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Study on Abrasive Particle Impact Modeling and Cutting Mechanism

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Abstract: Abrasive particles play a vital role in the impact on materials during abrasive waterjet cutting. To study the effect of particles on the cutting performance during abrasive waterjet cutting, the mostly irregular shape of the abrasive particles in the actual cutting process needs to be considered. In this paper, the particles are simplified as angular and circular particles. The method of Smoothed Particle Hydrodynamics (SPH) is used to simulate the process of particle impact targeting in abrasive jet cutting. Because the abrasive particle impact causes a large local deformation or removal of the surface of the target material, the traditional grid-based numerical method is not suitable for such problems; thus, the SPH method, which is suitable for the impact problem, is selected to establish the numerical model and solve it. In this paper, the fracture process of abrasive particles with different shapes of impact ductility and brittle target materials is studied by a numerical model. In the modeling process, abrasive particles are modeled as rigid bodies with material properties, the ductile materials is an aluminum alloy, the brittle material is quartz glass, which are simulated by changing the initial input conditions and particle shape, and the model is verified by experiments. The results show that the model successfully reproduces the collision process of particles during abrasive jet cutting, including the deformation mechanisms of plowing, fracture and crushing of the target.

Keywords: abrasive particles; Smoothed Particle Hydrodynamics; particle impact; surface erosion; cutting mechanism; impact simulation

1. Introduction

The impact process of abrasive water jets can be considered a material impact problem after coupling solid particles with fluid. The solid particles erode the surface of the workpiece and cause local damage, such as plastic deformation, tiny cracks, and material falling off. The workpiece surface is destroyed and removed under continuous erosion. The difference between abrasive water jet cutting technology and water jet cutting is mainly the influence of the abrasive particles. The cutting performance of the jet is enhanced after adding abrasive particles, the pressure required by the jet is reduced, and the cutting efficiency is increased. Therefore, the influence of abrasive particles is very important in the process of jet cutting.

Domestic and foreign researchers have performed much research on the impact cutting of abrasive particles, including Finnie, who first proposed the theory of surround cutting and

established a rigid particle impact model^[1]. Bitter further improved the cutting theory and then proposed the plastic target cutting model^[2]. The model divides the material removal process into two parts: erosion and wear at small and large angles. The study of Zeng and Kim is more inclined to the microscopic study of the target surface after cutting. Through a series of experimental means, such as scanning electron microscopy, material removal is caused by crystal cracking and viscous flow^[3]. The solid particle impact model is mainly used to study the erosion wear mechanism during abrasive cutting or to predict the influence of multiple particles on the surface morphology of the target workpiece^[4,5]. For the solid particle collision model, the FEM is usually used to calculate the deformation of the solid surface. Abrasive particles are modeled as rigid spheres or polyhedrons, ignoring the material properties related to abrasive deformation, and adopting the FEM is an effective method to solve the problem of solid mechanics. However, when the abrasive particles are treated, it causes large plastic deformation and even removes the material, which reduces the accuracy of the calculation. Hence, in this paper, the meshless SPH method is used in the research of abrasive particle impact. This method can avoid the drawbacks of mesh-based algorithms and has good adaptability to solve large deformation impacts. In modeling, Feng et al.^[6] visualized the abrasive particles in the jet by the SPH method, added the coupling relationship between abrasive particles and water and studied the interaction between water and abrasive particles in the process of abrasive water jet cutting. Dong et al.^[7] established a solid particle impact process model to study the removal form of ductile targets. The SPH method is superior to the traditional numerical method in simulating the moving interface and large deformation after impact. In this paper, parametric modeling of abrasive particles is carried out, and ductile and brittle target materials are selected. The abrasive particles and targets are composed of SPH particles with different properties. The impact parameters and shape parameters of abrasive particles are considered when the model is established, and the parameterized modeling of abrasive particles is carried out according to the shape and motion characteristics of the particles. At the same time, according to the mechanical behavior of the particle impact target, the motion control equation of material removal is described, and a suitable constitutive model is introduced to describe the deformation and failure behavior of the material. The deformation failure mechanism of different shapes of abrasive particles

impacting targets was obtained by comparing the experimental results with the simulation results.

2. Establishment of abrasive particle impact model

2.1 Abrasive impact process description

The abrasive water jet process is a complex fluid-solid coupling problem involving multiphase flow, fatigue failure, chemical corrosion and a series of physical and chemical problems. However, if the motion of abrasive particles during an abrasive water jet is simplified as the impact process of a single particle, the problem becomes a problem of impact mechanics, which in turn greatly simplifies the research process. Therefore, a single abrasive particle is modeled in our approach. The particle size of abrasive particles is generally $200\ \mu\text{m}\sim 300\ \mu\text{m}$; thus, particles are less affected by the surrounding fluid media during collisions. Assuming that the abrasive particle shape is irregular, we define the particle parameters, as shown in Figure 1. The incident velocity of this abrasive particle is v_i , and the angle between the incident velocity and the horizontal axis is α_i . The reflection velocity of abrasive particles after rebound is v_r , the angle between the reflection velocity and the horizontal direction is α_r , and the reflection angular velocity is ω_r . The geometric angle of the shape at the impact point of the abrasive particle is defined as A , its value is $0\sim 180^\circ$, the angle between the center of mass and the vertical direction is defined as the impact azimuth θ_i , and is defined as shown in Fig. 2. The range of the azimuth angle is $-90^\circ\sim 90^\circ$. Because the nozzle is perpendicular to the target material during the abrasive jet cutting process, according to the statistics and the experiments reported in the literature^[8], the azimuth is mostly in the range of $-45^\circ\sim 45^\circ$, which covers 83% of all impact particles. Therefore, the range of impact azimuths of abrasive particles is considered to be $-45^\circ\sim 45^\circ$.

Fig. 1 Characteristic parameters of irregular abrasive particles

Fig. 2 Range of azimuth angle of abrasive particle impact

2.2 Establishment of abrasive particle and target models

When the abrasive particles impact the target, because the hardness of the particles is much larger than that of the target, the crushing and deformation of the abrasive particles are not considered in the modeling. When modeling abrasive particles, according to the different impact forms, they are converted into three typical particle models, as shown in figure 4. The circle represents the smooth impact of abrasive particles on the target; the triangle represents the impact morphology when the geometric angle of abrasive particles is acute; the square represents the impact of abrasive particles whose geometric angle is greater than or equal to 90° .

Fig. 3 Equivalence model of abrasive particles

The cutting mechanism for angular particles and circular particles is different when they collide with the target material. Circular particles are more prone to extrusion of the target material, which leads to the material being "squeezed out". Angular particles result in a "excavate-fracture" removal, that is, the angular abrasive particle tip is inserted into the material surface. However, under the resistance of the target, the particles cannot continue to

penetrate and the material produces a back spin, so the particle tip leads to an excavation effect, and the excavated material breaks before the end of impact. Therefore, the process is directly accompanied by material removal.

