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Frictional contact problem of one dimensional hexagonal piezoelectric quasicrystals layer

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Abstract Based on three-dimensional (3D) general solutions for one-dimensional (1D) hexagonal piezoelectric quasicrystals (PEQCs), this paper studied the frictional contact problem of 1D-hexagonal PEQCs layer. The frequency response functions (FRFs) for 1D-hexagonal PEQCs layer are analytically derived by applying double Fourier integral transforms to the general solutions and boundary conditions, which are consequently converted to the corresponding influence coefficients (ICs). The conjugate gradient method (CGM) is used to obtain the unknown pressure distribution, while the discrete convolution-fast Fourier transform technique (DC-FFT) is applied to calculate the displacements and stresses of phonon and phason, electric potentials and electric displacements. Numerical results are given to reveal the influences of material parameters and loading conditions on the contact behavior. The obtained 3D contact solutions are not only helpful further analysis and understanding of the coupling characteristics of phonon, phason and electric field, but also provide a reference basis for experimental analysis and material development.

Keywords 1D-hexagonal piezoelectric quasicrystals, layer, frictional contact, DC-FFT

1 Introduction

In 1984, Shechtman [1] discovered the icosahedral phason with both long-range quasi-periodic translation order and long-range orientation order in the electron diffraction pattern of Al-Mn alloy particles formed by quenching, and therefore won the Nobel's Prize in 2011. Levine and Steinhardt named this material as quasicrystals (QCs) [2]. The discovery of QCs breaks through the traditional concept of dividing solids into crystalline and amorphous states. It is a significant progress in condensed matter physics and materials science. Because of its unique atomic arrangement structure,

QCs have high hardness, low adhesion, low friction coefficient and other excellent properties. QCs are widely used, including advanced surface modification materials such as thermal barrier coatings for aircraft engines, high-performance structural reinforcement materials, low-carbon maraging ultra-high strength steels, hydrogen storage materials and other new functional materials [3].

Piezoelectricity is an important property of QCs. By virtue of the piezoelectric effect, QCs has better sensor and actuator functions in the design of intelligent structures and systems. Fujiwara et al. [4] discovered the piezoelectric effect of QCs for the first time, and discussed the electronic structure and electron transport of QCs. Hu et al. [5] proposed the elastic theory framework of PEQCs, and pointed out that the piezoelectric effect of QCs is excited by phonon and phason field simultaneously. Rao et al. [6] obtained the number of independent nonvanishing piezoelectric coefficients. With the development of preparation technology of piezoelectric quasicrystals (PEQCs) and actual requirement to resist contact damage, the relevant researches on contact problem becomes more and more important. Scanning Probe/Piezoelectric response Force Microscope (PFM) is a very important technology, which can be used to test and analyze the properties of materials. These techniques all depend on the relevant 3D contact theory analysis of materials. In conclusion, the 3D contact analysis of PEQCs layer has significantly theoretical values, which also attracts many researchers' attention. Li and Liu[7] derived the character formulae of representation matrices of the piezoelectric property tensors for the 1D-QCs. Recently, Altay and Dokmeci [8] obtained 3D fundamental equations of PEQCs. The 3D general solution of 1D-hexagonal PEQCs according to the rigorous operator theory and Almansi theorem is given by Chen and Li [9, 10]. By using the same method, Gao et al. [11-13] derived the 3D elastic general solution of 2D-PEQCs. Based on the general solution of QCs, Li et al. [14] obtained the fundamental solution of 1D-hexagonal PEQCs in 2014. Based on the general solutions of QCs, Fan et al. [15] obtained the analytical solution of the contact problem of 2D-QCs by using the complex function method. Zhao and Li [16, 17] studied contact problems of QCs by using analytical function theory and dual integral equation theory. Based on Hankel transform, the frictionless contact problem of 1D and 2D-QCs half spaces are considered by Wu and Li [18, 19]. Using the trial and error method, Hou et al. [20, 21] studied the 3D fundamental solution of 1D hexagonal QCs layer. Li and Zhou [22] studied the fundamental solutions and frictionless contact problem in a semi-infinite space of 2D hexagonal PEQCs.

The study on 3D contacts would provide more general solutions and more accurate understanding to real-world problems [23,24]. However, little has been done on 3D contact problems of PEQCs layers. The semi-analytical models (SAMs), supported by the conjugate gradient method (CGM) and the fast Fourier transform (FFT) [25-27], appear to be efficient in solving 3D contact problems involving multifield coupling [28-31]. This paper reports the development of the frictional contact of 1D-

hexagonal PEQCs layer, based on the SAMs, by the first mathematical derivation of the closed-form FRFs for 1D-hexagonal PEQCs layer and creative extension of the numerical DC-FFT framework. The contact responses of displacements and stresses of phonon and phason, electric potentials and electric displacements are solved. The first set of numerical results are given to reveal the influences of material parameters and loading conditions on the contact behavior.

2 Problem description and formulation

2.1 Basic Equations

1D-hexagonal PEQCs are transversely isotropic materials in which the phonon, phason and electric field are coupled. Phonon is the quantum of elastic vibration. Phason is the local rearrangement of atoms in the cell based on Penrose. In the Cartesian coordinate system, it is assumed that the atoms are arranged periodically in the $x - y$ plane, and quasi periodic arrangement along $z -$ axis. Therefore, the constitutive equations for 1D-hexagonal PEQCs are [14]

$$\left\{ \begin{array}{l} \sigma_{xx} = C_{11}\varepsilon_{xx} + C_{12}\varepsilon_{yy} + C_{13}\varepsilon_{zz} + R_1 w_{zz} - e_{31}E_z \\ \sigma_{yy} = C_{12}\varepsilon_{xx} + C_{11}\varepsilon_{yy} + C_{13}\varepsilon_{zz} + R_1 w_{zz} - e_{31}E_z \\ \sigma_{zz} = C_{13}\varepsilon_{xx} + C_{13}\varepsilon_{yy} + C_{33}\varepsilon_{zz} + R_2 w_{zz} - e_{33}E_z \\ \sigma_{yz} = \sigma_{zy} = 2C_{44}\varepsilon_{yz} + R_3 w_{zy} - e_{15}E_y \\ \sigma_{xz} = \sigma_{zx} = 2C_{44}\varepsilon_{xz} + R_3 w_{zx} - e_{15}E_x \\ \sigma_{xy} = \sigma_{yx} = 2C_{66}\varepsilon_{xy} = (C_{11} - C_{12})\varepsilon_{xy} \\ H_{zz} = R_1\varepsilon_{xx} + R_1\varepsilon_{yy} + R_2\varepsilon_{zz} + K_1 w_{zz} - e'_{33}E_z \\ H_{zx} = 2R_3\varepsilon_{xz} + K_2 w_{zx} - e'_{15}E_x \\ H_{zy} = 2R_3\varepsilon_{yz} + K_2 w_{zy} - e'_{15}E_y \\ D_z = e_{31}\varepsilon_{xx} + e_{31}\varepsilon_{yy} + e_{33}\varepsilon_{zz} + e'_{33}w_{zz} + \xi_{33}E_z \\ D_x = 2e_{15}\varepsilon_{xz} + e'_{15}w_{zx} + \xi_{11}E_x \\ D_y = 2e_{15}\varepsilon_{yz} + e'_{15}w_{zy} + \xi_{11}E_y \end{array} \right. \quad (1)$$

where σ_{ij} , ε_{ij} and C_{ij} are the stresses, strains and elastic stiffness constants in the phonon field, respectively; H_{ij} , w_{ij} and K_i are the corresponding quantities in the phason field, respectively; D_i , E_i and ξ_{ij} are the electric displacements, electric fields and dielectric coefficients, respectively; R_i is the phonon-phason coupling elastic constant; e_{ij} and e'_{ij} are the piezoelectric coefficients.

In the phason and electric fields, there are similar geometric relations between the strain and displacement component. The generalized strain-displacement relations (geometric equations) are

$$\left\{ \begin{array}{l} \varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \varepsilon_{yy} = \frac{\partial u_y}{\partial y}, \varepsilon_{zz} = \frac{\partial u_z}{\partial z} \\ \varepsilon_{yz} = \varepsilon_{zy} = \frac{1}{2} \left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z} \right) \\ \varepsilon_{xz} = \varepsilon_{zx} = \frac{1}{2} \left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right) \\ \varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2} \left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right) \\ w_{zz} = \frac{\partial w_z}{\partial z}, w_{zx} = \frac{\partial w_z}{\partial x}, w_{zy} = \frac{\partial w_z}{\partial y} \\ E_z = -\frac{\partial \phi}{\partial z}, E_x = -\frac{\partial \phi}{\partial x}, E_y = -\frac{\partial \phi}{\partial y} \end{array} \right. \quad (2)$$

where u_i , w_z and ϕ are phonon displacement, phason displacement and electric potential, respectively.

