

# An Intrinsic Parameter Calibration Method for R-LAT System Based on CMM

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## Research Article

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## Title page

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**Abstract:** Rotary-laser automatic theodolite (R-LAT) system is a distributed large-scale metrology system, which provides parallel measurement in scalable measurement room without obvious precision losing. Each of R-LAT emits two nonparallel laser planes to scan the measurement space via evenly rotation, while the photoelectric sensors receive these laser planes signals and performs the coordinate calculation based on triangulation. The accurate geometric parameters of the two laser planes plays a crucial role in maintaining the measurement precision of R-LAT system. Practically, the geometry of the two laser plane, which is termed as intrinsic parameters, is unknown after assembled. Therefore, how to figure out the accurate intrinsic parameters of each R-LAT is a fundamental question for the application of R-LAT system. This paper proposed an easily operated intrinsic parameter calibration method for R-LAT system with adopting coordinate measurement machine. The mathematic model of laser planes and the observing equation group of R-LAT are established. Then, the intrinsic calibration is formulated as a nonlinear least square problem that minimize the sum of deviations of target points and laser planes, and the ascertain of its initial guess is introduced. At last, experience is performed to verify the effectiveness of this method, and simulations are carried out to investigate the influence of the target point configuration on the accuracy of intrinsic parameters.

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# An Intrinsic Parameter Calibration Method for R-LAT system based on CMM

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## 1 Introduction

Rotary-laser automatic theodolite (R-LAT) system is a distributed large-scale metrology system with adopting rule of triangulation, which also called indoor GPS by Nikon [1] and wMPS [2]. Compare with traditional large-scale metrology instruments like laser tracker (LT), photogrammetry, theodolite system, total station, CMM arm etc., the R-LAT system shows advantages in several points: (i) accomplish parallel measurement tasks; (ii) provide submillimeter measurement precision as 0.2mm+10ppm; (iii) the measurement volume is scalable with increasing more R-LATs and without losing measurement precision. Moreover, it can collaborate with other instruments to deal with complex measurement tasks. Nowadays, R-LAT system have been widely used in industrial manufacture such as aircraft fuselage assembly [3,4], AGV navigation [5,6], robot motion control

[7] and so on. Muelaner et al. [8-10] investigated the angular uncertainty of iGPS, Maisano et al. [11] studies the functional of iGPS, Schimitt et al. [12] evaluated its dynamic working performance via compare with LT. These studies show R-LAT system is capable to provide submillimeter precision measurement and promising application prospect in industrial manufacturing.

Essentially, accurate intrinsic and extrinsic parameters is the crucial fundament to ensure the work performance of R-LAT system. Particularly, the intrinsic parameters, which are fixed after assembled, directly decide the measurement precision of R-LAT system. Zhao et al. [13] proposed a calibration method for R-LAT system with adopting spatial coordinate control network. Coplanar equations are established by measuring the coordinate known target points, and the equation group is used to calculate both the intrinsic and extrinsic parameters. The target points are fixed on a huge mechanic frame and are calibrated by LT, however, it is inconvenient for field application and expensive. Similarly, photogrammetry system which is a distributed metrology system, intrinsic parameters calibration is necessary for the cameras. Plenty of studies have been devoted, especially, the flexible calibration method developed by Zhang [14] has been widely applied since its elegant operation that only a checkerboard is involved with combing several times measurement. This convenient calibration model could provide inspiration to easy the calibration for R-LAT system.

Therefore, this paper proposed a flexible intrinsic calibration method for R-LAT system with adopting a CMM which is easy to provide the target points and good at reducing cost. The rest part is arranged as follows: Section 2 briefly introduces the mathematic model of R-LAT system; Section 3 presents the intrinsic parameters calibration model and describes the ascertain method of its initial guess; Section 4 carries out experiment and simulations to verify the accuracy of this method and to investigate the influence of target distribution on the result accuracy.

## 2 Mathematic model of R-LAT system

R-LAT system consists of R-LATs, photoelectric sensors, data process unit, and relevant equipment like scale-bar, hand-held probe. As shown in Fig. 1(a), The R-LATs are distributed around the measurement object and the coordinates of measurement photoelectric sensors can be obtained via the rule of triangulation. During working process, as shown in Fig. 1(b), the head of each R-LAT emits two nonparallel laser planes, and it rotates around the axis with specified unique speed. Besides, each R-LAT emits an omnidirectional pulse laser as its time reference once reaching predefined phase. In the same time, in the common scanning room of multi-R-LAT, photoelectric sensors record their triggered time of laser planes. Subsequently, these time sequence data are transmitted to the data process unit for lateral processes like signal filtering, logic reconstruction, coordinate calculation and so on. As one key operation, the time moments for each  $k$ th R-LAT are identified as  $t_{k,0}$  for reference time,  $t_{k,1}$  and  $t_{k,2}$  for the two laser planes in sequence. As a result, according to the specified rotation speed  $n_k$  of each  $k$ th R-LAT, the rotation angle  $\theta_{k,1}$  and  $\theta_{k,2}$  of the two laser planes that go through the photoelectric sensor relative to the reference phase can be given

as follows:

$$\begin{cases} \theta_{n,1} = (t_{k,1} - t_{k,0})n_k\pi / 30 \\ \theta_{n,2} = (t_{k,2} - t_{k,0})n_k\pi / 30 \end{cases} \quad (1)$$

Obviously, the two rotation angles decide the geometry of the two laser planes for  $k$ th R-LAT, while the two planes generate an intersection line goes through the corresponding photoelectric sensor. This is the basic measurement model of one R-LAT for a photoelectric sensor. In this basis, all the R-LATs of one R-LAT system are performed lines that go through the same photoelectric sensor as shown in Fig. 1(a), establishing a spatial triangulation model. Consequently, the coordinate of the target photoelectric sensor can be figured out once the precise laser planes of all the R-LATs and the positions and orientations of all the R-LATs are known.