2.3 Shape modeling of abrasive particles

In this paper, triangles and squares are considered in the study of angular particles, while circular particles are used as nonangular particles. When describing the shape of the particle, the size of the particle is first determined, assuming that the height of the triangle particle is h and the side length of the square is a ; then, the center point coordinate position of the abrasive particles is determined. Because the particles studied in this paper are all assumed to be regularly shaped particles, the center, centroid and center of gravity of the particles are all coincident. Finally, the impact angle θ_i is determined for the particles. The SPH particle formation process is shown in Fig. 4.

Fig. 4 SPH Particle formation process

A is the center point of the abrasive particle, and its coordinates are (x_0, y_0) , then the coordinates of the impact vertex of the equilateral triangle are $a_1(x_1, y_1)$.

$$\cos\theta_i = \frac{|x_1 - x_0|}{\frac{2}{3}h} \quad (1)$$

$$\sin\theta_i = \frac{y_0 - y_1}{\frac{2}{3}h} \quad (2)$$

Therefore

$$x_1 = x_0 + \frac{2}{3}h \cdot \cos\theta_i \quad (3)$$

or

$$x_1 = x_0 - \frac{2}{3}h \cdot \cos\theta_i \quad (4)$$

$$y_1 = y_0 - \frac{2}{3}h \cdot \sin\theta_i \quad (5)$$

where x_1 is the horizontal coordinate value of the impact point, and y_1 is the vertical coordinate value of the impact point.

Hence, the coordinates of the impact vertex of the triangle particles incident on the left side of the center axis are $a_1(x_0 + \frac{2}{3}h \cdot \cos\theta_i, y_0 - \frac{2}{3}h \cdot \sin\theta_i)$. A particle incident on the right side of the center axis is $a_1(x_0 - \frac{2}{3}h \cdot \cos\theta, y_0 - \frac{2}{3}h \cdot \sin\theta)$. The other two vertices of the triangle are a_2 and a_3 , and the coordinate values are $a_2(x_2, y_2)$ and $a_3(x_3, y_3)$.

$$\cos(60^\circ - \theta_i) = \frac{x_1 - x_2}{\frac{2\sqrt{3}}{3}h} \quad (6)$$

$$\sin(60^\circ - \theta_i) = \frac{y_2 - y_1}{\frac{2\sqrt{3}}{3}h} \quad (7)$$

Therefore,

$$x_2 = x_1 - \left(\frac{\sqrt{3}}{3}\cos\theta_i + \sin\theta_i\right) \cdot h \quad (8)$$

$$y_2 = y_0 - \left(\frac{2 + \sqrt{3}}{3}\sin\theta_i + \cos\theta_i\right) \cdot h \quad (9)$$

For vertex a_2 ,

$$\sin(\theta_i - 30^\circ) = \frac{x_1 - x_3}{\frac{2\sqrt{3}}{3}h} \quad (10)$$

$$\cos(\theta_i - 30^\circ) = \frac{y_3 - y_1}{\frac{2\sqrt{3}}{3}h} \quad (11)$$

Therefore,

$$x_3 = x_1 - (\sin\theta_i - \frac{\sqrt{3}}{3} \cos\theta_i) \cdot h \quad (12)$$

$$y_3 = y_0 + (\frac{\sqrt{3}-2}{3} \sin\theta_i + \cos\theta_i) \cdot h \quad (13)$$

As a result, the coordinates of the other two vertices of an equilateral abrasive particle are $a_2(x_2, y_2)$ and $a_3(x_3, y_3)$, or $a_2((x_1 - (\frac{\sqrt{3}}{3} \cos\theta_i + \sin\theta_i) \cdot h, y_0 - (\frac{2+\sqrt{3}}{3} \sin\theta_i + \cos\theta_i) \cdot h)$ and $a_3((x_1 - (\sin\theta_i - \frac{\sqrt{3}}{3} \cos\theta_i) \cdot h, y_0 + (\frac{\sqrt{3}-2}{3} \sin\theta_i + \cos\theta_i) \cdot h)$.

For square particles, the coordinate of the impact vertex can be obtained when the input parameters) such as side length a , impact angle θ_i and center point coordinates $A_s(x_{s0}, y_{s0})$ are known.

$$\sin\theta_i = \frac{|x_{s1} - x_{s0}|}{\frac{\sqrt{2}}{2}a} \quad (14)$$

$$\cos\theta_i = \frac{y_{s0} - y_{s1}}{\frac{\sqrt{2}}{2}a} \quad (15)$$

Therefore,

$$x_{s1} = x_{s0} + \frac{\sqrt{2}}{2}a \cdot \sin\theta_i \quad (16)$$

or

$$x_{s1} = x_{s0} - \frac{\sqrt{2}}{2}a \cdot \sin\theta_i \quad (17)$$

$$y_{s1} = y_{s0} - \frac{\sqrt{2}}{2}a \cdot \cos\theta_i \quad (18)$$

where x_{s1} is the horizontal coordinate value of the impact point,, and y_{s1} is the vertical coordinate value of the impact point.

Hence, the coordinates of the impact vertex of the triangle particles incident on the left side of the center axis are $a_{s1}(\ x_{s0} + \frac{\sqrt{2}}{2}a \cdot \sin \theta_i, \ y_{s0} - \frac{\sqrt{2}}{2}a \cdot \cos \theta_i)$, A particle incident on the right side of the center axis is $a_{s1}(x_{s0} - \frac{\sqrt{2}}{2}a \cdot \sin \theta_i, \ y_0 - \frac{2}{3}h \cdot \sin \theta)$.

The other three vertices of a square abrasive particle a_{s2} , a_s , and a_{s4} , with coordinate values $a_{s2}(x_{s2}, y_{s2})$, $a_{s3}(x_{s3}, y_{s3})$ and $a_{s4}(x_{s4}, y_{s4})$.

