H}_k = \sum_{d=0}^{N_c-1} \mathbf{H}_d e^{-j\frac{2\pi k d}{K}} \quad (5)$$

3. Design of Hybrid Precoder/ Combiner Using SBL-KF

The maximization of the mutual information gives the optimal precoders $\mathbf{F}_{\text{BB},k}$ and $\mathbf{F}_{\text{RF}}$. The optimal precoder is achieved using Gaussian signalling over the mmWave MIMO channel and the similar work discussed in [2], [5], as given below

$$(\mathbf{F}_{\text{BB},k}^{\text{opt}}, \mathbf{F}_{\text{RF}}^{\text{opt}}) = \arg \max_{\mathbf{F}_{\text{BB}}, \mathbf{F}_{\text{RF}}} I(\mathbf{F}_{\text{BB},k}, \mathbf{F}_{\text{RF}}) \quad (6)$$

$$= \arg \max_{\mathbf{F}_{\text{BB},k}, \mathbf{F}_{\text{RF}}} \log_2 \left| \mathbf{I} + \frac{\rho}{N_s \sigma_n^2} \mathbf{H}_k \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB},k} \mathbf{F}_{\text{BB},k}^H \mathbf{F}_{\text{RF}}^H \mathbf{H}_k^H \right| \quad (7)$$

The direct maximization of the mutual information is difficult, as the limitation of non convex unit magnitude is associated with each of the RF precoder element $\mathbf{F}_{\text{RF}}$. $\mathbf{F}_{\text{opt},k} = \mathbf{V}(:, 1:N_s) \in \mathbb{C}^{N_T \times N_s}$ denotes the unconstrained ideal digital MIMO precoder. $\mathbf{F}_{\text{opt},k}$ is considered as submatrix and is formed from the first N_s columns of $\mathbf{V} \in \mathbb{C}^{N_T \times N_T}$. These can be obtained from the singular value decomposition (SVD) of $\mathbf{H}_k$. From the transmit array vectors, select the columns of $\mathbf{F}_{\text{RF}}$ RF precoder, which satisfy the constraint of unit magnitude elements and form a basis for the column space of the ideal digital MIMO precoder $\mathbf{F}_{\text{opt},k}$.

The maximization of mutual-information with respect to the precoder is approximated by the sparse matrix and it is formulated as below.

Let the quantized transmit array dictionary vector is defined as $\mathbf{A}_T = [\mathbf{a}_T(\theta_1^t), \mathbf{a}_T(\theta_2^t), \dots, \mathbf{a}_T(\theta_G^t)] \in \mathbb{C}^{N_T \times G}$ where the AoD-set $\Phi_T = \{\phi_g^t, \forall 1 \leq g \leq G\}$ spans the angular range $\phi_g^t \in [0, \pi]$ with $\cos(\phi_g^t) = \frac{2}{G}(g-1)-1, \forall 1 \leq g \leq G$.

The problem of optimal transmit precoder design for mmWave MIMO system is solved by approximating the optimal ideal precoder $\mathbf{F}_{\text{opt},k}$ and is discussed below

$$\hat{\mathbf{F}}_{\text{BB},k}^{\text{opt}} = \arg \min_{\hat{\mathbf{F}}_{\text{BB},k}} \left\| \mathbf{F}_{\text{opt},k} - \mathbf{A}_T \hat{\mathbf{F}}_{\text{BB},k} \right\|_F \quad (8)$$

$$\text{s.t. } \left\| \text{diag}(\hat{\mathbf{F}}_{\text{BB},k} \hat{\mathbf{F}}_{\text{BB},k}^H) \right\|_0 \leq K, \left\| \mathbf{A}_T \hat{\mathbf{F}}_{\text{BB},k} \right\|_F^2 = N_s \quad (9)$$

The first constraint $\left\| \text{diag}(\hat{\mathbf{F}}_{\text{BB},k} \hat{\mathbf{F}}_{\text{BB},k}^H) \right\|_0 \leq R$, arises due to the fact that $\hat{\mathbf{F}}_{\text{BB},k}^{\text{opt}} \in \mathbb{C}^{G \times N_s}$ can have only $N_T^{\text{RF}} \leq R$ nonzero rows with respect to the number of active RF chains. The second constraint $\left\| \mathbf{A}_T \hat{\mathbf{F}}_{\text{BB},k} \right\|_F^2 = N_s$, represents the total transmit power.