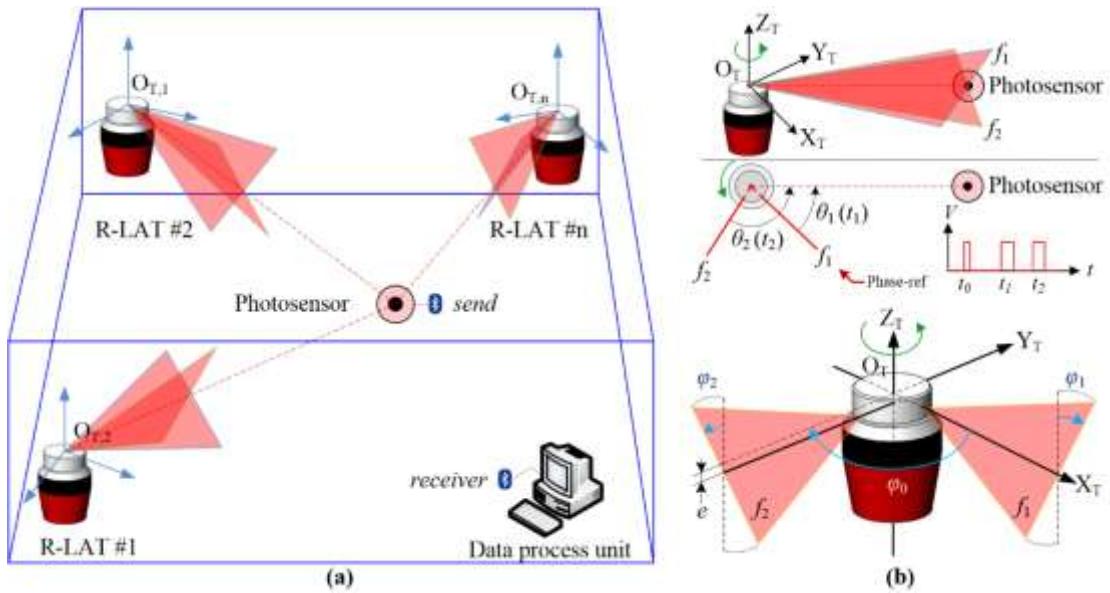


Fig. 1. The basic working model of R-LAT system: (a) the triangulation of R-LAT system, and (b) the laser plane model and the measurement time of one R-LAT.

Essentially, each R-LAT encodes the measurement space by the intersection spatial line of its two laser planes with corresponding rotation angles. Therefore, the two laser planes of one R-LAT need delicately design to ensure all the spatial lines are unique. As provide in Fig. 1(b), a Cartesian coordinate frame  $O_T-X_T-Y_T-Z_T$  is set on the head of R-LAT to describe the laser plane model, in which  $O_T$  is coincided with the emit center of laser planes,  $Z_T$ -axis is coincided with the rotation axis of R-LAT,  $X_T$ -axis is accorded with the references phase of R-LAT, and  $Y_T$ -axis is defined by right hand rule. Herein, the first laser plane  $f_1$  is set up by rotating  $X_T-O_T-Z_T$  with a non-zero angle  $\varphi_1$  around  $X_T$ -axis, while the second laser plane  $f_2$  is configured by rotating  $X_T-O_T-Z_T$  with a non-zero angle  $\varphi_2$  around  $X_T$ -axis and around  $Z_T$ -axis with  $\varphi_0$  in sequence. In practical, the second laser plane likely departs the origin  $O_T$  along  $Z_T$ -axis with a tiny  $e$  due to manufacture / assembly errors. As a result, the equations of the two laser planes are given as follows:

$$\begin{cases} f_1 : \cos(\varphi_1)y + \sin(\varphi_1)z = 0 \\ f_2 : -\sin(\varphi_0)x + \cos(\varphi_0)\cos(\varphi_2)y + \sin(\varphi_2)z + e = 0 \end{cases} \quad (2)$$

Obviously, the geometry of the two laser are defined by 4 parameters, i.e.,  $\varphi_0$ ,  $\varphi_1$ ,  $\varphi_2$ , and  $e$ , which is termed as the intrinsic parameters of one R-LAT.

### 3 Intrinsic parameter calibration model for R-LAT system

Undoubtedly, the intrinsic parameters of all the R-LATs play a crucial role in the coordinate measurement of R-LAT system. However, exact intrinsic parameters are unknown after R-LATs assembled, thereby calibrating the intrinsic parameters of R-LAT is the premise for R-LAT system. In practical, inspecting the rotation angles of laser planes of a R-LAT through custom-designing instruments is an option. However, it is expensive and delicate operations are required. Instead, practicable calibration operation like measuring some targets (photoelectric sensors) as laser tracker station-transfer [15] and measuring a calibration checkboard for camera calibration is appreciate with respect its good economic, easily operation, and short implement period.

