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Research Article

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Posted Date: July 12th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-650763/v1>

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GEOMETRICAL- AND TOOTH CONTACT ANALYSIS OF THE AREAS AND PERIMETERS ON THE ARCHIMEDEAN WORM WHEELS DEPENDING ON THE MODIFICATION OF THE AXIAL MODULE

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Abstract: With the knowledge of the recommendations from reference literatures - six types of Archimedean worm gear drives were designed with the modification of the axial module. After that computer aided models were generated. These pairs were loaded by the same torque to analyse the mechanical parameters on the tooth contact zone. Due to the geometrical establishment, three teeth are connected with the worm surface at the same time, that is why these teeth are analysed. Polygon method was used to approach the kinematical impression on the worm wheel surface. As a result, we were able to define the area- and perimeter percentages of these impressions on the worm wheel surface for each tooth. We also determined the correlation between the axial module and the analysed mechanical parameters supposing average values on the contact surfaces.

Keywords: worm, worm wheel, module, teeth, stress, deformation

INTRODUCTION

Worm gear drives are widely used in the world for different engineering constructions providing high transmission ratio and perpendicular shaft position between the elements. In this case more teeth are connected at the same time that is why higher torque could be transferred than other cases of spur gear and bevel gear [1, 2, 4 - 9, 20, 21, 22, 24].

Although the manufacturing process of the worm wheel is complicated, it could be generated by direct motion mapping. It means the geometry of the worm defines the cutting tool, which is called a hob, which manufactures the teeth of the worm wheel [1, 4, 25, 26, 27]. From geometrical point of view, the worm and the hob are the same. The difference between them are addendum and tooth width. We should also provide clearances during the tooth connection to avoid heat deformations [1, 4, 25, 26, 27].

F.L. Litvin found mathematical processes to determine and optimize the tooth contact areas in case of different gears. He designed special drives for different companies. His research group developed computer aided software for the geometrical designing and the localization of the contact area on toothed gears [6, 7, 23].

A. Fuentes Aznar and his research group deal with tooth contact analysis (TCA) which contains geometrical designing, computer aided modelling (CAD) and Finite Element Analysis (FEM) for standardized gear drives and special connecting elements [7]. The aim of the TCA is to determine the mechanical parameters on the tooth contact zone by different load forces and torques depending on the material type [7, 23].

I. Dudás researches the manufacturing development of the different worm gear drives. He also analyses these elements by mathematical way to provide the correct connection and

manufacturing process for them. The development stages of the grinding processes are important for the assurance of the better connection and the reduction of the wearing between the kinematic pairs [4].

T. Bercsey analysed the connection of the globoid worm and hyperbolic gear. On the other hand, he proved the application of the kinematic method for toroidal gears [2, 3].

I. Sz. Krivenko invented the cylindrical worm gear drives having arched profile in axial section. Based on his experience, he recommended a certain designing process for this construction. The advantage of this geometry is the convex-concave teeth connection which results better contact lines and area than in case of linear profile [8].

V. Goldfarb works on the geometrical development of spiroid- and cylindrical worm gears by mathematical and constructional way. He analyses the contact mechanical parameters by FEM for the geometrical localization [13, 14].

The journal of Modeling the double worm-face gears [9] presents the matrix - vectorial mathematical model of the double worm-face gear with cylindrical worm and a graphical modelling which is based on the specific geometrical characteristics accomplished by means of the Autodesk Inventor 3D modelling program.

In his journal, M.V. Chernets [10] presents a computational method of worm gears with Archimedean and involute worms. Engagement correction is presented, based on the contact pressures and wear of tooth of the wheel, as well as the gear life and the sliding speed in the engagement.

In their international journal K. Kawasaki and I. Tsuji [11] shows a machining method of large-sized cylindrical worm gears with Niemann profiles using a computer numerical control (CNC) machining center. The tooth contact pattern and transmission errors of large-sized worm gear pair with Niemann profiles are analysed before machining of the worm and worm wheel. After that, the machining conditions of the worm is determined by calculating each offset distance between the worm axis and the center axis of the end mill, and then the worm is machined by swarf cutting. That means machining by the side surface of the end mill.

In the Study on the influence of gear hobbing and shaft misalignments on the geometric interference of cylindrical worm gear set, [12] the author presents the results of the machine tool setting errors of the worm gear hobbing and its effects of the shaft misalignments of the worm gear set on the interference.