The design of hybrid combiner matrices $\mathbf{W}_{\text{RF}}$ and $\mathbf{W}_{\text{BB},k}$ is discussed, that minimizes the Minimum Mean Square Error (MMSE) between the received vectors $\hat{\mathbf{y}}$ and transmit vector $\mathbf{s}$, which is describe as

$$\hat{\mathbf{W}}_{\text{BB},k}^{\text{opt}} = \arg \min_{\hat{\mathbf{W}}_{\text{BB},k}} \left\| \mathbf{R}_{yy}^{1/2} (\mathbf{W}_{\text{MMSE},k} - \mathbf{A}_R \hat{\mathbf{W}}_{\text{BB},k}) \right\|_F \quad (10)$$

$$\text{s.t. } \left\| \text{diag}(\hat{\mathbf{W}}_{\text{BB},k} \hat{\mathbf{W}}_{\text{BB},k}^H) \right\|_0 \leq R \quad (11)$$

Where $\mathbf{R}_{yy} = E[\mathbf{y}_k \mathbf{y}_k^H] \in \mathbb{C}^{N_R \times N_R}$ and $\mathbf{W}_{\text{MMSE},k}^H = \frac{1}{\sqrt{\rho}} (\mathbf{F}_{\text{BB},k}^H \mathbf{F}_{\text{RF}}^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB},k} + \frac{\sigma_n^2 N_s}{\rho} \mathbf{I})^{-1} \mathbf{F}_{\text{BB},k}^H \mathbf{F}_{\text{RF}}^H \mathbf{H}_k^H \in \mathbb{C}^{N_s \times N_R}$ is

the optimum MMSE combiner. The $\mathbf{A}_R \in \mathbb{C}^{N_R \times G}$ denotes the G -quantized receive array response matrix. The optimal combiner is represented as $\mathbf{R}_{yy}^{1/2} \mathbf{W}_{\text{MMSE},k}$ and $\mathbf{R}_{yy}^{1/2} \mathbf{A}_R$ denotes the optimal combiner and receive array response respectively.

3.1. Design of Precoder using SBL- KF

The SBL-KF technique starts by assigning the precoder matrix $\hat{\mathbf{F}}_{\text{BB},k}[n]$

$$p(\hat{\mathbf{F}}_{\text{BB},k}[n]; \Gamma[n]) = \prod_{i=1}^G p(\hat{\mathbf{F}}_{\text{BB},k}[n](i); \gamma_{k,i}[n]) \quad (12)$$

Where $\Gamma_k[n] = \text{diag}(\gamma_{k,1}[n], \gamma_{k,2}[n], \dots, \gamma_{k,G}[n]) \in \mathbb{R}^{G \times G}$ denotes the hyper parameter matrix and $\gamma_{k,i}[n], \forall 1 \leq g \leq G$ denotes the hyperparameter associated with the i^{th} row of $\hat{\mathbf{F}}_{\text{BB}}$.

Let $\hat{\mathbf{F}}_{\text{BB},k}[n/n-1]$ denotes the predicted estimate and $\Sigma_{b,k}[n/n-1]$ denotes the associated error covariance matrix for the n^{th} filtering block.

Similarly, $\hat{\mathbf{F}}_{\text{BB},k}[n/n]$ represent the estimate block and $\Sigma_{b,k}[n/n]$ represent the error covariance matrix for the n^{th} filtering block. The variance error is denoted as σ_a^2 . The posterior density of $\hat{\mathbf{F}}_{\text{BB},k}[n]$ precoder matrix is evaluated as $p(\hat{\mathbf{F}}_{\text{BB},k(j)}[n] | \mathbf{F}_{\text{opt},k(j)}[n]; \Gamma_k) \sim \text{CN}(\mathbf{M}_{k(j)}[n], \Sigma_k[n])$. The mean vector and covariance matrix is shown in equation (13) and equation (14) respectively.

$$\mathbf{M}_k[n] = \frac{1}{\sigma_a^2} \sum_k \mathbf{A}_T^H \mathbf{F}_{\text{opt},k}[n] \quad (13)$$

$$\Sigma_k[n] = \left(\frac{1}{\sigma_a^2} A_T^H A_T + (\Gamma_k[n])^{-1} \right)^{-1} \quad (14)$$

The result of $\hat{F}_{BB,k}[n]$ precoder depends on the hyperparameter matrix $\Gamma_k[n]$. The reduced value of $\Gamma_k[n]$ decreases the result of precoder matrix. Substituting the value of $\Gamma_k[n] = \hat{\Gamma}_k^{(m-1)}[n]$ in the equation (14) to obtain the covariance matrix.