In the absence of body forces and free charges, the equilibrium equations can be expressed as

$$\left\{ \begin{array}{l} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0 \\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0 \\ \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = 0 \\ \frac{\partial H_{zx}}{\partial x} + \frac{\partial H_{zy}}{\partial y} + \frac{\partial H_{zz}}{\partial z} = 0 \\ \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0 \end{array} \right. \quad (3)$$

By substituting Eqs. (1) and (2) into Eq. (3), the following governing equations can be obtained

$$\begin{aligned} & (C_{11} \frac{\partial^2}{\partial x^2} + C_{66} \frac{\partial^2}{\partial y^2} + C_{44} \frac{\partial^2}{\partial z^2}) u_x + (C_{12} + C_{66}) \frac{\partial^2}{\partial x \partial y} u_y + (C_{13} + C_{44}) \frac{\partial^2}{\partial x \partial z} u_z \\ & + (R_1 + R_3) \frac{\partial^2}{\partial x \partial z} w_z + (e_{31} + e_{15}) \frac{\partial^2}{\partial x \partial z} \phi = 0 \\ & (C_{12} + C_{66}) \frac{\partial^2}{\partial x \partial y} u_x + (C_{66} \frac{\partial^2}{\partial x^2} + C_{11} \frac{\partial^2}{\partial y^2} + C_{44} \frac{\partial^2}{\partial z^2}) u_y + (C_{13} + C_{44}) \frac{\partial^2}{\partial y \partial z} u_z \\ & + (R_1 + R_3) \frac{\partial^2}{\partial y \partial z} w_z + (e_{31} + e_{15}) \frac{\partial^2}{\partial y \partial z} \phi = 0 \\ & (C_{13} + C_{44}) \frac{\partial}{\partial z} \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) + (C_{44} \Delta + C_{33} \frac{\partial^2}{\partial z^2}) u_z + (R_3 \Delta + R_2 \frac{\partial^2}{\partial z^2}) w_z + (e_{15} \Delta + e_{33} \frac{\partial^2}{\partial z^2}) \phi = 0 \\ & (R_1 + R_3) \frac{\partial}{\partial z} \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) + (R_3 \Delta + R_2 \frac{\partial^2}{\partial z^2}) u_z + (K_2 \Delta + K_1 \frac{\partial^2}{\partial z^2}) w_z + (e'_{15} \Delta + e'_{33} \frac{\partial^2}{\partial z^2}) \phi = 0 \end{aligned} \quad (4)$$

$$(e_{31} + e_{15}) \frac{\partial}{\partial z} \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) + (e_{15} \Delta + e_{33} \frac{\partial^2}{\partial z^2}) u_z + (e'_{15} \Delta + e'_{33} \frac{\partial^2}{\partial z^2}) w_z - (\xi_{11} \Delta + \xi_{33} \frac{\partial^2}{\partial z^2}) \phi = 0$$

where $\Delta = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$ is the plane Laplacian operator.

2.2 General solutions

Using the rigorous operator theory and generalized Almansi theorem [32], the 3D general solutions are [14]

$$\begin{aligned} u_x &= -\frac{\partial \psi_0}{\partial y} + \sum_{i=1}^4 \frac{\partial \psi_i}{\partial x}, u_y = \frac{\partial \psi_0}{\partial x} + \sum_{i=1}^4 \frac{\partial \psi_i}{\partial y}, \\ u_z &= \sum_{i=1}^4 \alpha_{1i} \frac{\partial \psi_i}{\partial z_i}, w_z = \sum_{i=1}^4 \alpha_{2i} \frac{\partial \psi_i}{\partial z_i}, \phi = \sum_{i=1}^4 \alpha_{3i} \frac{\partial \psi_i}{\partial z_i} \end{aligned} \quad (5)$$

where the potential function ψ_i satisfies the quasi Laplace equation

$$(\Delta + \frac{\partial^2}{\partial z_i^2}) \psi_i = 0 (i = 0, 1, 2, 3, 4) \quad (6)$$

with $z_i = s_i z$, $s_0 = \sqrt{C_{66}/C_{44}}$, s_1, s_2, s_3, s_4 are different eigen-roots with a positive real part satisfying

the characteristic equation $as^8 - bs^6 + cs^4 - ds^2 + e = 0$, $\alpha_{(k-1)i} = \frac{d_k - c_k s_i^2 + b_k s_i^4 - a_k s_i^6}{s_i (c_1 - b_1 s_i^2 + a_1 s_i^4)}$ ($k = 2, 3, 4$), and

the constants $a, b, c, d, e, a_i, b_i, c_i$ and d_i are given in Appendix A.

From Eqs. (1-3) and (5,6), we can easily derive the expressions of the generalized stress components as

$$\begin{aligned} \sigma_{xx} &= -2C_{66} \frac{\partial^2 \psi_0}{\partial x \partial y} + \sum_{i=1}^4 (\gamma_{0i} \frac{\partial^2}{\partial z_i^2} - 2C_{66} \frac{\partial^2}{\partial y^2}) \psi_i \\ \sigma_{yy} &= 2C_{66} \frac{\partial^2 \psi_0}{\partial x \partial y} + \sum_{i=1}^4 (\gamma_{0i} \frac{\partial^2}{\partial z_i^2} - 2C_{66} \frac{\partial^2}{\partial x^2}) \psi_i \\ \sigma_{zz} &= \sum_{i=1}^4 \gamma_{1i} \frac{\partial^2 \psi_i}{\partial z_i^2}, \sigma_{xy} = C_{66} (\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}) \psi_0 + \sum_{i=1}^4 2C_{66} \frac{\partial^2 \psi_i}{\partial x \partial y} \\ \sigma_{zx} &= -s_0 \rho_1 \frac{\partial^2 \psi_0}{\partial y \partial z_0} + \sum_{i=1}^4 \lambda_{1i} \frac{\partial^2 \psi_i}{\partial x \partial z_i}, \sigma_{zy} = s_0 \rho_1 \frac{\partial^2 \psi_0}{\partial x \partial z_0} + \sum_{i=1}^4 \lambda_{1i} \frac{\partial^2 \psi_i}{\partial y \partial z_i} \\ H_{zx} &= -s_0 \rho_2 \frac{\partial^2 \psi_0}{\partial y \partial z_0} + \sum_{i=1}^4 \lambda_{2i} \frac{\partial^2 \psi_i}{\partial x \partial z_i}, H_{zy} = s_0 \rho_2 \frac{\partial^2 \psi_0}{\partial x \partial z_0} + \sum_{i=1}^4 \lambda_{2i} \frac{\partial^2 \psi_i}{\partial y \partial z_i} \\ H_{zz} &= \sum_{i=1}^4 \gamma_{2i} \frac{\partial^2 \psi_i}{\partial z_i^2}, D_x = -s_0 \rho_3 \frac{\partial^2 \psi_0}{\partial y \partial z_0} + \sum_{i=1}^4 \lambda_{3i} \frac{\partial^2 \psi_i}{\partial x \partial z_i} \\ D_y &= s_0 \rho_3 \frac{\partial^2 \psi_0}{\partial x \partial z_0} + \sum_{i=1}^4 \lambda_{3i} \frac{\partial^2 \psi_i}{\partial y \partial z_i}, D_z = \sum_{i=1}^4 \gamma_{3i} \frac{\partial^2 \psi_i}{\partial z_i^2} \end{aligned} \quad (7)$$

in which λ_{ki} , γ_{ki} and ρ_i are given in Appendix A as well.

2.3 Frictional contact of 1D-hexagonal PEQCs layer

Fig. 1. 1D-hexagonal PEQCs layer subjected to normal force P_z , tangential force P_x , phason force H and electric charge Q

Fig. 1 illustrates the contact problem considered in which the 1D-hexagonal PEQCs indenter (a sphere of radius R) slides on the 1D-hexagonal PEQC layer along the x direction. The 1D-hexagonal PEQCs layer with finite thickness of h was deposited on the rigid grounding substrate. The normal force P_z , tangential force P_x , normal phason force H and electric charge Q are applied on the indenter, which lead to pressure p_z , shear traction p_x , phason force h_z and electric charge density q_z at the contact interface S (contact radius a). Obviously,

$$\int_S p_z dxdy = P_z, \int_S p_x dxdy = P_x, \int_S h_z dxdy = H, \int_S q_z dxdy = Q$$

Since there is no experimental study on how to apply the phason force on the QCs surface, but there are some artificial methods that can cause the disorder of QCs phase field [33,34]. To show the responses under the phason force from the perspective of theoretical study, we assume the indenter can induce not only the phason force but also the phason displacement, and the applied phason force is equivalent inside the contact region. Furthermore, in the sliding contact, friction follows the Coulomb friction law, so that $p_x = \mu p_z$, with μ for friction coefficient. The boundary conditions at the upper surface of 1D-hexagonal PEQCs layer can be written as [28]

$$\begin{aligned} \sigma_{zx} \big|_{z=0} &= -p_x = -\mu p_z, (x,y) \in S \\ \sigma_{zx} \big|_{z=0} &= 0, (x,y) \notin S \\ \sigma_{zy} \big|_{z=0} &= 0 \\ \sigma_{zz} \big|_{z=0} &= \begin{cases} -p_z, (x,y) \in S \\ 0, (x,y) \notin S \end{cases} \end{aligned} \quad (8)$$

$$H_{zz} \Big|_{z=0} = \begin{cases} -h_z, & (x,y) \in S \\ 0, & (x,y) \notin S \end{cases}$$

$$D_z \Big|_{z=0} = \begin{cases} -q_z, & (x,y) \in S \\ 0, & (x,y) \notin S \end{cases}$$

The lower surface of 1D-hexagonal PEQCs layer is perfectly bonded to a rigid grounded substrate, so that the boundary conditions at the lower surface are

$$u_x \Big|_{z=h} = 0, u_y \Big|_{z=h} = 0, u_z \Big|_{z=h} = 0, w_z \Big|_{z=h} = 0, \phi \Big|_{z=h} = 0 \quad (9)$$

In order to determine the potential functions ψ_i , the following direct double Fourier transforms and its inverse are

$$\begin{aligned} \hat{f}(m,n,z) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y,z) e^{-j(mx+ny)} dx dy \\ f(x,y,z) &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{f}(m,n,z) e^{j(mx+ny)} dm dn \end{aligned} \quad (10)$$

where $j = \sqrt{-1}$ without specification elsewhere; Hat “ $\hat{\cdot}$ ” denotes the double Fourier transform; m and n represent the transform variables with respect to the x and y directions, respectively.

Applying Fourier transforms to Eq. (6) results in the following exponential form of potential functions ψ_i

$$\hat{\psi}_i(m,n,z) = A_i(m,n) e^{-\alpha z_i} + \bar{A}_i(m,n) e^{\alpha z_i} \quad (i = 0, 1, 2, 3, 4) \quad (11)$$

where $\alpha = \sqrt{m^2 + n^2}$; $A_i(m,n)$ and $\bar{A}_i(m,n)$ are ten unknowns to be determined from the boundary conditions.