#### 3.1 CMM-assisted intrinsic parameter calibration

The measuring mathematic model of R-LAT system has indicated that, for each R-LAT, its two laser plane  $f_1$  and  $f_2$  formulate two coplanar equations with the rotation angle  $\theta_1$  and  $\theta_2$  that go through the target photoelectric sensor  $\mathbf{P}$  as below:

$$\begin{cases} [0, \cos(\varphi_1), \sin(\varphi_1), 0] \cdot \begin{bmatrix} \mathbf{R}_z(\theta_1) & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = 0 \\ [-\sin(\varphi_0), \cos(\varphi_0)\cos(\varphi_2), \sin(\varphi_2), e] \cdot \begin{bmatrix} \mathbf{R}_z(\theta_2) & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = 0 \end{cases} \quad (3)$$

Where  $[x_p, y_p, z_p, 1]^T$  is the coordinate of target photoelectric sensor  $\mathbf{P}$  in the R-LAT coordinate frame ( $O_T-X_TY_TZ_T$ ), and  $\mathbf{R}_z(\theta)$  is the rotation matrix around Z-axis given as follow:

$$\mathbf{R}_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

Accordingly, a coplanar equation group (the observing equation group) for one R-LAT can be established by measuring target photoelectric sensors that placed in several spatial positions. Clearly, one can figure out that the intrinsic parameters of the R-LAT are the only unknown variables of this coplanar equation group once the coordinate value of these target photoelectric sensors are known. Consequently, the intrinsic parameters can be identified by process the equation group as a nonlinear

least-square problem.

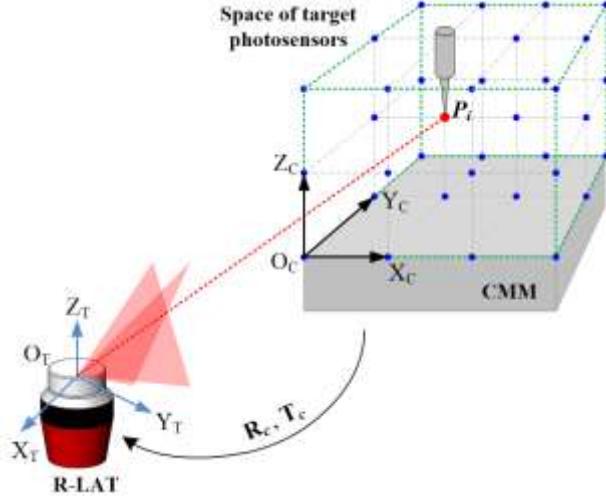


Fig. 2. The intrinsic parameters calibration model of R-LAT with a CMM.

In practical calibration process, the precise coordinate values of the target photoelectric sensor can be provided by precise coordinate measurement instrument like CMM, laser tracker and so on. As shown in Fig. 2, after installing the R-LAT on the ground, the target photoelectric sensor that attaches to the probe of a CMM is moved to  $M$  positions for measuring. Finally, an observing equation group composes of  $2M$  coplanar plane equations is obtained. However, it is noteworthy that the all the coordinate values of target photoelectric sensor are in the measurement coordinate frame of CMM ( $O_C-X_C-Y_C-Z_C$ ). They should be transformed into the R-LAT coordinate frame by a  $3 \times 3$  rotation matrix  $\mathbf{R}_c$  and a  $3 \times 1$  translation vector  $\mathbf{T}_c$  to satisfy the formulation as equation (3). As a result, the  $i$ th target photoelectric sensor position, of which the measurement coordinate value is  $\mathbf{P}_i = [x_i, y_i, z_i, 1]^T$  and the corresponding rotation angles are  $\theta_{i,1}$  and  $\theta_{i,2}$ , defines the observing equations as follow:

$$\begin{cases} [0, \cos(\varphi_1), \sin(\varphi_1), 0] \cdot \begin{bmatrix} \mathbf{R}_z(\theta_{i,1}) & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{R}_c & \mathbf{T}_c \\ 0 & 1 \end{bmatrix} \cdot \mathbf{P}_i = 0 \\ [-\sin(\varphi_0), \cos(\varphi_0)\cos(\varphi_2), \sin(\varphi_2), e] \cdot \begin{bmatrix} \mathbf{R}_z(\theta_{i,2}) & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{R}_c & \mathbf{T}_c \\ 0 & 1 \end{bmatrix} \cdot \mathbf{P}_i = 0 \end{cases} \quad (5)$$

For convenience, the translation vector  $\mathbf{T}_c$  is specified as  $[t_x, t_y, t_z]^T$ ; the rotation matrix  $\mathbf{R}_c$  is formulated by Euler angles (rotated  $\beta$  around z-axis, rotate  $\beta$  around y-axis, and rotated  $\alpha$  around x-axis in sequence) as follow:

$$\mathbf{R}_c = \begin{bmatrix} \cos(\beta)\cos(\gamma) & \sin(\alpha)\sin(\beta)\cos(\gamma) - \cos(\alpha)\sin(\gamma) & \cos(\alpha)\sin(\beta)\cos(\gamma) + \cos(\alpha)\sin(\gamma) \\ \cos(\beta)\sin(\gamma) & \sin(\alpha)\sin(\beta)\sin(\gamma) + \cos(\alpha)\cos(\gamma) & \cos(\alpha)\sin(\beta)\sin(\gamma) - \sin(\alpha)\cos(\gamma) \\ -\sin(\beta) & \sin(\alpha)\cos(\beta) & \cos(\alpha)\cos(\beta) \end{bmatrix} \quad (6)$$

Herein, the observing equation group includes 10 unknown variables, i.e., 4 intrinsic parameters of R-LAT ( $\varphi_0$ ,  $\varphi_1$ ,  $\varphi_2$ , and  $e$ ) and 6 elements of  $\mathbf{T}_c$  and  $\mathbf{R}_c$  ( $t_x$ ,  $t_y$ ,  $t_z$ ,  $\alpha$ ,  $\beta$ , and  $\gamma$ ). To ensure the unique solution is existed for this observing equation group, the equation number  $2M$  must no less than the

number of unknown variables 10, which denotes  $M \geq 5$ . Actually, due to the rotation error of R-LAT and the timing error of photoelectric sensor as well as the disturbances of environment, the  $i$ th target photoelectric sensor will deviate to the  $j$ th laser plane, and this error  $d_{ij}$  can be defined as:

$$\begin{cases} d_{i1} = [0, \cos(\varphi_1), \sin(\varphi_1), 0] \cdot \begin{bmatrix} \mathbf{R}_c & \mathbf{T}_c \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{R}_z(\theta_{i,1}) & 0 \\ 0 & 1 \end{bmatrix} \cdot \mathbf{P}_i - 0 \\ d_{i2} = [-\sin(\varphi_0), \cos(\varphi_0)\cos(\varphi_2), \sin(\varphi_2), e] \cdot \begin{bmatrix} \mathbf{R}_c & \mathbf{T}_c \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{R}_z(\theta_{i,2}) & 0 \\ 0 & 1 \end{bmatrix} \cdot \mathbf{P}_i - 0 \end{cases} \quad (7)$$

