Condition-based Maintenance Optimization for Gamma Deteriorating Systems under Performance-Based Contracting

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Title page

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ORIGINAL ARTICLE

Condition-based Maintenance Optimization for Gamma Deteriorating Systems under Performance-based Contracting

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Abstract: As a new form of support contract, performance-based contracting (PBC) has been extensively adopted in industry and manufacturing recent years. However, maintenance optimization problems under PBC have not received enough attention. In order to further expand the application of PBC in the field of maintenance optimization, we investigate the condition-based maintenance (CBM) optimization for gamma deteriorating systems under PBC. Considering the repairable single-component system subject to the gamma degradation process, this paper proposes a CBM optimization model with an objective of maximizing the profit and improving system performance at a lower cost under PBC. In the proposed CBM model, first inspection interval is considered to reduce the inspection frequency and the cost. Then, a particle swarm algorithm (PSO) and related solution procedure are presented in order to solve the multiple decision variables in our proposed model. Finally, a numerical example is provided to illustrate the applicability and effectiveness of our proposed model. Through comparing the proposed policy with the conventional one, the superiority of our proposed policy is proved, which can bring more profits to providers and improve performance. Sensitivity analysis is conducted to investigate the effect of corrective maintenance cost and time required for corrective maintenance to optimization policy. Comparative study is given to illustrate the necessity of distinguishing first inspection interval and repeat inspection interval.

Keywords: Performance-based contracting • Condition-based maintenance • Gamma process • Profit maximization • Inspection interval

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1 Introduction

Service-oriented is the innovation of an organization's capabilities and processes to shift from selling products to selling integrated products and services that deliver value in use [1,2]. As the global economy continues to be service-oriented, the role of operation and maintenance (O&M) has become increasingly important. This trend is particularly significant in the field of national defense, aviation, and energy [3]. For instance, in the U.S. defense budget for fiscal year 2020, O&M costs were \$292.7 billion, accounting for 41% of the total defense budget. Compared with fiscal year 2019, O&M costs increased by \$9.2 billion [4].

Conventionally, O&M services are implemented under material-based contracts (MBC) in which customers pay support providers according to the spare parts and labor consumed each time [5]. Because of its ease of implementation, MBC has been widely adopted for supporting equipment operation and maintenance in private and public sectors [6]. However, with the implementation of MBC, the service providers will expose the problem of insufficient innovation. This is because a majority part of the provider's revenue comes from the components, services, and consumables they sold. As the equipment ages and becomes obsolete, providers may profit from these individual products. In contrast, customers need to pay more support and maintenance costs [7-9].

"Performance-based contracting" (PBC) as a new form of support contract well addresses this challenge. Under PBC, customers pay for the outcomes delivered by providers rather than the individual components or services [10,11]. Likewise, providers' compensation is tied to successfully achieving required outcomes. If providers do not take actions and allow the product to obsolete, the consequences will be borne by themselves. PBC promotes providers to innovate in the form of

materials, technologies, processes, and policies.

PBC originated from the Performance-based Logistics (PBL) of the U.S. military. PBL is an outcome-based product support strategy that plans and delivers an integrated, affordable performance solution that optimizes weapon system readiness. PBL focuses on the ability of the support solution to affordably satisfy the war fighter's requirements, usually expressed in terms of availability, not on the consumption of resources or who performs the work [12]. In short, its essence is to purchase performance outcomes, rather than individual components or services. In the following discussion, PBL has the same meaning as PBC.

In recent years, with the wide application of PBC, the research on PBC has gradually increased. However, most existing studies on PBC concentrate on contracts [13,14], risks [15-18], incentives [19,20], and performance metrics [21]. Tan [14] developed a distribution-free contract that sets the guaranteed energy savings level based on the mean and the standard deviation of the energy savings and the profit-sharing ratio between the energy service companies (ESCO) and the energy user. Shang et al. [18] established a credit risk identification model to examine the credit status of Energy performance contracting (EPC) project clients (i.e., energy-using companies) in China based on rough set theory. Selviaridis et al. [20] conducted three in-depth case studies of contractual relationships, which exhibit differences in terms of how performance incentives are framed, i.e., using promotion, prevention, and "hybrid" frames, respectively. Akkermans et al. [21] described a design science research project that develops a novel approach to buyer-supplier contracting, using collaborative key performance indicators (KPIs). According to our observation, mathematical modelling and optimization approaches under PBC are rarely addressed.