For vertex a_{s2} :

$$\cos(45^\circ - \theta_i) = \frac{x_{s1} - x_{s2}}{a} \quad (19)$$

$$\sin(45^\circ - \theta_i) = \frac{y_{s2} - y_{s1}}{a} \quad (20)$$

Therefore,

$$x_{s2} = x_{s1} - \left(\frac{\sqrt{2}}{2} \sin \theta_i + \frac{\sqrt{2}}{2} \cos \theta_i \right) \cdot a \quad (21)$$

$$y_{s2} = y_{s0} - \frac{\sqrt{2}}{2} a \cdot \sin \theta_i \quad (22)$$

Similarly, the coordinates of the other vertices can be obtained as follows:

$$x_{s3} = x_{s1} - \sqrt{2}a \cdot \sin \theta_i \quad (23)$$

$$y_{s3} = y_{s0} + \frac{\sqrt{2}}{2} a \cdot \cos \theta_i \quad (24)$$

$$x_{s4} = x_{s1} + \left(\frac{\sqrt{2}}{2} \cos \theta_i - \frac{\sqrt{2}}{2} \sin \theta_i \right) \cdot a \quad (25)$$

$$y_{s4} = y_{s0} + \frac{\sqrt{2}}{2} a \cdot \sin \theta_i \quad (26)$$

As a result, the coordinates are $a_{s2}((x_{s1} - (\frac{\sqrt{2}}{2} \sin \theta_i + \frac{\sqrt{2}}{2} \cos \theta_i) \cdot a, \ y_{s0} - \frac{\sqrt{2}}{2} a \cdot \sin \theta_i))$,
 $a_{s3}((x_{s1} - \sqrt{2}a \cdot \sin \theta_i, \ y_{s0} + \frac{\sqrt{2}}{2} a \cdot \cos \theta_i))$, $a_{s4}((x_{s1} + (\frac{\sqrt{2}}{2} \cos \theta_i - \frac{\sqrt{2}}{2} \sin \theta_i) \cdot a, \ y_{s0} + \frac{\sqrt{2}}{2} a \cdot \sin \theta_i))$.

For circular abrasive particles, the impact point coordinates are $a_{c1}(x_{c0}, y_{c0} - d/2)$. When generating SPH particles, a series of uniform surface nodes are generated among the particles to solve the surface normal vector, as shown in Fig. 5. The surface normal vectors of the

nodes generated by the surface are defined as follows:

$$\vec{n}_{k+1} = \pm \left(\frac{y_{k+2} - y_k}{|\vec{x}_{k+2} - \vec{x}_k|}, -\frac{x_{k+2} - x_k}{|\vec{x}_{k+2} - \vec{x}_k|} \right) \quad (27)$$

where the nodes adjacent to $k+1$ are k and $k+2$, $\vec{x}_{k+2}=(x_{k+2}, y_{k+2})$, and $\vec{x}_k=(x_k, y_k)$.

Triangular and square abrasive particles can be integrated into angular particles, which can be triangulated when calculating the moment of inertia of the angular particles. The coordinates of the center of mass are obtained after triangle dissection of the angular particles.

The coordinate of the center of mass is $\vec{x}_c$

$$\vec{x}_c = \frac{\sum A_n \cdot \vec{x}_{nc}}{\sum A_n} \quad (28)$$

where $\sum A_m$ is the total area of angular particles, and $\vec{x}_{nc}$ is the centroid coordinate of subtriangle n after the split.

Direction of boundary particle generation

Fig. 5 Direction of boundary particle generation

Therefore, the mass and moment of inertia of angular abrasive particles are defined as follows:

$$\begin{cases} M = \sum m_n \\ I_z = \sum (i_{zn} + m_n(\vec{x}_{nc} - \vec{x}_c)^2) \end{cases} \quad (29)$$

$$i_{zn} = \frac{m_n}{36} (a_n^2 + b_n^2 + c_n^2) \quad (30)$$

where i_{zn} is the moment of inertia of the n th subtriangle around its centroid rotation axis. m_n is the mass of the n th subtriangle, and the three subtriangle sides are, a_n , b_n , and c_n .

The moment of inertia of the circular abrasive particles can be obtained by the following

formula:

$$I_z = \frac{m_c \cdot d^2}{8} \quad (31)$$

where m_c is the mass of the circular abrasive particle, and d is the diameter.

3. Brief introduction of the SPH method

3.1 Implementation of SPH method process

The SPH method is a meshless algorithm that does not need to use a mesh when describing the problem domain. Instead, it uses a series of SPH particles instead of meshing. The particles have their own physical properties, including mass, density, velocity, acceleration and energy, and move according to the conservation of the control equation. Compared with the meshless nodes in other meshless methods that only act as interpolation points, SPH particles also have material properties and motion based on internal interactions and external forces. Because SPH particles act as approximate points and have material properties, real-world conditions can be accurately represented^[9]. In the process of calculation, it takes two basic steps to convert the partial differential equation into the SPH equation, namely, kernel approximation and particle approximation. The kernel approximation approximates the field variables and their derivatives by using the information in the nearby regions Ω , while the particle approximation discretizes the integral function into a sum of finite particles in the Ω domain. The problem domain is shown in Fig. 6.

Fig. 6 Approximation of SPH particles in the two-dimensional problem domain

For continuous smooth functions, any point m in the domain can be expressed as

follows:

$$f(\mathbf{r}) = \int_{\Omega} f(\mathbf{r}') \delta(\mathbf{r} - \mathbf{r}') d\mathbf{r}' \quad (32)$$

where $\mathbf{r}$ is the space position vector, and $\delta(\mathbf{r} - \mathbf{r}')$ is the Dirac function.

$$\delta(\mathbf{r} - \mathbf{r}') = \begin{cases} \infty, & \mathbf{r} = \mathbf{r}' \\ 0, & \mathbf{r} \neq \mathbf{r}' \end{cases} \quad (33)$$

When solving this problem, it is difficult to solve the integral by a numerical method. Therefore, the $f(\mathbf{r})$ function is approximated by the finite integral form given by formula (34) on the domain Ω , that is, the kernel approximation of the function,

$$\langle f(\mathbf{r}) \rangle = \int_{\Omega} f(\mathbf{r}') W(\mathbf{r} - \mathbf{r}', h) d\mathbf{r}' \quad (34)$$

where h is the smoothing length.

A kernel function is compact, and the value of the function is 0 beyond $2h$ of the definition domain.

$$W(\mathbf{r} - \mathbf{r}', h) = 0 \quad (35)$$

where $|\mathbf{r} - \mathbf{r}'| \geq 2h$.

The kernel function is normalized as follows:

$$\int W(\mathbf{r} - \mathbf{r}', h) d\mathbf{r}' = 1 \quad (36)$$

According to formula (34), the spatial derivative of function $f(\mathbf{r})$ can be expressed as $\nabla \cdot f(\mathbf{r})$.

$$\langle \nabla \cdot f(\mathbf{r}) \rangle = \int_{\Omega} [\nabla \cdot f(\mathbf{r}')] W(\mathbf{r} - \mathbf{r}', h) d\mathbf{r}' = - \int_{\Omega} f(\mathbf{r}') \cdot \nabla W(\mathbf{r} - \mathbf{r}', h) d\mathbf{r}' \quad (37)$$

The particle approximation can be expressed as follows:

$$\langle f(\mathbf{r}) \rangle = \int f(\mathbf{r}') W(\mathbf{r} - \mathbf{r}', h) d\mathbf{r}' \approx \sum_{j=1}^N \frac{m_j}{\rho_j} f(\mathbf{r}_j) W(\mathbf{r} - \mathbf{r}_j, h_i) \quad (38)$$

Therefore,

$$\langle f(\mathbf{r}_i) \rangle = \sum_{j=1}^N \frac{m_j}{\rho_j} f(\mathbf{r}_j) W(\mathbf{r} - \mathbf{r}'_j, h_i) \quad (39)$$

where m_j is the mass of particle j , ρ_j is the density of particle j , and m_j/ρ_j is the volume of particle j .