In the Meshing theory of mismatched ZC1 worm drive, [15] the contact endpoints, the datum point, and the internal momentary contact points are all determined by solving their corresponding nonlinear equation sets iteratively. The numerical results of the relative principal curvature, the transmission error, and the drive ratio error of the worm drive are all obtained.

J. Sohn, N. Park [17] present the geometric interference of cylindrical worm gear drives using an oversized hob to cut the worm gear. The momentary contact line of a fully conjugated gear set becomes a momentary point of contact when an oversized hob is used.

An Academic writing [18] presents a novel point-contact hourglass worm drive consisting of an involute helical beveloid (IHB) gear and an oblique planar enveloping (OPE) hourglass worm – as a result of the increasing requirements of high-precision and high bearing capacity in industrial robot and aerospace technology.

I. GENERATION- AND THE GEOMETRY OF ARCHIMEDEAN WORMS

The worm surface is generated by such cutting tool which has straight-lined blade. This cutting edge is situated in the axial section of the worm [4, 6, 7, 20, 26, 27, 28]. The generating lines are in coordinate system C_p , which is connected to the edge of the tool. This coordinate system does rotational and linear motion at the same time, which generate helical surface of the worm

(Figure 1) [4, 6, 7, 20, 26, 27, 28]. The transformation matrix between the C_0 and C_{1R} systems is:

$$M_{1R,0} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & \pm p \cdot \theta \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Figure 1. Geometry of the cutting edge (a) and coordinate transformation (b)

Based on Figure 1, the normal unit vector is [7]:

$$\vec{n}_{1R}(\eta, \theta) = \frac{\partial \vec{r}_{1R}}{\partial \eta} \times \frac{\partial \vec{r}_{1R}}{\partial \theta} \quad (2)$$

The generated surface on the C_{1R} coordination system is:

$$\vec{r}_{1R}(\eta, \theta) = M_{1R,p}(\theta) \cdot \vec{r}_p(\eta) \quad (3)$$

In case of right-hand worm, side I surface is:

$$\left. \begin{aligned} x_{1RI} &= \eta \cdot \cos\alpha_{ax} \cdot \cos\theta \\ y_{1RI} &= \eta \cdot \cos\alpha_{ax} \cdot \sin\theta \\ z_{1RI} &= -\eta \cdot \sin\alpha_{ax} + \left(r_0 \cdot \tan\alpha_{ax} - \frac{p_x - s_{ax1}}{2} \right) + p \cdot \theta \end{aligned} \right\} \quad (4)$$

The surface unit normal vector of side I surface is:

$$\vec{n}_{1RI}(\eta, \theta) = (p \cdot \sin\theta + \eta \cdot \sin\alpha_{ax} \cdot \cos\theta) \cdot \vec{i} - (p \cdot \cos\theta - \eta \cdot \sin\alpha_{ax} \cdot \sin\theta) \cdot \vec{j} + \eta \cdot \cos\alpha_{ax} \cdot \vec{k} \quad (5)$$

In case of right-hand worm, side II surface is:

$$\left. \begin{aligned} x_{1RII} &= \eta \cdot \cos \alpha_{ax} \cdot \cos \theta \\ y_{1RII} &= \eta \cdot \cos \alpha_{ax} \cdot \sin \theta \\ z_{1RII} &= \eta \cdot \sin \alpha_{ax} - \left(r_0 \cdot \tan \alpha_{ax} - \frac{p_x - s_{ax1}}{2} \right) + p \cdot \theta \end{aligned} \right\} \quad (6)$$

The surface unit normal vector of side II surface is:

$$\begin{aligned} \vec{n}_{1RII}(\eta, \theta) &= (p \cdot \sin \theta - \eta \cdot \sin \alpha_{ax} \cdot \cos \theta) \cdot \vec{i} \\ &\quad - (p \cdot \cos \theta + \eta \cdot \sin \alpha_{ax} \cdot \sin \theta) \cdot \vec{j} + \eta \cdot \cos \alpha_{ax} \cdot \vec{k} \end{aligned} \quad (7)$$

If $y_{1R}=0$, the equations of the two sections are $x_{1R}=x_{1R}(\eta)$ and $z_{1R}=z_{1R}(\eta)$:

$$\left. \begin{aligned} x_{1R} &= \eta \cdot \cos \alpha_{ax} \\ y_{1R} &= 0 \\ z_{1R} &= \eta \cdot \sin \alpha_{ax} - \left(r_0 \cdot \tan \alpha_{ax} - \frac{p_x - s_{ax1}}{2} \right) \end{aligned} \right\} \quad (8)$$

or

$$\left. \begin{aligned} x_{1R} &= -\eta \cdot \cos \alpha_{ax} \\ y_{1R} &= 0 \\ z_{1R} &= \eta \cdot \sin \alpha_{ax} - \left(r_0 \cdot \tan \alpha_{ax} - \frac{p_x - s_{ax1}}{2} \right) + p \cdot \pi \end{aligned} \right\} \quad (9)$$