Finally, the hyperparameter vector $\gamma_{k,i}[n]$ for the m^{th} EM iteration is given as

$$\gamma_{k,i}^{(m)}[n] = \frac{1}{N_s} \left\| \mathbf{M}_{k,(i)}^{(m)}[n] \right\|^2 + \sum_{k,(i,i)}^{(m)}[n] \quad (15)$$

The convergence of SBL-KF based precoder matrix $\hat{F}_{BB,k}[n]$ is obtained from mean value and is represented as $\hat{F}_{BB,k}[n] = \mathbf{M}_k^{(m)}[n]$. The columns of the transmit array response matrix A_T gives the optimal precoder $F_{\text{opt},k}[n]$, which guarantees the sparsest solution of the matrix.

Online nature based SBL-KF scheme take advantage over offline nature.

The online nature SBL-KF scheme is initialized as

$$\hat{F}_{BB,k}[-1/-1] = 0, \sum_{b,k}[-1/-1] = \hat{\Gamma}_k^{(m)}[0], \text{ and } \hat{\Gamma}_k^{(0)}[0] = I_{G^2}, \forall n \quad (16)$$

Further, the hyper parameter matrix $\hat{\Gamma}_k^{(0)}[0]$ of the n^{th} block for the k^{th} subcarriers is initialized as $\hat{\Gamma}_k^{(0)}[n] = \hat{\Gamma}_k^{(0)}[n-1]$

SBL-KF for Precoder Design is shown in Algorithm 1.

Algorithm1: SBL-KF for Precoder Design

Input: $F_{\text{opt},k}$, A_T , Variance of the approximation error σ_a^2 , Pilot power P , Stopping Parameters $\epsilon = 10^{-6}$ and $m_{\text{max}} = 100$

Initialization: $\hat{F}_{BB,k}[-1/-1] = 0_{G^2 \times N_s}$, $\sum_{b,k}[-1/-1] = \hat{\Gamma}_k^{(m)}[0]$ and $\hat{\Gamma}_k^{(0)}[0] = I_{G^2}$

1. Obtain the approximation error value of precoder $\hat{F}_{BB,k}^{\text{opt}} = F_{\text{opt},k} - A_T \hat{F}_{BB,k}$

2. Obtain the estimate of the matrix $\hat{F}_{BB,k}[n/n]$

$$G_k[n] = \sqrt{P} \sum_{b,k} [n/n-1] A_T^H (R_v + P A_T \sum_{b,k} [n/n-1] A_T^H)^{-1}$$

$$F_{BB,k}[n/n] = F_{BB,k}[n/n-1] + G_k[n] \hat{F}_{BB,k}^{\text{opt}}$$

$$\sum_{b,k} [n/n] = (I - \sqrt{P} G_k[n] A_T) \sum_{b,k} [n/n-1]$$

Hyper parameter estimate: Set

$$\hat{\gamma}_{k,i}^{(0)}[0] = 1, \forall 1 \leq i \leq G \Rightarrow \hat{\Gamma}_k^{(0)}[0] = I$$

Set counter $k = -1$ and $\hat{\Gamma}_k^{(-1)}[0] = 0$

While $\| \hat{\gamma}_k^{(m+1)}[n] - \hat{\gamma}_k^{(m)}[n] \|_2 > \epsilon$ and $m < m_{\text{max}}$ **do**

$$\Sigma_k[n] = \left(\frac{1}{\sigma_a^2} A_T^H A_T + (\Gamma_k[n])^{-1} \right)^{-1}$$

$$\mathbf{M}_k[n] = \frac{1}{\sigma_a^2} \sum_k [n] \mathbf{A}_T^H \hat{\mathbf{F}}_{\text{BB},k}^{\text{opt}} [n]$$

for $i = 0, 1, \dots, G$ **do**

$$\gamma_{k,i}^{(m)}[n] = \frac{1}{N_s} \left\| \mathbf{M}_{k,(i)}^{(m)}[n] \right\|^2 + \sum_{k,(i,i)}^{(m)} [n]$$

