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Biswaroop Mukherjee (✉ MAIL.BISWAROOP@GMAIL.COM)

Massachusetts Inst of Tech-MIT <https://orcid.org/0000-0002-7155-5412>

Airlia Shaffer

Massachusetts Institute of Technology

Parth Patel

Massachusetts Institute of Technology

Zhenjie Yan

Massachusetts Institute of Technology

Cedric Wilson

Massachusetts Institute of Technology

Valentin Crepel

Massachusetts Institute of Technology

Richard Fletcher

Massachusetts Institute of Technology <https://orcid.org/0000-0002-3650-2446>

Martin Zwierlein

Massachusetts Institute of Technology

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Crystallization of Bosonic Quantum Hall States

Biswaroop Mukherjee, Airlia Shaffer, Parth B. Patel, Zhenjie Yan,
Cedric C. Wilson, Valentin Crépel, Richard J. Fletcher, Martin Zwierlein
*MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics, and Department
of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
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The dominance of interactions over kinetic energy lies at the heart of strongly correlated quantum matter, from fractional quantum Hall liquids [1], to atoms in optical lattices [2] and twisted bilayer graphene [3]. Crystalline phases often compete with correlated quantum liquids, and transitions between them occur when the energy cost of forming a density wave approaches zero. A prime example occurs for electrons in high magnetic fields, where the instability of quantum Hall liquids towards a Wigner crystal [4–11] is heralded by a roton-like softening of density modulations at the magnetic length [7, 8, 12–14]. Remarkably, interacting bosons in a gauge field are also expected to form analogous liquid and crystalline states [15–26]. However, combining interactions with strong synthetic magnetic fields has been a challenge for experiments on bosonic quantum gases [24–26]. Here, we study the purely interaction-driven dynamics of a Landau gauge Bose-Einstein condensate [27] in and near the lowest Landau level (LLL). We observe a spontaneous crystallization driven by condensation of magneto-rotons [7, 8, 12], excitations visible as density modulations at the magnetic length. Increasing the cloud density smoothly connects this behaviour to a quantum version of the Kelvin-Helmholtz hydrodynamic instability, driven by the sheared internal flow profile of the rapidly rotating condensate. At long times the condensate self-organizes into a persistent array of droplets, separated by vortex streets, which are stabilized by a balance of interactions and effective magnetic forces.

When electrons are placed in a magnetic field, their kinetic energy is quenched. The single particle states form discrete, highly degenerate Landau levels, and correspond to wavepackets localized to the magnetic length ℓ_B . In the presence of interactions between electrons, owing to the absence of kinetic energy, one naturally expects the formation of a Wigner crystal of periodicity $\sim \ell_B$ [4–6, 9–11, 28]. Famously however, the interplay of the macroscopic degeneracy and interactions instead typically favours the strongly correlated fractional quantum Hall liquids, which host fractional charges, anyonic exchange statistics, and topologically protected transport properties [1]. The tendency to crystallize is still apparent in a pronounced minimum in the collective excitation spectrum at wavevectors $k \sim 1/\ell_B$ [7, 8, 12–14]. In analogy with the roton minimum in ^4He , also considered a precursor of solidification [29], these excitations are called magneto-rotons [7, 8, 13, 14].

The fate of interacting bosons in the presence of a gauge

field is of fundamental importance in the classification of topological states of matter [20, 21]. Quantum Hall states [15, 16, 22, 23], exotic vortex lattices [17] and vortex-free states under extreme fields [19] were predicted. Quantum phase transitions between such states were found to be signaled by the softening of a roton-like collective mode [18].

Bosonic quantum gases in artificial magnetic fields [24–26] have been generated via spin-orbit coupling [30–32], phase imprinting in lattices [33–40], and by rotation of the trapped gas [41–43]. The latter approach uses the analogy between the Lorentz force on a charged particle in a magnetic field, and the Coriolis force on a massive particle in a frame rotating at frequency Ω , giving $\omega_c = 2\Omega$ and $\ell_B = \sqrt{\hbar/(m\omega_c)}$ as the rotational analog of the cyclotron frequency and the magnetic length, respectively.