Nowadays, maintenance is widely recognized as an essential business function and a critical element of asset management. Organizations increasingly realize that they can improve efficiency and reliability by planning maintenance interventions in more effective way [22]. Therefore, an increasing number of preventive maintenance (PM) is implemented. Traditionally, PM takes the form of system overhaul or unit replacement based on operating time, which is often referred to as time-based maintenance (TBM) [23]. Different from TBM policies that are developed based on historical failure data, CBM is a maintenance approach that emphasizes on combining data-driven reliability models with sensor data collected from monitored operating systems to develop strategies for condition monitoring and maintenance [24]. With the used of emerging technologies such as Micro Electro Mechanical System (MEMS), various sensors, and wireless tele-communication, CBM has developed rapidly and CBM modelling attracts an

increasingly attention [25]. Alaswad and Xiang [24] provided a review of CBM literatures with emphasis on mathematical modelling and optimization approaches. They focused on important aspects of the CBM, such as optimization criteria, inspection frequency, maintenance degree, solution methodology and so on. For systems subject to continuous deterioration, we commonly used stochastic deterioration models, such as Wiener process, Gamma process, and Inverse process. When degradation is in the form of cumulative damage, Gamma process is an appropriate model, which has been extensively studied in CBM models [26,27]. However, to our best knowledge, the work related to CBM maintenance optimization for gamma deteriorating systems under PBC is rarely reported.

In order to fill the gap mentioned above, this paper investigates the condition-based maintenance optimization for gamma deteriorating systems under PBC. The main objective is to determine the optimal decision variables (i.e., inspection interval and preventive maintenance threshold), which maximize the profit and improve system performance at a lower cost. Compared with the existing works, a stepwise linear revenue function is adopted to correlate the availability with the support provider's profit. In order to illustrate the effectiveness of PBC, we compare the proposed policy with conventional policy (i.e., cost minimization). Then, a sensitivity analysis of corrective maintenance cost and time required for corrective maintenance is conducted. Finally, we conduct a comparative study of considering first inspection interval or not, which is involved in our proposed model and rarely considered in previous models.

The rest of the paper is organized as follow. Section 2 provides a brief literature review of the mathematical modelling and optimization approaches under PBC. Section 3 presents the degradation process with random effects, problem description, and model assumptions. Section 4 develops a condition-based maintenance optimization model for gamma deteriorating systems under PBC. The solution algorithm and related procedure are presented in Section 5. A numerical example is provided in Section 6, which validates the applicability and effectiveness of our proposed model. The conclusion and future research directions are presented in Section 7.

2 Literature Review

In the past ten years, research interests on mathematical modelling and optimization approaches under PBC have been gradually increasing. In this section, we review the papers related to mathematical modelling and optimization approaches of maintenance support under PBC.

In Table 1, we have review 18 literature related to mathematical modelling and optimization approaches of

maintenance support under PBC. These papers are ranked according to the publishing time. We mainly summarize them in application domains, solution algorithm, first inspection interval considered or not, and characteristic feature.

With respect to the application domains, we divide it into spares inventory and maintenance optimization. From Table 1, we observe that 12 of the reviewed papers [28-39] focused on the spares inventory. Spare parts are common inventory stock items, which are needed to maintenance equipment and the cost of spare parts takes a large share of product lifecycle cost [40]. For this reason, in order to obtain the optimal stock keeping units that maximizing a company's profit subject to a collection of performance constraints, PBC is applied and optimization models are developed. By contrast, only 6 of the reviewed papers [41-46] related to maintenance optimization under PBC.

In regard to the solution algorithms, one-third of the reviewed papers did not mention the solution algorithm. 10 of the reviewed papers adopted the traditional solution algorithms, such as simulation [31,32,34,38,43], allocation algorithm [29,37], gradient descent [34], selection algorithm [37], discrete algorithm [42], and coordinate search algorithm [45]. Only 2 reviewed papers applied heuristic intelligent algorithm, they are artificial bee colony (ABC) algorithm [44] and particle swarm optimization (PSO) algorithm [46], respectively. For low-dimensional problems, traditional solution algorithm is relatively simple and effective. However, for high-dimensional problems, such as maintenance optimization problems with multiple decision variables, adopting traditional solution algorithm will take a lot of time and the accuracy of the solution is relatively low. In contrast, the heuristic intelligent algorithms have high operating efficiency, and are less affected by the dimensionality of the problem [47,48]. Therefore, the application of heuristic intelligent algorithm is needed for complicated maintenance optimization problems under PBC.