The contact point between the connecting elements could be calculated by the Fundamental Law of gear-tooth action [6, 7]:

$$\vec{n}_{1R} \cdot \vec{v}_{1R} = 0 \quad (10)$$

The formula (10) could also be used to determine surface of the worm wheel if the worm surface is known [1, 4 - 7, 22, 23, 25, 27].

II. DESIGN- AND MODELLING PROCESS

Based on the recommendations of the reference literature [4 - 8, 13, 14, 16, 20, 21, 27] six types of Archimedean worm gear drives are designed by different axial modules. All the other input parameters are unchanged (f^* , c^* , r^* , z_1 , z_2 , L , b_2 , α_n , j). The geometrical calculations are created by GearTeq software (Table 1).

Table 1. The calculated parameters of the designed worm gear drives

The parameters of the worm gear drive	I.	II.	III.	IV.	V.	VI.
Axial module (m_{ax}) [mm]	6.5	7	8	9	10	11
Diametral pitch (p_d) [mm]	3.907	3.628	3.175	2.822	2.54	2.309
Normal diametral pitch (p_{dn}) [mm]	4.022	3.751	3.314	2.978	2.712	2.497

Normal module (m_n) [mm]	6.315	6.77	7.662	8.527	9.363	10.168
Worm lead of thread (p_x) [mm]	61.3	66	75.4	84.8	94.2	103.7
Circular pitch of the worm-wheel (t_0) [mm]	20.4	22	25.1	28.3	31.4	34.6
Lead angle (γ_m) [mm]	13.7	14.7	16.7	18.6	20.6	22.4
Addendum coefficient (f')	1					
Clearance coefficient (c^*)	0.2					
Fillet coefficient (r')	0.3					
Number of threads (z_1)	3					
Number of teeth of the worm-wheel (z_2)	28					
Direction of the wheel	right hand					
Pitch diameter of the worm (d_{01}) [mm]	80					
Pitch diameter of the worm-wheel (d_{02}) [mm]	182	196	224	252	280	308
Outside diameter of the worm (d_{a1}) [mm]	93	94	96	98	100	102
Outside diameter of the worm-wheel (d_{a2}) [mm]	199.8	214.9	245.1	275.3	305.5	335.7
Center distance (a) [mm]	131	138	152	166	180	194
Face width of the worm-wheel (b_2) [mm]	50					
Worm length (L) [mm]	200					
Pressure angle (α_{ax}) [°]	20.5	20.6	20.8	21	21.2	21.5
Normal pressure angle (α_n) [°]	20					
Addendum of the worm (h_{a1}) [mm]	6.5	7	8	9	10	11
Addendum of the worm-wheel (h_{a2}) [mm]	6.5	7	8	9	10	11
Dedendum of the worm (h_{f1}) [mm]	7.6	8.2	9.3	10.	11.7	12.8
Dedendum of the worm-wheel (h_{f2}) [mm]	7.6	8.2	9.3	10.5	11.7	12.8
Tooth thickness of the worm (S_{ax1}) [mm]	10.2	11	12.6	14.1	15.7	17.3
Tooth thickness of the worm-wheel (S_{ax2}) [mm]	9.2	10	11.6	13.1	14.7	16.3
Fillet radius (r) [mm]	1.9	2.1	2.4	2.7	3	3.3
Backlash (j) [mm]	1					
Transmission ratio (i)	3/28					
Diameter ratio (q)	10					11

Figure 2. The geometrical parameters of the worm and the worm wheel

After all the calculations, the CAD models of the elements could be generated by SolidWorks software (Figure 2). The GearTeq and the SolidWorks are interoperable that is why the geometrical parameters of the elements could be imported relatively easily to the SolidWorks for CAD modelling and assembly settings.

III. ANALYSIS OF THE CONTACT SURFACES OF THE WORM WHEEL BY TCA

Based on the geometrical facts, three worm wheel teeth are connected with the worm at the same time. Each connected teeth are analyzed. The material type of the elements is structural steel [28]. Four coordinate systems are needed for the analysis:

- one rotational coordinate system for the worm,
- one rotational coordinate system for the worm wheel,
- two coordinate systems for the analyzed tooth connection (linear and normal system).