Signatures of physics near the lowest Landau level have been observed in rotating Bose gases [41, 42]. In recent work at MIT, condensates have been prepared directly in the lowest Landau gauge wavefunction using geometric squeezing [27]. In this mean-field quantum Hall regime [16], all bosons occupy a single wavefunction, whose subsequent dynamics subject to a gauge field can be studied, offering a microscopic insight into the individual building blocks of quantum Hall systems. An advantage of rotation is that the interactions between atoms are decoupled from the induced gauge potential, in contrast to other methods for which the effective magnetic field appears within a dressed-atom picture, leading to additional unwanted interaction terms [44].

Here, we directly observe the evolution of an interacting Bose-Einstein condensate occupying a single Landau gauge wavefunction in the LLL [45]. We find that the Landau gauge condensate is unstable under the influence of interactions, exhibiting spontaneous growth of a snaking mode leading to a persistent density wave order at the magnetic length ℓ_B as illustrated in Fig. 1. At the heart of this crystallization is the coupling between the relative momentum and spatial overlap of two particles in a gauge field. This lowers the interaction energy cost of populating higher-momentum states, and leads to the dynamical instability of the lowest (Goldstone) collective excitation branch [18, 46]. The ensuing proliferation of excitations at momenta near $\hbar/l_B$ can be viewed as condensation of magneto-rotons, in analogy to the Wigner crystal instability of quantum Hall systems [4–11]. Condensation at non-zero momentum has been predicted in superfluid helium above a critical velocity [47, 48]. Roton-like excitations and instabilities in Bose-Einstein condensates were induced via cavity mediated interactions [49, 50], spin-orbit

FIG. 1. Spontaneous crystallization of an interacting Bose-Einstein condensate in an artificial magnetic field. (a) In the laboratory frame, the condensate freely rotates in a circularly symmetric harmonic trap at the trapping frequency ω . Occupied states in the energy spectrum are sketched (L_z : angular momentum). (b) In the rotating frame, the condensate experiences an effective magnetic field B but no scalar potential. The energy spectrum is flattened into Landau levels (k : momentum along y). Only the $k = 0$ Landau gauge wavefunction is occupied. (c) The irrotationality of the condensate in the laboratory frame imposes a sheared velocity profile in the rotating frame which is dynamically unstable towards a periodic density modulation. Motion with momentum $\hbar k$ along the y -direction is tied to sideways displacement of the wavefunction along x . The reduced overlap of $|k| > 0$ states with the $k = 0$ condensate lowers the interaction energy cost of collective excitations, leading to spontaneous population of $\pm k$ pairs whose interference with the condensate results in a density modulation. (d) This dynamical instability is reflected in a (Goldstone) collective excitation branch which is imaginary across a range of wavevectors, shown by a red line. The spectrum shown is calculated for a condensate in the LLL, for which the interaction energy gn provides the only relevant energy scale and the magnetic length $\ell_B = 1.6 \mu\text{m}$ sets the length scale. Here g is the mean-field coupling constant, and $n = n_{2D}(0)$ is the peak 2D density. (e) Absorption images of the evolution of the condensate density in the rotating frame, displaying a snake-like instability and the formation of droplet arrays. Here, the cloud width $R = 2.34 \ell_B$, and the frames are taken in intervals of 2.5 cyclotron periods ($2\pi/\omega_c = 5.6 \text{ ms}$). The zoom-in reveals vortex streets between adjacent droplets, indicating counterflow at their interface.

coupling [51, 52], shaken optical lattices [53, 54], driven interactions [55] and dipolar interactions [56–58]. These instabilities were tightly connected to evidence for supersolidity, the simultaneous existence of spatial and superfluid order [50, 52, 59–62]. In our case, the instability to density-

101 wave order arises purely from the interplay of contact inter-
102 actions and a gauge field. No external drive is present, nor is
103 there any residual scalar potential in the rotating frame. The
104 absence of kinetic energy in the LLL directly implies that the
105 crystallization rate is set solely by the interaction energy of
106 the gas.

107 By increasing the condensate density such that many Lan-
108 dau levels become populated, we observe a crossover from
109 LLL behaviour to a hydrodynamic instability driven by the
110 sheared internal velocity profile. Analogous phenomena are
111 ubiquitous throughout hydrodynamics, from the diocotron in-
112 stability in charged plasmas [63] and fragmentation of elec-
113 tron beams [64], to the Kelvin-Helmholtz instability in atmo-
114 spheric and astrophysical systems [65, 66]. In the context of
115 superfluids, for which the circulation is quantized, the Kelvin-
116 Helmholtz instability has been detected in liquid helium [67],
117 and theoretically predicted at the boundary between counter-
118 flowing condensates [68, 69]. In our quantum hydrodynamic
119 setting, we directly observe streets of quantized vortices sepa-
120 rating emergent droplets, revealing the quantum nature of the
121 instability at the most microscopic level.

