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Adaptive Finite-time Dynamic Surface Neural Network Control of an Uncertain Robot with Output Constraint and Input Saturation

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Abstract— In this study, a finite-time dynamic surface neural network control is developed for an uncertain n-link robot subject to input saturation and output constraints. First, a barrier Lyapunov function and a hyperbolic tangent function are applied to solve the system constraints using a dynamic surface control. Subsequently, a radial basis function neural network is utilized to handle system uncertainties. Then, a finite-time filter is employed in the design to achieve the fast convergence and a Nussbaum function is employed to optimize the design process. Finally, the simulation results show that the dynamic tracking error is proved to converging to zero, and the proposed control method is effective and never violates the constraints.

Index Terms — Neural network, Input saturation, Output constraints, Nussbaum function, Dynamic surface control, Finite-time.

I. INTRODUCTION

In recent years, the application scenarios of manipulator systems are constantly expanding, so the research on them is increasingly deepening [1]–[3]. Due to different application scenarios, the robot system is generally subjected to constraints [4]–[8], such as dead zone [9], saturation [10], [11], hysteresis [12], [13], specified performance [14]–[16], and so forth. Saturation as a common nonlinear characteristics is broadly found in the actuator of physical systems because of the upper limit of motor torques [17]–[21]. Ignoring saturation effects can lead to system performance degradation and even instability. Output constraints widely exist in consideration of safety or performance specifications [22]. Violating output constraints will result in output performance degradation and even bring serious consequences. Therefore, the synthetic influence of output constraint and input saturation in the robot system should be considered during the process of control design.

To address the output constraint or input saturation, many researchers have proposed diverse solutions

in recent years [23]–[30]. To list some examples, in [24], [26], a saturation function was employed to describe the mathematical model of input saturation. In [27], the dead zone nonlinearity was applied to replace the saturation nonlinearity in multi-agent systems, and the hyperbolic tangent function was applied to the control law design of a flexible manipulator with input and rate constraints [30], which can effectively solve the problem that the sharp corner of saturation function was not differentiable. In [31], [32], the constrained system was transformed into the unconstrained case using the system transformation technique and the system remained to be stable. A barrier Lyapunov function (BLF) was adopted to deal with the control problems of state or output constrained systems in [2], [8], [33]–[36]. However, the above methods only resolved the issue of output constraint or input saturation and cannot be applied to the robot system simultaneously affected by output constraint and input saturation. In specific applications, constraints generally exist in multiple forms, which will pose an increased challenge to the control design and analysis.

Since the backstepping control is able to handle diverse nonlinearities during the design [37]–[42], it has become a common design aid for the control of various nonlinear systems [30], [43]–[46]. The main idea is to split the high-order system into several subsystems, and then use Lyapunov method to design the appropriate virtual control law for each subsystem to achieve the overall control object, but the biggest problem is that “term explosion” will occur in the process of frequent derivation, which makes the control law design difficult. In order to solve this issue, the dynamic surface control (DSC) was presented in [47] by designing a filter to obtain a first-order derivative approximation of the input signal based on the definition of derivative, which greatly reduced the complexity of the design process. In [48], a radial basis function neural network (RBFNN) based adaptive control approach was presented with a dynamic surface technique for stochastic nonlinear pure-feedback constrained systems. In [49], [50], an adaptive neural network based DSC was developed for nonlinear saturated systems. In [51], an adaptive DSC was presented for hypersonic vehicles with dead zone. In [52], based on fuzzy control and DSC, a dynamic-scaling adaptive fuzzy tracking controller was constructed to cope with unknown nonlinearities. However, the above literatures were confined to the fixed-time DSC of nonlinear systems with input constraints, and these schemes are ineffective for the finite-time convergence [53]–[57] DSC of the robot system with output constraint and input saturation.

In this study, we tend to develop a finite-time adaptive neural network DSC for an n-link rigid robot system with output constraint and input saturation. Compared with the existing work, the main contributions are:

- (i) A hyperbolic tangent function and a Nussbaum function are introduced to tackle the input saturation

in the robot system, and the Moore Penrose inverse term and BLF are adopted to guarantee no transgression of output constraint.