For all the laser coplanar equations, the object function can be defined as:

$$F = \min \left( \sum_{i=1}^M \sum_{j=1}^2 d_{i,j}^2 \right) \quad (8)$$

The 10 unknown variables can be figured out by minimizing the object function with adopting Levenberg-Marquardt (L-M) method [16] with considers its excellent performance in solve nonlinear least-square problem. In practical, looking forward to improve the accuracy of the solution, the observing equation group is overdefined by measuring plenty of target photoelectric sensor.

### 3.2 Initial guess ascertaining

Appropriate initial guess is significant to solving nonlinear least square problem, which ensures the iterative searching is converge to the true solution instead of trapping in local optimum. In fact, it is not easy to get a reasonable initial guess for the unknown variables of equation group (7). Particularly, the rotation matrix  $\mathbf{R}_c$  and translation vector  $\mathbf{T}_c$  are hard to assess due to  $\mathbf{X}_T$ -axis of each R-LAT is invisible, even the approximate rotation angles of laser planes can be measured manually while the specification deviation  $e$  is given as the designed/measured value directly.

In the basis of the working principle of R-LAT system, one can find similarity between the R-LATs and cameras in photogrammetry, in which the 3D points are mapped to a 2D formulation with lacking the depth information. Accordingly, identifying the  $\mathbf{R}_c$  and  $\mathbf{T}_c$  of one R-LAT in global coordinate frame is same as the classical Perspective-n-Point (PnP) question [17-19]. For this reason, direct linear transformation (DLT) method [20] can be used to gain the initial value of  $\mathbf{R}_c$  and  $\mathbf{T}_c$  with considering the advantages like unique solution and lower computation cost. The operation is organized as three steps: (i) transforming the measurement model of R-LAT to a camera; (ii) performing the DLT method; (iii) extracting the initial values from estimated  $\mathbf{R}_c$  and  $\mathbf{T}_c$ . The details are performed as follows:

(i) As shown in Fig. 3, the frame of the R-LAT is set as coincides with the camera coordinate frame, while the image plane goes through point  $[0, 1, 0]^T$  and perpendiculars to the  $\mathbf{Y}_T$ -axis. The direction of the  $\mathbf{X}$ -axis and  $\mathbf{Z}$ -axis of the image plane coordinate frame ( $O_I-X_IY_I$ ) is accordance with the R-LAT coordinate frame.

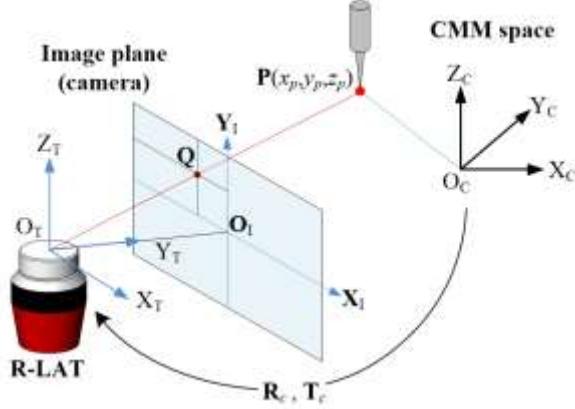


Fig. 3. The transformation model between R-LAT and camera.

According to this camera model, the ray  $\mathbf{L}_i$  from  $\mathbf{O}_T$  to the  $i$ th spatial target  $\mathbf{P}_i$  intersects the image plane at  $\mathbf{Q}_i$ . With omits the tiny deviation  $e$ , the ray vector  $\mathbf{L} = [l_x, l_y, l_z]^T$  can be identified as the cross product of the normal vectors  $\mathbf{N}_1$  and  $\mathbf{N}_2$  of the two laser plane that go through the corresponding target  $\mathbf{P}$ :

$$\mathbf{L} = \frac{\mathbf{N}_1 \times \mathbf{N}_2}{|\mathbf{N}_1 \times \mathbf{N}_2|} \quad (9)$$

Where normal vector  $\mathbf{N}_1$  and  $\mathbf{N}_2$  are given as follows:

$$\begin{cases} \mathbf{N}_1 = [0, \cos(\varphi_1), \sin(\varphi_1)] \cdot \mathbf{R}_z(\theta_1) \\ \mathbf{N}_2 = [-\sin(\varphi_0), \cos(\varphi_0)\cos(\varphi_2), \sin(\varphi_2)] \cdot \mathbf{R}_z(\theta_2) \end{cases} \quad (10)$$

Meanwhile, the coordinate value  $[u, v]$  for point  $\mathbf{Q}$  in image plane coordinate system can be given as below:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -l_x/l_y \\ l_z/l_y \end{bmatrix} \quad (11)$$

(ii) Herein, all the target points  $\mathbf{P}$  and their corresponding 2D coordinate values  $[u, v]$  on image plane are known. Obviously, to identify the rotation matrix  $\mathbf{R}_c$  and translation vector  $\mathbf{T}_c$  is a PnP question as below:

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = [\mathbf{R}_c \quad \mathbf{T}_c] \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix} \quad (12)$$

The results, i.e.,  $\mathbf{R}_c$  and  $\mathbf{T}_c$ , can be figured out with adopting DLT method directly.