Regarding first inspection interval, to our best knowledge, there has never been a research considering the first inspection interval in maintenance optimization under PBC. First inspection interval was firstly proposed by Jia and Christer [49]

for modelling functional checking procedures in Reliability Centered Maintenance (RCM). They have verified the effectiveness of the first inspection interval. The fly in the ointment is that they did not consider the downtime in their numerical example.

With respect to the characteristic feature in maintenance optimization, it can be known from Table 1 that PBC has applied mathematical statistical theory [41,43,45], stochastic deterioration model [42], delay-time-based maintenance model [44], degradation-threshold-shock model [46] and so on. At present, the popularity of CBM in the research community and industrial applications relies heavily on the development of stochastic deterioration models [24]. Nevertheless, there is only one paper [42] that studies the application of stochastic deterioration models under PBC. So, the research on stochastic deterioration models under PBC need to be further strengthened and developed.

Based on the above discussions, we can conclude that:

1. Few studies have applied PBC in maintenance optimization. There is still an urgent need to study the maintenance optimization problem under PBC.
2. The application of emerging heuristic intelligent algorithms in maintenance optimization under PBC is relatively rare. With the in-depth study of maintenance optimization problems under PBC, traditional algorithms cannot meet the requirements of efficiency and accuracy.
3. As an effective approach, first inspection interval has not been considered in the maintenance optimization under PBC so far.
4. As an important part of CBM, current research on stochastic deterioration model in the maintenance optimization under PBC is far from enough.

Consequently, this paper attempts to investigate the condition-based maintenance optimization for gamma deteriorating systems under PBC. Meanwhile, we not only adopt the PSO algorithm, but also consider the first inspection interval.

Table 1 Overview of application domains, solution algorithm, first inspection interval considered or not, and characteristic feature by the main reviewed contributions

Ref.	Application domain	Solution algorithm	Consider first inspection interval?	Characteristic Feature
Kim et al. (2007)	Spares inventory	-	-	Principal-agent model
Nowicki et al. (2008)	Spares inventory	Allocation algorithm	-	METRIC
Mirzahosseini and Piplani (2011)	Spares inventory	-	-	METRIC; Queueing theory; Markov chain
Jin and Wang (2012)	Spares inventory; Reliability	Simulation	-	Reliability design; Spare parts logistic
Jin and Tian (2012)	Spares inventory; Reliability	Simulation	-	Reliability design; Spare parts logistic
Mirzahosseini and Piplani (2013)	Spares inventory; Reliability	-	-	METRIC; Reliability design; Spare parts logistic
Zhang et al. (2014)	Spares inventory; Reliability	Simulation	-	Reliability design; Spare parts logistic
Jin et al. (2015)	Spares inventory; Reliability	Gradient Descent	-	METRIC; Game theory
Mirzahosseini et al. (2016)	Spares inventory; Reliability	-	-	METRIC; Reliability design; Spare parts logistic
Riccardo et al. (2016)	Spares inventory	Quick Request selection algorithm; Local Department Kit allocation algorithm	-	METRIC
Qiu et al. (2017)	Maintenance optimization	-	No	Mathematical statistical theory; Virtual age model
Xiang et al. (2017)	Maintenance optimization	Discrete algorithm	No	Stochastic deterioration model
Hur et al. (2018)	Spares inventory	Runge-Kutta methods; Discrete event simulation	-	Markov chain
Patra et al. (2019)	Spares inventory; Reliability	-	-	Principal-agent model; Time-series model
Wang et al. (2019)	Maintenance optimization	Simulation	No	Mathematical statistical theory
Yang et al. (2019)	Maintenance optimization	ABC algorithm	No	Delay-time-based maintenance model
Li et al. (2020)	Maintenance optimization	Coordinate search algorithm	No	Mathematical statistical theory
Wang et al. (2020)	Maintenance optimization	PSO algorithm	No	Degradation-threshold-shock model
This paper	Maintenance optimization	PSO algorithm	Yes	Stochastic deterioration model

3 Problem Description and Assumptions

3.1 Degradation process with random effects

Generally, degradation is a physical or chemical process in which the internal materials of the system suffer a gradual change under external stresses. This degradation process can be viewed as damage to a system and will accumulate over time which eventually lead to a product failure when the accumulated damage reaches a failure threshold [27]. Figure 1 shows the illustration of the process of degradation failure. Where $X(t)$ denotes the performance degradation at time t , L_f denotes the threshold of the failure, and t_f represents the time when $X(t)$ arrive L_f .