Applying Fourier transforms to Eqs. (5) and (7), combined with Eq. (11), the transformed displacements and stresses of phonon and phason field, electric potential and electric displacement expressed as

$$\begin{aligned} \hat{u}_x &= jnA_0 e^{-\alpha z_0} + jn\bar{A}_0 e^{\alpha z_0} - \sum_{i=1}^4 jmA_i e^{-\alpha z_i} - \sum_{i=1}^4 jm\bar{A}_i e^{\alpha z_i} \\ \hat{u}_y &= -jmA_0 e^{-\alpha z_0} - jm\bar{A}_0 e^{\alpha z_0} - \sum_{i=1}^4 jnA_i e^{-\alpha z_i} - \sum_{i=1}^4 jn\bar{A}_i e^{\alpha z_i} \\ \hat{u}_z &= -\sum_{i=1}^4 \alpha_{1i} \alpha A_i e^{-\alpha z_i} + \sum_{i=1}^4 \alpha_{1i} \alpha \bar{A}_i e^{\alpha z_i} \\ \hat{w}_z &= -\sum_{i=1}^4 \alpha_{2i} \alpha A_i e^{-\alpha z_i} + \sum_{i=1}^4 \alpha_{2i} \alpha \bar{A}_i e^{\alpha z_i} \\ \hat{\phi} &= -\sum_{i=1}^4 \alpha_{3i} \alpha A_i e^{-\alpha z_i} + \sum_{i=1}^4 \alpha_{3i} \alpha \bar{A}_i e^{\alpha z_i} \end{aligned}$$

$$\begin{aligned}
\mathcal{E}_{xx} &= 2C_{66}mnA_0e^{-\alpha z_0} + \sum_{i=1}^4 (\gamma_{0i}\alpha^2 + 2C_{66}n^2)A_i e^{-\alpha z_i} + 2C_{66}mn\bar{A}_0e^{\alpha z_0} + \sum_{i=1}^4 (\gamma_{0i}\alpha^2 + 2C_{66}n^2)\bar{A}_i e^{\alpha z_i} \\
\mathcal{E}_{yy} &= -2C_{66}mnA_0e^{-\alpha z_0} + \sum_{i=1}^4 (\gamma_{0i}\alpha^2 + 2C_{66}m^2)A_i e^{-\alpha z_i} - 2C_{66}mn\bar{A}_0e^{\alpha z_0} + \sum_{i=1}^4 (\gamma_{0i}\alpha^2 + 2C_{66}m^2)\bar{A}_i e^{\alpha z_i} \\
\mathcal{E}_{zz} &= \sum_{i=1}^4 \gamma_{1i}\alpha^2 A_i e^{-\alpha z_i} + \sum_{i=1}^4 \gamma_{1i}\alpha^2 \bar{A}_i e^{\alpha z_i} \\
\mathcal{E}_{xy} &= C_{66}(n^2 - m^2)A_0e^{-\alpha z_0} - \sum_{i=1}^4 2C_{66}mnA_i e^{-\alpha z_i} + C_{66}(n^2 - m^2)\bar{A}_0e^{\alpha z_0} - \sum_{i=1}^4 2C_{66}mn\bar{A}_i e^{\alpha z_i} \\
\mathcal{E}_{zx} &= -jns_0\rho_1\alpha A_0e^{-\alpha z_0} + \sum_{i=1}^4 jm\lambda_{1i}\alpha A_i e^{-\alpha z_i} + jns_0\rho_1\alpha \bar{A}_0e^{\alpha z_0} - \sum_{i=1}^4 jm\lambda_{1i}\alpha \bar{A}_i e^{\alpha z_i} \\
\mathcal{E}_{zy} &= jms_0\rho_1\alpha A_0e^{-\alpha z_0} + \sum_{i=1}^4 jn\lambda_{1i}\alpha A_i e^{-\alpha z_i} - jms_0\rho_1\alpha \bar{A}_0e^{\alpha z_0} - \sum_{i=1}^4 jn\lambda_{1i}\alpha \bar{A}_i e^{\alpha z_i} \\
\mathcal{H}_{zx} &= -jns_0\rho_2\alpha A_0e^{-\alpha z_0} + \sum_{i=1}^4 jm\lambda_{2i}\alpha A_i e^{-\alpha z_i} + jns_0\rho_2\alpha \bar{A}_0e^{\alpha z_0} - \sum_{i=1}^4 jm\lambda_{2i}\alpha \bar{A}_i e^{\alpha z_i} \\
\mathcal{H}_{zy} &= jms_0\rho_2\alpha A_0e^{-\alpha z_0} + \sum_{i=1}^4 jn\lambda_{2i}\alpha A_i e^{-\alpha z_i} - jms_0\rho_2\alpha \bar{A}_0e^{\alpha z_0} - \sum_{i=1}^4 jn\lambda_{2i}\alpha \bar{A}_i e^{\alpha z_i} \\
\mathcal{H}_{zz} &= \sum_{i=1}^4 \gamma_{2i}\alpha^2 A_i e^{-\alpha z_i} + \sum_{i=1}^4 \gamma_{2i}\alpha^2 \bar{A}_i e^{\alpha z_i} \\
\mathcal{B}_x &= -jns_0\rho_3\alpha A_0e^{-\alpha z_0} + \sum_{i=1}^4 jm\lambda_{3i}\alpha A_i e^{-\alpha z_i} + jns_0\rho_3\alpha \bar{A}_0e^{\alpha z_0} - \sum_{i=1}^4 jm\lambda_{3i}\alpha \bar{A}_i e^{\alpha z_i} \\
\mathcal{B}_y &= jms_0\rho_3\alpha A_0e^{-\alpha z_0} + \sum_{i=1}^4 jn\lambda_{3i}\alpha A_i e^{-\alpha z_i} - jms_0\rho_3\alpha \bar{A}_0e^{\alpha z_0} - \sum_{i=1}^4 jn\lambda_{3i}\alpha \bar{A}_i e^{\alpha z_i} \\
\mathcal{B}_z &= \sum_{i=1}^4 \gamma_{3i}\alpha^2 A_i e^{-\alpha z_i} + \sum_{i=1}^4 \gamma_{3i}\alpha^2 \bar{A}_i e^{\alpha z_i}
\end{aligned} \tag{12}$$

Applying Fourier transforms to Eqs. (8) and (9), we have

$$\begin{aligned}
ns_0\rho_1(A_0 - \bar{A}_0) - m\lambda_{11}(A_1 - \bar{A}_1) - m\lambda_{12}(A_2 - \bar{A}_2) - m\lambda_{13}(A_3 - \bar{A}_3) - m\lambda_{14}(A_4 - \bar{A}_4) &= -j\alpha^{-1}\mu\mathcal{E}_z \\
ms_0\rho_1(A_0 - \bar{A}_0) + n\lambda_{11}(A_1 - \bar{A}_1) + n\lambda_{12}(A_2 - \bar{A}_2) + n\lambda_{13}(A_3 - \bar{A}_3) + n\lambda_{14}(A_4 - \bar{A}_4) &= 0 \\
\gamma_{11}(A_1 + \bar{A}_1) + \gamma_{12}(A_2 + \bar{A}_2) + \gamma_{13}(A_3 + \bar{A}_3) + \gamma_{14}(A_4 + \bar{A}_4) &= -\alpha^{-2}\mathcal{E}_z \\
\gamma_{21}(A_1 + \bar{A}_1) + \gamma_{22}(A_2 + \bar{A}_2) + \gamma_{23}(A_3 + \bar{A}_3) + \gamma_{24}(A_4 + \bar{A}_4) &= -\alpha^{-2}\mathcal{H}_z \\
\gamma_{31}(A_1 + \bar{A}_1) + \gamma_{32}(A_2 + \bar{A}_2) + \gamma_{33}(A_3 + \bar{A}_3) + \gamma_{34}(A_4 + \bar{A}_4) &= -\alpha^{-2}\mathcal{B}_z \\
nA_0e^{-\alpha s_0h} + n\bar{A}_0e^{\alpha s_0h} - \sum_{i=1}^4 mA_i e^{-\alpha s_ih} - \sum_{i=1}^4 m\bar{A}_i e^{\alpha s_ih} &= 0 \\
mA_0e^{-\alpha s_0h} + m\bar{A}_0e^{\alpha s_0h} + \sum_{i=1}^4 nA_i e^{-\alpha s_ih} + \sum_{i=1}^4 n\bar{A}_i e^{\alpha s_ih} &= 0 \\
\sum_{i=1}^4 \alpha_{1i}A_i e^{-\alpha s_ih} - \sum_{i=1}^4 \alpha_{1i}\bar{A}_i e^{\alpha s_ih} &= 0
\end{aligned} \tag{13}$$

$$\begin{aligned}\sum_{i=1}^4 \alpha_{2i} A_i e^{-\alpha_{s_i} h} - \sum_{i=1}^4 \alpha_{2i} \bar{A}_i e^{\alpha_{s_i} h} &= 0 \\ \sum_{i=1}^4 \alpha_{3i} A_i e^{-\alpha_{s_i} h} - \sum_{i=1}^4 \alpha_{3i} \bar{A}_i e^{\alpha_{s_i} h} &= 0\end{aligned}$$

The following results can be obtained by solving the above equations

$$\begin{aligned}A_0 &= jn\mu\bar{p}_z / [\alpha^3 s_0 \rho_1 (1 + e^{-2\alpha s_0 h})], \quad \bar{A}_0 = -jn\mu\bar{p}_z / [\alpha^3 s_0 \rho_1 (1 + e^{2\alpha s_0 h})] \\ A_1 &= (-jm\mu\bar{p}_z / \alpha^3 + \bar{q}_z^{(3)} t_{04}^{(3)} / (t_{34}^{(3)} \alpha^2) + q_1^{(4)} t_{03}^{(4)} / t_{23}^{(4)} + p_1^{(5)} t_{02}^{(5)} / t_{12}^{(5)}) / t_{01}^{(6)} \\ A_2 &= -(t_{11}^{(5)} A_1 + p_1) / t_{12}^{(5)}, \quad A_3 = -(\sum_{i=1}^2 t_{2i}^{(4)} A_i + q_1) / t_{23}^{(4)}, \quad A_4 = -(\sum_{i=1}^3 t_{3i}^{(3)} A_i + \bar{q}_z \alpha^{-2}) / t_{34}^{(3)} \\ \bar{A}_1 &= -\sum_{i=1}^4 A_i e^{-\alpha(s_i + s_1)h} - \sum_{i=2}^4 \bar{A}_i e^{\alpha(s_i - s_1)h}, \quad \bar{A}_2 = -\sum_{i=1}^4 a_{1i}^{(2)} A_i / \bar{a}_{11}^{(2)} \\ \bar{A}_3 &= -(\sum_{i=1}^4 a_{2i}^{(1)} A_i + \bar{a}_{21}^{(1)} \bar{A}_2) / \bar{a}_{22}^{(1)}, \quad \bar{A}_4 = -(\sum_{i=1}^4 a_{3i} A_i + \sum_{i=1}^2 \bar{a}_{3i} \bar{A}_{i+1}) / \bar{a}_{33}\end{aligned} \quad (14)$$

in which $p_1, q_1, t_{ki}^{(n)}, a_{ki}^{(n)}$ and $\bar{t}_{ki}^{(n)}, \bar{a}_{ki}^{(n)}$ are given in Appendix B.

