Measurements of the Planck length from a Ball-Clock without Knowledge of Newton's Gravitational Constant G or the Planck Constant

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Abstract

In this paper we show how one can extract the Planck length from ball with a built-in stopwatch with no knowledge of the Newtonian gravitational constant or the Planck constant. This is remarkable as until recently it has been assumed one cannot find the Planck length without knowledge of Newton's gravitational constant. This method of measuring the Planck length should also be of great interest to not only physics researchers but also to physics teachers and students as it conveniently demonstrates that the Plank length is directly linked to gravitational phenomena, not only theoretically, but practically. To demonstrate that this is more than a theory we report 100 measurements of the Planck length using this simple approach. We will claim that, despite the mathematical and experimental simplicity, our findings could be of great importance in better understanding the Planck scale, as our findings strongly support the idea that to detect gravity is to detect the effects from the Planck scale indirectly.

Keywords: Planck length, Planck units, Newton's gravitational constant, Planck constant, Compton wavelength.

Max Planck, in 1899 and 1906, [1, 2] assumed there were three important universal constants, the speed of light, the Planck constant and the Newtonian gravitational constant. Based on these he used dimensional analysis and predicted there had to be a very fundamental length; $l_p = \sqrt{\frac{G\hbar}{c^3}}$, time unit; $t_p = \sqrt{\frac{G\hbar}{c^5}}$ and mass; $m_p = \sqrt{\frac{\hbar c}{G}}$. Still, what were these Planck units related to? Max Planck said little about this, except he called them natural units. He had also not been the first to suggest natural units. Stoney [3] had, in 1883, already suggested natural units derived from the elementary charge, the Newtonian gravitational constant and the speed of light. However, today among most physicists the Planck units are considered more important than the Stoney units.

The Planck units were in no way immediately accepted as something important, and it was mainly the Planck constant (quanta of energy) and Planck's law that made Max Planck very famous in physics and won him a Nobel Prize. The Planck units are today considered to potentially play an important role in a "final" unified theory. Already by 1916 in one of his papers Einstein [4] had laid out his general relativity theory, indicating that a quantum gravity theory was the next natural step to an even better understanding of gravity, or in his own words:

> "*Because of the intra-atomic movement of electrons, the atom must radiate not only electromagnetic but also gravitational energy, if only in minute amounts. Since, in reality, this cannot be the case in nature, then it appears that the quantum theory must modify not only Maxwell's electrodynamics but also the new theory of gravitation.* (Einstein 1916, p. 696)."

However, Einstein said little or nothing about how the quantum theory should cause modifications in gravity theory. Eddington [5], in 1918, is likely to have been the first to suggest that the Planck length must play a central role in new quantum gravity theory, or in his own words:

> *There are three fundamental constants of nature which stand out as pre-eminent. The velocity of light, $300 \cdot 10^{10}$ C.G.S. units ; dimensions LT^{-1}. The quantum, $6.55 \cdot 10^{-27}$, ML^2T^{-1} C.G.S. Units. The constant of gravitation, $6.66 \cdot 10^{-8}$; $M^{-1}L^2T^{-2}$. From these we can construct a fundamental unit of length whose value is 4×10^{-33} cm. There are other natural units of length, the radii of the positive and negative unit electric charges, but these are of an altogether higher order of magnitude. But it is*

evident that this length must be the key to some essential structure. It may not be an unattainable hope that someday a clearer knowledge of the process of gravitation may be reached.

And it is clear that this is the Planck length that Eddington here refers to. However, this "speculation" that gravity likely had to be linked to the Planck length was by no means easily accepted. For example, Bridgman [6] in 1931 ridiculed Eddington for his speculative claim, and he himself basically claimed the Planck units were more of a mathematical artifact coming out of dimensional analysis rather than something liked to something physical. Today most physicists, especially those working with gravity and quantum gravity theories, [7–11] seem to agree with Eddington that the Planck length must likely play central role in a quantum gravity — in addition, superstring theorists assume the Planck length is important for their theory. Still, there are others [12] that are more on the Bridgman side and think of the Planck units more like a mathematical artifact, or at least something we never can measure except from deriving it through dimensional analysis. However, recently we [13] have shown how one can measure the Planck length independent of G and $\hbar$ using a Newton force spring, although this method is somewhat more complicated than shown here. Here, we will extend our analysis to measure the Planck length with a ball with a built-in stopwatch — this makes it particularly easy to perform such an experiment, even high school physics students can now get a better feel of how one can extract the Planck length from gravity phenomena.

