Kinematics Model Approach to Energy Efficiency in Sustainable Manufacturing

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Abstract

As the energy efficiency is a worldwide topic for sustainability in sustainable manufacturing, the industry ought to undertake the responsibility in a low-energy manner. For energy efficiency, this paper proposed a novel lightweight Non-holonomic Spherical Underactuated Robot (NSUR) with lower energy consumption and fewer actuators than traditional robotics. The kinematics of a robot refers the motion analysis, such as coordinates, velocity, and acceleration. However, current researches always focus on the holonomic full-actuated robot. This paper is devoted to a kind of novel NSUR and its theoretical kinematics analysis model. After NSUR was introduced, the kinematic representation of NSUR was discussed. A pair inverse and forward kinematics analysis of NSUR was put forward in detail. The kinematic analysis of joint of underactuated robot, as the application of NSUR, is applied to explain and verify this methodology. The measurements and comparisons of energy efficiency between NSUR and traditional robotics are also derived in detail.

Keywords: *energy efficiency; non-holonomic robotics; underactuated robotics; kinematics analysis; sustainable manufacturing.*

1. Introduction

Energy is considered to be critical in societal and industrial activities, especially in intelligent manufacturing and robotics [1]. As the energy consumption is a recent global concern in sustainable manufacturing, it should take the responsibility through a low energy cost approach to clean energy [2], a low carbon footprint approach based on low carbon emissions [3], a low-product environmental footprint approach to sustainable assessment [4-5], optimize model-driven energy efficiency improvement methods [6-7]. Current methods for low-energy could be basically classified into to increase energy efficiency and to reduce energy consumption in sustainable manufacturing.

Energy efficiency is a hot topic in the sustainable world [8]. Energy efficiency is simply the ratio of energy services out to energy input [9]. Energy efficiency refers to adoption of a specific technology that reduces overall energy consumption without changing relevant behaviour [10]. Energy efficiency concerns the technical ratio between the quantity of primary or final energy consumed and the maximum quantity of energy services obtainable heating, lighting, cooling, mobility, and others, whilst end-use energy saving addresses the reduction of final energy consumption, through energy efficiency improvement or behavioural change [11].

Given the close relationship between the characteristics of different energy sources and energy efficiency, many researchers are looking for ways to improve energy efficiency from the energy source itself. The first generation of biofuels produced by fermentation of food materials such as sugar and starch are mainly biodiesel and bioethanol. The oxygen content of ethanol is beneficial to reduce emissions, and biodiesel could be used as a solvent for ethanol and diesel. The

second generation biofuels produced from non-food raw materials such as wooden rods and other agricultural residues, corn and straw fibers are mainly butanol and 2,5-Dimethylfuran (DMF). N-butanol has the advantages of high energy density, low volatility, good hydrophilicity, etc. It is less corrosive to the fuel system and more suitable for transportation and storage [12]. DMF has a higher octane than ethanol and a higher energy density than n-butanol, and the production cost can be reduced by more than 60% [13].

The reduction of energy consumption through improvements in energy efficiency has become an important goal to increase energy supply security, and to reduce the emissions of CO₂ and other pollutants caused by power generation [14-16]. In this field, there are many approaches, such as lightweight material [17], lightweight design [18], co-evolutionary algorithm [19], low-carbon manufacturing [20], structural optimization [21], digital twin-driven sustainable manufacturing [22] and new structural solutions for low-energy cost. New structural solution of mechanical products could be the most effective and essential approach to low-energy cost [23]. This paper proposed a new structural solution of robotics for low-energy cost, i.e. a novel Non-holonomic Spherical Underactuated Robotics (NSUR) with fewer actuators than its Degrees of Freedom (DOFs).

The robot is one of the most fundamental elements in advanced engineering. Contrast with the dynamics of robotics, the kinematics is involved in the motion of robots, such as the position [24], velocity and acceleration [25]. The kinematics of robots [26] includes forward kinematics and inverse kinematics. The forward kinematics is the kinematic equations of the position and orientation of a robot's end effector as a function of its joint angles [27]. In the inverse

kinematics, it computes the joint parameters that achieve a specified position and orientation of the end effector [28].

The holonomic and non-holonomic robots are concepts from the constraint and system in classical mechanics discipline [29]. The holonomic constraint is the relationship between the position variables and possibly time, which could be expressed in certain equations to describe the system. The constraint is non-holonomic, if it could not meet the above requirements. For example, the motion of a particle constrained to lie on the surface of a sphere is subject to a holonomic constraint, but if the particle is able to fall off the sphere under the influence of gravity, the constraint becomes non-holonomic.

A robot could be classified into underactuated robot, full-actuated robot, and redundant-actuated robot, based on the relationship between the number of its actuators and its DOFs [30]. The underactuated robot's number of actuators is smaller than its DOFs with advantages, such as light weight, low consumption [31], such as differential robots [32], compliant robots [33], triggered robots, and passive robots [34]. It is becoming new direction of robotics with widespread concerns in space-in-orbit robots [35], underwater robots [36], highly-flexible robots, and parallel robots [37], etc.

Generally speaking, researchers often concentrate on traditional full-actuated robot in engineering. However, it seems more feasible for a fewer actuators' robot with the advantage of non-holonomic constraints. It is essential to achieve the kinematics model of NSUR. The relationship between the actuator and the output mechanism could be figured out with this kinematics method including both inverse and forward analysis specifically. It is certain that there are several kinematics divergences between traditional holonomic robots and NSUR.

Considering a unique free-joint angle as a variable robot drive, it is necessary to collect the values of driving-joint angle so that the forward kinematics of the NSUR could be derived correctly. Similarly, the value of the passive joint and the detecting sensor are equivalent.

The remainder of this paper carries out some other works. Section 2 introduces a novel non-holonomic underactuated spherical robotics. Section 3 provides kinematic representation of the novel NSUR. Section 4 provides forward kinematics analysis and inverse theoretical kinematics analysis of the novel NSUR, and the kinematic analysis is further used to explain the proposed methodology. Section 5 makes vital valuations and comparisons of energy efficiency between traditional robotics. Section 6 summarizes this research.

2.Introduction of a novel NSUR for energy efficiency

This paper proposes a novel kind of NSUR, as shown in Fig. 1.

Fig. 1. NSUR with 3 free-rollers

This kind of robot consists of 3 free-rollers and a globular body. These free-rollers are evenly distributed with rolling contact on the inner surface of the

globular body with 120° . All the axes of these free-rollers are coplanar through the center of the globular body.

It takes advantage of rolling contact with certain pressure to maintain the relative-rolling connection among the inner-surface of globular body and free-rollers. Therefore, it could enhance the accuracy. It could be also seen that this new kind of robot could save volume of spatial structures, and enhances the kinematics accuracy of NSUR's constraint condition. The institutional load could be totally dispersed to these free-rollers with the result of better bearing performance. Two of 3 free-rollers are selected to be mechanism drives, which could approximately achieve 3-DOF and compose a 3-free-roller NSUR.

As a new type of robot configuration, NSUR mainly achieves energy efficiency improvement through three-wheel non-holonomic spherical driving and under-actuated configuration reduction. The non-holonomic spherical drive can reduce the set of paths covered between the two reachable poses while ensuring that the position of the reachable space point is not reduced. On the one hand, the reduction of path set can eliminate inappropriate path planning schemes, so as to improve the decision-making efficiency of mechanism motion planning and achieve the purpose of reducing the unstable running time and energy consumption of the mechanism. On the other hand, the decrease of feasible path is accompanied by the increase of new reaction forces in joints, which could be effectively used to eliminate the actuator. In this way, the underactuated configuration could be used to reduce hardware complexity, so as to reduce joint load and energy consumption. The underactuated configuration of NSUR is embodied in two aspects. First, inspired by the reduction of the set of non-holonomic spherical drive paths, the built-in three-wheel design satisfies the

condition that the number of drives is less than the degree of freedom, which can improve the flexibility of spherical motion. Second, the NSUR consisting of a power-driven branch and a non-complete spherical drive branch can realize three degrees of freedom movement of the platform with two driving sources, which greatly reduces the number of parts of the mechanism. In general, NSUR provides a new idea to improve energy efficiency, that is, to achieve reduction of the mechanism through the non-holonomic spherical driving constraint under the parallel configuration.

3. Kinematic representation of NSUR for energy efficiency

The constraints of NSUR have unique structural features and their kinematics model is also very peculiar. In Figure 1, the angular displacement of the i_{th} free-roller is θ_i , and the axis direction of the 1_{st} free-roller is the Y-axis. The X-axis is vertical to the Y-axis and on the plane of the fixed stable plane (plane S), and the globular body could rotate around the axis of the plane S and over the sphere centre. Define the angular displacement of the axis θ_o , the angle of the positive direction and the X-axis β . The angle φ is between the 1_{st} and 2_{nd} free-roller, is called the structural angle as 120°.