- (ii) An adaptive neural network DSC with a finite-time filter is designed to approximate the unknown dynamic model, improve the system robustness, ensure a good trajectory tracking performance, and make the output of the filter track the input signal in a finite-time.
- (iii) The Lyapunov method is employed to demonstrate the stability of the system, and all the trajectory tracking errors will converge to zero.

II. PROBLEM FORMULATION

Lemma 1. [2] If the function $V(t) \geq 0$ is a continuous function for $\forall t \in R$ with a bounded $V(0)$, we have:

$$\dot{V}(t) \leq -c_1 V(t) + c_2 \quad (1)$$

where $c_1, c_2 > 0$ are constants.

Lemma 2. The follow inequality holds for any vectors $x, y \in R^n$:

$$x^T y \leq \frac{\epsilon^p \|x\|^p}{p} + \frac{\|y\|^q}{q \epsilon^q} \quad (2)$$

where $\epsilon > 0, p > 1, q > 1$.

Lemma 3. [57] For the filter

$$\begin{aligned} \dot{x}_d + \alpha(x_d - x_e) + \beta(x_d - x_e)^{q/p} &= 0 \\ x_d(0) &= x_e(0) \end{aligned} \quad (3)$$

where x_d is the output signal, x_e is the input signal, α and β are positive constant, p, q are odd numbers and $p > q > 0$. If above conditions are true, then for any input signal $x_e \in [0, +\infty]$, the output signal can follow the input signal in a finite-time with the convergence upper bound satisfying the following:

$$t = \frac{p}{\alpha(p-q)} \ln \frac{\alpha(x_d(0) - x_{e,\max})^{(p-q)/p} + \beta}{\beta}. \quad (4)$$

Based on the Lagrangian function, the mathematical model of the n-linked robot under study is formulated as follows

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = u(v) - J(q)^T f(t) \quad (5)$$

where $q, \dot{q}, \ddot{q} \in R^n$ denote the position, velocity and acceleration vectors, respectively, $u(v) \in R^n$ is

the input of the system, and $v \in R^n$ is the intermediate variable. $M(q), C(q, \dot{q}) \in R^{n \times n}$, and $G(q) \in R^n$ represent the inertia matrix, Centripetal and Coriolis torques matrix, and gravitational force vector, respectively, with $M(q)$ being a positive definite matrix. $J(q)$ denotes the nonsingular Jacobian matrix, and $f(t) \in R^n$ denotes the vector of external disturbance.

Let $x_1 = q$ and $x_2 = \dot{q}$, then we obtain the following translation

$$\begin{aligned}\dot{x}_1 &= \dot{x}_2 \\ \dot{x}_2 &= M^{-1}[u(v) - J(x_1)^T f(t) - C(x_1, x_2)\dot{x}_1 - G(x_1)].\end{aligned}\tag{6}$$

For the convenience of the following text, we abbreviate notations of $M(q)$, $C(q, \dot{q})$, $G(q)$, $J(q)$, and $f(t)$ as M , C , G , J , and f , respectively.

Property 1: The matrix $M(q)$ is symmetric and positive definite.

Property 2: The matrix $\dot{M}(q) - 2C(q, \dot{q})$ is skew-symmetric.

In this study, the robot system subjected to the input saturation is considered and the saturation limit is $u_M > 0$ satisfying $|u(v)| \leq u_M$. Hence, the hyperbolic tangent smoothing function is exploited for approximating the saturated nonlinearity designed as

$$u(t) = g(v) = u_M \tanh\left(\frac{v(t)}{u_M}\right) = u_M \frac{e^{v(t)/u_M} - e^{-v(t)/u_M}}{e^{v(t)/u_M} + e^{-v(t)/u_M}},\tag{7}$$

where $v(t)$ is an intermediate variable, and we design auxiliary systems as

$$\dot{v} = -cv + \omega,\tag{8}$$

with $c > 0$, then the control design is translated into the design of ω .