Accordingly, the Fourier transform solutions for the corresponding displacements and stresses of phonon and phason field, electric potential and electric displacement are established by substituting the coefficients $\{A_0, A_1, A_2, A_3, A_4, \bar{A}_0, \bar{A}_1, \bar{A}_2, \bar{A}_3, \bar{A}_4\}$ into Eq. (12).

The transformed solution $\bar{h}_z(m, n, z)$ of the layers is function of the transformed pressure $\bar{p}_z$, phason force $\bar{h}_z$, and electric charge density $\bar{q}_z$, which can be further expressed as

$$\bar{h}_z(m, n, z) = \bar{G}_{p_z}^z(m, n, z) \bar{p}_z(m, n) + \bar{G}_{h_z}^z(m, n, z) \bar{h}_z(m, n) + \bar{G}_{q_z}^z(m, n, z) \bar{q}_z(m, n) \quad (15)$$

where $\bar{G}_q^z$ denotes the frequency response functions (FRFs) response of p excited by load q . Take surface normal displacement u_z as an example, $\bar{G}_{h_z}^z$ represents the FRFs of u_z caused by q_z .

The transformed solution is analytically expressed in closed forms in the Fourier-frequency domain, as shown in Eq. (15), which should be further integrated to be converted into the space domain from the Fourier-frequency domain. Applying the inverse Fourier transforms to Eq. (15) with respect to the frequency numbers m and n , lead to

$$u_z(x, y, z) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\bar{G}_{p_z}^z \bar{p}_z + \bar{G}_{h_z}^z \bar{h}_z + \bar{G}_{q_z}^z \bar{q}_z) e^{j(mx+ny)} dm dn \quad (16)$$

It is difficult to solve the complicated integration in Eq. (16) analytically; however, this solution is for the space-domain convolutions between surface excitations and mechanical responses. Therefore, DC-FFT is employed to solve the convolutions in the present contact problems.

2.4 DC-FFT algorithm

In the Fourier domain, the FRFs represents the material response of the Dirac delta functions (a generalized unit point load at the coordinate origin). Therefore, by letting $\hat{p}_z=1(\hat{h}_z=0, \hat{q}_z=0)$, $\hat{h}_z=1(\hat{p}_z=0, \hat{q}_z=0)$, or $\hat{q}_z=1(\hat{p}_z=0, \hat{h}_z=0)$ respectively in (12), the components of the FRFs can be obtained analytically as

$$\begin{bmatrix} \hat{G}_{zz} \\ \hat{G}_{vz} \\ \hat{G}_{\phi} \end{bmatrix} = \begin{bmatrix} \hat{G}_{zz}^{\hat{p}_z=1} & \hat{G}_{zz}^{\hat{h}_z=1} & \hat{G}_{zz}^{\hat{q}_z=1} \\ \hat{G}_{vz}^{\hat{p}_z=1} & \hat{G}_{vz}^{\hat{h}_z=1} & \hat{G}_{vz}^{\hat{q}_z=1} \\ \hat{G}_{\phi}^{\hat{p}_z=1} & \hat{G}_{\phi}^{\hat{h}_z=1} & \hat{G}_{\phi}^{\hat{q}_z=1} \end{bmatrix}, \begin{bmatrix} \hat{G}_{zz} \\ \hat{G}_{vz} \end{bmatrix} = \begin{bmatrix} \hat{G}_{zz}^{\hat{p}_z=1} & \hat{G}_{zz}^{\hat{h}_z=1} & \hat{G}_{zz}^{\hat{q}_z=1} \\ \hat{G}_{vz}^{\hat{p}_z=1} & \hat{G}_{vz}^{\hat{h}_z=1} & \hat{G}_{vz}^{\hat{q}_z=1} \end{bmatrix} \quad (17)$$

Once the FRFs are given, the components of continuous Fourier transforms of influence coefficients (ICs) can be solved by multiplying the FRFs and the shape functions of Fourier transforms [26]

$$\begin{bmatrix} \hat{K}_{zz} & \hat{K}_{vz} & \hat{K}_{\phi} & \hat{K}_{zz} & \hat{K}_{vz} & \hat{K}_{\phi} \end{bmatrix}^T = \begin{bmatrix} \hat{G}_{zz} & \hat{G}_{vz} & \hat{G}_{\phi} & \hat{G}_{zz} & \hat{G}_{vz} & \hat{G}_{\phi} \end{bmatrix}^T Y(m, n) \quad (18)$$

in which Y is the shape function given as follows

$$Y(x, y) = \begin{cases} 1 & |x| \leq \Delta x/2, |y| \leq \Delta y/2 \\ 0 & \text{other} \end{cases} \quad (19)$$

$$\hat{Y}(m, n) = \frac{4 \sin(m\Delta x/2) \sin(n\Delta y/2)}{mn}$$

The discrete Fourier transforms of ICs are calculated from continuous ICs with the aliasing treatment [35,36]

$$\hat{K} = \frac{1}{\Delta x \Delta y} \sum_{r_x=-AL}^{r_x=AL} \sum_{r_y=-AL}^{r_y=AL} \hat{K} \left(\frac{2\pi}{M_e \Delta x} k - \frac{2\pi}{\Delta x} r_x, \frac{2\pi}{N_e \Delta y} l - \frac{2\pi}{\Delta y} r_y \right) \quad (20)$$

$$(-M_e/2 \leq k \leq M_e/2, -N_e/2 \leq l \leq N_e/2)$$

where Δx and Δy are the mesh sizes along the x and y directions, respectively; AL represents the level of the aliasing control; $M_e = 2^\gamma M$ and $N_e = 2^\gamma N$ denote the refinement mesh numbers of the original mesh numbers M and N ; $\hat{K}$ is the discrete Fourier transform form of ICs. Then, the DC-FFT algorithm is implemented to process pressure, phason force, electric charge by means of the corresponding components of the ICs. The normal displacements and stresses of phonon and phason, electric potential and electric displacement are given by the following inverse fast Fourier transform (IFFT), respectively.

$$u_z = IFFT(\hat{K}^{u_z} \cdot \begin{bmatrix} \hat{p}_z & \hat{h}_z & \hat{q}_z \end{bmatrix}^T) = IFFT(\hat{K}_{p_z}^{u_z} \hat{p}_z + \hat{K}_{h_z}^{u_z} \hat{h}_z + \hat{K}_{q_z}^{u_z} \hat{q}_z)$$

$$w_z = IFFT(\hat{K}^{w_z} \cdot \begin{bmatrix} \hat{p}_z & \hat{h}_z & \hat{q}_z \end{bmatrix}^T) = IFFT(\hat{K}_{p_z}^{w_z} \hat{p}_z + \hat{K}_{h_z}^{w_z} \hat{h}_z + \hat{K}_{q_z}^{w_z} \hat{q}_z)$$

$$\begin{aligned}
\phi &= IFFT(\hat{K}^\phi \cdot [\hat{p}_z \quad \hat{h}_z \quad \hat{q}_z]^T) = IFFT(\hat{K}_{p_z}^\phi \hat{p}_z + \hat{K}_{h_z}^\phi \hat{h}_z + \hat{K}_{q_z}^\phi \hat{q}_z) \\
\sigma_{zz} &= IFFT(\hat{K}^{\sigma_{zz}} \cdot [\hat{p}_z \quad \hat{h}_z \quad \hat{q}_z]^T) = IFFT(\hat{K}_{p_z}^{\sigma_{zz}} \hat{p}_z + \hat{K}_{h_z}^{\sigma_{zz}} \hat{h}_z + \hat{K}_{q_z}^{\sigma_{zz}} \hat{q}_z) \\
H_z &= IFFT(\hat{K}^{H_z} \cdot [\hat{p}_z \quad \hat{h}_z \quad \hat{q}_z]^T) = IFFT(\hat{K}_{p_z}^{H_z} \hat{p}_z + \hat{K}_{h_z}^{H_z} \hat{h}_z + \hat{K}_{q_z}^{H_z} \hat{q}_z) \\
D_z &= IFFT(\hat{K}^{D_z} \cdot [\hat{p}_z \quad \hat{h}_z \quad \hat{q}_z]^T) = IFFT(\hat{K}_{p_z}^{D_z} \hat{p}_z + \hat{K}_{h_z}^{D_z} \hat{h}_z + \hat{K}_{q_z}^{D_z} \hat{q}_z)
\end{aligned} \tag{21}$$

The above formulation shows the DC-FFT algorithm for the 1D-hexagonal PEQCs layer, which can also be applied to the indenter by letting the layer thickness to infinite.