At the point where the free-roller is contacted to the body, the velocity of the globular body is perpendicular to the stationary plane going through the free-roller's axis and the center of the body. It is available that:

$$R \sin \beta \dot{\theta}_o = r \dot{\theta}_1 \quad (1)$$

$$-R \sin(\varphi - \beta) \dot{\theta}_o = r \dot{\theta}_2 \quad (2)$$

where r is the radius of the inner wheel, R is the spherical radius.

Thus, it could be found that if β is a constant, in other words, the globular body rotates around a fixed axis in the plane S, which is continuously at an angle to the X-axis, Eq(1) and (2) could be integrated. This constraint becomes a holonomic constraint. In the contrast, for the non-holonomic spherical structure, the globular body could be circled around the axis in the plane S, β is constantly changing, leading to Eq(1) and (2) on the time is not integral. This kind of mathematical constrains that are not integral promptly is a non-holonomic constraint. This indicates that β is constantly changing so that the constraint is called as a non-holonomic constraint.

In actual, the constraints of NSUR are partially similar to the holonomic spherical constraints, which could also achieve 3 rotational DOFs, from a random initial posture to one terminal posture, both sharing the same bit space. However, there are some differences between them. For a holonomic spherical constraint, any movement in the bit space that is not accessible to the unreachable space is possible. For a non-holonomic spherical mechanism, the path could not only intersect the unreachable space, but also measure up the non-holonomic condition. That is the movement of the NSUR is not arbitrary in the bit space.

The constraints of NSUR could achieve 3 rotational DOFs by reaching an arbitrary terminal posture from an initial posture by:

1) For P_A and P_B incompletely in the plane S, link P_A and P_B in a straight line, take the intersecting lines through the plane perpendicular to the line and over the center of the sphere and plane S as the axis, revolve around the axis to access P_B .

2) For P_A and P_B both lie in the plane S, the same method is applied. P_A and P_B are connected into a straight line, the intersecting lines could be taken

through the plane perpendicular to the line and over the center of the sphere and the plane S as the axis, and rotate around the axis to reach an attitude of P_B.

Therefore, the constraint of NSUR is the same as the holonomic spherical constraint, which could achieve 3 DOFs, from an initial attitude to any terminal attitude.

The differences among the several path selections could be illustrated by a simple example, as follows.

Two points P_A and P_B are in the plane S. For the constraints of NSUR, first, connect P_A and P_B to a straight line, then take the vertical line and the center of the plane and the plane S intersection line as the axis, around the axis rotation could reach P_B.

For the holonomic spherical constraint, this path around Z-axis rotation could shift the globular body position from P_A to P_B. However, the non-holonomic spherical restraint robot could not follow this path, that is, non-holonomic constraints could also access the spherical z-axis rotation DOF, but the whole path between the points is reduced.

Furthermore, the kinematic characteristics of this NSUR could be obtained when Eq(1) is divided by Eq(2).

$$-\sin \beta / \sin(\varphi - \beta) = \dot{\theta}_1 / \dot{\theta}_2 \quad (3)$$

Assuming that the free-rollers 1 and 2 are treated as drive wheels, and θ_1, θ_2 is known.

1) when $\varphi \neq 180^\circ$

β could be obtained by the formula (3), θ_0 could be calculated by β substituted into the formula (1). That is, the rotation axis and the spinning speed of the globular body could be determined, the globular body would be

unconstrained and could be rotated around the plane S and through any axis of the center of the sphere, along with the change in the value of the angular velocity of the input driving free-rollers 1, 2 and the change of $\dot{\theta}_1 / \dot{\theta}_2$.

2) when $\varphi = 180^\circ$

$$\sin(\varphi - \beta) = -\sin \beta \quad (4)$$

$$-\sin \beta / \sin(\varphi - \beta) = -\sin \beta / -\sin \beta = 1 \quad (5)$$

That is, the size of β is irrelevant with $\dot{\theta}_1, \dot{\theta}_2$, β could be transformed between $0-180^\circ$, and could not be determined. This is a two-free-roller robot.

From the above analysis, while $\varphi \neq 180^\circ$, it could be found that the robot shown in Figure 1 that the two drive wheels could be driven at any speed around the plane S by any axis on the plane S and through of the center of the ball. In other word, the three DOFs of the globular body could be controlled just through two drives, and the number of drives is less than the DOFs. In fact, this structure is called as an underactuated robot.

Of course, this is only a simple form of the constraints of NSUR in underactuated institutions. It could also be applied in a variety of robots in the form of spherical constraints to produce underactuated robots, especially in parallel robots. It requires studying the kinematic controllability of various underactuated structures. Therefore, it is very important to carry out the correct formulation of the kinematic characteristics of the NSUR's constraints.

In the existing literature, the kinematic characteristics of the NSUR's constraints are reduced to the complex motion around X and Y axes. Let's talk about this simplicity. It could be analyzed by P_A and P_B all in the plane S in Eq(2).

Assuming that the compound motion is ϕ around the X-axis and the rotation τ around the Y-axis, the D-H representation method is shown as follows.

$$R(x, \phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix}, \quad R(y, \tau) = \begin{bmatrix} \cos \tau & 0 & \sin \tau \\ 0 & 1 & 0 \\ -\sin \tau & 0 & \cos \tau \end{bmatrix} \quad (6)$$

The transformation matrix T of the motion is

$$T = R(y, \tau)R(x, \phi) = \begin{bmatrix} c\tau & 0 & s\tau \\ 0 & 1 & 0 \\ -s\tau & 0 & c\tau \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix} = \begin{bmatrix} c\tau & s\tau s\phi & s\tau c\phi \\ 0 & c\phi & s\phi \\ -s\tau & c\tau s\phi & c\tau c\phi \end{bmatrix} \quad (7)$$

where c represents cosine function, s represents sine function.

According to 2) P_A and P_B both in the plane S, spherical position from P_A to P_B , the transformation of the array could also be simplified to rotate around Z-axis.

$$R(z, \theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (8)$$

At this time, $T=R(z, \theta)$, then $\phi=0^\circ$, $\tau=0^\circ$. The conditions meet when the globular body could not move. Obviously, this simplification is unreasonable, because it could not correctly represent the globular body pose changes. Therefore, a new simplified method is necessary to represent the pose changes of the globular body in the non-holonomic constraint.

The position of the globular body could be defined as β and θ_o . The pose change of the globular body should be appeared in the NSUR's constraint as long as these two parameters of the transformation matrix are represented. Pose changes could be broken down into two sequential rotary motion. First, the globular body rotates β around the Z-axis to reach the required position of the

shaft. At this position, the axis of rotation is the current X-axis. And then θ_o around the axis of rotation, that is, the rotating angle θ_o around the current X-axis.

$$R(z, \beta) = \begin{bmatrix} \cos \beta & -\sin \beta & 0 \\ \sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R(x, \theta_o) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_o & -\sin \theta_o \\ 0 & \sin \theta_o & \cos \theta_o \end{bmatrix} \quad (9)$$

Because the rotation is θ_o around the current X-axis, so the required transformation matrix is available by left-multiplication method, that is:

$$\begin{aligned} T_1 &= R(z, \beta) R(x, \theta_o) = \begin{bmatrix} \cos \beta & -\sin \beta & 0 \\ \sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_o & -\sin \theta_o \\ 0 & \sin \theta_o & \cos \theta_o \end{bmatrix} \\ &= \begin{bmatrix} \cos \beta & -\sin \beta \cos \theta_o & \sin \beta \sin \theta_o \\ \sin \beta & \cos \beta \cos \theta_o & -\cos \beta \sin \theta_o \\ 0 & \sin \theta_o & \cos \theta_o \end{bmatrix} \end{aligned} \quad (10)$$

The above deduction provides a new idea of simplifying the complex kinematics of non-holonomic spherical restraints. In this way, the constraint of NSUR is regarded as two independent rotating joints, and the posture transformation is presented by the rotation transformation matrix by the D-H method.

4. Kinematics analysis of the novel NSUR for energy efficiency

4.1 Establishment of the space coordinate system

Since the axis of joint rotation of NSUR intersects at a point, the directions of each joint sub-coordinate system could be determined more quickly and conveniently in the spatial coordinate system, thus improving the kinematic

solution efficiency of NSUR. As it is presented in Figure 2, the output platform $V_1V_2V_3$ is connected to a fixed triangular platform $U_1U_2U_3$ by two concentric 3R-series branch $U_1W_1V_1$ and $U_2W_2V_2$, and a ball pair S. The rotation axis of all the kinematic chains of the robot is converged at the center of the ball pair S. The pair S could be decomposed for that it is difficult to express with the D-H parameter method. The ball pair has 3 rotation DOFs, it is equivalent to three series of rotation vice. The three axes of rotation reach a point, which is concentric. Similarly, the pair S is approximate to a branch of concentric 3R series. For making follow-up description and resume kinematics model convenient, it is simplified by the corresponding virtual chain $U_3W_3V_3$. It would make it easier to comprehend the established kinematic model.