Our method will rely on the fact that we know the Compton wavelength of the gravity object from which we will measure the gravity effects. First, we start out by measuring the Compton wavelength based on Compton scattering as described by Compton [14] in 1923. Compton scattering consists of shooting photons at an electron and measuring the wavelength of the photon before $\lambda_{\gamma,1}$ and after $\lambda_{\gamma,1}$ it hits the electron as well as the angle between the ingoing and outgoing photon (θ)

$$\lambda_e = \frac{\lambda_{\gamma,2} - \lambda_{\gamma,1}}{1 - \cos\theta} \tag{1}$$

.

It is important to note that this way of measuring the Compton wavelength of the electron requires no knowledge of the Planck constant. We could alternatively have found the Compton wavelength from the electron mass in kilograms (kg), but this would in general require knowledge of the Planck constant. This is given by the well-known Compton wavelength formula $\lambda = \frac{h}{mc}$; however, we will stick to the first method that is independent of knowledge of the Planck constant. Next, we will find the Compton wavelength of a proton. We then utilize the fact that the Compton wavelength is inversely linearly proportional to the cyclotron frequency. This because the cyclotron frequency is given by

$$\omega = \frac{qB}{m} \tag{2}$$

and since the charge of a proton and electron is the same, the relative cyclotron frequency of a proton relative to an electron is equal to the Compton wavelength ratio; that is to say, we have

$$\frac{\omega_e}{\omega_P} = \frac{\frac{qB}{m_e}}{\frac{qB}{m_P}} = \frac{m_P}{m_e} = \frac{\lambda_e}{\lambda_P} \approx 1836.15 \tag{3}$$

This method has been used to accurately find the proton electron mass ratio [15], which is identical to the Compton wavelength ratio. If we have now measured the electron Compton wavelength by Compton scattering, we simply need to divide this by the cyclotron frequency ratio to find the Compton wavelength of the proton. Theories about the Compton wavelength of the proton go long back at least to Levitt [16] in 1958, who claimed

Most of the experimental lengths concerning the fundamental forces in the nucleus are integral multiples of the Compton wavelength, λ_0, of the proton, where $\lambda_0 = \frac{h}{m_0 c} = 1.32 \times 10^{-13}$ cm (which we can thus call a 'Compton'). These lengths include the effective range of nucleon interactions, and the amplitudes of dispersion.

The author uses notation λ_0 for the Compton wavelength of the rest mass, but we have for the rest-mass Compton wavelength simply used notation λ. We also note increased interest in the Compton wavelength of the proton, which has recently for example been suggested to also be directly related to the proton radius [17]. Some will possibly protest here and claim only elementary particles like the electron have a Compton wavelength, and not protons. We to a large degree agree with such an argument, but even if a composite particle or even a large composite mass does not have one single Compton wavelength, they ultimately consist of many elementary particles with a Compton wavelength. The aggregates of the individual Compton wavelength in any mass are given by the relation below (see also [13])

$$\lambda = \frac{1}{\sum_i^n \frac{1}{\lambda_i}} \tag{4}$$

Since we already know how to find the Compton wavelength of a proton (without knowledge of h) all we now need to do to find the Compton wavelength of, for example, a handful of matter is to count the number of atoms in that clump of matter. To count the number of atoms in matter goes long back in the history of physics, at least to Avogadro's number [18]. It is not easy, but we can indeed count the number of atoms very accurately today, for example one of the new kg definition suggestions was rooted in counting atoms in silicon spheres [19–21]. But then what about really large masses such as the Earth, how can we count the number of atoms in the Earth? Here we can take advantage of the fact that the ratio of the Compton wavelength in two masses is always linearly proportional to almost any two gravitational measurements performed on the two objects after correcting for distances to the center of the gravity objects. For example, if we know the gravitational acceleration from a small silicon sphere and the gravitational acceleration field of the Earth, then we know that the Compton wavelength in the Earth is related to

$$\frac{g_1 R_1^2}{g_2 R_2^2} = \frac{\lambda_2}{\lambda_1} \tag{5}$$

To predict the gravitational acceleration from the mass of the Earth, in standard physics we need to know G since we have $g = \frac{GM}{R^2}$, but we can measure the gravitational acceleration of the Earth, for example at the surface of the Earth without any knowledge of G. This, for example, we can do by dropping a ball from height H to the ground and measuring how long this took, and based on this we know g from the following formula (this is well known)

$$g = \frac{2H}{T^2} \tag{6}$$

.