122 To analyze the instability, consider the condensate in the
123 frame rotating at the frequency ω of the isotropic harmonic
124 trap, where it experiences a synthetic magnetic field but no
125 scalar potential (see Fig. 1a,b), and thus evolves under the
126 Hamiltonian

$$\hat{H} = \int d^2r \hat{\Psi}^\dagger \left[\frac{(\hat{\mathbf{p}} - q\mathbf{A})^2}{2m} + \frac{g}{2} \hat{\Psi}^\dagger \hat{\Psi} \right] \hat{\Psi}. \quad (1)$$

Here $\hat{\Psi}^\dagger(\mathbf{r})$ is the bosonic field operator, $\hat{\mathbf{p}}$ is the canonical momentum, q and $\mathbf{A}$ are the charge and vector potential in the equivalent magnetic problem, and g is the two-dimensional mean-field coupling constant [45]. Geometric squeezing prepares a translationally-invariant condensate most conveniently described within the Landau gauge $q\mathbf{A} = (0, m\omega_c x)$ [27, 45] for which the Hamiltonian becomes

$$\hat{H} = \int d^2r \hat{\Psi}^\dagger \left[\frac{\hat{p}_x^2}{2m} + \frac{1}{2} m \omega_c^2 \left(\hat{x} - \frac{\hat{p}_y \ell_B^2}{\hbar} \right)^2 + \frac{g}{2} \hat{\Psi}^\dagger \hat{\Psi} \right] \hat{\Psi}. \quad (2)$$

127 Cyclotron motion of the atoms is reflected in an effective har-
128 monic oscillator along the x -direction of frequency $\omega_c = 2\omega$,
129 whose non-interacting energy states correspond to different
130 Landau levels (see Fig. 1b). Each level is macroscopically
131 degenerate since it costs no energy to translate the centers
132 of cyclotron orbits. Initially, the y -momentum of all atoms
133 is zero, and their cyclotron motion centred at $x = 0$ with a
134 two-dimensional number density $n_{2D}(x)$. The condensate has
135 uniform phase and thus features a sheared velocity profile
136 $\mathbf{v} = -q\mathbf{A}/m = (0, -\omega_c x)$ proportional to the vector poten-
137 tial (see Fig. 1c). We parameterize the crossover from LLL
138 to hydrodynamic behaviour by the ratio $\frac{gn}{\hbar\omega_c}$ of the conden-
139 sate's mean-field energy $\sim gn$ to the Landau level spacing
140 $\hbar\omega_c$, giving a measure for the number of occupied Landau lev-

els [27, 41]. Here $n = n_{2D}(0)$ is the peak density. In our experiment $\frac{gn}{\hbar\omega_c}$ varies from 0.6 to 7.3, corresponding to a central filling fraction $n\ell_B^2$ of 50 and higher, meaning the condensate lies within the mean-field quantum Hall regime [15, 16, 43].

The dynamical instability illustrated in Fig. 1 can be understood in the low- and high-density limits as follows. When $gn \lesssim \hbar\omega_c$, the condensate is restricted to the LLL and shows a Gaussian transverse density profile with a $1/e$ radius of ℓ_B [27, 46]. A Bogoliubov analysis around this state generically results in a Hamiltonian of the form [18]

$$\hat{H}_{\text{LLL}} = \sum_{k>0} A_k \left(\hat{a}_k^\dagger \hat{a}_k + \hat{a}_{-k}^\dagger \hat{a}_{-k} \right) + B_k \left(\hat{a}_k^\dagger \hat{a}_{-k}^\dagger + \hat{a}_k \hat{a}_{-k} \right), \quad (3)$$