(iii) Finally, according to equation (6), the components of the Euler angles ( $\alpha$ ,  $\beta$ , and  $\gamma$ ) can be extracted from the rotation matrix  $\mathbf{R}_c$  as below:

$$\begin{cases} \alpha = \text{atan } 2(m_{32}, m_{33}) \\ \beta = \text{atan } 2(-m_{31}, \sqrt{m_{32}^2 + m_{33}^2}) \\ \gamma = \text{atan } 2(m_{21}, m_{11}) \end{cases} \quad (13)$$

Where  $m_{ij}$  is the element of matrix  $\mathbf{R}_c$  at  $i$ th row  $j$ th column.

### 3.3 Procedure of intrinsic parameters calibration with CMM for R-LAT

With combing the aforementioned intrinsic parameters calibration model and the initial guess

ascertaining method, the complete calibration procedure is organized as shown in Fig. 4.

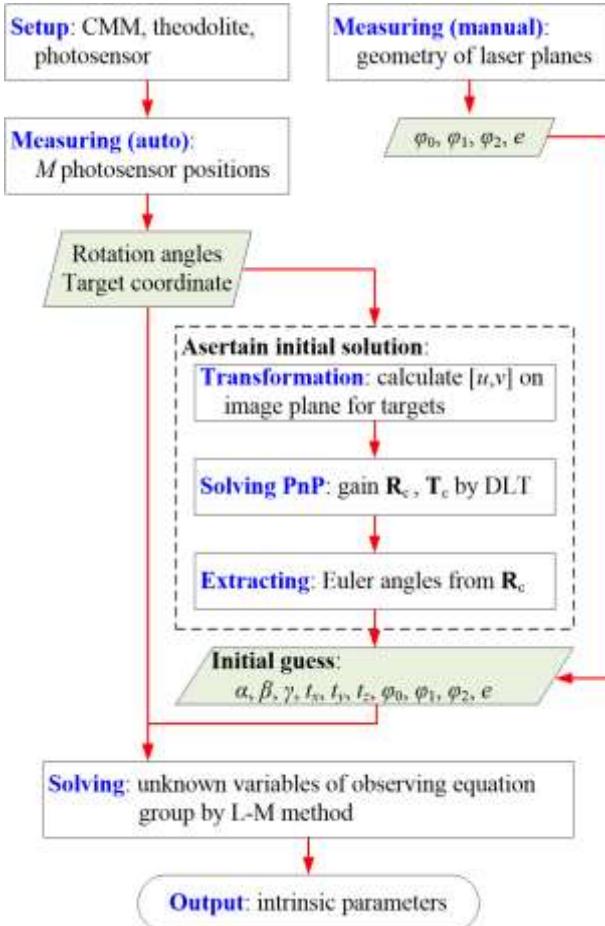


Fig. 4. The procedure of intrinsic parameters calibration.

#### 4 Examples and investigation on calibration precision

The proposed method is used to calibrate three R-LATs, then a scale-bar is measured by R-LAT system that composed by these R-LATs to evaluate the accuracy of this method. In addition, numerical calibration simulations are carried out to investigate the influence of target position distribution on the accuracy of calibrated result.

##### 4.1 Calibration experiment

As shown in Fig. 5(a), 100 target points were sampled in 400mm×400mm×400mm by a CMM (type: Hexagon Global 575, working room 500mm×700mm×500mm) use the same photoelectric sensor for three R-LATs respectively. The measured data of these target points established a least-square problem, and it was solved by L-M method. The calibrated intrinsic parameters of the R-LATs are demonstrated in table. 1. With substituting the result of intrinsic parameters into equation (7), the distances of all target points that deviate to the two laser planes were figured out as in Fig. 6(a) respectively. The maximum deviations for both two laser plane are no more than 0.08mm. It indicates the result intrinsic parameters are accurately defined the laser planes of these R-LATs.

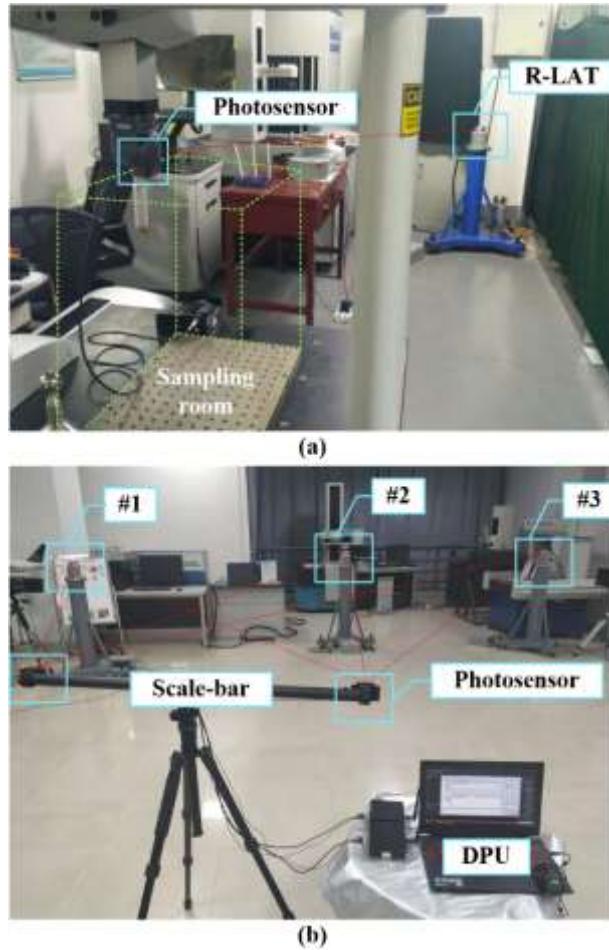


Fig. 5. Experiment: (a) the intrinsic parameters calibration for one R-LAT by CMM, and (b) scale-bar measurement by R-LAT system.