Fig. 2. Non-holonomic underactuated robot joint

According to the D-H method, the non-holonomic underactuated robot joint is shown in Fig.2. $O_i - X_iY_iZ_i (i = A - L)$, character i represents the corresponding joint, including the joint of the branches and three joints

corresponding to the virtual chain for the ball pair. Due to the axis of the joint of non-holonomic and underactuated robot joint are not in parallel, the origin of nine coordinate systems could be coincident with O . Here, the first joint axis of the first branch is established. The origin of $O_i - X_i Y_i Z_i (i = F)$ is O . The Z_F -axis coincides to the axis of the joint. The direction of the X_F -axis is the normal direction of the plane Z and Z_F . $X_F = \pm Z_F \times Z$, Y_F is determined by the right-hand spiral rule, and $Y_F = \pm X_F \times Z_F$.

By the same way, the other coordinates could be found. The $O - XYZ$ is placed on the fixed platform. The Z -axis is upright and the X -axis direction is the normal direction of the plane defined by the Z -axis and Z_F -axis, named as $X = Z \times Z_F$. The flexible-moving coordinate system $O - X'Y'Z'$ is defined by the movement extremity and the origin coincides with the origin one. The Z' -Axis is upright and the X' -axis direction is the normal direction of the plane determined by the Z' -axis and the Z -axis.

At the initial position, the static coordinate system goes along with the moving-platform coordinate system. The non-holonomic underactuated parallel robot of exoskeleton robot could be analyzed by D-H method, the D-H parameter is associated with the connecting rod j adjacent to the link i , that is $\theta_{AF}, \theta_{BA}, \theta_{CB}, \theta_{DC}, \theta_{ED}, \theta_{FE}, \theta_{FH}, \theta_{GF}, \theta_{HG}, \theta_{IG}, \theta_{JI}, \theta_{KJ}, \theta_{CK}, \theta_{KL}$ and θ_{LC} . Corresponding to the angle among the plane of the adjacent connecting rod, and the angle among the connecting rod from the rotation of the subject axis to the ball looks positive counterclockwise. By the characteristics of the spherical body, the whirl of the auxiliary axis are remitted to the center of the ball O , that is:

$$a_i = 0 \quad (i = A-L)$$

$$d_{AF}=d_{BA}=d_{CB}=d_{DC}=d_{ED}=d_{FE}=d_{FH}=d_{GF}=d_{HG}=d_{IG}=d_{JI}=d_{KJ}=d_{CK}=d_{KL}=d_{LC}=0$$

$$\alpha_A = \alpha_E = \alpha_I = \alpha_1; \alpha_B = \alpha_D = \alpha_J = \alpha_2, \alpha_F = \alpha_G = \alpha_H = \alpha_3, \alpha_C = \alpha_K = \alpha_L = \alpha_4$$

Fixed in the i_{th} link local coordinate system is the center of the ball point. The Z-axis is in the direction of the axis of rotation (where Z_B and Z_K coincide, Z_E and Z_H coincide). The Y_i -axis ($i=A-L$) is located in the i_{th} connecting rod on the plane and on the side of the solid rod, X_i ($i=A-L$) axis is perpendicular to the plane of the corresponding link to constitute the right hand rectangular coordinate system. The transformation matrix between adjacent and link coordinate system is:

$$T_{ij} = Rot(z, \theta_{ij}) Trans(0, 0, d_{ij}) Trans(a_i, 0, 0) Rot(x, \alpha_i) \quad (11)$$

$(i = A \sim L, j = A \sim L, i \neq j)$

4.2 Inverse kinematics analysis

There is a method of obtaining an inverse solution of a non-holonomic underactuated parallel robot joint. Two input angles θ_{AF} and θ_{EH} of the drive sources of parallel robot joint of the NUSR could be calculated by three known posture angles of the movement extremity (ψ , θ , ϕ). The axial unit vectors u_i, w_i, v_i are the positional representations of the joints in the fixed coordinate system O-XYZ, respectively.

1) Axis direction unit vectors of fixed platform u_1, u_2, u_3

According to Fig.2:

$$u_1 = (0, -\sin \beta_1, -\cos \beta_1)^T \quad (12)$$

$$u_i = R_{\delta_i} u_1 = (\sin \delta_i \sin \beta_1, -\cos \delta_i \sin \beta_1, -\cos \beta_1)^T \quad (13)$$

In the above formula:

$$\mathbf{R}_{\delta_i} = \begin{pmatrix} \cos \delta_i & -\sin \delta_i & 0 \\ \sin \delta_i & \cos \delta_i & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \delta_1 = 0, \quad \delta_2 = 120^\circ, \quad \delta_3 = 240^\circ \quad (14)$$

2) Intermediate axis unit vectors $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3$

Because the non-holonomic underactuated robot joint is a spherical robot, it is of great significance to solve kinematics by the middle joint axis vector $(\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3)$ of the three branches in the institution.

(1) the vector cosine of $\mathbf{w}_1$

Create an auxiliary coordinate system here, as shown in Fig.3.

Fig.3. Vector w_1

It is built in $O-XYZ$. The auxiliary Z_F -axis is the OZ_{11} -axis, the Z -axis of the static coordinate system is OZ -axis, the auxiliary coordinate system Z_B -axis is the OZ_{12} -axis. Therefore, the angle among the OZ -axis and the OZ_{11} -axis is $(\pi - \beta_1)$, the angle among the OZ_{11} -axis and the OZ_{12} -axis is α_1 , the outer angle of the OZ_{12} -axis is θ_1 . Through the geometric relationship, three vector cosine w_{1x}, w_{1y}, w_{1z} of $\mathbf{w}_1$ could be drawn.

$$w_{1x} = s_{23}s_2 = \sin \alpha_1 \sin \theta_1 \quad (15)$$

$$\begin{aligned} w_{1y} &= -(s_{12}c_{23} + c_{12}s_{23}c_2) = -[\sin(\pi - \beta_1)\cos \alpha_1 + \cos(\pi - \beta_1)\sin \alpha_1 \cos \theta_1] \\ &= -\sin \beta_1 \cos \alpha_1 + \cos \beta_1 \sin \alpha_1 \cos \theta_1 \end{aligned} \quad (16)$$

$$\begin{aligned} w_{1z} &= c_{12}c_{23} - s_{12}s_{23}c_2 = \cos(\pi - \beta_1)\cos \alpha_1 - \sin(\pi - \beta_1)\sin \alpha_1 \cos \theta_1 \\ &= -\cos \beta_1 \cos \alpha_1 - \sin \beta_1 \sin \alpha_1 \cos \theta_1 \end{aligned} \quad (17)$$

In the above formula, s_{23}, c_{23} is the abbreviation of the sine and cosine of the angle of the OZ₁₁-axis and the OZ₁₂-axis, s_{12}, c_{12} is the abbreviation of the sine and cosine of the angle of the OZ₁₁-axis and the OZ-axis, s_2, c_2 is the abbreviation of the sine and cosine of the outer angle of the OZ-axis.

$$\mathbf{w}_1 = \begin{pmatrix} w_{1x} \\ w_{1y} \\ w_{1z} \end{pmatrix} = \begin{pmatrix} \sin \alpha_1 \sin \theta_1 \\ -\sin \beta_1 \cos \alpha_1 + \cos \beta_1 \sin \alpha_1 \cos \theta_1 \\ -\cos \beta_1 \cos \alpha_1 - \sin \beta_1 \sin \alpha_1 \cos \theta_1 \end{pmatrix} \quad (18)$$

(2) the vector cosine of $\mathbf{w}_2$

Similarly,

$$\begin{aligned} \mathbf{w}_2 &= \mathbf{R}_{\delta_i} \mathbf{w}_1'' \\ &= \begin{pmatrix} \cos \delta_i \sin \alpha_1 \sin \theta_2 + \sin \delta_i \sin \beta_1 \cos \alpha_1 - \sin \delta_i \cos \beta_1 \sin \alpha_1 \cos \theta_2 \\ \sin \delta_i \sin \alpha_1 \sin \theta_2 - \cos \delta_i \sin \beta_1 \cos \alpha_1 + \cos \delta_i \cos \beta_1 \sin \alpha_1 \cos \theta_2 \\ -\cos \beta_1 \cos \alpha_1 - \sin \beta_1 \sin \alpha_1 \cos \theta_2 \end{pmatrix} \end{aligned} \quad (19)$$

where $\delta_2 = 120^\circ$.