Figure 1 The process of degradation failure

The system degradation process is usually stochastic so that it is an effective approach to describe the uncertainty in the system degradation process using random process. In order to facilitate mathematical processing, the commonly used stochastic degradation models are mostly homogeneous and stable independent incremental processes, such as Wiener process and Gamma process. The Wiener process is appropriate in describing a degradation which shows increment/decrement of deterioration over time (non-monotonic). However, Gamma process is more suitable in modelling monotonically increasing (or decreasing) degradation [24], such as the degradation is in the form of cumulative damage. In this paper, we consider a system that deteriorates over time and the state of the system can be described by a gamma process with random effects.

In mathematical terms, the gamma process is defined as follows. Denote that random variables X has a gamma distribution with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$, and its probability density function is given by

$$Ga(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} I_{(0, \infty)}(x) \quad (1)$$

where $\Gamma(\alpha) = \int_0^\infty u^{\alpha-1} e^{-u} du$, and $I_A(x) = 1$ for $x \in A$ and $I_A(x) = 0$ for $x \notin A$.

Furthermore, let $\alpha(t)$ be a non-decreasing, right-continuous, real-valued function for $t \geq 0$, with $\alpha(0) = 0$. The gamma process with shape function $\alpha(t) > 0$ and scale parameter $\beta > 0$ is a continuous stochastic process $\{X(t), t \geq 0\}$. The process has the following properties:

- (1) $X(0) = 0$;
- (2) $X(t + \Delta t) - X(t) \sim Ga(t; \alpha(\Delta t), \beta)$ for all $\Delta t > 0$ and $t \geq 0$;
- (3) $X(t)$ has independent increments.

When shape parameter $\alpha(t)$ is a linear function of time t , the random process is a stationary gamma process. In this paper, we assume $\alpha(t) = at$, then the probability density function can be expressed as

$$f(x; at, \beta) = \frac{\beta^{at}}{\Gamma(at)} x^{at-1} e^{-\beta x} \quad (2)$$

and the distribution function is given by

$$F(x; at, \beta) = \frac{\beta^{at}}{\Gamma(at)} \int_0^x u^{at-1} e^{-\beta u} du \quad (3)$$

3.2 Problem Description

Condition-based maintenance is one of the most effective maintenance strategies to deal with degradation failure [50]. It refers to a maintenance strategy that collects and assesses real-time information and recommends maintenance decisions based on the current condition of the system. At present, CBM is mostly based on periodic inspections, which is shown in Figure 2. Where T_1 and T denote inspection intervals, L_p denotes preventive maintenance threshold, T_i , T_p , T_f represent time required for inspection, preventive maintenance, and corrective maintenance, respectively.

Figure 2 Periodic inspection policy of degradation process

The design of this type of policy has mainly focused on (1) inspection interval and (2) preventive maintenance threshold. Different from traditional inspection models, we consider the first inspection interval, T_1 , because the probability of the cumulative deterioration level above the preventive maintenance threshold is relatively small in the early period. Thus, it is necessary to set the first inspection interval in order to reduce the number of inspections and the cost.

Generally, cost is the mainly consideration when determining the inspection interval and preventive maintenance threshold. The existing maintenance optimization models often use the optimization criterion that minimizing system maintenance cost rate but ignoring performance. Sometimes when the maintenance cost rate is minimized the system reliability measures are very low, which will not be acceptable in practice. In this paper, we deal with an optimization model of condition-based maintenance for gamma deteriorating system under PBC. Different from the existing optimization models, PBC motivates service providers to implement effective maintenance policies in order to boost profits and improve system performance at a lower cost [32]. For this reason, how to find the optimal first inspection interval, repeat inspection interval, and preventive maintenance threshold to achieve the objectives of PBC is what we need to study.

3.3 Model Assumptions

The modelling assumptions used in this paper are as follows:

(1) A repairable single-component system subject to the gamma degradation process is investigated. Let $X(t)$ denote the performance degradation of time t and satisfy the following conditions: (a) when $t=0$, $X(0)=0$, the system is in health state; (b) the increments of degradation are non-negative and independent.