The axis position of the linear system is the same as that of the rotational system of the worm wheel. The linear- and normal systems are adopted on the middle point of the surface of the worm wheels that we are currently analyzing.

Dense meshing is applied on the analyzed contact area (Figure 3). The dimension of the element is 0.4 mm. Automatic meshing is selected for the outside area. The friction coefficient is $\mu=0.01$ on the contact zone.

Figure 3. Tooth connection of the elements

All degrees of freedom are fixed on the worm wheel. Rotation around the axis of rotation of the worm is permitted, other degrees of freedom are also fixed. The worm is loaded by 300 Nm torque around the axis of rotation. The same load is used for every drive.

3.1. Normal stress analysis

The normal stress is interpreted perpendicularly to the surface of the worm wheel [19, 28]. We analyze the average stress results and the geometrical shape of the impressions on the surfaces of the worm wheels for each connected tooth. Figure 4 shows the result of the 1st tooth.

c) $m=8$ mm

d) $m=9$ mm

e) $m=10$ mm

f) $m=11$ mm

Figure 4. Normal stress results on the 1st tooth

The received contact impressions are defined by a lot of peripheral points which are imported to the GeoGebra mathematical software. This software is a complex mathematical software having a lot of operation possibilities. These points are connected by lines. As a result we receive polygons whose area and perimeter could be calculated by mathematical way. Figure 5 shows these polygons.

Figure 5. The structure of the stress impressions on the 1st tooth

Figure 6 shows the result of the normal stress on the 2nd tooth while Figure 7 shows the polygons of the impressions.

a) $m=6.5$ mm

b) $m=7$ mm

c) $m=8$ mm

d) $m=9$ mm

e) $m=10$ mm

f) $m=11$ mm

Figure 6. Normal stress results on the 2nd tooth

Figure 7. The structure of the stress impressions on the 2nd tooth

a) m=6.5 mm

b) m=7 mm

c) m=8 mm

d) m=9 mm

e) m=10 mm

f) m=11 mm

Figure 8. Normal stress results on the 3rd tooth

Figure 8 shows the results of normal stress on the 3rd tooth while Figure 9 shows the polygons of the impressions.

Figure 9. The structure of the stress impressions on the 3rd tooth

Figure 10. The area percentages of the impressions of the normal stress

The geometrical areas of the surfaces of the worm wheels are determined by polygon method. Figure 10 shows the comparison of the contact impressions and the areas of the surfaces of the worm wheels in percentage. The area percentages are continuously decreasing while there is an increase in the axial module from $m=7$ mm. The highest percentages are on the second tooth since it is totally in contact. The results on the third tooth are the lowest to $m=9$ mm axial module. After that the results on the 3rd tooth are higher than on the 2nd tooth ($m=10$ mm and $m=11$ mm). Due to the modifications of the modules, the areas and the geometrical shapes of the surfaces of the worm wheels were modified.

Figure 11 shows the perimetrical percentage. The results are constantly decreasing depending on the increase of the axial module. The highest percentages are on the 2nd tooth upto $m=9$ mm axial module. After that the highest results are on the 3rd teeth.

The lowest results are on the 3rd tooth upto m=7 mm axial module. After that the lowest results are on the 1st teeth.

Based on Figure 12 , the average normal stress results are fluctuating and increasing depending on axial module. The highest results on the 1st and 2nd teeth are always fluctuating depending on the axial module.

Figure 11. The perimetrical percentages as a result of the normal stress

Figure 12. The average normal stress results depending on the axial module

3.2. Normal deformation analysis

The normal deformation is analyzed perpendicularly to the surface of the worm wheel [19, 28]. It was also analyzed for each connecting tooth of the worm wheel.

a) $m=6.5$ mm

b) $m=7$ mm

c) $m=8$ mm

d) $m=9$ mm

e) $m=10$ mm

f) $m=11$ mm

Figure 13. Results of normal deformation on the 1st tooth

The effects of the deformation zones are saved into the GeoGebra software. We set lines to the shape of the zones which connect the border points of the deformation zones (Polygon method). Similarly to the stress analyses, we receive polygons, of which area and perimeter could be calculated by mathematical way. Figure 13 and 14 show the results in case of the 1st tooth.