The spatial derivative of the field function at particle i is approximately

$$\langle \nabla \cdot f(\mathbf{r}_i) \rangle = \rho_i \sum_{j=1}^N m_j \left[\frac{f(\mathbf{r}_j)}{\rho_j^2} + \frac{f(\mathbf{r}_i)}{\rho_i^2} \right] \cdot \nabla_i W_{ij} \quad (40)$$

3.2 Smooth length

In large deformation impact problems particles can be separated. When the smoothing length is constant, it is possible to increase the distance between particles. When the smoothing length reaches a certain value, the interaction between particles disappears. If the smoothing length is compressed, the particles may enter the adjacent computational domain, which leads to a significant reduction in computational speed. To solve this problem, according to the average density, the smoothing length is modified as follows:

$$h = h_0 \left(\frac{\rho_0}{\rho} \right)^{\frac{1}{d}} \quad (41)$$

where h_0 is the initial smoothing length, ρ_0 is the initial density, ρ is the density, and d is the dimension of the computational problem.

The smoothing length can also be adjusted by using the time derivative of the smoothing function.

$$\frac{dh}{dt} = -\frac{1}{d} \frac{h}{\rho} \frac{d\rho}{dt} \quad (42)$$

3.3 SPH equations for solids

By using the mass conservation equation, the Navier-Stokes equation of solid media can be expressed in tensor form:

$$\frac{d\rho}{dt} = -\rho \frac{\partial v^\beta}{\partial x^\beta} \quad (43)$$

The momentum conservation equation is defined as follows:

$$\frac{dv^\alpha}{dt} = \frac{1}{\rho} \frac{\partial \sigma^{\alpha\beta}}{\partial x^\beta} \quad (44)$$

where ρ is the solid medium density, v^β is the velocity vector, x^β is the position vector, and t is the time.

The SPH equations for solids can be expressed as^[11]

$$\left\{ \begin{array}{l} \frac{d\rho_i}{dt} = \rho_i \sum_{j=1}^N \frac{m_j}{\rho_j} v_{ij}^\beta \cdot \frac{\partial W_{ij}}{\partial x_i^\beta} \\ \frac{dv_i^\alpha}{dt} = - \sum_{j=1}^N m_j \left[\frac{p_i + p_j}{\rho_i \rho_j} - \frac{\mu_i \varepsilon_i^{\alpha\beta} + \mu_j \varepsilon_j^{\alpha\beta}}{\rho_i \rho_j} + \Pi_{ij} \right] \frac{\partial W_{ij}}{\partial x_i^\beta} \end{array} \right. \quad (45)$$

where W is the smooth kernel function, μ is Poisson's ratio, ε is the strain, and Π_{ij} is the artificial viscosity term.

In solid media, stress can be expressed as the sum of hydrostatic pressure and stress deviation^[12],

$$\sigma^{\alpha\beta} = -p\delta^{\alpha\beta} + S^{\alpha\beta} \quad (46)$$

where $S^{\alpha\beta}$ is the stress bias. The Jaumann rate is substituted into the stress bias, which can be expressed as follows:

$$\dot{S}^{\alpha\beta} = 2G \left(\dot{\varepsilon}^{\alpha\beta} - \frac{1}{3} \delta^{\alpha\beta} \dot{\varepsilon}^{\gamma\gamma} \right) + S^{\alpha\gamma} R^{\beta\gamma} + S^{\beta\gamma} R^{\alpha\gamma} \quad (47)$$

where G is the shear modulus, and $\dot{\varepsilon}$ is the strain rate tensor, which can be expressed as follows:

$$\dot{\varepsilon}^{\alpha\beta} = \frac{1}{2} \left(\frac{\partial v^\alpha}{\partial x^\beta} + \frac{\partial v^\beta}{\partial x^\alpha} \right) \quad (48)$$

where above, $R^{\alpha\beta}$ is the rotation rate tensor, which can be expressed as follows:

$$R^{\alpha\beta} = \frac{1}{2} \left(\frac{\partial v^\alpha}{\partial x^\beta} - \frac{\partial v^\beta}{\partial x^\alpha} \right) \quad (49)$$

Assume that the material obeys the von Mises yield criterion. When the equivalent stress (I_2) is greater than the yield stress (Y_{JC}) of the material, the stress deviation is corrected to

$$S^{\alpha\beta} = S^{\alpha\beta} Y_{JC} / I_2 \quad (50)$$

where the equivalent stress can be expressed as follows:

$$I_2 = \sqrt{3/2 S^{\alpha\beta} S^{\alpha\beta}} \quad (51)$$

3.4 Abrasive particle movement

When the abrasive particles impact the target material, the velocity can be decomposed into the translational velocity and the rotational velocity, and the conservation equation of the abrasive particles can be expressed as follows:

$$\begin{cases} \frac{dV_j^\alpha}{dt} = \frac{F_j^\alpha}{M_j} \\ \frac{dW_j^\alpha}{dt} = \frac{T_j^\alpha}{I_j} \\ \frac{dX_j^\alpha}{dt} = V_j^\alpha \end{cases} \quad (52)$$

where j is the retrieved j th abrasive particle, V_j^α is the translational velocity, W_j^α is the rotational velocity, X_j^α is the central position of the particle, M_j is the mass of the j th particle, I_j is the moment of inertia of the j th particle, F_j^α is the resultant force on the j th particle, and T_j^α is the action time.

In this model, each abrasive particle is associated with several discrete SPH nodes, and each SPH node carries field variables, including mass and velocity. The equations of motion of these nodes are defined as follows:

$$v_{j-i}^\alpha = V_i^\alpha + W_i^\alpha \times (x_{j-i}^\alpha - X_i^\alpha) \quad (53)$$

where j is the j th node in the i th particle, and x_{j-i}^α is the position vector of the node.

For the nodes close to the interface, two or three particles of the material can be included^[38]. The common method to address this situation is to solve all particles in the region by control. The material properties of these SPH particles are not considered. Accordingly, the total force F_j^α and moment T_j^α acting on the j th rigid body can be expressed as follows:

$$\begin{cases} F_j^\alpha = \sum_{i=1}^N m_{i-j} \frac{dv_{i-j}^\alpha}{dt} \\ T_j^\alpha = \sum_{i=1}^N m_{i-j} \frac{dv_{i-j}^\alpha}{dt} \times (x_{i-j}^\alpha - X_j^\alpha) \end{cases} \quad (54)$$

where i and j are the labels of SPH nodes and particles, m_{i-j} is the mass of the i th node from the j th particle, and $\frac{dv_{i-j}^\alpha}{dt}$ is the acceleration caused by the surrounding continuous phase in the i th node circle.