2.5 Contact conditions and numerical modeling

Now the contact problem of the indenter sliding on the 1D-hexagonal PEQCs layer can be solved, as shown in Fig. 1. The mechanical loadings at the contact interface S can be represented by pressure p_z , shear traction p_x with $p_x = \mu p_z$, phason force h_z and electric charge density q_z . Therefore, the following contact conditions apply to such a contact problem

$$\begin{aligned}
\bar{u}_{z1} + \bar{u}_{z2} &= g - h_0 \\
p_z > 0 \& \ g = 0 \Rightarrow \forall (x, y) \in S \\
p_z = 0 \& \ g > 0 \Rightarrow \forall (x, y) \notin S
\end{aligned} \tag{22}$$

where $\bar{u}_{z1}$ and $\bar{u}_{z2}$ are the surface normal displacement for the indenter and layer, respectively; g is the gap between the indenter and layer; h_0 is the profile of indenter.

Pressure p_z in Eq. (22) can be solved with the conjugate gradient method (CGM) described in Ref. [25].

3 Results and discussion

In the numerical process for solving the problem shown in Fig. 1, the fixed computational domain of $4a_0 \times 4a_0 \times h$ with $256 \times 256 \times 64$ grids in the x, y and z directions is employed, where a_0 is the maximum contact radius, which will be discussed later. The material properties are given in Table 1.

Table 1. The material constants of 1D-hexagonal PEQCs [14]. C_{ij}, K_i and R_i are in GPa, e_{ij} and e'_{ij} are in Cm^{-2} and ξ_{ij} are in $10^{-12} \text{C}^2 \text{N}^{-1} \text{m}^{-2}$

C_{11}	C_{12}	C_{13}	C_{33}	C_{44}	C_{66}
150	100	90	130	50	25
R_1	R_2	R_3	K_1	K_2	e_{31}
-1.5	1.2	1.2	0.18	0.30	-0.16
e_{33}	e_{15}	ξ_{11}	ξ_{33}	e'_{15}	e'_{33}

0.347	-0.138	82.6	90.3	-0.16	0.35
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3.1 Model verification

The correctness of the model is verified by comparison the 1D hexagonal PEQCs layer contact solutions of $h \rightarrow \infty$, $e_{ij}, e'_{ij}, \xi_{ij} \rightarrow 0$ with that of Wu et al. [18]. For the problem of 1D hexagonal PEQCs layer indented by a rigid insulating sphere with radius $R=1\text{mm}$ under $P_z=12\text{N}$, $h=10a_0$, $e_{ij}, e'_{ij}, \xi_{ij} = 10^{-30}$, the subsurface phonon normal stress, shear stress and phason stress are identical to those obtained by Wu et al. [18], as shown in Figs. 2(a, b, c).

Fig. 2. The contour of subsurface phonon normal stress $\sigma_{zz}(r, 0, z)$ (a), shear stress $\sigma_{zr}(r, 0, z)$ (b) and phason stress $H_z(r, 0, z)$ (c) distribution

Fig. 3. The surface pressure $p_z(x, 0, z)/p_0$ and electric potential $\phi(x, 0, z)/\phi_0$ distribution

The model is further verified through degenerating our solutions from “PEQCs layer” to “PE half space” by taking “ $h \rightarrow \infty, K_i, R_i \rightarrow 0$ ”. For the problem of the PZT-4 half space indented by a rigid insulating sphere with radius $R=1\text{mm}$ under normal force $P_z=12$, maximum contact radius $a_0=0.9086\sqrt[3]{cP_zR}$, contact pressure $p_0=P_z/(\pi a_0^2)$, electric potential $\phi_0=P_z \cdot 10^{-2}$, according to Ref. [37], we can easy to calculate $c=4.06 \times 10^{-12}$, the surface normal stress and electric potential are identical to those obtained by Ding et al. [37], as shown in Fig. 3.

Fig. 4. The indentation depth d and indentation load P_z, P_x, H, Q curves for 1D hexagonal PEQCs layer $h = 10a_0$ (a) and $h = a_0, R = 1\mu\text{m}$ (b)

The relationship between indentation depth $d = u_z(0, 0, z)$ and indentation load P_z, P_x, H, Q is shown as in Fig. 4, where radius $R = 1\mu\text{m}$ (or $200\mu\text{m}$), friction coefficient $\mu=0$ (or 0.5), layer thickness $h = a_0$ (or $10a_0$), normal force P_z ranging from 0 to 10mN , phason force $H=0$ (or 2mN) and electric charge $Q=0$ (or 1mC). It can be seen that in order to achieve the same indentation depth, the sphere with larger radius needs more force. The 1D hexagonal PEQCs layer $h = a_0$ needs more force than $h = 10a_0$. The indentation force of rigid ball is larger than that of the corresponding 1D hexagonal PQCS ball. Fig. 4 also reveals that increasing phason force have a negligible influence on the indentation depth. For 1D-hexagonal PEQCs layer $h = 10a_0$, increasing friction coefficient μ and electric charge Q will increase the indentation depth. However, the opposite tendency is true for 1D-hexagonal PEQCs layer $h = a_0$. In addition, when both the indenter and layer are 1D hexagonal PEQCs, the friction force has a greater influence on the indentation depth. These curves are in accordance with the law of pressure variation.

3.2 Effect of parameters

The problem of the sliding contact between the 1D-hexagonal PEQCs layer and the 1D-hexagonal PEQCs sphere is investigated, where the applied force $P_z = 0.2\text{mN}$, phason force $H = 2\text{mN}$, sphere radius $R = 1\mu\text{m}$, layer thickness $h = 0.5a_0$ (or $1.0a_0$ or $2.0a_0$), the friction coefficient $\mu=0$ (or 0.3 or 0.5 or 0.8) and electric charge $Q=0$ (or 1mC or 4mC or 8mC). For the convenience of display, the numerical results are normalized by the referenced parameters of maximum contact pressure p_0 , contact radius a_0 , phason displacement w_0 and electric potential ϕ_0 . For 1D-hexagonal PEQCs half

space, according to Ref. [38], we can calculate the maximum indentation depth $\delta_0 = a_0^2/R$, $a_0 = \sqrt[3]{3\pi\xi_{11}RP_z/4}$, $p_0 = 2a_0/(\pi^2 \xi_{11}R)$, $w_0 = \xi_{12}\delta_0/\xi_{11}$, $\phi_0 = \xi_{13}\delta_0/\xi_{11}$, $\xi_{11} = 3.3779 \times 10^{-12} \text{ m}^2/\text{N}$, $\xi_{12} = 1.1381 \times 10^{-11} \text{ m}^2/\text{N}$ and $\xi_{13} = 2.4084 \times 10^{-3} \text{ m}^2/\text{C}$.

Fig. 5. The pressure $p_z(x, 0, 0)/p_0$ distribution under different loading conditions

Fig. 6. The contour of surface pressure $p_z(x, y, 0)/p_0$ (a) and subsurface phonon stress $\sigma_{zz}(x, 0, z)/p_0$ (b) distribution under $\mu=0.5, Q=1\text{mC}, h=1.0a_0$

Fig. 7. The phason displacement $w_z(x, 0, 0)/w_0$ distribution under different loading conditions

Fig.8. The contour of surface phason displacement $w_z(x, y, 0)/w_0$ (a) and subsurface $w_z(x, 0, z)/w_0$ (b) distribution under $\mu=0.5, Q=1\text{mC}, h=1.0a_0$

Fig. 9. The electric potential $\phi(x, 0, 0)/\phi_0$ distribution under different loading conditions

Fig. 10. The contour of surface electric potential $\phi(x, y, 0)/\phi_0$ (a) and subsurface $\phi(x, 0, z)/\phi_0$ (b) distribution under $\mu = 0.5, Q = 1\text{mC}, h = 1.0a_0$

Figs. 5-10 show the dimensionless surface (subsurface) distributions for the pressure, phonon stress, phason displacement and electric potential, respectively. It can be found that friction coefficient

does not affect the contact area, but the shape of the pressure distribution is changed. The maximum values of pressure, phonon stress, phason displacement and electric potential are shifted away from the sliding direction. In addition, when the electric charge Q is larger, then the pressure and electric potential become larger, and the phason displacement becomes smaller. The increase of Q value leads to larger value of the maximum pressure and electric potential, smaller value of the maximum phason displacement. Which reveals that the contact interface appears “harder” when a larger electric charge Q is applied to the material. The peak phason displacement increases with an increasing of layer thickness h . A thicker layer leads to a smaller peak value of the contact pressure and electric potential. The increase of layer thickness h pushes the peak pressure, electric potential, and phason displacement closer to the peak value in half space case. Moreover, the electric potential and phason displacement extend out of the contact region and do not decrease to zero even at the edges of the contact regions of both surfaces. The surface pressure, electric potential and phason displacement decrease with an increasing of the distance far away from the contact surface.

(a) $Q = 1\text{mC}$, $h = 1.0a_0$

(b) $\mu = 0.5$, $h = 1.0a_0$

(c) $\mu = 0.5$, $Q = 1\text{mC}$

(d)

Fig. 11. The phason stress $H_z(0,0,z)/p_0$ distribution under different loading conditions (a, b, c) and the contour of subsurface phason stress $H_z(x,0,z)/p_0$ distribution under $\mu=0.5, Q=1\text{mC}, h=1.0a_0$ (d)

Fig. 12. The electric displacement $D_z(0,0,z)$ distribution under different loading conditions (a, b, c) and the contour of subsurface electric displacement $D_z(x,0,z)$ distribution under $\mu=0.5, Q=1\text{mC}, h=1.0a_0$ (d)

Fig. 13. The von Mises stress $\sigma_s(0,0,z)/(\sqrt{3}p_0)$ distribution under different loading conditions (a, b, c) and the contour of subsurface von Mises stress $\sigma_s(x,0,z)/(\sqrt{3}p_0)$ distribution under $\mu=0.5, Q=1\text{mC}, h=1.0a_0$ (d)

Figs. 11 and 12 plot the diagrams of the subsurface phason stresses and electric displacements under different loads. The larger friction coefficient μ is, the smaller subsurface phason stresses are, and the greater electric displacements are. Besides, with the increase of Q value, the phason stresses increase gradually, and electric displacements decrease. In general, subsurface phason stresses and electric displacements decrease with the increase of layer thickness h . Meanwhile, the position of maximum phason stresses and electric displacements deviates slightly from the contact surface. It can be seen from Fig. 13 that the von Mises stresses increase with an increasing of the friction coefficient μ . The increase of μ causes the location of the maximum von Mises stress to move towards the contact surface. It is clear that friction has significant effect on the von Mises stress, but influence of electric charge Q can be negligible. In addition, the von Mises stresses becomes smaller with an increasing of h . The stresses show obvious distortion due to friction. This distortion is somewhat alleviated by an increasing of layer thickness h .