(3) the vector cosine of $\mathbf{w}_3$

Similarly,

$$\mathbf{w}_3 = \begin{pmatrix} \cos \delta_i \sin \alpha_1 \sin \theta_3 + \sin \delta_i \sin \beta_1 \cos \alpha_1 - \sin \delta_i \cos \beta_1 \sin \alpha_1 \cos \theta_3 \\ \sin \delta_i \sin \alpha_1 \sin \theta_3 - \cos \delta_i \sin \beta_1 \cos \alpha_1 + \cos \delta_i \cos \beta_1 \sin \alpha_1 \cos \theta_3 \\ -\cos \beta_1 \cos \alpha_1 - \sin \beta_1 \sin \alpha_1 \cos \theta_3 \end{pmatrix} \quad (20)$$

where $\delta_3 = 240^\circ$.

3) The position of the movement extremity

The following shows the coordinate relationship of the movement extremity with the Euler angles, and $O-X_0Y_0Z_0$ coincides with $O-XYZ$, as shown in Fig. 4.

Fig. 4. The position of moving extremity

First, there is a horizontal plane π_1 through the center which intersects the equatorial plane $\pi_0(OX_3Y_3Z_3)$ with the straight line D_1D_2 . Rotate ψ around Z_0 -axis to make X_1 -axis coincides with OD_1 and get to the location $O-X_1Y_1Z_1$ from the $O-X_0Y_0Z_0$. Then turn θ around the X_1 -axis, make the axis $Y_2 \in \pi_0$ and get to the position $O-X_2Y_2Z_2$. After that, turn ϕ around the Z_2 -axis and reach the location $O-X_3Y_3Z_3$ in the plane π_0 . At this moment, it coincides with $O-X'Y'Z'$ of the movement extremity. Therefore, the angle ψ, θ, ϕ is defined as the azimuth of the movement extremity, $\psi = \angle XOD_1$, $\theta = \angle ZOZ_3$, $\phi = \angle D_1OX_3$. They are a group of independent parameters, which could lead to the relative posture of the above sphere in the inertial coordinates always. The entire transformation process is:

$$(\text{OX}_0\text{Y}_0\text{Z}_0) \xrightarrow[\psi]{Z_0} (\text{OX}_1\text{Y}_1\text{Z}_1) \xrightarrow[\theta]{X_1} (\text{OX}_2\text{Y}_2\text{Z}_2) \xrightarrow[\phi]{Z_2} (\text{OX}_3\text{Y}_3\text{Z}_3)$$

Therefore, the posture conversion matrixs among between the moving-platform $O-X'Y'Z'$ and $O-XYZ$ is:

$$\begin{aligned} Q_{\psi,\theta,\phi} &= Q_{Z_0,\psi} = Q_{X_1,\theta} = Q_{Z_2,\phi} \\ &= \begin{pmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{pmatrix} \begin{pmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c\psi c\phi - s\psi c\theta s\phi & -c\psi s\phi - s\psi c\theta c\phi & s\psi s\theta \\ s\psi c\phi + c\psi c\theta s\phi & -s\psi s\phi + c\psi c\theta c\phi & -c\psi s\theta \\ s\theta s\phi & s\theta c\phi & c\theta \end{pmatrix} \end{aligned} \quad (21)$$

wherein, $c\psi = \cos \psi$, $s\psi = \sin \psi$, the rest is similar.

4) Axial unit vector $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ of the movement extremity joint

It could be seen from the figure that the axial unit vector ($\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$) of the movement extremity joint coincides with z_{13}, z_{23}, z_{33} . In $O-XYZ$, it is expressed as ${}^Q\mathbf{v}_1, {}^Q\mathbf{v}_2, {}^Q\mathbf{v}_3$. In $O-X'Y'Z'$, it is expressed as ${}^P\mathbf{v}_1, {}^P\mathbf{v}_2, {}^P\mathbf{v}_3$.

Through the spherical robot of the geometric relationship:

$${}^P\mathbf{v}_i = \mathbf{R}_{\delta_i}(0, -\sin \beta_2, \cos \beta_2)^T \quad (22)$$

$$\mathbf{v}_i = {}^Q\mathbf{v}_i = Q_{\psi,\theta,\phi} {}^P\mathbf{v}_i \quad (23)$$

From the Eq(23), axis unit vector ($\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$) of the movement extremity known moving platform could be calculated by the azimuth ψ, θ, ϕ in the movement extremity.

5) Inverse kinematics

The following could be drawn due to the symmetrical structure and the angle of each branch intermediate axis and the movable platform axis is α_2 .

$$\mathbf{w}_i \cdot \mathbf{v}_i = \cos \alpha_2 \quad (i=1,2,3) \quad (24)$$

To facilitate the calculation,

$$\mathbf{v}_i = (v_{ix}, v_{iy}, v_{iz})^T \quad (i=1,2,3) \quad (25)$$

Substitute (25), (20), (19), (18) into (24), the following could be obtained:

$$\begin{aligned} \cos \alpha_2 = & (\cos \delta_i \sin \alpha_1 \sin \theta_i + \sin \delta_i \sin \beta_1 \cos \alpha_1 - \sin \delta_i \cos \beta_1 \sin \alpha_1 \cos \theta_i) v_{ix} + \\ & (\sin \delta_i \sin \alpha_1 \sin \theta_i - \cos \delta_i \sin \beta_1 \cos \alpha_1 + \cos \delta_i \cos \beta_1 \sin \alpha_1 \cos \theta_i) v_{iy} + \\ & (-\cos \beta_1 \cos \alpha_1 - \sin \beta_1 \sin \alpha_1 \cos \theta_i) v_{iz} \end{aligned} \quad (26)$$

Sort it out:

$$A_i \sin \theta_i + B_i \cos \theta_i + C_i = 0 \quad (i=1, 2, 3) \quad (27)$$

where

$$A_i = \cos \delta_i \sin \alpha_1 v_{ix} + \sin \delta_i \sin \alpha_1 v_{iy}$$

$$B_i = -\sin \delta_i \cos \beta_1 \sin \alpha_1 v_{ix} + \cos \delta_i \cos \beta_1 \sin \alpha_1 v_{iy} - \sin \beta_1 \sin \alpha_1 v_{iz}$$

$$C_i = \sin \delta_i \sin \beta_1 \cos \alpha_1 v_{ix} - \cos \delta_i \sin \beta_1 \cos \alpha_1 v_{iy} - \cos \beta_1 \cos \alpha_1 v_{iz} - \cos \alpha_2$$

Analysis the unknown parameters:

$$A_i \sin \theta_i + B_i \cos \theta_i + C_i = 0 \quad (i=1, 2, 3);$$

$$\delta_1 = 0, \quad \delta_2 = 120^\circ, \quad \delta_3 = 240^\circ;$$

β_1 is a structural parameter, that is a determined parameter before the design of the robot, the known parameter;

$\alpha_A = \alpha_1$ is a bar body parameter, the known parameter;

$\alpha_B = \alpha_2$ is a bar body parameter, the known parameter;

β_2 is a structural parameter, that is a determined parameter before the design of the robot, the known parameter;

$\theta_1, \theta_2, \theta_3$ are the outer corners, are unknown parameters;

In the three equations, $\theta_1, \theta_2, \theta_3$ are unknown. They are the known parameters. Therefore, the inverse of the underactuated robot could be obtained by solving this equation group.

4.3 Forward kinematics analysis

The following would apply the analytical geometry method to find the inverse solution, firstly, this paper establishes a kind of Cartesian coordinate system on the rotation matrix, as shown in Fig. 5.

1. The rotation matrix of the branch platform to the platform

1) The rotation matrix at a fixed platform

$O - X_F Y_F Z_F$ at the branching station could be overlapped to $O - XYZ$

with the following steps:

(a) Rotate θ_1 around the Z -axis, X_F coincide with X , $\theta_1 = 0$;

(b) Rotate $\pi - \beta_1$ along the X -axis, so that Z_F coincides with Z ,

β_1 is the known structural parameters.