end for

end while

Output: $\hat{\mathbf{F}}_{\text{BB},k}[n] = \mathbf{M}_k^{(m)}[n]$

3.2. Design of Combiner using SBL-KF

Similar to the SBL-KF based precoder design, here, the design of combiner using SBL-KF is discussed. The posterior density of $\hat{\mathbf{W}}_{\text{BB}}[n]$ can be evaluated as $p(\hat{\mathbf{W}}_{\text{BB},(j)}[n] | \mathbf{R}_{yy}^{1/2} \mathbf{W}_{\text{MMSE},(j)}[n]; \Gamma[n]) \sim \text{CN}(\mathbf{M}_{(j)}[n], \Sigma[n])$, with

$$\mathbf{M}[n] = \frac{1}{\sigma_a^2} \sum [n] (\mathbf{R}_{yy}^{1/2} \mathbf{A}_R^H) (\mathbf{R}_{yy}^{1/2} \mathbf{W}_{\text{MMSE}}[n]) \quad \text{and} \quad \Sigma[n] = \left(\frac{1}{\sigma_a^2} \mathbf{R}_{yy}^{1/2} \mathbf{A}_R^H \mathbf{A}_R + (\Gamma[n])^{-1} \right)^{-1}$$

Where $\mathbf{M}^{(m)}[n]$ denotes the mean value and $\Sigma^{(m)}[n]$ denotes the covariance matrix. This can be obtained from (13) and (14) by setting $\Gamma_k[n] = \hat{\Gamma}_k^{(m-1)}[n]$. Finally, the estimate of $\gamma_{k,i}[n]$ in the m^{th} EM iteration is derived as

$$\gamma_{k,i}^{(m)}[n] = \frac{1}{N_s} \left\| \mathbf{M}_{k,(i)}^{(m)}[n] \right\|^2 + \sum_{k,(i,i)}^{(m)} [n]$$

On convergence, the SBL-KF estimate of the matrix $\hat{\mathbf{W}}_{\text{BB},k}[n]$ is obtained as $\hat{\mathbf{W}}_{\text{BB},k}[n] = \mathbf{M}_k^{(m)}[n]$. SBL-KF scheme is used to design the optimal combiner and is shown in Algorithm 2.

The SBL-KF scheme is initialized as

$$\hat{\mathbf{W}}_{\text{BB},k}[-1/-1] = 0, \sum_{b,k} [-1/-1] = \hat{\Gamma}_k^{(m)}[0], \text{ and } \hat{\Gamma}_k^{(0)}[0] = \mathbf{I}_{G^2}, \forall n$$

Then, the hyper parameter matrix $\hat{\Gamma}_k^{(0)}[0]$ of the n^{th} block for the k^{th} subcarriers is initialized as

$$\hat{\Gamma}_k^{(0)}[n] = \hat{\Gamma}_k^{(0)}[n-1]$$

Algorithm2: SBL-KF for Combiner Design

Input: $\mathbf{R}_{yy}^{1/2} \mathbf{W}_{\text{MMSE},k}$, $\mathbf{R}_{yy}^{1/2} \mathbf{A}_R$, Variance of the approximation error σ_a^2 , Pilot power P , Stopping Parameters $\epsilon = 10^{-6}$ and $m_{\text{max}} = 100$

Initialization: $\hat{\mathbf{W}}_{\text{BB},k}[-1/-1] = 0_{G^2 \times N_s}$, $\sum_{b,k} [-1/-1] = \hat{\Gamma}_k^{(m)}[0]$ and $\hat{\Gamma}_k^{(0)}[0] = \mathbf{I}_{G^2}$

1. Obtain the approximation error value of combiner $\hat{\mathbf{W}}_{\text{BB},k}^{\text{opt}} = \arg \min_{\hat{\mathbf{W}}_{\text{BB},k}} \left\| \mathbf{R}_{yy}^{1/2} (\mathbf{W}_{\text{MMSE},k} - \mathbf{A}_R \hat{\mathbf{W}}_{\text{BB},k}) \right\|_F$

2. Obtain the estimate of the matrix $\hat{\mathbf{W}}_{\text{BB},k}[n/n]$

$$\mathbf{G}_k[n] = \sqrt{P} \sum_{b,k} [n/n-1] \mathbf{R}_{yy}^{1/2} \mathbf{A}_R^H (\mathbf{R}_v + P \mathbf{R}_{yy}^{1/2} \mathbf{A}_R \sum_{b,k} [n/n-1] \mathbf{R}_{yy}^{1/2} \mathbf{A}_R^H)^{-1}$$

$$\mathbf{W}_{\text{BB},k}[\mathbf{n}/\mathbf{n}] = \mathbf{W}_{\text{BB},k}[\mathbf{n}/\mathbf{n}-1] + \mathbf{G}_k[\mathbf{n}]\hat{\mathbf{W}}_{\text{BB},k}^{\text{opt}}$$

$$\sum_{\mathbf{b},k}[\mathbf{n}/\mathbf{n}] = (\mathbf{I} - \sqrt{\mathbf{P}}\mathbf{G}_k[\mathbf{n}]\mathbf{R}_{\text{yy}}^{1/2}\mathbf{A}_{\text{R}})\sum_{\mathbf{b},k}[\mathbf{n}/\mathbf{n}-1]$$