3.5 Constitutive Model of Target Material

When modeling, the accuracy of the description of the plastic behavior of the target material directly affects the accuracy of the calculation. Target materials mainly consider ductile materials and brittle materials, and in the study of this paper these two kinds of materials are considered elastoplastic materials.

Ductile materials and brittle materials do not have the same impact erosion mechanism. The ductile material is dominated by plastic deformation, and the surface of materials is cut and peeled after the high-speed jet impact, while the fracture of brittle materials is through cracks and crack propagation.

(1) Model of ductile materials

For engineering, the most commonly used material constitutive model is the Johnson-Cook model based on experimental data^[13]. The Johnson-Cook constitutive model

has a good predictive power when describing the mechanical behavior of metal materials in the process of large deformation and high strain rate cutting^[14]. As a result, the Johnson-Cook viscoplastic model is used, which can be expressed as follows:

$$\sigma_y = (A + B(\bar{\varepsilon}^p)^n)(1 + C \ln \dot{\varepsilon}_e^*)(1 - (T^*)^m) \quad (55)$$

$$\bar{\varepsilon}^p = \varepsilon^p / \varepsilon_0 \quad (56)$$

$$T^* = (T - T_r) / (T_m - T_r) \quad (57)$$

where σ_y is the yield stress of the material, A, B, C, m, and n are all the relevant parameters of the material, $\bar{\varepsilon}^p$ is the equivalent plastic strain, $\dot{\varepsilon}_e^*$ is the dimensionless plastic strain rate, T^* is the homologous temperature, ε^p is the plastic strain, ε_0 is the reference strain rate, the T is the actual temperature, T_r is the room temperature, and T_m is the melting point of the material.

The Johnson-Cook fracture criterion is adopted for the failure fracture behavior of the target material^[15]. In the Johnson-Cook failure model, the failure strain ε_f is expressed as follows:

$$\varepsilon_f = f(\sigma^*, T^*, \dot{\varepsilon}_e^*) = [D_1 + D_2 \exp(D_3 \sigma^*)][1 + D_4 \ln \dot{\varepsilon}_e^*][1 + D_5 T^*] \quad (58)$$

$$\sigma^* = \frac{\sigma_m}{\sigma_{eq}} \quad (59)$$

where $D_1 \sim D_5$ is the material performance parameter, σ_m is the average value of the main stress, and σ_{eq} is the equal effect force.

For the high-speed impact of abrasive particles on ductile material, to calculate the isotropic pressure of the material, the Mie-Gruneisen equation is used to simulate the impact effect of high-speed abrasive particles on the target material. The state equation is expressed as follows^[16]:

$$p = \frac{\rho_0 C_0^2 \eta \left(1 + \left(1 - \frac{\Gamma_0}{2}\right) \eta\right)}{(1 - (S_a - 1) \eta)} + \rho_0 \Gamma_0 e \quad (60)$$

where ρ_0 is the reference density (kg/m^3), Γ_0 is the Mie-Gruneisen equation constant,

S_a is the linear Hugoniot coefficient, and e is the internal energy per unit mass.

In the selection of ductile target material, a titanium alloy (TC4) is selected, and its material parameters are shown in Table 1.

Table 1 Material parameters of Ti-6Al-4V

Parameters	Symbol	Value
Density (kg/m) ³	ρ	4430
Elastic modulus (GPa)	E	110
Poisson's ratio	μ	0.33
Reference strain rate (s) ⁻¹	ε_0	4×10^{-4}
Room temperature (K)	T_r	293
Melting point (K)	T_m	1878
Johnson-Cook model parameters	A	1060 MPa
	B	1090 MPa
	C	0.0117
	n	0.884
	m	1.1
Johnson-Cook failure model parameters	D_1	-0.09
		0.27
	D_2	0.48
		0.014
	D_3	3.87
	D_4	
	D_5	

(2) Model of brittle materials

For brittle materials, the most commonly used model is the Johnson-Holmquist constitutive model^[17]. Model failure is defined as follows:

$$\sigma^* = [A(1 - D) + BP^{*N}](1 + C \ln \dot{\varepsilon}^*) \quad (61)$$

where σ^* is the equivalent strength (MPa), A is the cohesive strength (MPa), D is the

damage factor, B is the pressure hardening factor, P^* is the dimensionless HEL pressure, $P^* = P/P_{HEL}$, N is the stress hardening index, C is the strain rate coefficient, and $\dot{\epsilon}^*$ is the equal effect rate (s^{-1}).

The damage factor D represents the amount of damage of the target material. When D equals 0, it means that the target material is not damaged, and when D equals 1, it means that the target material begins to break. The expression is defined as follows:

$$D = \frac{\sum(\Delta\epsilon_p + \Delta\mu_p)}{\epsilon_p^f + \mu_p^f} \quad (62)$$

where $\Delta\epsilon_p$ is the increment of equal plastic deflection strain, $\Delta\mu_p$ is the increment of plastic volume strain, ϵ_p^f is the equivalent plastic deflection strain, and μ_p^f is the plastic volume strain. In the damage relationship of the JH 2 model, T represents the maximum tensile strength, D_1 and D_2 are the damage constants, and E_{fmin} represents the minimum plastic strain variable before fracture. where f_c is the static uniaxial compressive strength, parameter A represents the normalized cohesive strength, B is the normalized pressure hardening coefficient, C is the strain rate coefficient, σ^* is the normalized equivalent strength, and P is the static pressure.

In the JH₂ model, impacted by the abrasive particles, the target material is first compressed, and then it is stretched and peeled off when it reaches the limit state. The two processes are analyzed independently. In the compression stage, it is divided into an elastic zone, plastic transition zone and compact compression material zone, while in the process of tensile spalling, there is only the damage factor elastic zone.

In the elastic compression stage, the volume change of the brittle material is linear with the pressure, namely,

$$P = K\mu \quad (63)$$

where K is the bulk modulus (MPa), μ is the volume strain, $\mu = \rho/\rho_0 - 1$, ρ is the density of brittle materials (kg/m^3), and ρ_0 is the initial density of the brittle material.

In the plastic transition stage, some parts of the brittle materials have already produced nonrecoverable damage, that is, there are microcracks, and the pressure can be expressed as

follows:

$$P = P_c + K_c(\mu - \mu_c) \quad (64)$$

$$K_c = \frac{P_l - P_c}{\mu_l - \mu_c} \quad (65)$$

where P_c is the pressure at the impact point (MPa), P_l is the pressure at the material compaction point (MPa), μ_c is the volume strain at the impact point, and μ_l is the volume strain at the material compaction point.