Fig. 14. Effect of normalized layer thickness h/a_0 on the maximum dimensionless u_z/a_0 , p_z/p_0 , w_z/w_0 , ϕ/ϕ_0 and $\sigma_s/\sqrt{3} p_0$ distribution under $\mu=0.5$, $Q=1\text{mC}$

Fig. 14 reveals that the contact behaviors affected by layer thickness h/a_0 . For sufficiently large h/a_0 , the maximum dimensionless u_z/a_0 , p_z/p_0 , and $\sigma_s/\sqrt{3} p_0$ do not change their value along with the increasing layer thickness, and the layer problem becomes a half-space problem. However, w_z/w_0 and ϕ/ϕ_0 approach certain asymptotic values for larger h/a_0 , the phason and electric responses penetrate far deep from the contact surface, therefore, the influence of substrate can not be neglected.

The sensitivity function suggested by Makagon et al. [39] was employed to evaluate the coupling effects of material properties on contact behavior V . The dimensionless sensitivities below are

$$\text{numerically calculated with step length } 0.01f^0 : S(f) = \frac{V \Big|_{f=1.01f^0} - V \Big|_{f=0.99f^0}}{0.02V \Big|_{f=f^0}}, \text{ where } f \text{ is a selected}$$

material constant with f^0 being its reference value.

Fig. 15. Sensitivities of contact behaviors, including u_z , p_z , w_z and ϕ , to 17 independent material

parameters under $\mu=0.5, Q=1\text{mC}, h=1.0a_0$

Fig. 15 reveals the strong influence of the elastic stiffnesses $C_{11}, C_{13}, C_{33}, C_{44}$ on u_z, p_z, w_z and ϕ , while the influence of C_{12} is weak. The phonon-phason coupling elastic constants R_i , phason elastic stiffness constants K_i , dielectric coefficient ξ_{33} , piezoelectric coefficients e_{33} and e'_{ij} have the great influence on w_z and ϕ , while e_{31}, e_{15} and ξ_{11} have little effect. R_i promote the increase of w_z and ϕ , while K_i are the opposite trend. The above results are consistent with the physical nature of the problem.

3.3 Model extension

Now, we extend the model to the contact behavior involving indenters of different shapes. The sliding contact between the 1D-hexagonal PEQCs layer with an ellipsoidal 1D-hexagonal PEQC punch is investigated, where the applied force $P_z = 0.2\text{mN}$, major and minor radii $R_x = 2\mu\text{m}$ and $R_y = 1\mu\text{m}$, layer thickness $h = 2a_0$, the friction coefficient $\mu=0.5$, phason force $H = 2\text{mN}$ and electric charge $Q = 1\text{mC}$. From Figs. 16-19, we draw the following conclusions:

Fig. 16. The contour of contact surface pressure $p_z(x, y, 0)/p_0$ (a) and subsurface phonon stress $\sigma_{zz}(x, 0, z)/p_0$ (b) distribution

Fig. 17. The contour of surface phason displacement $w_z(x, y, 0)/w_0$ (a) and subsurface $w_z(x, 0, z)/w_0$ (b) distribution

Fig. 18. The contour of surface electric potential $\phi(x, y, 0)/\phi_0$ (a) and subsurface $\phi(x, 0, z)/\phi_0$ (b) distribution

Fig. 19. The contour of subsurface von Mises stresses $\sigma_s(x, 0, z)/(\sqrt{3}p_0)$ (a), electric displacement $D_z(x, 0, z)$ (b) and phason stress $H_z(x, 0, z)/p_0$ (c) distribution

Compared with the spherical indenter, it is more difficult for the ellipsoidal indenter to press in 1D-hexagonal PEQCs layer. The location of the maximum contact pressure, phonon stress, phason displacement, electric potential, von Mises stresses, phason stress and electric displacement is closer to the surface. The electric charge, friction coefficient and layer thickness still strongly influence contact behavior. For example, the value of von Mises stress decreases with the increase of layer thickness h . The location of the maximum von Mises stress is somewhat affected by the surface friction.

4 Conclusions

In this paper, the frictional contact problem of 1D-hexagonal PEQCs layer has been solved. The 3D general solutions for the problem of 1D-hexagonal PEQCs are obtained by using rigorous operator theory and Almansi theorem. The first analytical FRFs of displacements and stresses of phonon and phason, electric potentials and electric displacements are obtained by applying double Fourier integral transforms. The influences of material properties, surface pressure, phason force, electric charge and friction are numerically investigated by DC-FFT.

The major advantage of the current algorithm is high calculation efficiency, which can solve complicated contact problems in a few minutes. Because the influence coefficients are independent of indenter shapes, the proposed semi-analytical model can be conveniently implemented to various indenters, such as flat-ended rectangular, circular, and elliptical punches, spherical and ellipsoidal indenters, cylinders of a finite length, multi-connection contacts and multi-domain contacts. It is not only convenient for surface geometry treatment, but also can deal with complicated surface topographic features such as rough surfaces. Therefore, the model has a wide application prospect in the

design of multi-functional engineering components.

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Appendix A

The constants a, b, c, d and e involved in Eqs. (5,6) are listed as follows:

$$\begin{aligned}
 a &= C_{44}[\xi_{33}(R_2^2 - C_{33}K_1) - C_{33}e_{33}'^2 - K_1e_{33}'^2 + 2R_2e_{33}e_{33}'] \\
 b &= \xi_{11}C_{44}(R_2^2 - C_{33}K_1) - \xi_{33}[C_{11}C_{33}K_1 + C_{11}R_2^2 + C_{13}^2K_1 + 2C_{13}C_{44}K_1 - 2C_{13}R_2(R_1 + R_3) - C_{33}C_{44}K_2 \\
 &\quad + C_{33}(R_1 + R_3)^2 - 2C_{44}R_1R_2] - C_{11}(C_{33}e_{33}'^2 + R_1e_{33}'^2 - 2R_2e_{33}e_{33}') + C_{13}e_{33}'^2(C_{13} + 2C_{44}) + 2C_{13}K_1e_{33}(e_{15} \\
 &\quad + e_{31}) - 2C_{13}e_{33}'[(R_1 + R_3)e_{33} + R_2(e_{15} + e_{31})] - C_{33}K_1(e_{15} + e_{31})^2 + 2C_{33}e_{33}'[(R_1 + R_3)(e_{15} + e_{31}) - C_{44}e_{15}'] \\
 &\quad - C_{44}K_2e_{33}'^2 + 2C_{44}(K_1e_{31}e_{33} - R_1e_{33}e_{33}' - R_2e_{31}e_{33}' + R_2e_{33}e_{15}') + [R_2(e_{15} + e_{31}) + e_{33}(R_1 + R_3)]^2 \\
 c &= \xi_{11}C_{11}(R_2^2 - C_{33}K_1) + \xi_{11}C_{13}K_1(C_{13} + 2C_{44}) - 2\xi_{11}C_{13}R_2(R_1 + R_3) + \xi_{11}C_{33}(R_1^2 - C_{44}K_2) \\
 &\quad + \xi_{11}C_{33}R_3(2R_1 + R_3) - 2\xi_{11}C_{44}R_1R_2 - \xi_{33}C_{11}(C_{33}K_2 + C_{44}K_1 - 2R_2R_3) + \xi_{33}C_{13}(C_{13}K_2 + 2C_{44}K_2 \\
 &\quad - 2R_1R_3 - 2R_3^2) + \xi_{33}C_{44}R_1^2 - C_{11}(2C_{33}e_{15}'e_{33}' + C_{44}e_{33}'^2 + 2K_1e_{15}e_{33} + K_2e_{33}'^2) + 2C_{11}R_2(e_{15}'e_{33}' + e_{33}'e_{15}') \\
 &\quad + 2e_{33}'[C_{11}R_3e_{33}' + e_{15}'(C_{13}^2 + 2C_{13}C_{44})] + 2C_{13}(K_1e_{15} + K_2e_{33})(e_{15} + e_{31}) - 2C_{13}R_1(e_{15}'e_{33}' + e_{33}'e_{15}') \\
 &\quad - 2C_{13}R_2e_{15}'(e_{15} + e_{31}) - 2C_{13}R_3(2e_{15}'e_{33}' + e_{31}'e_{33}' + e_{33}'e_{15}') - C_{33}C_{44}e_{15}'^2 - C_{33}K_2(e_{15} + e_{31})^2 \\
 &\quad + 2C_{33}R_1e_{15}'(e_{15} + e_{31}) + 2C_{33}R_3e_{15}'(e_{15} + e_{31}) - C_{44}K_1e_{31}'^2 + 2C_{44}e_{31}(K_2e_{33} + R_1e_{33}') - 2C_{44}e_{15}'(R_1e_{33} \\
 &\quad + R_2e_{31}) + 2R_1^2e_{15}e_{33} - 2R_1R_2e_{15}'(e_{15} + e_{31}) + 2R_1R_3e_{15}e_{33} - 2R_3e_{31}[(R_1 + R_3)e_{33} - R_2(e_{15} + e_{31})] \\
 d &= -\xi_{11}C_{11}(C_{33}K_2 + C_{44}K_1 - 2R_2R_3) + \xi_{11}C_{13}^2K_2 + 2\xi_{11}C_{13}(C_{44}K_2 - R_1R_3 - R_3^2) + \xi_{11}C_{44}R_1^2 \\
 &\quad - \xi_{33}C_{11}(C_{44}K_2 - R_3^2) - C_{11}(C_{33}e_{15}'^2 + 2C_{44}e_{15}'e_{33}' + K_1e_{15}'^2) - 2C_{11}e_{15}(K_2e_{33} - R_2e_{15}' - R_3e_{33}') \\
 &\quad + 2C_{11}R_3e_{33}'e_{15}' + C_{13}^2e_{15}'^2 + 2C_{13}C_{44}e_{15}'^2 + 2C_{13}K_2e_{15}(e_{15} + e_{31}) - 2C_{13}R_1e_{15}'e_{15}' \\
 &\quad - 2C_{13}R_3e_{15}'(2e_{15} + e_{31}) - C_{44}K_2e_{31}'^2 + 2R_1e_{31}(C_{44}e_{15}' - R_3e_{15}') + R_1^2e_{15}'^2 + R_3^2e_{31}'^2 \\
 e &= C_{11}(R_3^2 - C_{44}K_2 - C_{44}e_{15}'^2 - K_2e_{15}'^2 + 2R_3e_{15}e_{15}')
 \end{aligned} \tag{A1}$$