where $\hat{a}_k$ is the annihilation operator for a particle with momentum $\hbar k$ along the y -direction. This Hamiltonian describes pairs of modes $\pm k$, with natural frequency $A_k/\hbar$ and coupled by a pair-creation operator of strength B_k which corresponds to a two-mode squeezing interaction in the language of quantum optics [70]. In a non-rotating uniform condensate, $A_k = \frac{\hbar^2 k^2}{2m} + gn$ and $B_k = gn$ [71] and hence pair-creation is always weaker than the mode energy, leading to stable excitations [72]. However, the effective magnetic field profoundly changes this picture. First, in the LLL there is no kinetic energy contribution to A_k . Second, as illustrated in Fig. 1c, the coupling between momentum and position means that states with $k \neq 0$ have a reduced overlap with the condensate and a correspondingly lower interaction energy. One finds [18] $A_k = gn [2 \exp(-k^2 \ell_B^2 / 2) - 1] / \sqrt{2}$ and $B_k = gn \exp(-k^2 \ell_B^2 / 2) / \sqrt{2}$, and the resulting dispersion $\varepsilon_k = \sqrt{|A_k|^2 - |B_k|^2}$ is shown in Fig. 1d. The spectrum is imaginary for an entire range of wavevectors $k > 0$ beyond the zero-energy Goldstone mode at $k = 0$, indicating dynamical instability of the Goldstone branch and correlated exponential growth of $\pm k$ pairs of these modes. Their interference with the $k = 0$ condensate results in a density modulation (see Fig. 1c). The fastest growth occurs at a wavevector $\sim 1/\ell_B$ giving a spatial modulation wavelength $\sim 2\pi$ times the magnetic length. This mode eventually becomes macroscopically occupied, corresponding to condensation of magneto-rotons and yielding a density modulation contrast of order unity. Crucially, since interactions provide the only energy scale in the LLL, the instability growth rate is determined purely by the interaction energy gn .

In the high-density limit where $gn \gg \hbar\omega_c$, a hydrodynamic description neglecting quantum pressure is valid. In this regime, the condensate initially exhibits a Thomas-Fermi density profile $n_{2D} \propto 1 - x^2/R_{\text{TF}}^2$ where $R_{\text{TF}} = \sqrt{\frac{2gn}{m\omega_c^2}} = \sqrt{\frac{2gn}{\hbar\omega_c}} \ell_B$ [46, 73]. The Coriolis force $2m\mathbf{v} \times \boldsymbol{\Omega}$ on each fluid element resulting from the shear flow $\mathbf{v} = (0, -\omega_c x)$ perfectly balances the local gradient of mean-field energy, resulting in an inhomogeneous equilibrium density despite the absence of any scalar potential. Our hydrodynamic stability analysis about this equilibrium state reveals a dynamical snaking instability of the cloud [46], in analogy to

FIG. 2. Structure factor and lengthscale of the emergent crystal. (a) The static structure factor, S_k , measured once the density modulation has reached steady-state for condensates with initial widths $R/\ell_B = 2.58, 1.75, 1.59, 1.28$, and 1.22 (top to bottom). The prominent peak reflects the periodic modulation of the cloud density. (b) Corresponding images of the steady-state crystal, illustrating the decrease in the modulation lengthscale with falling condensate density. (c) Dependence of the dominant modulation wavevector, k_{max} , on the cloud width, R/ℓ_B . The LLL and hydrodynamic results are indicated by the star and dashed line respectively (see text). The solid line shows the prediction of our Bogoliubov analysis [46], which shows excellent agreement with our data with no free parameters.

the Kelvin-Helmholtz instability of counterflow in fluid layers [65, 66], and the diocotron instability of charged plasmas and electron beams [63, 64]. The absence of quantum pressure means that the Thomas-Fermi radius and cyclotron frequency provide the only lengthscale and rate. Within the hydrodynamic analysis the instability develops at a wavevector set by the condensate width, as in the LLL, but at a density-independent rate $\propto \omega_c$ in striking qualitative contrast to the growth rate in the LLL.