The time of the material compaction stage is very short. After the compaction stage is complete, the microcracks begin to expand, and then staggered complex cracks are formed, which leads to spalling of the materials. The expression for pressure is defined as follows:

$$P = K_1\bar{\mu} + K_2\bar{\mu}^2 + K_3\bar{\mu}^3 \quad (66)$$

$$\bar{\mu} = \frac{\mu - \mu_l}{\mu + \mu_l} \quad (67)$$

where K_1 , K_2 and K_3 are material constants.

The equation of state for brittle materials can be expressed as follows:

$$\begin{cases} P = K_1\mu + K_2\mu^2 + K_3\mu^3 & \mu > 0 \\ P = K_1\mu & \mu \leq 0 \end{cases} \quad (68)$$

where K_1 , K_2 and K_3 are the correlation constants of the material and their values are measured by the plate impact experiment.. $\mu = \rho/\rho_0 - 1$, where ρ is the current density and ρ_0 is the initial density.

Glass was selected as the brittle target, and its specific material properties are shown in Table 2.

Table 2 Material parameters of glass

Parameters	Symbol	Value
Density (kg/m ³)	ρ	2530
Shear modulus (GPa)	E	30.4
JH2 model	A	0.93
	B	0.88
		0.003

Damage model parameters	C	0.35
	M	0.77
	N	
	D_1	0.053
	D_2	0.85
State equation parameters	K_1	45.4 GPa
		138 GPa
	K_2	290 GPa
	K_2	

3.6 Fracture and contact algorithms

The contact effects between SPH particles with different material properties can be achieved by applying contact forces to the SPH particles. In figure 3, when the distance between the injected SPH particles (water jet particles or abrasive particles) and the coal particles reaches twice the smoothing length, contact forces are generated on the relevant SPH particles. The radius of the SPH particle support area Ω is twice the smoothing length^[18]. The contact function is defined as follows:

$$\phi(x_i) = \sum_j^N \frac{m_j}{\rho_j} K \left[\frac{W(r_{ij})}{W(\Delta d)} \right]^n \quad (69)$$

where N is the number of particles of different materials in the supporting domain Ω of particle i , as shown in Fig. 7, and the value of N is 3. When calculating the contact force of the particle, only N_1 , N_2 , and N_3 are included as SPH particles in the support domain Ω . When x_a and x_b belong to the same kind of material, the value of the kernel function is 0. The values of K and n should be determined according to the working conditions. The K value is similar to the penalty stiffness values in finite element contact and is related to the material properties and impact velocity.

The potential function is similar to the repulsive force to avoid the tension instability. It has the following characteristics: 1) the value of the function inside the object is 0; b) the value is usually positive; c) the value of the potential function increases with decreasing

particle spacing.

The SPH scheme is used to discretize the gradient of the contact potential function. The contact forces acting on the particles are obtained as follows:

$$f_c(x_i) = \sum_j^N \frac{m_j}{\rho_j} \frac{m_i}{\rho_i} K n \frac{W(r_{ij})^{n-1}}{W(\Delta p_a)^n} \nabla_{x_i} W(r_{ij}) \quad (70)$$

After introducing the contact force into the SPH momentum equation, the results are obtained as follows:

$$\frac{dv_i^\alpha}{dt} = \sum_{j=1}^N m_j \left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_j^{\alpha\beta}}{\rho_j^2} - \Pi_{ij} + R_{ij} f_{ij}^n \right) \frac{\partial W_{ij}}{\partial x_i^\beta} - \frac{f_c^\alpha(x_i)}{m_i} \quad (71)$$

The smoothing length of the SPH particles at the fracture point decreases when calculating the fracture zone. This method reduces the interaction between SPH particles of the fracture zone and other SPH particles. However, reducing the smoothing length of invalid particles leads to a decrease in the time step and an increase in the calculation time. Therefore, when the critical value is reached, the particles are considered invalid, and fracture is assumed to occur at the position related to the particles. Then, the particles at that location are removed from the SPH calculation. However, to ensure the mass and momentum conservation of the whole jet system, the mass and momentum of the invalid particles that are not involved in the calculation are still retained.

Fig. 7 Particle contact diagram

In Fig.8, the particles j identified by red dots are invalid particles and are removed from

the list of adjacent particles associated with particle i . Therefore, although the mass and momentum are still constantly updated, the SPH calculations for particles j are not performed. When the target is in a compression state, the target can still bear the load. Therefore, although the particles associated with the damage variables have reached the fracture critical value, these particles are still used for SPH calculations under compression conditions.

Fig. 8 Schematic diagram of failure particles in the computational domain

3.7 Artificial viscous forces

When solving the problem of high-speed impact, a large amount of energy accumulation is generated in the impact region, which result in nonphysical oscillation in the calculation. The shock wave discontinuities in the impact region do not occur in real physics, but in a very small transition region, the material is very thin, the number of molecules is very small, and the size of the region is generally the average free path of several molecules. If there is no artificial viscosity term in the governing equation, the SPH solution presents significant nonphysical oscillations or fluctuations. This is because impact and discontinuity always exist under initial conditions, and the viscous force is not eliminated without artificial viscosity. Therefore, if there is no damping force in the momentum equation represented by equation (45), it causes unreasonable numerical oscillation or fluctuation problems.

In this paper, Monaghan's artificial viscosity form is used, and this artificial viscosity is the most widely used impact problem. It can not only convert kinetic energy into heat energy and provide essential dissipation of the collision surface but also prevent the nonphysical penetration of particles when they are close to each other. The specific expression is defined as follows^[19]:

$$\Pi_{ij} = \begin{cases} \frac{-\alpha_{\Pi} \bar{c}_{ij} \phi_{ij} + \beta_{\Pi} \phi_{ij}^2}{\bar{\rho}_{ij}}, & \mathbf{v}_{ij} \cdot \mathbf{x}_{ij} < 0 \\ 0, & \mathbf{v}_{ij} \cdot \mathbf{x}_{ij} \geq 0 \end{cases} \quad (72)$$

where

$$\phi_{ij} = \frac{h_{ij} \mathbf{v}_{ij} \cdot \mathbf{x}_{ij}}{|\mathbf{x}_{ij}|^2 + \varphi^2} \quad (73)$$

$$\bar{c}_{ij} = \frac{1}{2}(c_i + c_j) \quad (74)$$

$$\bar{\rho}_{ij} = \frac{1}{2}(\rho_i + \rho_j) \quad (75)$$

$$h_{ij} = \frac{1}{2}(h_i + h_j) \quad (76)$$

where α_{Π} and β_{Π} are the Monaghan-type artificial viscosity constants, and $\varphi = 0.1h_{ij}$ is used to prevent numerical divergence when particles are close to each other. That is, when the $\mathbf{x}_{ij}$ term in the denominator is zero, the calculation is unstable. The following ranges are used for the values of α_{Π} and β_{Π} ^[20]. When simulating free surface flow, α_{Π} is taken as 0.01; when dealing with solid mechanics problems, α_{Π} is taken as 2.5; and when dealing with impact problems, α_{Π} is taken as 1. The parameter β_{Π} is used to address the particle penetration problem at a high Mach number, while in the process of abrasive water jet impact, the value of the jet velocity is 100 m/s ~300 m/s, which corresponds to a low Mach number, so the value is 1.