The rotation matrix of $O - X_F Y_F Z_F$ at the platform 1.

$${}^O_F \mathbf{R} = Rot(z, \theta_1) \cdot Rot(x, \pi - \beta_1)$$

$$= \begin{pmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & d(\pi - \beta_1) & -s(\pi - \beta_1) \\ 0 & s(\pi - \beta_1) & d(\pi - \beta_1) \end{pmatrix} \quad (28)$$

$$= \begin{pmatrix} c\theta_1 & -s\theta_1 d(\pi - \beta_1) & s\theta_1 s(\pi - \beta_1) \\ s\theta_1 & c\theta_1 d(\pi - \beta_1) & -c\theta_1 s(\pi - \beta_1) \\ 0 & s(\pi - \beta_1) & d(\pi - \beta_1) \end{pmatrix}$$

Fig. 5. A branch link coordinate system

2) The rotation matrix of the Cartesian coordinate system corresponding to the middle of branch 1

The $O-X_A Y_A Z_A$ in the intermediate of a branch could be overlapped with $O-X_F Y_F Z_F$ by the following steps:

(a) Rotate $\alpha_A = \alpha_1$ around the X_F -axis, and then Z_A coincides with Z_F ;

(b) Rotate θ_{AF} along the Z_F -axis (θ_{AF} is the angle between X_F the angle X_A , that is, the branch of a fixed platform 1 at the rotation angle, the joint for the driving joint, so θ_{AF} is known), X_F is coincident with X_A .

$O-X_A Y_A Z_A$ is the rotation matrix relative to $O-X_F Y_F Z_F$.

$$\begin{aligned}
{}^F_A \mathbf{R} &= Rot(x, \alpha_1) \cdot Rot(z, \theta_{AF}) \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\alpha_1 & -s\alpha_1 \\ 0 & s\alpha_1 & c\alpha_1 \end{pmatrix} \begin{pmatrix} c\theta_{AF} & -s\theta_{AF} & 0 \\ s\theta_{AF} & c\theta_{AF} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c\theta_{AF} & -s\theta_{AF} & 0 \\ c\alpha_1 s\theta_{AF} & c\alpha_1 c\theta_{AF} & -s\alpha_1 \\ s\alpha_1 s\theta_{AF} & s\alpha_1 c\theta_{AF} & c\alpha_1 \end{pmatrix}
\end{aligned} \tag{29}$$

3) The rotation matrix of the related coordinate system at the branching platform 1

$O-X_B Y_B Z_B$ could be overlapped with $O-X_A Y_A Z_A$ by the following steps:

(a) Circle $\alpha_B = \alpha_2$ around the X_A -axis, so that Z_B coincides with Z_A ;

(b) Circle θ_{BA} along the Z_A -axis (θ_{BA} for the angle X_B between the angle X_A , that is, the middle of the branch 1 at the joint rotation angle, the joint is a passive joint, θ_{BA} as unknown parameters), so that X_B is coincident to X_A .

The rotation matrix is achieved by $O-X_B Y_B Z_B$ relative to $O-X_A Y_A Z_A$:

$$\begin{aligned}
{}^A_B \mathbf{R} &= Rot(x, \alpha_B) \cdot Rot(z, \theta_{BA}) \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\alpha_2 & -s\alpha_2 \\ 0 & s\alpha_2 & c\alpha_2 \end{pmatrix} \begin{pmatrix} c\theta_{BA} & -s\theta_{BA} & 0 \\ s\theta_{BA} & c\theta_{BA} & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned} \tag{30}$$

$$= \begin{pmatrix} c\theta_{BA} & -s\theta_{AF} & 0 \\ c\alpha_2 s\theta_{BA} & c\alpha_2 c\theta_{BA} & -s\alpha_2 \\ s\alpha_2 s\theta_{BA} & s\alpha_2 c\theta_{BA} & c\alpha_2 \end{pmatrix}$$

4) The rotation matrix of $O-X'Y'Z'$ and $O-X_B Y_B Z_B$

$O-X'Y'Z'$ could be overlapped with $O-X_B Y_B Z_B$ by the following steps:

(a) Circle around the axis, for the known structural parameters, so coincide with;

(b) Rotate θ_{CB} along the Z_B -axis (Identically, θ_{CB} for the angle X' between the angle X_B , that is, rotation angle at the joint of branch 1 moving platform 1, the joint is a passive joint, θ_{CB} as unknown parameters), so that X' is coincident to X_B .

So the rotation matrix is achieved by $O-X'Y'Z'$ with $O-X_B Y_B Z_B$:

$$\begin{aligned} {}^B\mathbf{R} &= Rot(x, \beta_2) \cdot Rot(z, \theta_{CB}) \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\beta_2 & -s\beta_2 \\ 0 & s\beta_2 & c\beta_2 \end{pmatrix} \begin{pmatrix} c\theta_{CB} & -s\theta_{CB} & 0 \\ s\theta_{CB} & c\theta_{CB} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (31) \\ &= \begin{pmatrix} c\theta_{CB} & -s\theta_{CB} & 0 \\ c\beta_2 s\theta_{CB} & c\beta_2 c\theta_{CB} & -s\beta_2 \\ s\beta_2 s\theta_{CB} & s\beta_2 c\theta_{CB} & c\beta_2 \end{pmatrix} \end{aligned}$$

The rotation matrix of the platform $O-X'Y'Z'$ to the fixed platform $O-XYZ$ at the branching 1 is:

$$\begin{aligned}
{}^O\mathbf{R}_1 &= {}^O\mathbf{R}_F {}^F\mathbf{R}_A {}^A\mathbf{R}_B {}^B\mathbf{R} \\
&= \begin{pmatrix} c\theta_{AF} & -s\theta_{AF} & 0 \\ c\alpha_1 s\theta_{AF} & c\alpha_1 c\theta_{AF} & -s\alpha_1 \\ s\alpha_1 s\theta_{AF} & s\alpha_1 c\theta_{AF} & c\alpha_1 \end{pmatrix} \cdot \begin{pmatrix} c\theta_{AF} & -s\theta_{AF} & 0 \\ c\alpha_1 s\theta_{AF} & c\alpha_1 c\theta_{AF} & -s\alpha_1 \\ s\alpha_1 s\theta_{AF} & s\alpha_1 c\theta_{AF} & c\alpha_1 \end{pmatrix} \cdot \begin{pmatrix} c\theta_{BA} & -s\theta_{BA} & 0 \\ c\alpha_2 s\theta_{BA} & c\alpha_2 c\theta_{BA} & -s\alpha_2 \\ s\alpha_2 s\theta_{BA} & s\alpha_2 c\theta_{BA} & c\alpha_2 \end{pmatrix} \cdot \begin{pmatrix} c\theta_{CB} & -s\theta_{CB} & 0 \\ c\beta_2 s\theta_{CB} & c\beta_2 c\theta_{CB} & -s\beta_2 \\ s\beta_2 s\theta_{CB} & s\beta_2 c\theta_{CB} & c\beta_2 \end{pmatrix}. \quad (32)
\end{aligned}$$

2. The rotation matrix of the movement extremity to the fixed platform at branching 2

1) The rotation matrix at the virtual chain 2

$O-X_H Y_H Z_H$ could be transformed with $O-XYZ$ by the following steps:

Establish another reference $O-X''Y''Z''$, O is coincided with the center of the sphere, the Z'' -axis coincides with the static Z -axis, and the X'' -axis is perpendicular to the plane defined by the Z'' -axis and the Z_H -axis. It is referred as $X'' = Z'' \times Z$.

$O-X''Y''Z''$ is rotated 120° around the static coordinate system to coincide with the static coordinate system. $O-X_H Y_H Z_H$ is first subjected to by the following transformation, $O-X_H Y_H Z_H$ is coincided with $O-X''Y''Z''$.

(1) Rotate θ_1 around the Z -axis, X_H is coincide with X'' , $\theta_1 = 0$.

(2) Rotate $\pi - \beta_1$ along the X -axis, Z_H is coincided with Z'' , β_1 is known as structural parameters.

$O-X''Y''Z''$ rotates 120° could be coincided with the original $O-XYZ$. The rotation matrix is generated by $O-X_H Y_H Z_H$ at the platform 2.

$${}^O_H\mathbf{R} = \mathbf{R}_{\delta_2} Rot(z, \theta_1) \times Rot(x, \pi - \beta_1) \quad (33)$$

$$\begin{aligned}
\mathbf{R}_{\delta_2} &= \begin{pmatrix} \cos \delta_2 & -\sin \delta_2 & 0 \\ \sin \delta_2 & \cos \delta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \delta_2 = 120^\circ \\
{}^0_H \mathbf{R} &= \begin{pmatrix} \cos \delta_2 & -\sin \delta_2 & 0 \\ \sin \delta_2 & \cos \delta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & d(\pi - \beta_1) & -s(\pi - \beta_1) \\ 0 & s(\pi - \beta_1) & d(\pi - \beta_1) \end{pmatrix} \\
&= \begin{pmatrix} \cos \delta_2 & -\sin \delta_2 & 0 \\ \sin \delta_2 & \cos \delta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\theta_1 & -s\theta_1 d(\pi - \beta_1) & s\theta_1 s(\pi - \beta_1) \\ s\theta_1 & c\theta_1 d(\pi - \beta_1) & -c\theta_1 s(\pi - \beta_1) \\ 0 & s(\pi - \beta_1) & d(\pi - \beta_1) \end{pmatrix} \quad (34)
\end{aligned}$$

2) The rotation matrix of the Cartesian coordinate system is corresponding to the middle of the chain 2. $O - X_D Y_D Z_D$ in the middle of the branch 2 could be overlapped with $O - X_H Y_H Z_H$ by the following steps:

- (a) Circle $\alpha_E = \alpha_1$ around the axis X_H , so that Z_D coincides with Z_H ;
- (b) Circle θ_{EH} along the axis Z_H (θ_{EH} for the angle between the joint X_H and X_D , that is, the joint rotation angle at the branch 2, which is known to drive the joints θ_{EH} is known), so that X_H is coincident with X_D .