(2) The inspection of the system is discrete inspection and the state of the system can only be detected through inspection. All inspections are viewed as perfect. The first inspection interval is T_1 and the repeated inspection interval is T .

(3) If $X(t) < L_p$, the system continues to work until the next inspection.

(4) If $L_p < X(t) < L_f$, the system will be preventively repaired. The preventive repair is assumed to be imperfect. More specifically, it is assumed that the system is return to an as-good-as new state with probability p and remains in the same state with probability $q=1-p$.

(5) If $L_f < X(t)$, the system is failure and corrective maintenance will be implemented. Corrective maintenance is assumed to be perfect, which can restore the system to the as-good-as new state. Furthermore, the failure is a non-obvious failure which can only be found through inspections, so the system will continue to operate when a failure happens until the inspection.

Notations

$X(t)$	Cumulative amount of deterioration
$F(x; \alpha t, \beta)$	Cumulative distribution function of $X(t)$
$f(x; \alpha t, \beta)$	Probability density function of $X(t)$
L_p	Preventive maintenance threshold
L_f	Failure threshold
C_i	Inspection cost
C_p	Preventive maintenance cost
C_f	Corrective maintenance cost
T_i	Inspection time
T_p	Time required for preventive maintenance
T_f	Time required for corrective maintenance
T_{up}	Uptime during a renewal cycle
T_{down}	Downtime during a renewal cycle
A	Expected average availability
EC	Expected cost rate per unit time
ER	Expected revenue rate per unit time
EP	Expected profit rate per unit time

4 Maintenance Optimization Modelling under Performance-Based Contracting

4.1 Optimization Model of Performance-based Contracting

The purpose of this section is to calculate and optimize the expected profit rate of the system under PBC. PBC is a typical profit centered policy, which considers the performance and operation cost of the product. In this paper, the performance is measured by the expected average availability, A , because most products in defense and industry need higher system stability and longer operation time. On the other hand, the operation cost is measured by the expected cost rate per unit time, EC .

Before formulating the profit function, we need to express the relationship between the profit rate and availability with the help of revenue function. As it is easy to implement and can clearly express the relationship between the profit and availability, the stepwise linear revenue function has been widely applied to judge the

maintenance decision, the schematic diagram of stepwise linear revenue is shown in Figure 3, which can be expressed as follow:

$$ER = \begin{cases} 0, & A < A_{\min} \\ \theta + \pi(A - A_{\min}), & A \geq A_{\min} \end{cases} \quad (4)$$

where θ denotes the fixed part of the revenue, $\pi(A - A_{\min})$ represents the performance-based incentive. If $A < A_{\min}$, suppliers could not obtain any revenue. If $A \geq A_{\min}$, suppliers can get fixed revenue and incentive.

Figure 3 Stepwise linear revenue function

Let EP represents the expected profit rate per unit time to the service providers. Considering the expected cost rate and the expected revenue rate, the maintenance optimization model with the objective of maximizing the expected profit rate can be formulated as:

$$\begin{aligned} \max EP(T_1, T, L_p) &= ER(T_1, T, L_p) - C(T_1, T, L_p) \\ (T_1^*, T^*, L_p^*) &= \arg \max \{EP(T_1, T, L_p)\} \\ \text{s.t.} \quad &\begin{cases} 0 < T < T_1 < T_{\max} \\ 0 < L_p < L_f \end{cases} \end{aligned} \quad (5)$$

where $T_{\max}$ is the possible time constraint imposed on the inspection interval.

For comparison purposes, a benchmark model with the objective of minimizing cost is proposed.

$$\begin{aligned} \min EC(T_1, T, L_p) \\ (T_1^*, T^*, L_p^*) &= \arg \min \{EC(T_1, T, L_p)\} \\ \text{s.t.} \quad &\begin{cases} 0 < T < T_1 < T_{\max} \\ 0 < L_p < L_f \end{cases} \end{aligned} \quad (6)$$

4.2 Calculation of Renewal Probabilities

In order to obtain EC and A in the expected profit rate function, we need to calculate the probabilities of preventive renewal and corrective renewal first.