The constants a_i, b_i, c_i and d_i involved in Eqs. (5,6) are listed as follows:

$$\begin{aligned}
 a_1 &= -\xi_{33}(C_{13}K_1 + C_{44}K_1 - R_1R_2 - R_2R_3) - (C_{13} + C_{44})e_{33}'^2 - K_1e_{33}(e_{15} + e_{31}) + [(R_1 + R_3)e_{33} + R_2(e_{15} + e_{31})]e_{33}' \\
 b_1 &= -\xi_{11}[(C_{13} + C_{44})K_1 - (R_1 + R_3)R_2] - \xi_{33}[(C_{13} + C_{44})K_2 - R_1R_3 - R_3^2] - 2e_{15}'e_{33}'(C_{13} + C_{44}) \\
 &\quad - (K_1e_{15} + K_2e_{33})(e_{15} + e_{31}) + R_1(e_{15}'e_{33}' + e_{33}'e_{15}') + R_2e_{15}'(e_{15} + e_{31}) + R_3(2e_{15}'e_{33}' + e_{31}'e_{33}' + e_{33}'e_{15}') \\
 c_1 &= -\xi_{11}[(C_{13} + C_{44})K_2 - (R_1 + R_3)R_3] - (C_{13} + C_{44})e_{15}'^2 - K_2e_{15}(e_{15} + e_{31}) + (R_1e_{15} + 2R_3e_{15} + R_3e_{31})e_{15}' \\
 a_2 &= -C_{44}(\xi_{33}K_1 + e_{33}'^2)
 \end{aligned}$$

$$\begin{aligned}
b_2 &= \xi_{33}[(R_1+R_3)^2 - C_{11}K_1 - C_{44}K_2] - C_{11}e_{33}'^2 - C_{44}(\xi_{11}K_1 + 2e_{15}'e_{33}') - K_1(e_{15}+e_{31})^2 \\
&\quad + 2e_{33}'(e_{15}+e_{31})(R_1+R_3) \\
c_2 &= \xi_{11}[(R_1+R_3)^2 - C_{11}K_1 - C_{44}K_2] - C_{44}e_{15}'^2 - C_{11}(\xi_{33}K_2 + 2e_{15}'e_{33}') - K_2(e_{15}+e_{31})^2 \\
&\quad + 2e_{15}'(e_{15}+e_{31})(R_1+R_3) \\
d_2 &= -C_{11}(\xi_{11}K_2 + e_{15}'^2) \\
a_3 &= C_{44}(\xi_{33}R_2 + e_{33}'e_{33}') \\
b_3 &= \xi_{11}C_{44}R_2 + \xi_{33}(C_{11}R_2 - C_{13}R_1 - C_{13}R_3 - C_{44}R_1) + C_{11}e_{33}'e_{33}' \\
&\quad - (C_{13}e_{33}' + R_1e_{33}')(e_{15}+e_{31}) - C_{44}(e_{31}e_{33}' - e_{33}'e_{15}') + R_2(e_{15}+e_{31})^2 - R_3e_{33}'(e_{15}+e_{31}) \\
c_3 &= \xi_{11}[C_{11}R_2 - C_{13}(R_1+R_3) - C_{44}R_1] + C_{11}(\xi_{33}R_3 + e_{15}'e_{33}' + e_{33}'e_{15}') \\
&\quad - C_{13}e_{15}'(e_{15}+e_{31}) - C_{44}e_{31}e_{15}' - R_1e_{15}'^2 - (R_1-R_3)e_{15}e_{31} + R_3e_{31}^2 \\
d_3 &= C_{11}(\xi_{11}R_3 + e_{15}'e_{15}') \\
a_4 &= C_{44}(R_2e_{33}' - K_1e_{33}') \\
b_4 &= C_{11}(R_2e_{33}' - K_1e_{33}') + C_{13}K_1(e_{15}+e_{31}) - C_{13}e_{33}'(R_1+R_3) + C_{44}(K_1e_{31} - K_2e_{33} - R_1e_{33}' + R_2e_{15}') \\
&\quad - R_2(e_{15}+e_{31})(R_1+R_3) + (R_1+R_3)^2e_{33}' \\
c_4 &= C_{11}(R_2e_{15}' + R_3e_{33}' - K_1e_{15}' - K_2e_{33}') + C_{13}K_2(e_{15}+e_{31}) - C_{13}e_{15}'(R_1+R_3) \\
&\quad + C_{44}(K_2e_{31} - R_1e_{15}') + R_1^2e_{15}' + R_1R_3(e_{15}-e_{31}) - R_3^2e_{31} \\
d_4 &= C_{11}(R_3e_{15}' - K_2e_{15}')
\end{aligned} \tag{A2}$$

In order to define the constants in Eqs. (7), the following auxiliary constants λ_{ki} , γ_{ki} and ρ_i are introduced as:

$$\begin{aligned}
\gamma_{0i} &= s_i(C_{13}\alpha_{1i} + R_1\alpha_{2i} + e_{31}\alpha_{3i}) - C_{11} \\
\gamma_{1i} &= s_i(C_{33}\alpha_{1i} + R_2\alpha_{2i} + e_{33}\alpha_{3i}) - C_{13} \\
\gamma_{2i} &= s_i(R_2\alpha_{1i} + K_1\alpha_{2i} + e_{33}'\alpha_{3i}) - R_1 \\
\gamma_{3i} &= s_i(e_{33}\alpha_{1i} + e_{33}'\alpha_{2i} + \xi_{33}\alpha_{3i}) - e_{31} \\
\lambda_{1i} &= C_{44}(\alpha_{1i} + s_i) + R_3\alpha_{2i} + e_{15}\alpha_{3i} \\
\lambda_{2i} &= R_3(\alpha_{1i} + s_i) + K_2\alpha_{2i} + e_{15}'\alpha_{3i} \\
\lambda_{3i} &= e_{15}(\alpha_{1i} + s_i) + e_{15}'\alpha_{2i} - \xi_{11}\alpha_{3i} \\
\rho_1 &= C_{44}, \rho_2 = R_3, \rho_3 = e_{15}
\end{aligned} \tag{A3}$$

Appendix B

The intermediate variables $p_1, q_1, t_{ki}^{(n)}, a_{ki}^{(n)}$ and $\bar{t}_{ki}^{(n)}, \bar{a}_{ki}^{(n)}$ in Eqs. (14) are:

$$\begin{aligned}
t_{0i} &= -\gamma_{0i} - \gamma_{04}e^{-\alpha(s_i+s_4)h} (i=1-4), \bar{t}_{0i} = \gamma_{0i} - \gamma_{04}e^{\alpha(s_i-s_4)h} (i=1-3) \\
t_{ki} &= \gamma_{ki} - \gamma_{k4}e^{-\alpha(s_i+s_4)h} (k=1-3, i=1-4), \bar{t}_{ki} = \gamma_{ki} - \gamma_{k4}e^{\alpha(s_i-s_4)h} (k=1-3, i=1-3) \\
a_{ki} &= (\alpha_{ki} + \alpha_{k4})e^{-\alpha s_i h} (k=1-3, i=1-4), \bar{a}_{ki} = (-\alpha_{ki} + \alpha_{k4})e^{\alpha s_i h} (k=1-3, i=1-3)
\end{aligned}$$