Hyper parameter estimate: Set

$$\hat{\gamma}_{k,i}^{(0)}[0] = 1, \forall 1 \leq i \leq G \Rightarrow \hat{\Gamma}_k^{(0)}[0] = \mathbf{I}$$

Set counter $\mathbf{m} = -1$ and $\hat{\Gamma}_k^{(-1)}[0] = 0$

While $\|\hat{\gamma}^{(k+1)}[\mathbf{n}] - \hat{\gamma}^{(k)}[\mathbf{n}]\|_2 > \epsilon$ and $\mathbf{m} \mid < \mathbf{m}_{\text{max}}$ **do**

$$\Sigma_k[\mathbf{n}] = \left(\frac{1}{\sigma_a^2} \mathbf{R}_{\text{yy}}^{1/2} (\mathbf{A}_{\text{R}}^H \mathbf{A}_{\text{R}}) + (\Gamma_k[\mathbf{n}])^{-1} \right)^{-1}$$

$$\mathbf{M}_k[\mathbf{n}] = \frac{1}{\sigma_a^2} \Sigma_k[\mathbf{n}] \mathbf{R}_{\text{yy}}^{1/2} \mathbf{A}_{\text{R}}^H \hat{\mathbf{W}}_{\text{BB},k}^{\text{opt}}[\mathbf{n}]$$

for $i = 0, 1, \dots, G$ **do**

$$\gamma_{k,i}^{(m)}[\mathbf{n}] = \frac{1}{N_s} \left\| \mathbf{M}_{k,(i,i)}^{(m)}[\mathbf{n}] \right\|^2 + \sum_{k,(i,i)}^{(m)}[\mathbf{n}]$$

end for

end while

Output: $\hat{\mathbf{W}}_{\text{BB},k}[\mathbf{n}] = \mathbf{M}_k^{(m)}[\mathbf{n}]$

3.3. Active RF chains at the precoder and combiner design

The ordered estimates of the hyperparameters are denoted as $\hat{\gamma}_{r_1} \geq \hat{\gamma}_{r_2} \geq \dots \hat{\gamma}_{r_G}$. The two-step procedure is followed to obtain baseband precoder matrix $\mathbf{F}_{\text{BB},k}$ from $\hat{\mathbf{F}}_{\text{BB},k}$. In the first step, $G - R$ rows of $\hat{\mathbf{F}}_{\text{BB},k}$ matrix is removed with respect to the indices $r_{R+1}, r_{R+2}, \dots, r_G$ of hyperparameter estimates to obtain the matrix $\hat{\mathbf{F}}_{\text{BB},k} \in \mathbb{C}^{R \times N_s}$. The second step, $R - N_T^{\text{RF}}$ rows of the matrix $\hat{\mathbf{F}}_{\text{BB},k}$ is removed with respect to the hyperparameters to obtain the baseband precoder matrix $\mathbf{F}_{\text{BB},k} \in \mathbb{C}^{N_T^{\text{RF}} \times N_s}$. The transmit RF precoder $\mathbf{F}_{\text{RF}}$ is extracted by $\mathbf{A}_{\text{T}}$ transmit array vector by retaining the N_T^{RF} columns. The proposed SBL-KF scheme is select the active RF chains based on the estimated hyperparameter vector. Finally, the $\mathbf{F}_{\text{BB},k}$ is normalize to

satisfy the power constraint as $\mathbf{F}_{\text{BB},k} \leftarrow \frac{\sqrt{N_s}}{\|\mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB},k}\|_{\text{F}}} \mathbf{F}_{\text{BB},k}$. The same procedure is followed to

obtain the baseband combiners $\mathbf{W}_{\text{BB},k}$ from $\hat{\mathbf{W}}_{\text{BB},k}$, the RF combiners $\mathbf{W}_{\text{RF}}$ is extracted from the

receive array vector A_R by retaining the N_R^{RF} columns. The $W_{BB,k}$ is normalize to satisfy the

$$\text{power constraint as } W_{BB,k} \leftarrow \frac{\sqrt{N_s}}{\|W_{RF} W_{BB,k}\|_F} W_{BB,k}.$$

4. Results

Simulation results of the proposed SBL-KF based hybrid precoder/ combiner scheme is discussed and compared with the existing schemes such as SBL, SOMP, GHP, OMP and LS for mmWave MIMO-OFDM systems.