III. CONTROL DESIGN

A. Adaptive Neural Dynamic Surface Controller Design

Step 1: First, a position error is defined as

$$z_1 = x_1 - y_d.\tag{9}$$

Then, the first virtual control variable a_1 is introduced and a second error variable is defined as $z_2 = x_2 - a_1$. We choose

$$a_1 = -K_1 z_1 + \dot{y}_d\tag{10}$$

where K_1 is the gain matrix satisfying $K_1 = K_1^T > 0$. We choose the first Lyapunov function as

$$V_1 = \frac{1}{2} \sum_{i=1}^n \log \frac{b_{1i}^2}{b_{1i}^2 - z_{1i}^2}. \quad (11)$$

The differentiation of V_1 yields

$$\dot{V}_1 = \sum_{i=1}^n \left(-\frac{K_{1ii} z_{1i}^2}{b_{1i}^2 - z_{1i}^2} + \frac{z_{1i} z_{2i}}{b_{1i}^2 - z_{1i}^2} \right). \quad (12)$$

Step 2: From (10), we obtain the following

$$\dot{a}_1 = -K_1 \dot{z}_1 + \ddot{y}_d. \quad (13)$$

The derivative of z_2 is expressed as

$$\begin{aligned} \dot{z}_2 &= M^{-1} [u(v) - J(x_1)^T f(t) \\ &\quad - C(x_1, x_2) \dot{x}_1 - G(x_1)] - \dot{a}_1. \end{aligned} \quad (14)$$

Now, a new error signal z_3 is given as

$$z_3 = g(v) - a_{2c}. \quad (15)$$

$\dot{z}_2$ is rewritten as

$$z_2 = M^{-1} [z_3 + y_2 + a_{2d} - J^T f - Cx_2 - G] - \dot{a}_1. \quad (16)$$

Invoking the Moore Penrose inverse yields

$$z_2^T (z_2^T)^+ = \begin{cases} 0, & z_2 = [0, 0, \dots, 0]^T \\ 1, & \text{Otherwise} \end{cases} \quad (17)$$

Then, we design the virtual control law a_{2d} as

$$\begin{aligned} a_{2d} &= -K_2 z_2 - (z_2^T)^+ \sum_{i=1}^n \frac{z_{1i} z_{2i}}{b_{1i}^2 - z_{1i}^2} \\ &\quad + Ca_1 + G + M\dot{a}_1 + J^T f, \end{aligned} \quad (18)$$

where K_2 is the gain matrix with $K_2 = K_2^T > 0$.

Since M , C , G , and f are uncertain, we use the RBFNN to approximate the unknown system parameters.

a_{2d} is then changed as

$$a_{2d} = -K_2 z_2 - (z_2^T)^+ \sum_{i=1}^n \frac{z_{1i} z_{2i}}{b_{1i}^2 - z_{1i}^2} - \text{sgn}(z_2^T) \odot J^T \bar{f} + \hat{W}^T S(Z), \quad (19)$$

where $\hat{W}$ is the estimated value of the weight vector W^* with the estimated error defined as $\tilde{W} = W^* - \hat{W}$, and $S(Z)$ is the basis function vector.

Invoking (18) and (19), we have $\hat{W}^T S(Z) = Ca_1 + G + M\dot{a}_1$. Then, we can further derive $W^* S(Z)$ as

$$\begin{aligned} W^* S(Z) &= \hat{W}^T S(Z) - \varepsilon, \\ &= Ca_1 + G + M\dot{a}_1 - \varepsilon, \end{aligned} \quad (20)$$

with $Z = [x_1^T, x_2^T, a_1^T, \dot{a}_1^T]$ being input variables of neural networks and ε being the approximation error. And we design the updating law as

$$\dot{\hat{W}}_i = -\Gamma_i [S_i(Z) z_{2,i} + \sigma_i \hat{W}_i] \quad (21)$$

where Γ_i is the constant gain matrix, and $\sigma_i > 0$ is a small positive constant.

Consider the second Lyapunov function candidate as

$$V_2 = V_1 + \frac{1}{2} z_2^T M(x_1) z_2 + \frac{1}{2} \sum_{i=1}^n \tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i. \quad (22)$$

Differentiating (22) leads to

$$\begin{aligned} \dot{V}_2 &\leq - \sum_{i=1}^n \frac{K_{1i} z_{1i}^2}{b_{1i}^2 - z_{1i}^2} - z_2^T (K_2 - \frac{1}{2} I_{n \times n}) z_2 \\ &\quad + z_2^T y_2 + z_2^T z_3 - \sum_{i=1}^n \frac{\sigma_i}{2} \|\tilde{W}_i\|^2 \\ &\quad + \sum_{i=1}^n \frac{\sigma_i}{2} \|W_i^*\|^2 + \frac{1}{2} \|\bar{\varepsilon}\|^2. \end{aligned} \quad (23)$$