From a small mass where we been able to count the number of atoms, for example a silicon sphere, we can measure the gravitational acceleration field with a Cavendish apparatus based on the following relation

$$g = \frac{2\pi^2 L\theta}{T^2} \tag{7}$$

where θ is the angle of the arm when it is deflected, and L is the distance between the two small balls in the apparatus, and T is the oscillation time. Pay attention to the fact that no G or $\hbar$ are also needed to find the gravitational acceleration field in the Cavendish apparatus. It is often said that the Cavendish apparatus was developed to measure G, however this is not the case. A Cavendish apparatus can be used to measure G, but Cavendish in 1798 did not use or measure the gravitational constant G. What Cavendish did was to use the apparatus to find the gravitational effects from a uniform mass that he could know the density of, in his case lead balls. When he knew this, he could find the density of the Earth relative to this. The gravity constant was actually first introduced in 1873 by Cornu and Baill [22] and was partly needed because one had redefined the mass standard in terms of kg; they actually used notation f for the gravity constant, while Boys [23] in 1994 introduced today's notation G. Whether one uses f or G notation is naturally only a purely cosmetic question. Anyway, the main point here is that we can measure gravitational acceleration with no knowledge of G, and if we have counted the number of atoms (protons and neutrons) in the reference mass, for example a silicon sphere, then we can find the Compton wavelength of even a planet or a sun without any knowledge of G or $\hbar$. The Compton wavelength of the Earth is approximately $\lambda_E \approx 3.70 \times 10^{-67}$ m, and the reduced Compton wavelength is this number divided by 2π, which gives approximately $\bar{\lambda}_E \approx .5.89 \times 10^{-68}$ m. Again, this is not a physical single Compton wavelength, but corresponds to the aggregate of the Compton wavelength of all elementary particles making up the mass of the Earth, see formula 4.

It is well known that one can measure g by measuring the velocity of a ball from a drop height H — as looked at in the section above. Traditionally one needs two synchronized clocks to do this: one clock that measures when the ball is dropped from point H, and one clock measuring when the ball hits the ground or passes a point below H. That is to say an external observer frame observes the time it takes for the object to fall, and therefore to be accelerated by the gravitational acceleration field. In recent years this has become much easier to do as we have instead incorporated a stopwatch inside the ball itself, in other words a proper observer. What is known as a gravity ball or g-ball is a ball with a built-in stopwatch that starts at the moment one drops the ball and stops when the ball hits something, such as the ground. Such a ball costs about 30 USD at the time of writing, and it has about one hundredth of a second precision. If we know the Compton wavelength of the Earth, then we can use such a ball to measure the Planck length in a very simple way. This new device makes it very easy to measure the gravitational acceleration field that again is given by

$$g = \frac{2H}{T^2} \tag{8}$$

as we also have $g = \frac{GM}{R^2}$ and in addition we can solve the Planck length formula, $l_p = \sqrt{\frac{G\hbar}{c^3}}$ with respect to G, this gives

$$G = \frac{l_p^2 c^3}{\hbar} \tag{9}$$

To make the gravity constant a function of the Planck length would naturally only lead to a circular unsolvable problem if we cannot find the Planck length independent of knowledge of G, something we soon will show is possible. Next, we can solve the Compton 1923 wavelength formula with respect to the mass, this gives

$$m = \frac{h}{\lambda}\frac{1}{c} \tag{10}$$

the gravitational acceleration is given by

$$g = \frac{Gm}{R^2} = \frac{\frac{l_p^2 c^3}{\hbar}\frac{h}{\lambda}\frac{1}{c}}{R^2} = c^2\frac{l_p^2}{\bar{\lambda}R^2} \tag{11}$$

as we can measure g without knowing G or m, we can now solve this equation with respect to l_p, this gives:

$$l_p = \frac{R}{c}\sqrt{g\bar{\lambda}} \tag{12}$$

Next we replace g with $g = \frac{2H}{T^2}$, this gives

$$l_p = \frac{R}{cT}\sqrt{2H\bar{\lambda}} \tag{13}$$

That is to say, we can now measure the Planck length from dropping a ball with a built-in stopwatch, as T is the drop time and H is the height of the drop; furthermore, R is the radius of the Earth and c is the speed of light that we can look up or measure independently. Figure 1 shows a picture of our drop-ball with built-in stopwatch.