In this paper, we assume the system is renewed when a preventive maintenance is performed perfectly or the time that the cumulative deterioration level at the inspection is above the failure threshold. It is noticed that first inspection interval, T_1 , is greater than subsequent inspection intervals, T .

4.2.1 Probability of Preventive Renewal

Now we derive the probability of preventive maintenance perfectly performed at the λ^{th} inspection and denote this probability by p_λ ($\lambda=1,2,3,\dots$). At the first inspection, if the cumulative deterioration level is greater than the preventive maintenance threshold L_p but lower than the failure threshold L_f , the preventive maintenance is performed perfectly with probability p . The probability of having a perfect preventive maintenance at the first inspection is

$$p_1 = \Pr\{L_p < X(T_1) < L_f\} \times p \quad (7)$$

Perfect PM at the second inspection ($\lambda=2$) includes two scenarios. One is the case where PM is needed and performed perfectly at the second inspection but not needed at the first inspection. Another possibility is that PM at first inspection is as-bad-as-old and the system did not fail during the second inspection interval, meanwhile, it is preventively repaired perfectly at the second inspection. Thus, we have the probability of perfect PM at the second inspection as follows

$$\begin{aligned} p_2 &= \Pr\{X(T_1) < L_p \text{ \& } L_p < X(T_1 + T) < L_f\} \times p \\ &\quad + \Pr\{L_p < X(T_1) < L_f \text{ \& } L_p < X(T_1 + T) < L_f\} \\ &\quad \times (1 - p)p \end{aligned} \quad (8)$$

PM performed perfectly at the third inspection ($\lambda=3$) has three possibilities. One is the case that PM is not needed until the third inspection (shown in Figure 4(a)). The second one is that PM is not needed until the second inspection, but the PM at the second inspection is not done perfectly, and requires PM at the third inspection (shown in Figure 4(b)). The last one is that the system needs PM at both first and second inspections but neither PM is perfect, the system still needs PM at the third inspection (shown in Figure 4(c)). The probability of PM at the third inspection is given by adding these probabilities together.

$$\begin{aligned} p_3 &= \Pr\{X(T_1 + T) < L_p \text{ \& } L_p < X(T_1 + 2T) < L_f\} \\ &\quad \times p + \Pr\{X(T_1) < L_p \text{ \& } L_p < X(T_1 + T) < L_f \text{ \& } L_p \\ &\quad < X(T_1 + 2T) < L_f\} \times (1 - p)p + \Pr\{L_p < X(T_1) \\ &\quad < L_f \text{ \& } L_p < X(T_1 + 2T) < L_f\} \times (1 - p)^2 p \end{aligned} \quad (9)$$

Now we derive a general probability of a PM performed perfectly at the λ^{th} inspection as follow:

$$p_\lambda = \begin{cases} \Pr\{L_p < X(T_1) < L_f\} \times p & \lambda = 1 \\ \Pr\{X(T_1) < L_p \& L_p < X(T_1 + T) < L_f\} \times p + \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1 + T) < L_f\} \times (1-p)p & \lambda = 2 \\ \Pr\{X(T_1 + (\lambda-2)T) < L_p \& L_p < X(T_1 + (\lambda-1)T) < L_f\} \times p + \sum_{n=2}^{\lambda-1} \Pr\{X(T_1 + (n-2)T) < L_p \& L_p < X(T_1 + (n-1)T) < L_f \& L_p < X(T_1 + (\lambda-1)T) < L_f\} \times (1-p)^{\lambda-n} p + \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1 + (\lambda-1)T) < L_f\} \times (1-p)^{\lambda-1} p & \lambda > 2 \end{cases} \quad (10)$$

where

$$\begin{aligned} & \Pr\{X(T_1 + (\lambda-2)T) < L_p \& L_p < X(T_1 + (\lambda-1)T) < L_f\} \\ &= \int_0^{L_p} f(u, \alpha(T_1 + (\lambda-2)T), \beta) \int_{L_p-u}^{L_f-u} f(v, T, \beta) dv du \end{aligned} \quad (11)$$

$$\begin{aligned} & \Pr\{X(T_1 + (n-2)T) < L_p \& L_p < X(T_1 + (n-1)T) < L_f \& L_p < X(T_1 + (\lambda-1)T) < L_f\} \\ &= \int_0^{L_p} f(u, \alpha, T_1 + (n-2)T, \beta) \int_{L_p-u}^{L_f-u} f(v, \alpha T, \beta) \\ & \quad \times F(L_f - u - v, \alpha(\lambda - n)T, \beta) dv du \end{aligned} \quad (12)$$