From these arguments, for all condensate densities we anticipate an emergent density modulation with a lengthscale set by the width of the initial cloud. For a quantitative analysis, from our experimental images (see

Fig. 1e) we obtain the static structure factor $S_k \equiv |n_k|^2/N$, where $n_k = \int dy n_{1D}(y) e^{-iky}$ is the Fourier transform of the one-dimensional number density $n_{1D}(y)$ [74] and $N = \int dy n_{1D}(y)$. In Fig. 2a we show examples of S_k obtained once the density modulation has fully developed, which show a well-defined peak at a wavevector $k_{\max}$. We attribute the much smaller secondary peak at $2k_{\max}$ to the contiguous traces of condensate linking adjacent droplets. In Fig. 2c we show $k_{\max}$ as a function of the condensate density, which is parameterized by the ratio R/ℓ_B where R is the full-width-at-half-maximum of the initial cloud divided by $2\sqrt{\log 2}$. This normalization is chosen such that $R/\ell_B \rightarrow 1$ for vanishing gn , while in the high-density limit $R/\ell_B = \sqrt{gn/(\hbar\omega_c \log 2)}$. At all densities, we indeed find an instability lengthscale of order the cloud width, $k_{\max} \sim 1/R$. The star indicates the LLL prediction $k_{\max} = 0.98/\ell_B$ and the dashed line shows the hydrodynamic result $k_{\max} = 0.95/R$ [46] neglecting quantum pressure. The solid line presents $k_{\max}$ that we obtain from a numerical solution of the Bogoliubov equations [46], showing excellent agreement with the data without any free parameters.

While the cloud width sets the instability lengthscale in both the LLL and hydrodynamic regimes, the growth rate shows qualitatively different behaviour. In Fig. 3a, we show S_k as a function of time for several different condensate densities. In addition to the decrease in the instability lengthscale at lower densities, we observe a concurrent reduction of the growth rate. At each wavevector we fit the time-evolution of the structure factor with the theoretically expected function $S_k(t) = A \cosh(2\Gamma t)$ [46], and extract the instability growth rate $\Gamma(k)$. This is reported in Fig. 3b, along with the imaginary component of the corresponding Bogoliubov spectrum which shows good agreement without any free parameters. We note that the experimental data also reveal some growth in S_k at higher wavevectors than the unstable region predicted by the linear Bogoliubov analysis. We attribute this to non-linear effects, and have performed numerical simulations of the Gross-Pitaevskii (GP) equation, finding that these exhibit the same behaviour [46].

We capture the typical crystallization rate corresponding to a particular condensate density by the growth rate of the dominant instability, $\Gamma(k_{\max})$, and in Fig. 3c plot this as a function of R/ℓ_B . When $R/\ell_B \gg 1$ the rate is density-independent and consistent with our hydrodynamic result $\Gamma = 0.14\omega_c$, shown by the dashed line. However, for lower interaction energies the gas enters the LLL where gn provides the only energy scale. We observe a concurrent slowing down of the instability, and the data approach the LLL prediction $\Gamma = 0.21gn/\hbar$ indicated by a dotted line. At all densities, the data show good agreement with the rate obtained from our Bogoliubov analysis, reported as the solid line.

After its initial hyperbolic growth, S_k reaches a steady-state as shown in the inset of Fig. 3c. The emergent crystal is long-lived, with each droplet stabilized by a balance of the outward mean-field pressure and an inwards Coriolis force. This arises from the circulating flow within each droplet which is

FIG. 3. Instability growth dynamics. (a) The temporal evolution of the static structure factor, $S_k(t)$, for condensates with different initial widths which reveals a density-dependence of both the modulation lengthscale and growth rate. (b) The measured instability growth rate, Γ , as a function of wavevector. The solid line shows the rate obtained from our Bogoliubov analysis [46] and captures the data well with no free parameters. (c) The instability growth rate at the dominant unstable wavevector as a function of the condensate width. At high densities we find good agreement with the density-independent hydrodynamic rate $\Gamma = 0.14\omega_c$ (dashed line). As the density falls, we observe a crossover to the LLL scaling $\Gamma = 0.21gn/\hbar$ (dotted line). Solid line: Bogoliubov analysis [46]. The inset shows $S_k(t)$ at $k_{\max}$ for condensates in the hydrodynamic regime (dark red) and the LLL (light red), along with the corresponding fits used to extract the rate (see text).

imposed by the gauge field, and is evident from vortices intersecting adjacent droplets (see Fig. 1e). The counterflow speed at the interface of two droplets of radius R is $\sim \omega_c R$, giving a gradient of $m\omega_c R/\hbar$ in the relative phase and a vortex spacing of $2\pi\ell_B^2/R$. Adjacent droplets are therefore separated by $\sim (R/\ell_B)^2$ vortices. In the limit of classical hydrodynamics this number is large and the quantization of circulation is irrelevant, while in the LLL adjacent droplets are separated by