Table.1 The calibrated intrinsic parameters of three R-LATs.

R-LAT No.	$\varphi_0$ (°)	$\varphi_1$ (°)	$\varphi_2$ (°)	e (mm)
#1	-31.750	32.048	89.957	-38.653
#2	-30.070	33.990	89.599	25.907
#3	-28.737	32.565	89.514	31.502

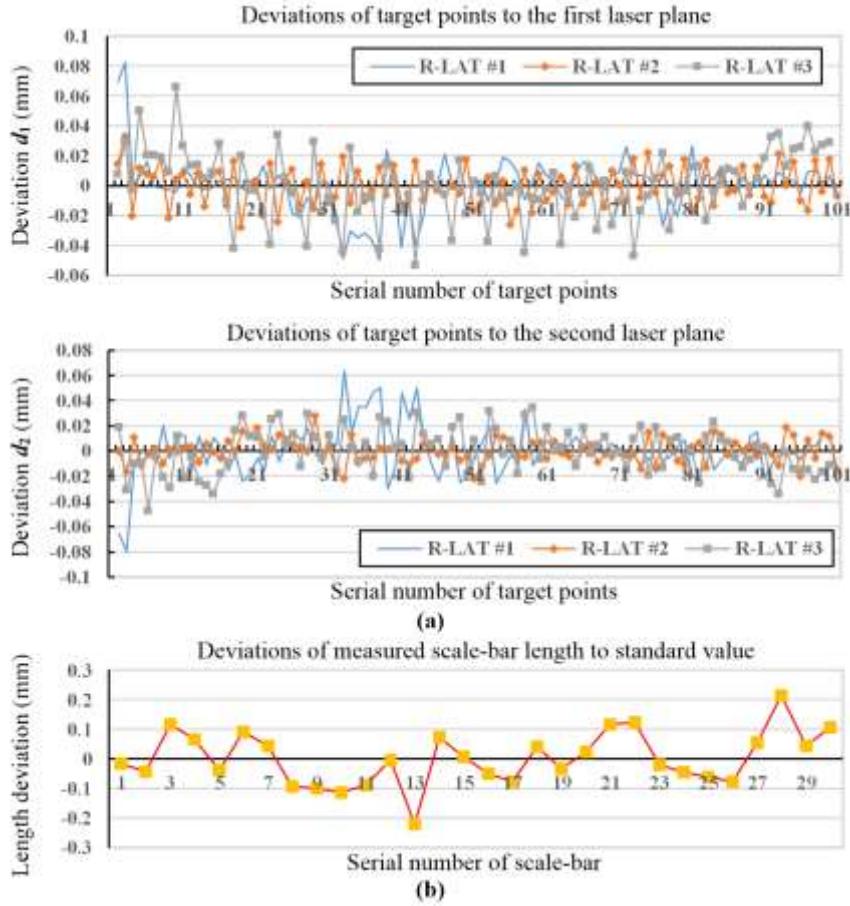


Fig. 6. The deviations of two laser plane for all the target points: (a) deviations of target point to the two laser planes, and (b) the measurement length deviation of scale-bar.

Furthermore, as provided in Fig. 5(b), a scale-bar with calibrated length  $969.467 \pm 0.005$  mm was measured at 30 positions by a R-LAT system that composed by aforementioned three R-LATs after delicately extrinsic calibration. The deviations of the measured lengths are shown in Fig. 6(b). The maximum length deviation is no larger than 0.22mm, which shows the same precision grade as iGPS provided by Nikon [1]. The result proofs this calibration model is effect in industrial application.

In addition, one can find that the calibrated axial eccentricities  $e$  of these R-LATs are deviated seriously to the desired 0 mm. Several factors might contribute to it: (i) the deflections of the laser planes, which are produce since the laser beams are not align to the axis of the cylindrical prisms; (ii) machining and assembly errors of the two laser devices. Besides, the vibration of the head of the R-LAT also leads to measurement errors rather than the simplified rotation angle errors.

#### 4.2 Influence of the target distribution on calibration precision

The spatial configuration of target points probably affects the precision of calibrated result since it decides the laser planes in the observe equation group. Hence, figure out the influence of the target points configuration on the calibration result will provide optimal guidance for intrinsic parameter calibration of R-LAT and benefit the work performance of R-LAT system.

Since the truth values of the intrinsic parameters of every R-LAT are never known, the deviation is difficult to evaluate. Instead, numerical calibration simulations are adopted taking into account to

the truth values are known. The measurement data for numerical calibration simulations is generated as following steps: (i) sampling desired target points with known coordinate value in CMM coordinate frame; (ii) identifying the truth rotation angles of the two laser planes for each target point according to equation (3) with known truth intrinsic parameters and rotation matrix as well as translation vector; (iii) taking account of the rotation speed fluctuation of the R-LAT, these nominal rotation angles are deliberately distorted by adding zero-mean normally distribute error with known variance ( $\sigma=3$  arc second). Then the measurement rotation angles and corresponding target points are used in intrinsic parameter calibration.

The number of target points and the distributed spatial size of the target points are the two mainly factors decide the observing equation group. Therefore, two groups of simulations were carried out to investigate their influence on the calibrated results. All the simulations shared the same specification truth intrinsic parameters i.e.,  $\varphi_0=90^\circ$ ,  $\varphi_1=-30^\circ$ ,  $\varphi_2=30^\circ$ , and  $e=0.8\text{mm}$ , and the layout of the R-LAT and CMM is illustrated as in Fig. 7(a) while the sampling space was designed as a cube according to the working space of CMM as shown in Fig. 7(b).