$$\begin{aligned}
t_{ki}^{(1)} &= t_{ki} - \bar{t}_{k3} a_{3i} / \bar{a}_{33} \quad (k=0-3, i=1-4), \bar{t}_{ki}^{(1)} = \bar{t}_{ki} - \bar{t}_{k3} \bar{a}_{3i} / \bar{a}_{33} \quad (k=0-2, i=1, 2) \\
a_{ki}^{(1)} &= a_{ki} - \bar{a}_{k3} a_{3i} / \bar{a}_{33} \quad (k=1, 2, i=1-4), \bar{a}_{ki}^{(1)} = \bar{a}_{ki} - \bar{a}_{k3} \bar{a}_{3i} / \bar{a}_{33} \quad (k=1, 2, i=1, 2) \\
t_{ki}^{(2)} &= t_{ki}^{(1)} - \bar{t}_{k2}^{(1)} a_{2i}^{(1)} / \bar{a}_{22}^{(1)} \quad (k=0-3, i=1-4), \bar{t}_{k1}^{(2)} = \bar{t}_{k1}^{(1)} - \bar{t}_{k2}^{(1)} \bar{a}_{21}^{(1)} / \bar{a}_{22}^{(1)} \quad (k=0-3) \\
a_{1i}^{(2)} &= a_{1i}^{(1)} - \bar{a}_{12}^{(1)} a_{2i}^{(1)} / \bar{a}_{22}^{(1)} \quad (i=1-4), \bar{a}_{11}^{(2)} = \bar{a}_{11}^{(1)} - \bar{a}_{12}^{(1)} \bar{a}_{21}^{(1)} / \bar{a}_{22}^{(1)} \\
t_{ki}^{(3)} &= t_{ki}^{(2)} - \bar{t}_{k1}^{(2)} a_{1i}^{(2)} / \bar{a}_{11}^{(2)} \quad (k=0-3, i=1-4) \\
t_{ki}^{(4)} &= t_{ki}^{(3)} - t_{k4}^{(3)} t_{3i}^{(3)} / t_{34}^{(3)} \quad (k=0-2, i=1-3) \\
t_{ki}^{(5)} &= t_{ki}^{(4)} - t_{k3}^{(4)} t_{2i}^{(4)} / t_{23}^{(4)} \quad (k=0, 1, i=1, 2) \\
t_{01}^{(6)} &= t_{01}^{(5)} - t_{02}^{(5)} t_{11}^{(5)} / t_{12}^{(5)} \\
p_1 &= \frac{q_0}{\alpha^2} - \frac{q_0 t_{14}^{(3)}}{t_{34}^{(3)} \alpha^2} - q_1 t_{13}^{(4)} / t_{23}^{(4)}, \quad q_1 = \frac{q_0}{\alpha^2} - \frac{q_0 t_{24}^{(3)}}{t_{34}^{(3)} \alpha^2}
\end{aligned} \tag{B1}$$

References

1. Shechtman D. G., Blech I. A., Gratias D., Cahn, J.: Metallic phason with long-range orientational order and no translational symmetry. *Physical Review Letters*. **53(20)**, 1951-1953 (1984)
2. Levine D., Steinhardt P. J.: Quasicrystals: A New Class of Ordered Structures. *Physical Review Letters*. **53(26)**, 2477-2480 (1984)
3. Dubois J. M.: New prospects from potential applications of quasicrystalline materials. *Materials Science & Engineering A*. **294(none)**, 4-9 (2000)
4. Fujiwara T., Trambly de Laissardière G., Yamamoto S.: Electronic Structure and Electron Transport in Quasicrystals. *Materials Science Forum*. **150**, 387-394 (1994)
5. Hu C. Z., Wang R. H., Ding D. H., Yang, W.: Piezoelectric effects in quasicrystals. *Physical Review B*. **56(5)**, 2463 (1997)
6. Rao K. R. M., Rao P. H., Chaitanya B. S. K.: Piezoelectricity in quasicrystals: A group-theoretical study. *Pramana*. **68(3)**, 481-487 (2007)
7. Li C. L., Liu Y. Y.: The physical property tensors of one-dimensional quasicrystals. *Chinese Physics*. **13(006)**, 924-931 (2004)
8. Altay G., Cengiz D. M.: On the fundamental equations of piezoelectricity of quasicrystalline media. *International Journal of Solids and Structures*. **49(23)**, 3255-3262 (2012)
9. Chen W. Q., Ma Y. L., Ding H. J.: On three-dimensional elastic problems of one-dimensional hexagonal quasicrystal bodies. *Mechanics Research Communications*. **31(6)**, 633-641 (2004)
10. Li X. Y., Deng H.: On 2D Green's functions for 1D hexagonal quasi-crystals. *Physica B: Condensed Matter*. **430**, 45-51 (2013)
11. Gao Y., Zhao B. S.: A general treatment of three-dimensional elasticity of quasicrystals by an operator method. *Physica Status Solidi (B)*. **243(15)**, 13 (2006)
12. Wang X., Pan E.: Analytical solutions for some defect problems in 1D hexagonal and 2D

- octagonal quasicrystals. *Pramana*. **70(5)**, 911-933 (2008)
13. Gao Y., Ricoeur A.: Three-dimensional Green's functions for two-dimensional quasi-crystal bimetals. *Proceedings of The Royal Society A*. **467(2133)**, 2622-2642 (2011)
 14. Li X. Y., Li P. D., Wu T. H., Zhu Z. W.: Three-dimensional fundamental solutions for one-dimensional hexagonal quasicrystal with piezoelectric effect. *Physics Letters A*. **378(10)**, 826-834 (2014)
 15. Zhou W. M., Fan T. Y.: Contact Problem in Decagonal Two-Dimensional Quasicrystal. *Journal of Beijing Institute of Technology*. **10(1)**, 51-55 (2001)
 16. Zhao X. F., Li X., Ding S. H.: Two kinds of contact problems in three-dimensional icosahedral quasicrystals. *Applied Mathematics and Mechanics(English Edition)*. **36(12)**, 1569-1580 (2015)
 17. Zhao X. F., Li X., Ding S. H.: Two kinds of contact problems in dodecagonal quasicrystalline materials of point group 12mm. *Acta Mechanica Solida Sinica*. **29(2)**, 167-177 (2016)
 18. Wu Y. F., Chen W. Q., Li X. Y.: Indentation on one-dimensional hexagonal quasicrystals: general theory and complete exact solutions. *Philosophical Magazine*. **93(8)**, 858-882 (2013)
 19. Li X. Y., Wu F., Wu Y. F., Chen W. Q.: Indentation on two-dimensional hexagonal quasicrystals. *Mechanics of Materials*. **76(sep.)**, 121-136 (2014)
 20. Hou P. F., Chen B. J., Zhang Y.: An accurate and efficient analytical method for 1D hexagonal quasicrystal coating under the tangential force based on the Green's function. *International Journal of Mechanical Sciences*. **131**, 982-1000 (2017)
 21. Hou P. F., Chen B. J., Zhang Y.: An accurate and efficient analytical method for 1D hexagonal quasicrystal coating based on Green's function. *Zeitschrift für angewandte Mathematik und Physik*. **68(4)**, 95 (2017)
 22. Li C., Zhou Y. T.: Fundamental solutions and frictionless contact problem in a semi-infinite space of 2D hexagonal piezoelectric QCs. *ZAMM: Ztschrift fur angewandte Mathematik und Mechanik*, (2019)
 23. Zhang X., Wang Q., Katharine L. Harrison, Scott A. Roberts, Stephen J. Harris: Pressure-Driven Interface Evolution in Solid-State Lithium Metal Batteries. *Cell Reports Physical Science*. **1**, 100012 (2020)
 24. Zhang X., Wang Q., Katharine L. Harrison, Katherine Jungjohann, Brad L. Boyce, Scott A. Roberts, Peter M. Attia, Stephen J. Harris: Rethinking how external pressure can suppress dendrites in lithium metal batteries. *Journal of The Electrochemical Society*. **166**, 3639-3652 (2019)
 25. Polonsky I. A., Keer L. M.: A numerical method for solving rough contact problems based on the multi-level multi-summation and conjugate gradient techniques. *Wear*. **231(2)**, 206-219 (1999)

26. Liu S., Wang Q.: Studying Contact Stress Fields Caused by Surface Traction With a Discrete Convolution and Fast Fourier Transform Algorithm. *Journal of Tribology*. **124**(1), 36-45 (2002)
27. Liu S., Wang Q., Liu G.: A versatile method of discrete convolution and FFT (DC-FFT) for contact analyses. *Wear*. **243**, 101–111 (2000)
28. Zhang X., Wang Z., Shen H., Wang Q.: Frictional contact involving a multiferroic thin film subjected to surface magnetoelectroelastic effects. *International Journal of Mechanical Sciences*. **131**, 633-648 (2017)
29. Zhang X., Wang Z. J., Shen H. M., Wang Q.: An efficient model for the frictional contact between two multiferroic bodies. *International Journal of Solids and Structures*. **130**, 133-152 (2018)
30. Zhang X., Wang Q.: A SAM-FFT based model for 3D steady-state elastodynamic frictional contacts. *International Journal of Solids and Structures*. **170**, 53-67 (2019)
31. Zhang X., Wang Q., He T.: Transient and steady-state viscoelastic contact responses of layer-substrate systems with interfacial imperfections. *Journal of the Mechanics and Physics of Solids*. **145**, 104170 (2020)
32. Ding H. J., Chen B., Liang J.: General solutions for coupled equations for piezoelectric media. *International Journal of Solids & Structures*. **33**(16), 2283-2298 (1996)
33. Jarić M. V., Nelson D. R.: Diffuse scattering from quasicrystals. *Physical Review B*. **37**(9), 4458–4472 (1988)
34. Coddens G., Bellissent R., Calvayrac Y., Ambroise J. P.: Evidence for Phason Hopping in Icosahedral AlFeCu Quasi-Crystals. *Europhysics Letters (EPL)*. **16**(3), 271–276 (1991)
35. Wang Q. , Sun L., Zhang X., Liu S., Zhu D.: FFT-Based Methods for Computational Contact Mechanics. *Frontiers in Mechanical Engineering*. **6**, 61 (2020)
36. Wang Z., Yu C., Wang Q.: An efficient method for solving three-dimensional fretting contact problems involving multilayered or functionally graded materials. *International Journal of Solids & Structures*. **66**, 46-61 (2015)
37. Ding H. J., Hou P. F., Guo F. L.: The elastic and electric fields for three-dimensional contact for transversely isotropic piezoelectric materials. *International Journal of Solids & Structures*. **37**(23), 3201-3229 (2000)
38. Chen W., Pan E., Wang H., Zhang C.Z.: Theory of indentation on multiferroic composite materials. *Journal of the Mechanics & Physics of Solids*. **58**(10), 1524-1551 (2010)
39. Makagon A., Kachanov M., Kalinin S. V., Karapetian E.: Indentation of spherical and conical punches into piezoelectric half-space with frictional sliding: Applications to scanning probe microscopy. *Physical Review B*. **76**(6) (2007)