Figure 14. The resulted structure of deformation in case of the 1st tooth

Figure 15 shows the result of the normal deformation on the 2nd tooth. Figure 16 shows the resulted polygons of the deformation zones.

a) m=6.5 mm

b) m=7 mm

c) m=8 mm

d) m=9 mm

e) $m=10$ mm

f) $m=11$ mm

Figure 15. Results of normal deformation in case of the 2nd tooth

Figure 16. The resulted structure of deformation in case of the 2nd tooth

Figure 17 shows the result of normal deformation on the 3rd tooth. Figure 18 shows the resulted polygons of the deformation zones.

a) $m=6.5$ mm

b) $m=7$ mm

Figure 17. Results of normal deformation in case of the 3rd tooth

Figure 18. The structure of deformation impressions on the 3rd teeth

With the knowledge of the resulted areas due to normal deformation and the whole surfaces of the teeth of the worm wheels, the area percentages could be determined (Figure 19). The highest results are received on the 1st tooth due to the maximum loads. This result decreases up to $m=8$ mm axial module. After that, it starts increasing. The result on the 2nd and 3rd teeth are continuously fluctuating depending on the axial module.

The area percentage of the $m=6.5$ mm and $m=7$ mm axial module are higher on the 2nd tooth than on the 3rd tooth. After that, the results on the 3rd tooth are the 2nd highest.

Figure 19. The resulted area percentages due to normal deformation

Figure 20. The resulted perimetrical percentages due to normal deformation

Figure 20 shows the perimetrical percentages between the impressions of the normal deformations and the whole surfaces of the worm wheels. The highest results are on the 1st tooth which decreases up to $m=8$ mm axial module. After that, it starts increasing.

The perimetrical percentages of the $m=6.5$ mm and $m=7$ mm axial module are higher on the 2nd tooth than on the 3rd tooth. After that, the results on the third tooth are the second highest. The results on the 2nd tooth are continuously fluctuating depending on the axial module. The results on the 3rd tooth are increasing depending on the axial module.

Figure 21. The average result due to normal deformation depending on the axial module

The average normal deformation values are also calculated (Figure 21). The highest results are received on the 1st tooth which decrease up to $m=9$ mm axial module. After that, it starts increasing. The highest result on the 1st tooth is in case of $m=6.5$ mm axial module.

The results on the 2nd tooth is fluctuating and continuously increasing. It is lower than the results on the 3rd tooth except in case of $m=6.5$ mm axial module.

The results on the 3rd tooth are continuously increasing depending on the axial module but it is always lower than the results on the 1st tooth.

3.3. Axial- and radial deformation analysis

The average deformations on the connecting teeth are also determined and divided into axial and radial directions on the surfaces of the worm wheels (Figure 22 and 23).

Figure 22. Axial module –deformation diagram in axial direction

Based on Figure 22, the following statements could be defined:

- the lowest results are on the 2nd tooth. It is continuously increasing from m=7 mm axial module.
- the results on the 1st tooth are increasing up to m=8 mm axial module. After that, they are decreasing. The highest result is in case of m=8 mm axial module.
- the results on the 3rd tooth are continuously increasing depending on the axial module.
- the results on the 3rd tooth are always higher than on the 2nd tooth.

Based on Figure 23, the following statements could be defined:

- the highest results are on the 1st tooth. This function is fluctuating. The highest result is in case of m=6.5 mm axial module.
- the results of the 2nd tooth are also fluctuating. The highest results are in case of m=6.5 mm axial module.
- the results of the 3rd tooth are continuously decreasing from m=7 mm axial module.
- the lowest results are on the 3rd tooth.

Figure 23. Axial module –deformation diagram in radial direction

CONCLUSION

The main property of the Archimedean worm is the linear profile which is generated on the axial section of the worm by straight-lined blade. This geometry is widely used for different gear boxes providing good load transmission and high transmission ratio. That is why the TCA researches are important for the geometrical analysis of them. The mechanical parameters could be defined before the manufacturing and the real application.

The aim of this study is to show the correlation between the changing mechanical parameters depending on the axial module. All of the initial parameters are the same except the axial modules which are selected by prescribed gear standards. With the knowledge of the constructional references, all the other geometrical parameters are determined by the GearTeq. The generation of this cylindrical worm surface is determined by mathematical way so the contact points and the surfaces of the worm wheel could also be determined.

CAD models were created by SolidWorks software. It is important to assemble them according to the tooth connection.

Since three teeth of the worm wheel are connected to the worm at the same time - we did our TCA for each of them.