The antennas at the transmitter and at the receiver are assumed as $N_T = 32$ and $N_R = 32$, respectively. The number of spatially active rays L is distributed uniformly and that is given as $L \sim U(L_{min}, L_{max})$. The minimum value of active path $L_{min} = 6$, maximum value of active path $L_{max} = 12$ and average value of active path $L_{avg} = 9$ is considered for simulation. The total number of RF chains is considered as $R=12$ and the active RF chain is assumed as $N_T^{RF}=N_R^{RF}=9$. In the frequency domain, the size of the FFT block used is $K=N=16$ and the frame length used is $N=16$.

N_s denotes the number of data streams and select the data streams under the condition of $N_s = \min(L_{min}, N_T^{RF}) = 6$. The path gains are represented as $CN(0, 1)$ random variables. Pulse shaping filter $p(\tau)$ with a roll-off factor value of 0.6 is used in the simulation.

The inter spacing of array antenna size is fixed, it's defined as half of the wavelength $d_T = d_R = \frac{\lambda}{2}$. The grid size $G=64$ is selected from the set of AoA/ AoD value. $\frac{P}{\sigma_n^2}$ denotes the Signal to

Noise Power Ratio (SNR). The initialization of hyperparameters as $\gamma_i^{(0)}=1, \forall 1 \leq i \leq G$. The value of hyperparameter threshold γ_{th} , the stopping parameter ϵ and the maximum number of EM iterations m_{max} is assumed as 10^{-4} , 10^{-9} and 50, respectively.

Fig 2. Performance of Spectral Efficiency vs SNR for proposed and existing schemes

The spectral efficiency performance for the proposed scheme is shown in Fig.2. The SBL-KF scheme based hybrid precoder/ combiner design achieve increased spectral efficiency when compared to the existing schemes. The proposed scheme result is close to the ideal optimal precoder and also gives the lower approximation error with feasible selection of data streams via active RF chains.

NMSE versus SNR performance for proposed scheme are plotted in Fig. 3. The NMSE performance of the SBL-KF based hybrid precoder/ combiner design is close to the ideal digital precoder and it gives lower NMSE value compared to the other schemes.

Number of paths vs NMSE for proposed and existing schemes is plotted in Fig.4. When the number of path is increases, the channel unknown parameters gets increased, thus the performance of NMSE increases for All schemes. NMSE vs SNR for various RF chain for proposed work is shown in Fig.5. Employing more number of RF chain at the transceiver gives the low estimation error in hybrid precoder/combiner systems.

The approximation errors $e = \|\mathbf{F}_{\text{opt}} - \mathbf{A}_T \hat{\mathbf{F}}_{\text{BB}}\|_F$ vs. Number of paths (L) for SBL-KF based hybrid precoder/ combiner and compared with existing scheme is plotted in Fig.6. From the Fig. 6, the SBL-KF scheme achieves a lower approximation error when compared to the existing schemes based on hybrid precoder/ combiner design. From the value of angular grid Φ , the AoA/ AoDs are selected; with this the hybrid precoder design attains the approximation error close to zero. The proposed scheme result is close to the optimal digital precoder $\mathbf{F}_{\text{opt}}$.

Fig 3. Performance of NMSE vs SNR for proposed and existing schemes

Fig 4. Performance of Number of paths vs NMSE for proposed and existing schemes

Fig 5. Performance of proposed scheme for various RF chains

Fig 6. Performance of approximation error vs SNR for proposed and existing schemes

5. Conclusion

In the frequency domain approach, SBL-KF based hybrid precoder/ combiner design for mmWave MIMO systems is discussed. The proposed method achieves increased spectral efficiency and it is closer to the ideal digital MIMO precoder/ combiner method. The proposed method performs better compared to the LS, OMP, SOMP and SBL schemes. Active RF chain is selected by initiating the hyperparameter method by selecting the active RF chain, which reduce the required RF chain and leads to reduce the power consumption in mmWave MIMO systems.

Declarations

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Authors' contributions SBL-KF based hybrid precoder / combiner design for mmWave MIMO channel is presented in the frequency domain.

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