Overall, the rotation matrix is generated by $O - X_D Y_D Z_D$ relative to the coordinate system.

$$\begin{aligned}
{}^H_D \mathbf{R} &= Rot(x, \alpha_1) \cdot Rot(z, \theta_{EH}) \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\alpha_1 & -s\alpha_1 \\ 0 & s\alpha_1 & c\alpha_1 \end{pmatrix} \begin{pmatrix} c\theta_{EH} & -s\theta_{EH} & 0 \\ s\theta_{EH} & c\theta_{EH} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (35) \\
&= \begin{pmatrix} c\theta_{EH} & -s\theta_{EH} & 0 \\ c\alpha_1 s\theta_{EH} & c\alpha_1 c\theta_{EH} & -s\alpha_1 \\ s\alpha_1 s\theta_{EH} & s\alpha_1 c\theta_{EH} & c\alpha_1 \end{pmatrix}
\end{aligned}$$

3) The rotation matrix of the Cartesian coordinate system corresponding to the chain 2

$O-X_C Y_C Z_C$ moving platform could be overlapped with $O-X_D Y_D Z_D$ by the following steps:

(a) Rotate $\alpha_D = \alpha_2$ around X_D , Z_C coincides with Z_D

(b) Rotate θ_{DE} along the Z_D -axis (θ_{DE} for the angle X_D between the angle X_C , that is, the joint rotation angle at the middle of the branch 2, the joint is the passive joints, θ_{DE} is as unknown parameters), X_D coincidence X_C .

From that, the rotation matrix is achieved by $O-X_C Y_C Z_C$ as

$$\begin{aligned} {}^D_C \mathbf{R} &= Rot(x, \alpha_2) \cdot Rot(z, \theta_{DE}) \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\alpha_2 & -s\alpha_2 \\ 0 & s\alpha_2 & c\alpha_2 \end{pmatrix} \begin{pmatrix} c\theta_{DE} & -s\theta_{DE} & 0 \\ s\theta_{DE} & c\theta_{DE} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c\theta_{DE} & -s\theta_{DE} & 0 \\ c\alpha_2 s\theta_{DE} & c\alpha_2 c\theta_{DE} & -s\alpha_2 \\ s\alpha_2 s\theta_{DE} & s\alpha_2 c\theta_{DE} & c\alpha_2 \end{pmatrix} \end{aligned} \quad (36)$$

4) The rotation matrix of the movement extremity $O-X'Y'Z'$ to $O-X_C Y_C Z_C$

$O-X'Y'Z'$ could be overlapped with $O-X_C Y_C Z_C$ by the following steps:

(a) rotate $\delta_2 = 120^\circ$ around the Z' -axis, the direction of X' is coincident with $Z' \times Z_C$;

(b) Rotate β_2 around the X_C -axis, β_2 is for the known structural parameters, Z' is coincident with $Z' \times Z_C$;

(c) Rotate θ_{CD} along the Z_B -axis (θ_{CD} is for the angle X' between the angle X_C , that is, rotation angle at moving platform joint at branch 2, the joint is a passive joint, θ_{CD} is as unknown parameters), X' is coincident with X_C .

So the rotation matrix is generated clearly by $O-X'Y'Z'$ relative to $O-X_cY_cZ_c$.

$${}^c\mathbf{R} = Rot(x, \beta_2) \cdot Rot(z, \theta_{CB}) \mathbf{R}_{\delta_2}$$

$$\text{where } \mathbf{R}_{\delta_2} = \begin{pmatrix} \cos \delta_2 & -\sin \delta_2 & 0 \\ \sin \delta_2 & \cos \delta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \delta_2 = 120^\circ$$

$$\begin{aligned} {}^c\mathbf{R} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\beta_2 & -s\beta_2 \\ 0 & s\beta_2 & c\beta_2 \end{pmatrix} \begin{pmatrix} c\theta_{CB} & -s\theta_{CB} & 0 \\ s\theta_{CB} & c\theta_{CB} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \delta_2 & -\sin \delta_2 & 0 \\ \sin \delta_2 & \cos \delta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c\theta_{CB} & -s\theta_{CB} & 0 \\ c\beta_2 s\theta_{CB} & c\beta_2 c\theta_{CB} & -s\beta_2 \\ s\beta_2 s\theta_{CB} & s\beta_2 c\theta_{CB} & c\beta_2 \end{pmatrix} \begin{pmatrix} \cos \delta_2 & -\sin \delta_2 & 0 \\ \sin \delta_2 & \cos \delta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned} \quad (37)$$

The rotation matrix is achieved by the platform $O-X'Y'Z'$ to the platform $O-XYZ$ at Branch 2.

$$\begin{aligned} {}^Q\mathbf{R}_2 &= {}^Q\mathbf{R}_H^H \mathbf{R}_D^D \mathbf{R}_C^C \mathbf{R} \\ &= \begin{pmatrix} \cos \delta_2 & -\sin \delta_2 & 0 \\ \sin \delta_2 & \cos \delta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\theta_1 & -s\theta_1 d(\pi - \beta_1) & s\theta_1 s(\pi - \beta_1) \\ s\theta_1 & c\theta_1 d(\pi - \beta_1) & -c\theta_1 s(\pi - \beta_1) \\ 0 & d(\pi - \beta_1) & d(\pi - \beta_1) \end{pmatrix} \\ &\quad \begin{pmatrix} c\theta_{EH} & -s\theta_{EH} & 0 \\ c\alpha_1 s\theta_{EH} & c\alpha_1 c\theta_{EH} & -s\alpha_1 \\ s\alpha_1 s\theta_{EH} & s\alpha_1 c\theta_{EH} & c\alpha_1 \end{pmatrix} \begin{pmatrix} c\theta_{DE} & -s\theta_{DE} & 0 \\ c\alpha_2 s\theta_{DE} & c\alpha_2 c\theta_{DE} & -s\alpha_2 \\ s\alpha_2 s\theta_{DE} & s\alpha_2 c\theta_{DE} & c\alpha_2 \end{pmatrix} \\ &\quad \begin{pmatrix} c\theta_{CB} & -s\theta_{CB} & 0 \\ c\beta_2 s\theta_{CB} & c\beta_2 c\theta_{CB} & -s\beta_2 \\ s\beta_2 s\theta_{CB} & s\beta_2 c\theta_{CB} & c\beta_2 \end{pmatrix} \begin{pmatrix} \cos \delta_2 & -\sin \delta_2 & 0 \\ \sin \delta_2 & \cos \delta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned} \quad (38)$$

3. The nonholonomic spherical pair corresponds to the rotation matrix of the theoretical chain 3

The theoretical chain 3, the branch 1 and the method could not be applied to obtain its rotation matrix. Here a NSUR's constraints kinematics could be modeled in Fig. 6.

In Fig.6, the globular body posture of a non-holonomic spherical restraining structure could be represented by β and θ_o . As long as the kinematic representation of these two parameters of the transformation matrix, the non-holonomic constraints in the globular body could be comprised in pose changes. Simultaneously, pose changes could be broken down into two rotary motion. First, the globular body rounds β , then it revolves around the current coordinate system of θ_o .

Fig. 6. Non-holonomic spherical sub-virtual chain coordinate system

In order to simplify the kinematic model, the non-holonomic spherical coordinate system is established in a coordinate system coincident with $O-XYZ$. In order to coincide the non-holonomic spherical sub-coordinate system with $O-XYZ$, it is necessary to adjust repeatedly the mechanical parameters of the non-holonomic spherical pair. The following is the specific structural coefficient requirement.