The main mechanical parameters which have to be analyzed in a TCA are the normal stress and normal deformation. We also made static structural analysis for the research. These results and geometrical effects on the contact zones are determined.

The effects of the pressure of the worm - on the surface of the worm wheel - are calculated by mathematical way. Polygons are used to determine these areas - of which geometries are determined by geometrical calculations (polygon method). The whole surfaces of the worm wheels are also calculated by the same way. Thus, the ratios of areas and perimeters between the whole surfaces and the connecting impressions are calculated in percentages for each connecting tooth in case of the normal deformation and normal stress analysis. That is why it is possible to show the correlations between the percentages and the axial modules.

Based on the average normal deformation and normal stress results, the functions and consequences are determined depending on the modification of the axial module.

The axial and radial deformations are also determined on the surfaces of the teeth with average values. The axial direction is parallel to the axis of rotation of the worm wheel. The radial direction is perpendicular to it. These results were also evaluated depending on the axial modules.

ACKNOWLEDGEMENT

Project no. **TKP2020-NKA-04** has been implemented with the support provided from the **National Research, Development and Innovation Fund of Hungary**, financed under the **2020-4.1.1-TKP2020** funding scheme.

This research was partly supported by the **János Bolyai Research Scholarship of the Hungarian Academy of Sciences**.

I, Dr. Sándor Bodzás, would like to express my great respect to **Prof. Dr. Tibor Bercsey** † (2020) (Budapest University of Technology and Economics, Budapest, Hungary) who provided plenty of consultations on the research topic of worm gear drives to me. He had several publications and wide experience in this topic. He was one of the reviewers of my Ph.D. dissertation who supported my work by his useful remarks.

FUNDING: not applicable

CONFLICTS OF INTEREST/COMPETING INTERESTS: not applicable

AVAILABILITY OF DATA AND MATERIAL: not applicable

CODE AVAILABILITY: not applicable

ETHICS APPROVAL: we approve it

CONSENT TO PARTICIPATE: we consent for the publishing

CONSENT FOR PUBLICATION: we consent for the publishing

AUTHORS' CONTRIBUTIONS: we consent for the publishing

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NOMENCLATURE

$\vec{v}_{1R}^{(12)}$	(mm/min ⁻¹)	Relative velocity vector
$\vec{n}_{1R}$		Normal vector of the surface in C _{1R} coordinate system
$\vec{r}_{1R}$		Placement vector of the moving points of the profile curve
$\vec{r}_P$		Position vector of the generating curve of the tool surface
$M_{1R,P}$		Coordinate transformation matrix (transforms C _P to C _{1R})
O _{1R} , O _P		Origins of the appropriate coordinate systems
C _{1R} (x _{1R} , y _{1R} , z _{1R})		Rotational coordinate system related to the driver gear wheel
C _P (x _P , y _P , z _P)		Tool coordinate system of generating curve
x, y, z	(mm)	Coordinates
η, θ		Internal parameters of the helicoidal surface
m _{ax}	(mm)	Axial module
p _d	(mm)	Diametral pitch
p _{dn}	(mm)	Normal diametral pitch
m _n	(mm)	Normal module
p _x	(mm)	Worm lead of thread
t ₀	(mm)	Circular pitch of the worm-wheel
γ _m	(mm)	Lead angle
f*		Addendum coefficient
c*		Clearance coefficient
r'		Fillet coefficient
z ₁		Number of threads
z ₂		Number of teeth of the worm-wheel
d ₀₁	(mm)	Pitch diameter of the worm
d ₀₂	(mm)	Pitch diameter of the worm-wheel
d _{a1}	(mm)	Outside diameter of the worm
d _{a2}	(mm)	Outside diameter of the worm-wheel
a	(mm)	Center distance
b ₂	(mm)	Face width of the worm-wheel
L	(mm)	Worm length
α _{ax}	(°)	Pressure angle
α _n	(°)	Normal pressure angle
h _{a1}	(mm)	Addendum of the worm
h _{a2}	(mm)	Addendum of the worm-wheel
h _{f1}	(mm)	Dedendum of the worm
h _{f2}	(mm)	Dedendum of the worm-wheel
S _{ax1}	(mm)	Tooth thickness of the worm
S _{ax2}	(mm)	Tooth thickness of the worm-wheel
r	(mm)	Fillet radius
j	(mm)	Backlash
i		Transmission ratio
q		Diameter ratio
M	(Nm)	Load torque

μ
TCA
CAD

Friction coefficient
Tooth Contact Analysis
Computer Aided Designing