Figure 1: The picture shows a ball with built-in stopwatch that starts when the ball is dropped (release of the button) and stops at impact, it has one hundredth of a second precision. We have used this ball to indirectly measure the Planck length one hundred times without knowledge of G as reported in Table 1.

We dropped this ball one hundred times from a height of approximately 2 meters. The time the ball used to fall this distance we read after each ball drop and inputted this time into formula 13. The formula also requires the reduced Compton wavelength of the Earth and this, as we have demonstrated in section 2, can be found independent of any knowledge of G or h. Table 1 reports the hundred ball-drop times and the corresponding Planck length we obtain from formula 13. Our average value from one hundred ball-drop measurements is 1.56×10^{-35} m. Almost all these values are somewhat lower than the CODATA 2019 value for the Planck length, which is 1.616255×10^{-35}. That our predictions from such measurements are lower is partly expected — first of all the formula we use is only valid for an ideal ball drop when the ball is not rotating at all during the drop and also that the drop is performed in vacuum. Both these effects give a longer time and lower Planck length estimate than under ideal conditions. We have merely performed the ball drops in a library room with normal room temperature. Air drag will slow the ball-drop time somewhat. After adjusting for air drag [24] we get an average estimated Planck length of approximately 1.59×10^{-35} meters. The theoretical time for the fall including air drag is given by the formula

$$t = \sqrt{\frac{m}{gk}} acosh(e^{Hk/m}) \tag{14}$$

where for our case the m is the kg mass of the clock-ball. This we measured on a kitchen weight to be 121 grams, further H is the height as before, 2 meters, and $k = \frac{1}{2}\rho C_d 4\pi r^2$, where r is the radius of the drop ball that we measured be 5 centimeters and C_d is the drag coefficient that is 0.47 for a ball, and ρ is the air density where we used 1.225 kg/m^3. So, to perform the adjustment we calculate the theoretical time for the drop in vacuum minus the theoretical time in the air and adjust our time numbers for this difference. This gives the adjusted Planck length. Our Planck length measure of 1.59×10^{-35} after air-drag adjustment is naturally still less accurate than the value given by CODATA 2019 which is from very accurate measurements with expensive apparatus that have been used to measure G, that again have been converted into the Planck length indirectly from dimensional analysis. Still, we found it interesting that we, with such very simple equipment, can measure the Planck length, which can easily be utilized in a classroom situation with minimal budget. More importantly, we have demonstrated that this can be done without any knowledge of G. Our way to find the Planck length is also very easy to perform, based on the fact that one already knows the Compton wavelength of the Earth. An interesting question is naturally why it is possible to measure the Planck length independent of G and $\hbar$? One possibility is that G is really a composite constant of the form $G = \frac{l_p^2 c^3}{\hbar}$ as has been suggested by [25], and that it is the Planck length and the speed of light (gravity) that are really important for gravity predictions and observations. The Planck constant we have seen cancels out with the Planck length in the mass, something discussed in more detail in [26]. This would still mean G is a universal gravitational constant, but that it consists of more fundamental constants. This is a new view and we naturally do not ask anyone to take this for granted, but rather to investigate it carefully. Recently we [26] have argued that G, h and c can be replaced with only c and l_p, but this and other questions we leave for further discussion. The main contribution from this paper is that one can very easily measure the Planck length quite accurately with a very simple and cheap device that is ideal for use in classroom settings. We hope this will encourage both researchers and students of physics to look closer at the Planck length, both from a theoretical and experimental stand point.