At the beginning, the Z-axis is perpendicular to the plane S, the axis of the free-roller is perpendicular to the Z-axis and the X-axis travels along the direction of the free-roller. So the free-roller is set up to ensure the link with the ball on the X-axis. The ultimate transformation matrix could be obtained by left multiplication due to θ_o of the current coordinate system.

$$\begin{aligned} {}^0_s\mathbf{R}_3 &= \mathbf{R}(Z, \beta) \mathbf{R}(X, \theta_o) = \begin{bmatrix} \cos \beta & -\sin \beta & 0 \\ \sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_o & -\sin \theta_o \\ 0 & \sin \theta_o & \cos \theta_o \end{bmatrix} \\ &= \begin{bmatrix} \cos \beta & -\sin \beta \cos \theta_o & \sin \beta \sin \theta_o \\ \sin \beta & \cos \beta \cos \theta_o & -\cos \beta \sin \theta_o \\ 0 & \sin \theta_o & \cos \theta_o \end{bmatrix} \end{aligned} \quad (39)$$

Analysis these independent parameters:

The conversion matrix between $O-X'Y'Z'$ and $O-XYZ$ is calculated by:

$$\begin{aligned} \mathbf{Q}_{\psi, \theta, \phi} &= \mathbf{Q}_{Z_0, \psi} \mathbf{Q}_{X_1, \theta} \mathbf{Q}_{Z_2, \phi} = \begin{pmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{pmatrix} \begin{pmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c\psi c\phi - s\psi c\theta s\phi & -c\psi s\phi - s\psi c\theta c\phi & s\psi s\theta \\ s\psi c\phi + c\psi c\theta s\phi & -s\psi s\phi + c\psi c\theta c\phi & -c\psi s\theta \\ s\theta s\phi & s\theta c\phi & c\theta \end{pmatrix} \end{aligned} \quad (40)$$

$c\psi = \cos \psi, s\psi = \sin \psi$, the rest is similar.

The orientation in the fixed platform is determined, so the transformation matrix of the appurtenant platform to the fixed platform is the same, so there are these equations:

$$\text{Branch 1:} \quad \mathbf{Q}_{\psi, \theta, \phi} = {}^0\mathbf{R}_1 = {}^0_F\mathbf{R}_A^F \mathbf{R}_B^A \mathbf{R}^B$$

$$\begin{aligned}
&= \begin{pmatrix} c\theta_{AF} & -s\theta_{AF} & 0 \\ c\alpha_1 s\theta_{AF} & c\alpha_1 c\theta_{AF} & -s\alpha_1 \\ s\alpha_1 s\theta_{AF} & s\alpha_1 c\theta_{AF} & c\alpha_1 \end{pmatrix} \begin{pmatrix} c\theta_{AF} & -s\theta_{AF} & 0 \\ c\alpha_1 s\theta_{AF} & c\alpha_1 c\theta_{AF} & -s\alpha_1 \\ s\alpha_1 s\theta_{AF} & s\alpha_1 c\theta_{AF} & c\alpha_1 \end{pmatrix} \\
&\begin{pmatrix} c\theta_{BA} & -s\theta_{AF} & 0 \\ c\alpha_2 s\theta_{BA} & c\alpha_2 c\theta_{BA} & -s\alpha_2 \\ s\alpha_2 s\theta_{BA} & s\alpha_2 c\theta_{BA} & c\alpha_2 \end{pmatrix} \begin{pmatrix} c\theta_{CB} & -s\theta_{CB} & 0 \\ c\beta_2 s\theta_{CB} & c\beta_2 c\theta_{CB} & -s\beta_2 \\ s\beta_2 s\theta_{CB} & s\beta_2 c\theta_{CB} & c\beta_2 \end{pmatrix}
\end{aligned} \quad (41)$$

Branch 2: $\mathbf{Q}_{\psi,\theta,\phi} = {}^0\mathbf{R}_2 = {}^0_H\mathbf{R}_D^H \mathbf{R}_C^D \mathbf{R}^C \mathbf{R}$

$$\begin{aligned}
&= \begin{pmatrix} \cos \delta_2 & -\sin \delta_2 & 0 \\ \sin \delta_2 & \cos \delta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\theta_1 & -s\theta_1 \mathcal{A}(\pi - \beta_1) & s\theta_1 \mathcal{A}(\pi - \beta_1) \\ s\theta_1 & c\theta_1 \mathcal{A}(\pi - \beta_1) & -c\theta_1 \mathcal{A}(\pi - \beta_1) \\ 0 & \mathcal{A}(\pi - \beta_1) & \mathcal{A}(\pi - \beta_1) \end{pmatrix} \\
&\begin{pmatrix} c\theta_{EH} & -s\theta_{EH} & 0 \\ c\alpha_1 s\theta_{EH} & c\alpha_1 c\theta_{EH} & -s\alpha_1 \\ s\alpha_1 s\theta_{EH} & s\alpha_1 c\theta_{EH} & c\alpha_1 \end{pmatrix} \begin{pmatrix} c\theta_{DE} & -s\theta_{DE} & 0 \\ c\alpha_2 s\theta_{DE} & c\alpha_2 c\theta_{DE} & -s\alpha_2 \\ s\alpha_2 s\theta_{DE} & s\alpha_2 c\theta_{DE} & c\alpha_2 \end{pmatrix} \\
&\begin{pmatrix} c\theta_{CB} & -s\theta_{CB} & 0 \\ c\beta_2 s\theta_{CB} & c\beta_2 c\theta_{CB} & -s\beta_2 \\ s\beta_2 s\theta_{CB} & s\beta_2 c\theta_{CB} & c\beta_2 \end{pmatrix} \begin{pmatrix} \cos \delta_2 & -\sin \delta_2 & 0 \\ \sin \delta_2 & \cos \delta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned} \quad (42)$$

Non-holonomic spherical sub-theoretical chain:

$$\begin{aligned}
\mathbf{Q}_{\psi,\theta,\phi} &= {}^0_S\mathbf{R}_3 = \mathbf{R}(z,\beta)\mathbf{R}(x,\theta_0) = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_0 & -\sin\theta_0 \\ 0 & \sin\theta_0 & \cos\theta_0 \end{bmatrix} \\
&= \begin{bmatrix} \cos\beta & -\sin\beta \cos\theta_0 & \sin\beta \sin\theta_0 \\ \sin\beta & \cos\beta \cos\theta_0 & -\cos\beta \sin\theta_0 \\ 0 & \sin\theta_0 & \cos\theta_0 \end{bmatrix}
\end{aligned} \quad (43)$$

Independent parameters in branch 1: $\theta_1, \beta_1, \alpha_A = \alpha_1, \theta_{AF}, \alpha_B = \alpha_2,$

$\theta_{BA}, \beta_2, \theta_{CB};$

X_F coincides with X , so $\theta_1 = 0$, the known parameters;

β_1 is the structural parameter, that is, the type of the robot determining

this parameter, the known parameter;

$\alpha_A = \alpha_1$ is the bar body parameter, the known parameter;

θ_{AF} is the angle between X_F and X_A . That is, a branch of a platform at the joint rotation angle, the joint for the drive joints, θ_{AF} is the known parameter;

$\alpha_B = \alpha_2$ is the bar body parameter, the known parameter;

θ_{BA} is the angle between X_B and X_A . That is, a branch of a platform at the joint rotation angle, the joint for the drive joints, θ_{AF} is the unknown parameter;

β_2 is the structural parameters, that is, the determined parameters in the design of the robot, the known parameters;

θ_{CB} is the angle between X' and X_B . That is, the rotation angle at branch 1 moving platform at the joint rotation angle, the joint for the passive joint, θ_{CB} is unknown parameters;

So only one of the two unknowns in branch 1 is θ_{BA} and θ_{CB} .

Branch 2: $\theta_1, \beta_1, \delta_2 = 120^\circ, \alpha_E = \alpha_1, \theta_{EH}, \alpha_D = \alpha_2, \theta_{DE}, \theta_{DE}, \beta_2, \theta_{CD}$;

X_F coincides with X , $\theta_1 = 0$, for the known parameters;

β_1 is the structural parameters, that is, the design of the robot to determine the parameters, the known parameters.

$\alpha_E = \alpha_1$ is the bar body parameters, the known parameters;

θ_{EH} is the angle between X_H and X_D . That is, the rotation angle at branch 2 set the platform at the joint, the joint for the drive joint, the known parameters;

$\alpha_D = \alpha_2$ is the bar body parameters, so the known parameters;

θ_{DE} is the angle between X_D and X_C . That is, the joint rotation angle at branch 2 in the middle, the joint for the passive joints, as unknown parameters;

β_2 is the structural parameters, that is, the design of the robot to determine the parameters, the known parameters;

θ_{CD} is the angle between X' and X_C . That is, branch 2 moving platform at the joint rotation angle, the joint for the passive joint, as unknown parameters;

θ_{DE} and θ_{CD} are only two unknowns in the chain 2;

β , θ_o are the non-holonomic spherical pair corresponds to the independent parameter of the virtual chain 3.