Step 3: Consider the Lyapunov function V_3 as

$$V_3 = V_2 + \frac{1}{2} z_3^T z_3. \quad (24)$$

In order to obtain the derivative of a_{2d} , the virtual control signal a_{2d} is designed by the finite-time first

order filter with small positive constants α_2 and β_2 as

$$\dot{a}_{2c} = -\alpha_2(a_{2c} - a_{2d}) - \beta_2(a_{2c} - a_{2d})^{q/p}, a_{2c}(0) = a_{2d}(0). \quad (25)$$

In this part, a Nussbaum function is adopted to optimize the design process. The specific forms are given as

$$\begin{aligned} N_i(\chi_i) &= \chi_i^2 \cos(\chi_i) \\ \dot{\chi}_i &= \gamma_{\chi i} z_{3i} \bar{\omega}_i \\ \omega_i &= N_i(\chi_i) \bar{\omega}_i \end{aligned} \quad (26)$$

where $\gamma_{\chi i} > 0$ is positive constant, and $\bar{\omega}$ is auxiliary control signal vector.

$\bar{\omega}$ is constructed as

$$\bar{\omega} = -K_3 z_3 - z_2 + p_{g^v} c v + \dot{a}_{2c}, \quad (27)$$

where K_3 is the gain matrix and $K_3 = K_3^T > 0$, $p_{g^v} = \text{diag}[\frac{\partial g_1}{\partial v_1}, \frac{\partial g_2}{\partial v_2} \dots]$.

Combining (24)-(27), we have

$$\begin{aligned} \dot{V}_3 &\leq -\sum_{i=1}^n \frac{K_{1i} z_{1i}^2}{b_{1i}^2 - z_{1i}^2} - z_2^T K_2 z_2 - z_3^T K_3 z_3 \\ &\quad - \sum_{i=1}^n \frac{\sigma_i}{2} \|\tilde{W}_i\|^2 + \sum_{i=1}^n \frac{\sigma_i}{2} \|W_i^*\|^2 + \frac{1}{2} \|\bar{\varepsilon}\|^2 \\ &\quad + \sum_{i=1}^n \frac{\dot{\chi}_i}{\gamma_{\chi i}} (p_{g_i^v} N_i(\chi_i) - 1) + \frac{1}{2} y_2^T y_2. \end{aligned} \quad (28)$$

B. Stability Analysis

For the error of first-order filter, we choose the Lyapunov candidate function as

$$V_4 = \frac{1}{2} y_2^T y_2. \quad (29)$$

Differentiating V_4 , then we have

$$\dot{V}_4 = -\alpha_2 y_2^T y_2 - \beta_2 \sum_{i=1}^n y_2(i)^{(p+q)/p} + y_2^T \eta_2, \quad (30)$$

with

$$|\dot{a}_{2d}| \leq \eta_2(z_1, z_2, \hat{W}_i, y_d, \dot{y}_d, \ddot{y}_d), \quad (31)$$

where η_2 is a nonnegative continuous function. According to Lemma 2, we have

$$\dot{V}_4 \leq -(\alpha_2 - \frac{1}{2})y_2^T y_2 + \frac{1}{2}\eta_2^T \eta_2. \quad (32)$$

In [57], the author pointed out that in DSC system, the first-order differential estimation error of filter to input signal is also very important for system stability. Then, we choose the Lyapunov function as

$$V_5 = \frac{1}{2}\xi_2^T \xi_2. \quad (33)$$

The derivative of V_5 gives

$$\begin{aligned} \dot{V}_5 &= -\alpha_2 \xi_2^T \xi_2 - \beta_2 \frac{q}{p} \sum_{i=1}^n y_2(i)^{(q-p)/p} \xi_2^T \xi_2 + \xi_2^T \zeta_2 \\ &\leq -(\alpha_2 - \frac{1}{2})\xi_2^T \xi_2 + \frac{1}{2}\zeta_2^T \zeta_2, \end{aligned} \quad (34)$$

with

$$|\ddot{a}_{2d}| \leq \zeta_2(z_1, z_2, \hat{W}_i, y_d, \dot{y}_d, \ddot{y}_d), \quad (35)$$

where ζ_2 is a nonnegative continuous function.