3.8 Time step

In this paper, the frog leaping method with low storage is used for time integration^[21]. When a time step is completed, the density, velocity and internal energy are advanced by half a step from the initial state, and the particle position is advanced by one step:

$$\rho_i^{1/2} = \rho_i^0 + \frac{\Delta t^1}{2} \frac{d\rho_i^0}{dt} \quad (77)$$

$$\mathbf{v}_i^{1/2} = \mathbf{v}_i^0 + \frac{\Delta t^1}{2} \frac{d\mathbf{v}_i^0}{dt} \quad (78)$$

$$\mathbf{x}_i^1 = \mathbf{x}_i^0 + \Delta t^1 \mathbf{v}_i^{1/2} \quad (79)$$

where Δt is the time step, ρ is the density of the particles, $\mathbf{v}$ is the velocity vector of the particle motion, $\mathbf{x}$ is the position vector of the particles, and the upper corner of each parameter represents the running time step.

In the overall calculation, to ensure that the particle density, velocity and internal energy of the half time step running parameters can be consistent with the position of the particle, these parameters advance forward half a time step at the beginning of each operation step to obtain the value of the integer step.

$$\rho_i^n = \rho_i^{n-1/2} + \frac{\Delta t^n}{2} \frac{d\rho_i^{n-1}}{dt} \quad (80)$$

$$\mathbf{v}_i^n = \mathbf{v}_i^{n-1/2} + \frac{\Delta t^n}{2} \frac{d\mathbf{v}_i^{n-1}}{dt} \quad (81)$$

where n is the time step label.

At the end of a time step operation, the density, velocity, internal energy, and position of the particles advance one time step:

$$\rho_i^{n+1/2} = \rho_i^{n-1/2} + \frac{\Delta t^n + \Delta t^{n+1}}{2} \frac{d\rho_i^n}{dt} \quad (82)$$

$$\mathbf{v}_i^{n+1/2} = \mathbf{v}_i^{n-1/2} + \frac{\Delta t^n + \Delta t^{n+1}}{2} \frac{d\mathbf{v}_i^n}{dt} \quad (83)$$

$$\mathbf{x}_i^{n+1} = \mathbf{x}_i^n + \Delta t^{n+1} \mathbf{v}_i^{n+1} \quad (84)$$

Considering the stability of the numerical calculation, each time step is limited according to the time step of viscous dissipation and external force. The expression is defined as follows:

$$\Delta t_{cv} = \min_i \left\{ \frac{h_i}{c_i + 0.6[\alpha_{\Pi} c_i + \beta_{\Pi} \max(\phi_{ij})]} \right\} \quad (85)$$

$$\Delta t_f = \min_i \left(\frac{h_i}{f_i} \right)^{\frac{1}{2}} \quad (86)$$

where c_i is the sound velocity of particle i in the material, h_i is the smoothing length of SPH particle i , α_Π and β_Π are artificial viscosity constants, f_i is the unit mass force acting on particle i , and the final time step is the minimum value in equations (85) and (86). The total number of steps of the impact process is determined by the actual process time of the impact phenomenon, and the total number of steps includes a complete process of particles impacting the target.

4 Numerical Results and Analysis

4.1 Numerical simulation

In this paper, the model parameters of angular particles (square and regular triangles) and circular particles are set to study the impact of different shapes of abrasive particles on the surface of ductile and brittle materials. The two-dimensional model is established by using the SPH code written in Fortran according to the basic principle outlined above. The target material is set as a rectangular target block with material properties, and the impact effect of different forms of abrasive particles is compared under the same conditions. The initial velocity of particle impact is 100 m/s, and the initial azimuth angles are 30° , 45° and 0° .

(1) Comparison of the results of the impact ductility material (TC4)

Fig. 9 Simulation results of triangular abrasive particles impacting ductile material

A V-shaped pit is formed when the abrasive particles (the impact azimuth angle is 0°) impact the target surface, and material accumulates on both sides of the impact pit. The simple vertical impact is not very effective for material removal because in the continuous impact process, the process of material accumulation to a failure value is slow. When the triangular abrasive particles impact the target with a certain impact azimuth angle, for example, when θ_i is equal to 30° , at the simulation time of $0\sim 45\ \mu\text{s}$, the target material accumulates on one side. When the abrasive particles cannot continue to penetrate the target body, the direction of the particle motion is deflected, which produces a certain angular velocity and it breaks away from the target; however, with increasing impact azimuth angle, when θ_i is equal to 45° , a single impact of an abrasive particle results in material surface removal due to the shallow impact extrusion area.

Fig. 10 Simulation results of square abrasive particles impacting ductility materials

Taking square abrasive particles as the impact body, the impact azimuth angle θ_i is 0° (vertical impact target) and 30° , and the impact speed is 100 m/s. At different times, the extracted collision effect is shown in Fig.10. For vertical incidence, at the same calculation time point, the pits generated by the impact on the target surface are shallow relative to the pits generated by the regular triangle abrasive particles, and the impact of the particles does not cause material removal and shedding. When the square abrasive particles impact the target at an azimuth angle of 30° , an area of unilateral accumulation is formed. When the target material in the accumulation area reaches the fracture limit of the material, the material is removed.

Fig. 11 Simulation results of circular abrasive particles impacting ductile material

The impact mode of the circular abrasive particles is relatively simple compared with that of the angular particles, and the simulation results of the vertical impact on target material are shown in Fig.11. The target material begins to form a stress concentration. With the increase in the calculation step, plastic deformation gradually occurs, and material accumulation is formed. Circular abrasive particles cannot effectively remove the surface material when impacting the target.

(2) Comparison of the results of impact on brittle materials (glass)

Glass was selected as the brittle target material in the simulation of the impact process, abrasive particles of different shapes are used to impact the target with different initial impact azimuth angles, and the impact velocity is set to 100 m/s. Fig. 12 shows that the removal method of the target material is different from that of the ductile material after being impacted. The brittle material mainly produces plastic deformation and breakage, and there is no obvious material accumulation or extrusion area. For the three shapes of abrasive particles, the cracks produced by the impact of regular triangular abrasive particles are the most obvious, and the longitudinal cracks are the deepest, followed by square abrasive particles and circular abrasive particles. For abrasive particles of the same shape, with increasing impact azimuth angle θ_i , the longitudinal depth of the stress concentration area becomes shallower, which is not conducive to material removal. For circular abrasive particles, the longitudinal depth of the stress concentration region produced by impacting brittle materials is deeper than that of impacting ductile materials.