First, the globular body rotates angle β around the z axis to reach the desired position of the shaft.

Hence, β , θ_o are unknown. At this point, the axis of rotation is the current coordinate system of the x-axis, and then around the axis of rotation angle θ_o , that is, around the current coordinate system of the x-axis angle θ_o .

$\psi = \angle XOD_1$ is the advance angle, $\theta = \angle ZOZ_3$ is the chapter angle, $\phi = \angle D_1OX_3$ is the rotation angle. These are a set of independent parameters, the posture could be uniquely determined by the sphere at any moment in the inertia coordinates, these three parameters are unknown parameters. In summary, the unknown parameter is $\psi = \angle XOD_1$, $\theta = \angle ZOZ_3$, $\phi = \angle D_1OX_3$, β , θ_o , θ_{DE} , θ_{CD} , θ_{BA} , θ_{CB} .

From the above discussion, it is meaningful for NSUR that the following conclusions could be drawn:

1. The quantity of non-holonomic spherical input constraints is less than the DOFs of the robot;
2. The DOF of the organization minus the quantity of non-holonomic spherical input constraints is the quantity of drives generated by the underactuated robot;
3. The drive position could only appear in pairs on a NSUR's free-roller, and the actual angle between the two drive free-rollers should not reach 180° and more, otherwise the robot would be uncontrollable practically.

5. Measurements and comparison of energy efficiency between the novel NSUR and traditional robot

In advanced engineering, the robots play an increasingly vital role with lots of energy. Power is an important measure of energy performance assessment of mechanical systems [38]. Due to the uncertainty of the position of the motion of NSUR, the power calculation is also quite complicated. There are various methods for calculating the output power of the entire mechanical system. This research introduces a method according to angular velocity and torque. An example of an underactuated spherical robot is listed and its kinematics is analyzed with virtual prototyping.

5.1 Power calculation for single connection pair

On the production line, the energy consumption of a robot happens not only the stable moving operation, but also other procedures, so it is meaningful to measure the average power consumption for a single connection pair.

The export power of NSUR is expressed as:

$$P = \frac{T_{\text{export}} \omega_{\text{export}}}{\eta} \quad (44)$$

Before measuring the entire non-holospheric system, it is necessary to analyze the kinematics of a single joint, i.e. the connection pair. The process of the single joint pair includes start-up, stable moving and stop phase. The times of these phases are defined as t_1 , t_2 , and t_3 , respectively. The torqueses of these phases are T_1 , T_2 , T_3 , respectively. And the angular velocities of these phases are ω_1 , ω_2 , and ω_3 , respectively.

The output torque in average can be calculated by the following formula:

$$T_{\text{avg}} = \sqrt{\frac{T_1^2 \cdot t_1 + T_2^2 \cdot t_2 + T_3^2 \cdot t_3}{t_1 + t_2 + t_3}} \quad (45)$$

The output angular velocity in average could be calculated as:

$$\omega_{\text{avg}} = \frac{\omega_1 \cdot t_1 + \omega_2 \cdot t_2 + \omega_3 \cdot t_3}{t_1 + t_2 + t_3} \quad (46)$$

5.2 Simulation of non-holonomic underactuated robot

A model of non-holonomic underactuated robot is set up by ADAMS software. It includes a floor fixed platform, a moving platform model which could be regarded as the system payload, two pairs of ball joints which are diagonally distributed, and tandem machinery composed of a series of rotating pairs.

In ADAMS, the setting parameters of the connection pairs and drive should be correct. The angular velocity of drive is set as 30 rad/s. Meanwhile, some physical constraints could be set to prevent branches from penetrating the model. Then, the motion time and step size of model ought to be defined.

In order to find out the kinematics of the non-holonomic underactuated robot, several sets of curves are selected to calculate the average power.

Fig. 7. The angular velocity of input driver

In Fig. 7. , it shows the angular velocity curve of a rotary pair which is connected to the output rod and the movement extremity during the 0 to 3 s rotation. The numerical average operation of the curve is 30.24 rad/s, which is basically consistent with the rated input angular velocity. It also demonstrates the correct of the model in another perspective.

Fig. 8. The angular velocity of platform

In Fig. 8, it shows the angular velocity curve of the movement extremity on the system. In Figure 8, after a short start-up, the angular velocity curve is stable over the range of 0.5 s to 3.0 s. The average value of the whole segment is approximately equal to 14.6 rad/s. From this aspect, compare the export angular velocity with the input angular velocity, it could be understood that the efficiency is approximately near to 50%, and this performance is excellent in many similar, parallel robot structures.

Fig. 9. Torque of input driver

In Fig. 9, it shows a torque curve of the joint at the drive. The non-holonomic underactuated robot has a defect that the moving characteristics are unstable due to the high DOFs, which is performed on the maximum near 2.17 s. The torque of the 0 to 3 s is averaged according to the measuring method in Section 5.1, and the input torque of this process could be obtained as 175.64 N*mm.

Fig. 10. Torque of three output mechanisms

In Fig. 10, it shows the torque curve of the three output rods in the underactuated robot during 0 to 3 s. At the similar time position, there is a pulse curve that is consistent with the maximum input torque. After the same performed data processing, and fitting three export torque curves together, the entire output

torque is 65.23 N*mm. After comparing the output torque and the input torque individually, the efficiency is 37.2%.

According to power equal to the multiplication of angular velocity and torque, the total energy efficiency of the non-holonomic underactuated robot is about 17.96%.

5.3 Comparison of power with industrial robots

The comparison object is the Fanuc robot M-1iA [39], whose parameters, such as the working range and the branch output condition, are similar to those of the underactuated robot in this paper.

This paper introduces two different evaluation methods of power. One is the ratio of load to mass, which excludes most mass of auxiliary mechanical structures. It also includes a comparison of the maximum motion range.

Table.1 Weight ratio of two mechanisms

	NSUR	Fanuc M-1iA
Payload (kg)	0.98	1
System quality removed of accessories (kg)	3.25	5.24
Ratio	30.15%	19.1%
Motion range (maximum)	27.06 rad/s	25.13 rad/s

From Table.1, it could be found that the total mass of the non-holonomic underactuated robot is lower and the load-to-weight ratio is increased by 11% when the payloads are approximate. Under the same working conditions, the maximum output speeds of the two robots are close.

The second comparison method is based on the physical definition of power and is characterized by the ratio of work to time. The input power of the

underactuated robot is estimated from multiplication of angular velocity and torque in Section 5.2. The output power is obtained by the multiplication of the effective load and the displacement in the unit of time. The ratio of the input and output power could be defined as an efficiency that describes the power transmission performance.

Table.2 Weight ratio of two mechanisms

	NSUR	Fanuc M-1iA
Payload (kg)	0.98	1
Vertical displacement (mm)	206.58	150
Drive time (s)	3	0.3
Export power (W)	0.673	5
Input power (W)	5.31	50
Energy efficiency	12.67%	10%

By the second comparison method, the power transmission efficiency of the non-holonomic underactuated robot is similar to that of the industrial robot widely used in market on the product level.

6. Conclusions

As energy is a recent global concern for sustainability, it is confronted with the responsibility of producing products with low-energy cost. This paper proposed a novel NSUR structure for robotics with fewer actuators than DOFs, and with lower energy consumption. After a novel NSUR was proposed, the inverse kinematics analysis and forward kinematic analysis of NSUR were obtained in detail. The measurements and comparisons of energy efficiency between NSUR and traditional robotics were also discussed. Based on the analysis and summary of the kinetics and energy efficiency of NSUR, this paper could

draw the following conclusions.

1. The features of the non-holonomic spherical drive configuration with fewer planned path sets could provide theoretical support for the improvement of energy efficiency and the reduction of hardware complexity of the underactuated robot.

2. The kinematics analysis of NSUR verifies the controllability of the new configuration and proves that the driving position must appear on the non-holonomic spherical driving roller in pairs.

3. Compared with the energy efficiency of traditional robots, NSUR has great advantages in energy efficiency improvement, especially in the case of insufficient energy supply in the deep sea, and could achieve low speed and stable operation.

Declarations

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Conflicts of interest

None.

Availability of data and material

Not applicable.

Code availability

Not applicable.

Authors' contributions

Bin He: Conceptualization, Methodology, Project administration, Supervision, Funding acquisition, Writing original draft;

Fuze Xu: Formal analysis, Data Curation, Investigation, Visualization, Validation, Writing - review & editing, Resources.

Pengchang Zhang: Formal analysis, Data Curation, Investigation, Validation, Writing - review & editing.

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