Choose the total Lyapunov candidate function as

$$V = V_3 + V_4 + V_5. \quad (36)$$

Differentiating V results in

$$\begin{aligned} \dot{V} &\leq -\sum_{i=1}^n \frac{K_{1iz_i^2}}{b_{1i}^2 - z_{1i}^2} - z_2^T K_2 z_2 - z_3^T K_3 z_3 \\ &\quad - \sum_{i=1}^n \frac{\sigma_i}{2} \|\tilde{W}_i\|^2 + \sum_{i=1}^n \frac{\sigma_i}{2} \|W_i^*\|^2 + \frac{1}{2} \|\bar{\varepsilon}\|^2 \\ &\quad + \sum_{i=1}^n \frac{\dot{\chi}_i}{\gamma_{\chi^i}} (p_{g_i^v} N_i(\chi_i) - 1) + (1 - \alpha_2) y_2^T y_2 \\ &\quad + (\frac{1}{2} - \alpha_2) \xi_2^T \xi_2 + \frac{1}{2} \eta_2^T \eta_2 + \frac{1}{2} \zeta_2^T \zeta_2. \end{aligned} \quad (37)$$

Let $\psi = \int_{\chi(0)}^{\chi(t)} (p_{g_i^v} N_i(s) - 1) ds$. First, for the Nussbaum function in the form of (26), if χ is a bounded function [58], ψ is bounded, which means that if its infinite integral exists, $\sum_{i=1}^n \frac{\dot{\chi}_i}{\gamma_{\chi^i}} (p_{g_i^v} N_i(\chi_i) - 1)$ is bounded. Then, we can find a constant O such that $\sum_{i=1}^n \frac{\dot{\chi}_i}{\gamma_{\chi^i}} (p_{g_i^v} N_i(\chi_i) - 1) \leq O$.

Based on the above analysis, we can obtain

$$\dot{V} \leq -\rho V + C, \quad (38)$$

where

$$\begin{aligned} \rho &= \min[\min(2K_{1i}), \min(\frac{\lambda_{\min}(2K_2)}{\lambda_{\max}(M)}), \min(2K_{3i}), \\ &\quad \min(\frac{\sigma_i}{\lambda_{\max}(\Gamma_i^{-1})}), 2(1 - \alpha_2), 2(\frac{1}{2} - \alpha_2)], \\ C &= \frac{1}{2}\eta_2^T \eta_2 + \frac{1}{2}\zeta_2^T \zeta_2 + O + \sum_{i=1}^n \frac{\sigma_i}{2} \|W_i^*\|^2 + \frac{1}{2} \|\bar{\varepsilon}\|^2, \end{aligned} \quad (39)$$

where $\lambda_{\min}(\bullet)$ and $\lambda_{\max}(\bullet)$ represent the minimum eigenvalue and the maximum eigenvalue of the matrix $(\bullet)$, respectively, and $\lambda(\bullet)$ is real. To ensure $\rho > 0$, α_2 must satisfy the following condition

$$2(\frac{1}{2} - \alpha_2) > 0. \quad (40)$$

Multiplying (38) by $e^{\rho t}$ yields

$$\frac{d}{dt} (V e^{\rho t}) \leq C e^{\rho t}. \quad (41)$$

Integrating the above inequality gives

$$V \leq \left(V(0) - \frac{C}{\rho} \right) e^{-\rho t} + \frac{C}{\rho} \leq V(0) + \frac{C}{\rho}. \quad (42)$$

Then we further have

$$\begin{aligned} \frac{1}{2} \sum_{i=1}^n \log \frac{b_{1i}^2}{b_{1i}^2 - z_{1i}^2} &\leq V(0) + \frac{C}{\rho} \\ \frac{1}{2} \|z_2\|^2 &\leq \frac{V(0) + \frac{C}{\rho}}{\lambda_{\min}(M)} \\ \frac{1}{2} \sum_{i=1}^n \left(\tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i \right) &\leq V(0) + \frac{C}{\rho}. \end{aligned} \quad (43)$$