FIG. 4. **Spontaneous breaking of translational symmetry.** (a) Images of the emergent crystal in three experimental iterations, along with the integrated one-dimensional density profiles $n_{1D}(y)$. The vertical dashed line shows the position of the centre-of-mass of the cloud, relative to which the modulation phase is random. (b) An image of the cloud averaged over 60 iterations, in which the density modulation is no longer visible. (c) The phase, ϕ , and visibility of the density modulation measured for multiple iterations of the experiment, for two different initial condensate densities. The phase is randomly distributed between 0 and 2π , indicating spontaneous breaking of the initial translational symmetry of the cloud. The visibility is defined as the contrast of the density modulation in the center of the cloud (shown as red arrows in (a)), relative to the largest visibility for each cloud width. The visibility appears largely independent of the phase chosen by the modulation.

a single vortex [18].

While the dynamical instability drives the growth of a density modulation, the initial seeding of the unstable mode must arise from thermal or quantum fluctuations in the gas density at $t = 0$ [46]. Since the phase of these fluctuations is random, this results in spontaneous breaking of the initial translational symmetry of the condensate. In Fig. 4 we show the phase and visibility of the density modulation observed in different iterations of our experiment. To account for small fluctuations in

the overall cloud position, we fit the one-dimensional density profile with a sinusoidal function modulated by a Gaussian envelope, and obtain the modulation phase ϕ relative to the centre-of-mass of the cloud. At all densities we find that the phase is uncorrelated between different experimental realizations, indicating spontaneous breaking of the initial translational symmetry.

The emergent crystallization observed here offers a pristine example of collective physics arising purely from the interplay of interparticle interactions and a gauge field. A natural immediate direction concerns the Goldstone mode associated with the spontaneous breaking of translational symmetry, corresponding to magneto-phonons in the droplet array [11]. This would be a remarkable instance of a propagating mode arising intrinsically from interactions, in the absence of any single-particle dynamics. While the densities in our experiment correspond to tens of atoms per flux quantum, our protocol can be straightforwardly extended to prepare clouds of lower filling fractions, which are expected to host beyond-mean-field, strongly correlated bosonic quantum Hall states [15–20, 22, 23].

Methods

Preparation of Landau gauge condensates We prepare condensates occupying a single Landau gauge wavefunction using the geometric squeezing protocol described in [27]. We begin with a condensate of $8.1(1) \times 10^5$ atoms of ^{23}Na in an elliptical time-orbiting-potential (TOP) trap [75], with a rms radial frequency $\omega = 2\pi \times 88.6(1)$ Hz, ellipticity 0.125(4), and axial frequency 2.8ω . We then rotate the ellipticity of the trap, ramping the rotation frequency from zero to ω . In the rotating frame, atoms experience both a synthetic magnetic field and a scalar saddle potential. Isopotential flow on this saddle, in analogy to the $\vec{E} \times \vec{B}$ Hall drift of electromagnetism, leads to elongation and contraction of the condensate along orthogonal directions and effecting unitary squeezing of the atomic density distribution [27]. We then turn off the saddle potential by setting the trap ellipticity to zero, which halts the outward flow of atoms. This results in an equilibrium, quasi-translationally-invariant condensate freely rotating at ω , which we allow to evolve for a variable time t . Finally, we obtain an absorption image of the in situ density distribution.

Imaging setup Our imaging resolution is sufficient to observe vortices in situ with a contrast of $\sim 60\%$ [27]. These have a characteristic size set by the healing length, which is ~ 300 nm in our system. This is significantly smaller than the quantum-mechanical ground state size of cyclotron orbits, set by the rotational analog of the magnetic length, $\ell_B = \sqrt{\hbar/(2m\omega)} = 1.6 \mu m$.

Coupling constant Given interaction energies close to the LLL, the axial motion at frequency 2.8ω is predominantly in its ground state. The coupling constant is then $g = \frac{4\pi\hbar^2 a}{m} \int dz |\phi(z)|^4$, where a is the three-dimensional s -wave scattering length, and $\phi(z)$ is the axial wavefunction with normalization $\int dz |\phi(z)|^2 = 1$.

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