Fig. 12 Simulation results of impact on brittle materials with abrasive particles of different shapes

5 Experimental results

To verify the accuracy of the numerical model and improve the SPH model, numerical simulations and experiments are carried out for the process of single abrasive particles impacting the target. The single particle impact experiment is realized by using the particle impact test bench. For different shapes of particles impacting on target material with different properties, a high-speed camera is used to capture and track the process dynamically. The main parameters in the experiment are as follows: (1) The size of square, regular triangle and circular particle is $a = h = d = 5 \text{ mm}$, respectively; (2) The particle impact velocity is 100 m/s ; (3) Figure 3 shows that when abrasive particles impact the target material in water, the main impact azimuth angle ranges from -45° to 45° (angular symmetry). Therefore, the impact azimuth angles are taken as 0° , 30° and 45° in the experimental process.

Fig. 13 Particle impact experimental device

In the experimental process, the particle impact experimental device is shown in Fig.13. The particles adopt a high-speed steel blade with high hardness. The dynamic process of the particle impact on the target is captured by a high-speed camera, as shown in Fig.14.

Fig. 14 Dynamic process of particle impact on target materials

To verify the accuracy of the SPH model, the calculation results of the single abrasive particle impacting the target are compared with the experimental results, as shown in Fig.15.

30° 45° The SPH models were used to simulate regular triangle, square particles, and then the titanium alloy (TC4) materials was impacted at 100 m/s, and the glass material was impacted by regular triangle, square and circular particles at the same speed. When the incident angle is 30° and 45° , the insertion depth is almost the same, but the simulation results show that the horizontal cutting amount is larger than the measured value, and the interpolation becomes larger as the angle increases. For the plastic deformation-dominated impact processes, Johnson-Cook parameters are based on a certain range of strain rates (less than 10^3). However, in the process of abrasive particle impact, the strain rate can reach 10^5 . Therefore, the yield limit obtained by numerical simulation is lower than the actual situation under this strain rate, which results in the calculated results being larger than the experimental results. When calculating the impact dependence on azimuth, the direction of stress concentration changes, so the displacement in the direction of stress concentration is greater

than in the actual situation. As a result, the impact dependence on azimuth angle is larger than the measured value for the horizontal displacement, and the difference is more obvious with increasing azimuth angle. A comparison of the simulation and experimental results of the impact deformation of ductile materials shows that the maximum error in the vertical direction is approximately 0.2 mm, the maximum error in the horizontal direction is approximately 0.5 mm, and the degree of coincidence between the simulation results curve and the experimental curve is high.

(a) Regular vertical triangle

(b) Regular triangle $\theta_i = 30^\circ$

(c) Regular triangle $\theta_i = 45^\circ$

(d) Vertical square

(e) Square $\theta_i = 30^\circ$

Fig. 15 Simulation and experimental comparison of impact on ductile materials

Fig.16 shows a comparison between the numerical simulation results and the experimental results for brittle materials. The experimental results in the figure are the dent profile of section 3 when the experimental drawing points are taken. By comparison, it can be found that the dent trajectory similarity is higher when the brittle material is impacted, but more broken points in the measured curves of angular particles lead to uneven curves, and the numerical simulation results are smoother. Because the behavior of the material in the impact process is mainly dynamic failure, and there are many factors affecting the fracture of brittle materials, only the fracture trend and morphology can be predicted. The impact form of circular particles is relatively simple, so the simulation results curves are more consistent with the measured curves. Generally, the error of the numerical simulation results for impacting brittle materials is larger than that for ductile materials, but it also has an allowable range of errors. Therefore, it can be concluded that the impact model of a single abrasive particle can simulate the impact on ductile and brittle materials well, and the simulation results are more accurate than those of other methods.

Fig. 16 Simulation and experimental comparison of impact on brittle materials

6. Conclusions

In this paper, the SPH method is used to construct the process model of abrasive particles impacting targets, and the model is verified by experiments. The conclusions are as follows:

(1) In this paper, the impact of abrasive particles in abrasive water jets is studied. The target is modeled as an elastic-plastic material, and the abrasive particles are modeled as rigid bodies with irregular shapes. The model can be used to study the impact process of microparticles and provide a new numerical model for the mechanism of abrasive water jets.

(2) Through the experiment of particle impact, the numerical simulation results are verified, and it is found that the accuracy of the numerical model is high, the impact process of a single particle can be simulated well, and the error of the dent curve formed by impact is less than 0.5 mm.

(3) The target materials with different material properties are impacted by circular, regular triangle and square abrasive particles. For ductile materials, a V-shaped pit is produced when the angular particles are vertically incident on the target surface, the materials accumulate on both sides of the impact pit, and the process of material accumulation to the failure value is slow. When the triangular abrasive particles impact the target at an azimuth angle of 30° , the target material accumulates on one side. When the abrasive particles cannot continue to penetrate the target, the movement direction of the particles is deflected, which produces a certain angular velocity and the particles detach from the target; however, with increasing impact azimuth angle, a single impact of an abrasive particle leads to the removal of surface material. When the square abrasive particles are incident vertically, at the same calculation time point, the pits produced by the impact on the target surface are shallower than those produced by the regular triangle, and the impact of the particles does not cause the material to be removed and shed. When the square particles impact the target at an azimuth angle of 30° , a single side accumulation is formed. When the target material in the accumulation area reaches the fracture limit of the material, material is removed. The impact mode of circular abrasive particles is simpler than those of the angular particles. When the circular abrasive particles impact the target material vertically, an initial stress concentration of the target material is formed. With increasing the calculation step, plastic deformation is gradually produced, and the accumulation of materials is formed. Circular abrasive particles cannot effectively remove the surface material when impacting the target.

Brittle materials differ from ductile materials in the way of removing material after being impacted; brittle materials mainly produce plastic deformation and breakage, and there is no obvious material accumulation and extrusion area. For the three shapes of abrasive particles, the cracks produced by the impact of regular triangular abrasive particles are the most obvious, and the longitudinal cracks are the deepest, followed by square abrasive particles and circular abrasive particles. For abrasive particles of the same shape, with increasing impact azimuth angle θ_i , the longitudinal depth of the stress concentration area becomes shallow, which is not conducive to material removal. For the circular abrasive particles, the longitudinal depth of the stress concentration region produced by the impact on brittle material is deeper than that of the impact on ductile material.

Declarations section

a. Funding (information that explains whether and by whom the research was supported)

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b. Conflicts of interest/Competing interests (include appropriate disclosures)

The authors declared that they have no conflicts of interest to the article entitled “Study on Abrasive Particle Impact Modeling and Cutting Mechanism”. We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted

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c. Availability of data and material (data transparency)

All data generated or analysed during this study are included in this published article.

d. Code availability (software application or custom code)

The code used or analysed during the current study are available from the corresponding author on reasonable request.

e. Ethics approval (include appropriate approvals or waivers)

Not applicable

f. Consent to participate (include appropriate statements)

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