Finally, we can obtain

$$\begin{aligned} \Omega_{z_1} &:= \left\{ z_{1i} \in \mathbb{R}^n \mid \|z_{1i}\| \leq \sqrt{k_{ai}^2 (1 - e^{-D})} \right\} \\ \Omega_{z_2} &:= \left\{ z_2 \in \mathbb{R}^n \mid \|z_2\| \leq \sqrt{\frac{D}{\lambda_{\min}(M)}} \right\} \\ \Omega_{\tilde{W}_i} &:= \left\{ \tilde{W}_i \in \mathbb{R}^n \mid \|\tilde{W}_i\| \leq \sqrt{\frac{D}{\lambda_{\min}(\Gamma_i^{-1})}} \right\} \end{aligned} \quad (44)$$

where $D = 2(V(0) + C/\rho)$, and the closed-loop error signals z_1 , z_2 , and $\tilde{W}_i$ will remain within the compact sets Ω_{z_1} , Ω_{z_2} , and $\Omega_{\tilde{W}_i}$, respectively. At this time, we conclude that all three signal errors will be maintained in closed sets, respectively, and all errors will converge to a neighborhood of zero under the proposed control with suitable parameter conditions.

IV. SIMULATIONS

A. Robot System

In this paper, the double joint rigid robot in [59] is used as the model, and the dynamical model description matrix of the robot system is defined as follows

$$M(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad (45)$$

$$C(q, \dot{q}) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \quad (46)$$

$$G(q) = \begin{bmatrix} G_{11} \\ G_{21} \end{bmatrix} \quad (47)$$

and

$$\begin{aligned} M_{11} &= m_1 l_{c1}^2 + m_2 (l_1^2 + l_{c2}^2 + 2l_1 l_{c2} \cos q_2) + I_1 + I_2 \\ M_{12} &= m_2 (l_{c2}^2 + l_1 l_{c2} \cos q_2) + I_2 \\ M_{21} &= m_2 (l_{c2}^2 + l_1 l_{c2} \cos q_2) + I_2 \\ M_{22} &= m_2 l_{c2}^2 + I_2 \\ C_{11} &= -m_2 l_1 l_{c2} \dot{q}_2 \sin q_2 \\ C_{12} &= -m_2 l_1 l_{c2} (\dot{q}_1 + \dot{q}_2) \sin q_2 \\ C_{21} &= m_2 l_1 l_{c2} \dot{q}_1 \sin q_2 \\ C_{22} &= 0 \\ G_{11} &= (m_1 l_{c2} + m_2 l_1) g \cos q_1 + m_2 l_{c2} g \cos (q_1 + q_2) \\ G_{21} &= m_2 l_{c2} g \cos (q_1 + q_2). \end{aligned} \quad (48)$$

TABLE I
PARAMETERS OF THE ROBOT

Parameter	Description	Value
m_1	Mass of link 1	2.00 kg
m_2	Mass of link 2	0.85 kg
l_1	Length of link 1	0.35 m
l_2	Length of link 2	0.31 m
I_1	Moment of inertia of link 1	$\frac{1}{4}m_1l_1^2\text{kgm}^2$
I_2	Moment of inertia of link 2	$\frac{1}{4}m_2l_2^2\text{kgm}^2$

The Jacobian matrix is written as

$$J(q) = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \quad (49)$$

and

$$\begin{aligned} J_{11} &= -l_1 \sin q_1 + l_2 \sin (q_1 + q_2) \\ J_{12} &= -l_2 \sin (q_1 + q_2) \\ J_{21} &= l_1 \cos q_1 + l_2 \cos (q_1 + q_2) \\ J_{22} &= l_2 \cos (q_1 + q_2) . \end{aligned} \quad (50)$$

Fig. 1. Physical model of the robotic system.

The specific parameters of the robotic system are listed in Table I, with providing the following initial states

$$q_1(0) = 0, q_2(0) = 1, \dot{q}_1(0) = 1, \dot{q}_2(0) = 0. \quad (51)$$

The expected tracking trajectory is set as $y_d = [\sin(t), \cos(t)]^T$ with $t \in [0, t_s]$ and $t_s = 20s$. The other conditions and parameters are taken as $K_1 = \text{diag}[20, 50]$, $K_2 = \text{diag}[20, 20]$, $K_3 = \text{diag}[20, 20]$, $c = 2$, and $k_1 = [0.5, 0.5]^T$.