Ball drop observation	Time measured	Planck length estimate	Ball drop observation	Time measured	Planck length estimate	Ball drop observation	Time measured	Planck length estimate
1	0.65	1.59E-35	34	0.66	1.56E-35	67	0.67	1.54E-35
2	0.65	1.59E-35	35	0.67	1.54E-35	68	0.67	1.54E-35
3	0.67	1.54E-35	36	0.67	1.54E-35	69	0.66	1.56E-35
4	0.67	1.54E-35	37	0.68	1.52E-35	70	0.65	1.59E-35
5	0.64	1.61E-35	38	0.66	1.56E-35	71	0.67	1.54E-35
6	0.66	1.56E-35	39	0.69	1.49E-35	72	0.66	1.56E-35
7	0.66	1.56E-35	40	0.67	1.54E-35	73	0.67	1.54E-35
8	0.64	1.61E-35	41	0.67	1.54E-35	74	0.66	1.56E-35
9	0.65	1.59E-35	42	0.67	1.54E-35	75	0.68	1.52E-35
10	0.65	1.59E-35	43	0.69	1.49E-35	76	0.69	1.49E-35
11	0.64	1.61E-35	44	0.64	1.61E-35	77	0.65	1.59E-35
12	0.66	1.56E-35	45	0.67	1.54E-35	78	0.67	1.54E-35
13	0.67	1.54E-35	46	0.65	1.59E-35	79	0.66	1.56E-35
14	0.65	1.59E-35	47	0.66	1.56E-35	80	0.68	1.52E-35
15	0.66	1.56E-35	48	0.67	1.54E-35	81	0.67	1.54E-35
16	0.69	1.49E-35	49	0.67	1.54E-35	82	0.67	1.54E-35
17	0.67	1.54E-35	50	0.64	1.61E-35	83	0.68	1.52E-35
18	0.67	1.54E-35	51	0.67	1.54E-35	84	0.67	1.54E-35
19	0.67	1.54E-35	52	0.69	1.49E-35	85	0.69	1.49E-35
20	0.68	1.52E-35	53	0.67	1.54E-35	86	0.68	1.52E-35
21	0.66	1.56E-35	54	0.65	1.59E-35	87	0.63	1.64E-35
22	0.66	1.56E-35	55	0.65	1.59E-35	88	0.65	1.59E-35
23	0.65	1.59E-35	56	0.66	1.56E-35	89	0.64	1.61E-35
24	0.67	1.54E-35	57	0.67	1.54E-35	90	0.65	1.59E-35
25	0.66	1.56E-35	58	0.68	1.52E-35	91	0.66	1.56E-35
26	0.66	1.56E-35	59	0.64	1.61E-35	92	0.66	1.56E-35
27	0.67	1.54E-35	60	0.67	1.54E-35	93	0.65	1.59E-35
28	0.65	1.59E-35	61	0.66	1.56E-35	94	0.65	1.59E-35
29	0.69	1.49E-35	62	0.67	1.54E-35	95	0.66	1.56E-35
30	0.65	1.59E-35	63	0.66	1.56E-35	96	0.66	1.56E-35
31	0.67	1.54E-35	64	0.67	1.54E-35	97	0.65	1.59E-35
32	0.67	1.54E-35	65	0.67	1.54E-35	98	0.67	1.54E-35
33	0.67	1.54E-35	66	0.67	1.54E-35	99	0.66	1.56E-35
						100	0.65	1.59E-35
	Average time :	0.664				Average	Planck length :	**1.56E-35**
	Air drag adjusted time :	0.640				Air drag adjusted	Planck length :	**1.59E-35**

Table 1: Hundred Planck length measured with a stop-clock ball. Drop height 2 meters, reduced Compton wavelength used for the Earth $5.89 \times 10^{-68} m$, be aware this can be found without knowledge of G or $\hbar$.

We have demonstrated that the Planck length can be found independent of G and h simply by using a ball with a built-in stowatch. This supports the idea that the Newton gravitational constant is likely a composite constant of the form $G = \frac{l_p^2 c^3}{\hbar}$ where $\hbar$ is needed to cancel out the Planck constant in the kg definition of mass. Our more direct (but still indirect) way of finding of the Planck length supports our theory that the detection of gravity is indeed the indirect detection of the Planck scale. We do not however assume this should or will be easily accepted. Still, such an idea should be of interest to the research community. After all early on it was even ridiculed that the Planck length had anything to do with gravity. Today however, most researchers in the field of quantum gravity think the Planck scale is essential, but they have just not figured out exactly how as yet. We hope this paper can encourage more researchers to consider the implications of why we can find the Planck length without any reliance on G. We also hope this simple way of measuring the Planck length can encourage a future generation of physics students to become interested in the Planck scale.

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Data Availability Statement

No data has been used for this paper, except the CODATA 2019 Planck length number that is publicly available at

https://physics.nist.gov/cgi-bin/cuu/Value?plkl