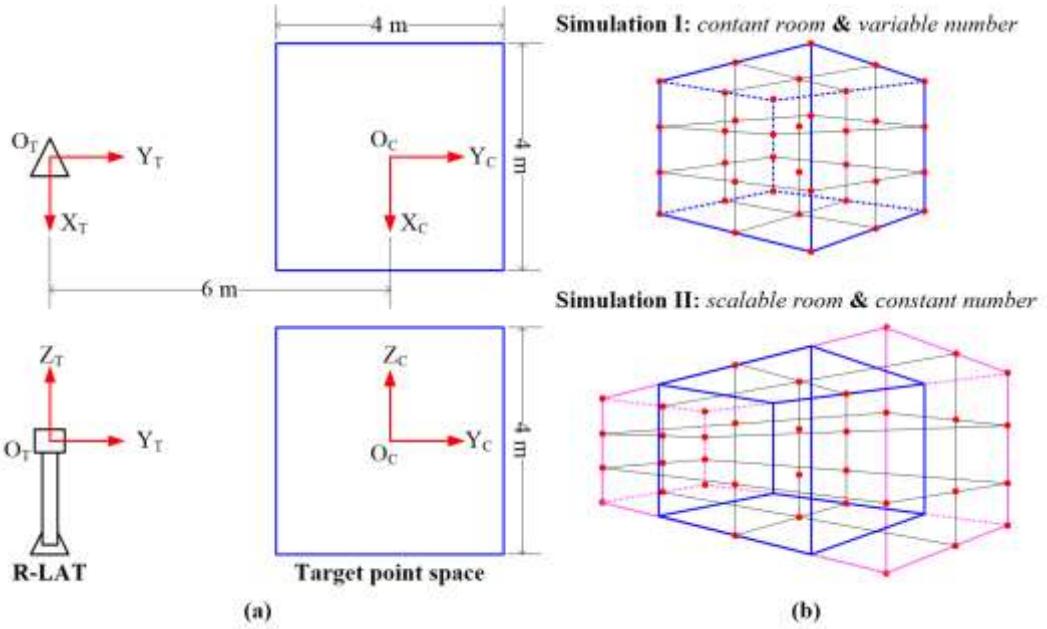


Fig. 7. the configurations of simulations for intrinsic parameters calibration: (a) the layout of the R-LAT and CMM, and (b) the spatial configuration of target points.

The first group of simulations was deployed with variable number of target points in constant sampling room. As provided in the upper of Fig. 7(b), within the  $4\text{m} \times 4\text{m} \times 4\text{m}$  room, the target points for every simulation were evenly sampled in every direction respectively with following the rule that the target count in one direction was set up as 3, 5, 7, 9, and 11 in sequence while in rest two directions the target count were constant to 5. Afterward, the intrinsic calibration is implemented 500 times for every configuration. The deviations between the estimated intrinsic parameters and the nominal values are figured out as in Figs. 8(a).

In contract, the second group of simulations arrange constant target point count in variable

sampling room. The sampling target count in the three direction are 5. With constant the center of sampling room, in each direction respectively the distance between two adjacent targets is set as 0.4m, 0.7m, 1.0m, 1.3m, and 1.6m in sequence. The intrinsic calibration is carried out 500 times for every configuration. The statistical analysis results of the estimated result compared to the nominal values are provided as in Figs. 8(b).