B. Model-based control

For the model-based (MB) control, we examine the MB control designed in (18), the parameters of the finite-time filters are $p = 15$, $q = 11$, $\alpha_2 = 50$, and $\beta_2 = 80$.

The simulation results under the MB control are described in Figs. 2~7. From Figs. 2~5, it can be seen that good position and velocity tracking performance are achieved, and the input v and saturation input $u(v)$ are shown in Figs. 6 and 7, which illustrate that the saturation constraint are never violated. In order to verify the efficacy of the designed control, we compare the backstepping control (BS) without output constraints and DSC control with the proposed finite-time DSC (FDSC), and the control law with input saturation based on backstepping control is designed as

$$v = \theta_1 - z_2 - c_3 z_3 - \frac{\partial a_2}{\partial x_2} \cdot \tanh \left(\frac{z_3 \odot \frac{\partial a_2}{\partial x_2}}{\varepsilon} \right) \left| M(x_1)^{-1} \right| \bar{f} \quad (52)$$

where $\theta_1 = \partial a_2 / \partial t$, and the error comparison of the three case are shown in Figs. 8 and 9. It can seen that after applying the output constraint, the DSC and FDSC can achieve a better performance in comparison with backstepping control, and the output error under FDSC can converge faster than that under DSC.

Fig. 2. x_{11} position trajectory and tracking error z_{11} .

Fig. 3. x_{12} position trajectory and tracking error z_{12} .

Fig. 4. x_{21} velocity trajectory and tracking error z_{21} .

Fig. 5. x_{22} velocity trajectory and tracking error z_{22} .

Fig. 6. Control input v and saturation input $u(v)$ for the first joint.

Fig. 7. Control input v and saturation input $u(v)$ for the second joint.

Fig. 8. Comparison of error z_{11} under three methods.

Fig. 9. Comparison of error z_{12} under three methods.

C. Adaptive Neural Network Control

For adaptive neural network control, a total of 256 nodes are chosen and the 8 centers of each layer node are selected in the area of $[-1, 1]$. The initial weights are $\hat{W}_{1,i} = 0, \hat{W}_{2,i} = 0, (i = 1, 2, 3 \dots 256)$. The variance of centers is set as $\eta_c^2 = 1, \sigma = [0.01, 0.01]^T, \Gamma_1 = 10I_{256 \times 256}$, and $\Gamma_2 = 10I_{256 \times 256}$.

Simulation results are plotted in Figs. 10~17 under control law (19) (27) and updating law (21). Figs. 10~13 display that x_1 and x_2 can successfully track the desirable trajectory. Figs. 14 and 15 depict

that the change of v and the input $u(v)$ is subjected to the input saturation. Figs. 16 and 17 show that the norms of the $\hat{W}_i$ and z_1 .

Fig. 10. x_{11} position trajectory and tracking error z_{11} .

Fig. 11. x_{12} position trajectory and tracking error z_{12} .

Fig. 12. x_{21} velocity trajectory and tracking error z_{21} .

Fig. 13. x_{22} velocity trajectory and tracking error z_{22} .

Fig. 14. Control input v and saturation input $u(v)$ for the first joint.

Fig. 15. Control input v and saturation input $u(v)$ for the second joint.

Fig. 16. Norms of the adaptation weights.

Fig. 17. Norms of the errors $\|z_1\|$.

D. Simulation Analysis

From simulation results, we can see that the presented control law (18), (19), and (27) can achieve a good trajectory tracking performance, the system constraints are never violated, compared with backstepping control (52), the complexity of the design is reduced, and the error of the system output converges faster by introducing a finite-time filter. For the control law (19), by introducing the neural network, the

unknown system dynamics model can be approximated only by updating one parameter, which reduces the complexity of design and has a good performance.

V. CONCLUSION

In this paper, an adaptive finite-time dynamic surface neural network control was presented for an uncertain manipulator system with unknown dynamics and constraints. The hyperbolic tangent function and BLF were employed to eliminate the constraints, and the RBF neural networks were applied to approximate the complicated robot dynamics. By utilizing the finite-time filter in the DSC, the system achieved the fast convergence. Finally, we concluded that the derived control was able to track a desired trajectory in finite-time, and the constraints were never violated.

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Conflict of Interest: We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

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