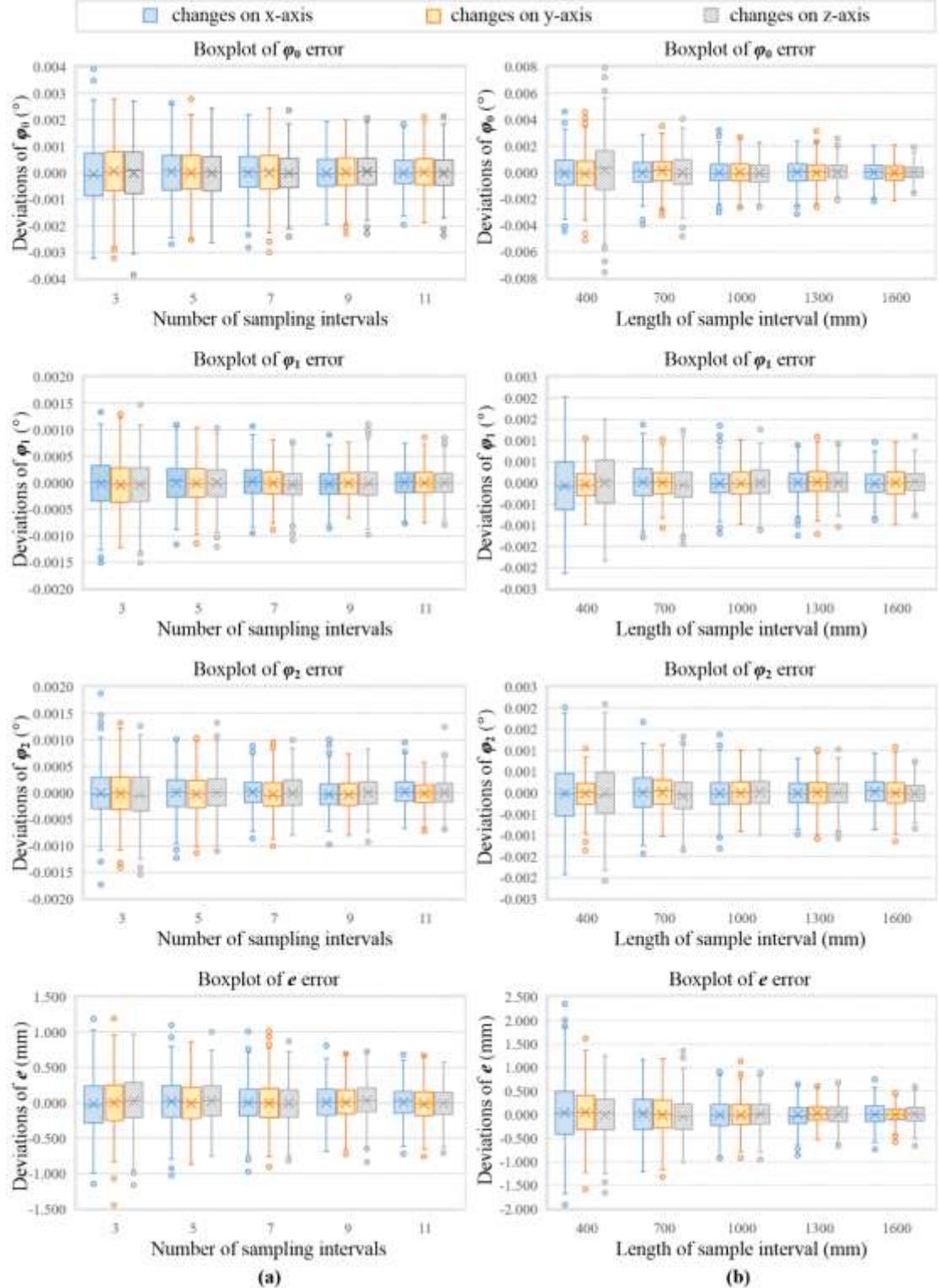


Fig. 8. Boxplot of the intrinsic parameters calibration errors against the target points set up in use:

(a) the errors of calibration results with changed number of target points, and (b) the errors of calibration results with scale changing of target points. On each box, the central mark is the median, the edges of the box are the 25% and 75% percentiles, the whisker extend to the most extreme data, the outliers are plotted individually.

As demonstrated in Fig. 8(a), the results of first simulation group indicate the estimated intrinsic parameters are even closing to the nominal value with increasing of the target point count. These trends are consistent in x, y, and z-axis direction. In addition, with taking the maximum deviations, the approximate relative accuracy of the four intrinsic parameters are 0.006%, 0.008%, 0.008%, 130% for  $\varphi_0$ ,  $\varphi_1$ ,  $\varphi_2$ , and  $e$ . Obviously, the angular components can be calibrated accurately, however, the axial deviation  $e$  exists a large uncertainty. The results of the second group simulation that shown in Fig. 8(b) point out the precisions of estimated intrinsic parameters are affected by the spatial layout of target points. The increases of interval length of adjacent target points in x-axis and z-axis direction provide great improvement on the precision of angular component of intrinsic parameters, however, the variation in y-axis shows less contribution. Meanwhile, the changing in all the directions provide consistent contributions on the component  $e$ .

In all, one can figure out the first group simulation performs higher precision than the second group simulation since more target points were used, which is agree with increasing the target point count benefits precision improving of intrinsic parameters. Meanwhile, the second group simulation shows, even fewer target points were adopted, enlarging the distribution range of target points along non-depth directions also contributes to intrinsic parameter precision promotion. Aiming to improve the calibration accuracy of intrinsic parameters, we suggest two manners: one is enlarge the room of target points, especially around the non-depth direction of R-LAT, where a spatial coordinate control network is established [13], the other one is increasing the target points number when the sampling is feasible in a small room. Besides, with considering the large relative errors of eccentricity  $e$  in the simulation, we also suggest a variant calibration method. The eccentricity  $e$  is not performed as an unknown variant but a constant input element which has been figured out by other inspect methods.

## 5 Conclusions

This study developed an intrinsic parameters calibration method for R-LAT system with adopting a CMM. The main conclusions are summarized as follows:

- (i) The measurement model of R-LAT system is established as coplanar equations of every R-LAT. In this basis, the 4 intrinsic parameters are defined to formulate the two laser plane of R-LAT with considering the axial deviation induced by assemble error.
- (ii) In intrinsic parameter calibration, the coordinate values of target points are provided by a CMM. All the coplanar equations of laser planes compose an observing equation group, which is formulated as a nonlinear least square problem to minimize the sum of deviations of target point to

corresponding laser plane. The unknown variables include 4 intrinsic parameters and 6 elements of the rotation matrix and translation vector between the coordinate frames of R-LAT and CMM.

(iii) An initial solute ascertaining method is given to improve the calculation of this nonlinear least square problem, which enhances the robustness of this calibration method. Through transforming the measurement model of one R-LAT into the image plane coordinate of a camera, figuring out the rotation matrix and translation vector is formulated as a PnP question and performed by the DTL method.

(iv) Both increasing the count of target points and positioning the target point widely along the non-depth direction of R-LAT improve the accuracy of intrinsic parameters. They are suitable for sampling target with a CMM and establishing a spatial coordinate control network by laser tracker. Furthermore, the investigations indicate that the angular components are easy to reach a higher accuracy lever rather than the eccentricity  $e$ .

## Declarations

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**Authors' contribution** Wenjun Su proposed the detailed method and carried out the formula derivation and theoretical analysis. Junkang Guo was responsible for the experimental work and part of data analysis. Zhigang Liu were involved in the discussion and significantly contributed to making the final draft of the article. Kang Jia proposed the research idea and technical scheme. All the authors read and approved the final manuscript.

**Data availability** The authors confirm that the data supporting the findings of this study are available within article.

## Compliance with ethical standards

**Competing interests** The authors declare that they have no competing interests.

**Ethical approval** Not applicable.

**Consent to participate** Not applicable.

**Consent to publish** Not applicable.

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