4.2.2 Probability of Corrective Renewal

Next, we derive the probability of corrective maintenance (CM) at the λ^{th} inspection and denote this probability by q_λ ($\lambda=1,2,3,\dots$). The probability of implementing a CM at the first inspection is

$$q_1 = \Pr\{X(T_1) > L_f\} \quad (13)$$

The necessity of CM at the second inspection ($\lambda=2$) also includes two possibilities. One is the case where PM is not needed at the first inspection, but the system failed between the first and the second inspections. The other is the case where the effect of PM at the first inspection is as-bad-as old and the cumulative deterioration level at the second inspection is above the failure threshold. Thus, the probability that CM is needed at the second inspection is given by

$$\begin{aligned} q_2 = & \Pr\{X(T_1) < L_p \& X(T_1 + T) > L_f\} + \Pr\{L_p < X(T_1) < L_f \& X(T_1 + T) > L_f\} \times (1-p) \end{aligned} \quad (14)$$

There are three scenarios where CM is needed at the third inspection ($\lambda=3$). One is that PM is not needed at either the first or the second inspection and the system fails between the second and the third inspection. The

second one is that the first time the system requires a PM is at the second inspection, but the PM at the second inspection is not done perfectly, and the system fails between the second and the third inspection. The last one is that the system is not perfectly repaired preventively at either first or the second inspection, and deteriorates to a “failed” state between the second and the third inspections. Therefore, we have the probability of CM being implemented at the third inspection as follow:

$$\begin{aligned} q_3 = & \Pr\{X(T_1 + T) < L_p \& X(T_1 + 2T) > L_f\} \\ & + \Pr\{X(T_1) < L_p \& L_p < X(T_1 + T) < L_f \\ & \& X(T_1 + 2T) > L_f\} \times (1-p) + \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1 + T) < L_f \& X(T_1 + 2T) > L_f\} \times (1-p)^2 \end{aligned} \quad (15)$$

There are four scenarios where CM is needed at the four inspection ($\lambda=4$). One is that PM is not needed before the fourth inspection and the system fails between the third and the fourth inspections (shown in Figure 5(a)). The second one is that the first time the system requires a PM is at the third inspection, but the PM at the third inspection is not done perfectly, the system fails between the third and the fourth inspections (shown in Figure 5(b)). The third one is that the first time the system requires a PM is at the second inspection, but the PM at both second and third inspection is imperfect, the system fails between the third and the fourth inspections (shown in Figure 5(c)). The last one is that the system is not perfectly repaired preventively at the first to the third inspection, and deteriorates to a “failed” state between the third and the fourth inspection (shown in Figure 5(d)). The probability of CM at the fourth inspection is given by adding these probabilities together.

$$q_4 = \Pr\{X(T_1 + 2T) < L_p \& X(T_1 + 3T) > L_f\} + \Pr\{X(T_1 + T) < L_p \& L_p < X(T_1 + 2T) < L_f \& X(T_1 + 3T) > L_f\} \\ \times (1-p) + \Pr\{X(T_1) < L_p \& L_p < X(T_1 + T) < L_f \& L_p < X(T_1 + 2T) < L_f \& X(T_1 + 3T) > L_f\} \times (1-p)^2 + \Pr\{L_p \\ < X(T_1) < L_f \& L_p < X(T_1 + 2T) < L_f \& X(T_1 + 3T) > L_f\} \times (1-p)^3 \quad (16)$$

The probability of CM at the λ^{th} inspection can be generalized as follows:

$$q_\lambda = \begin{cases} \Pr\{X(T_1) > L_f\} & \lambda = 1 \\ \Pr\{X(T_1) < L_p \& X(T_1 + T) > L_f\} + \Pr\{L_p < X(T_1) < L_f \& X(T_1 + T) > L_f\} \times (1-p) & \lambda = 2 \\ \Pr\{X(T_1 + T) < L_p \& X(T_1 + 2T) > L_f\} + \Pr\{X(T_1) < L_p \& L_p < X(T_1 + T) < L_f \& X(T_1 + 2T) > L_f\} \times (1-p) \\ + \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1 + T) < L_f \& X(T_1 + 2T) > L_f\} \times (1-p)^2 & \lambda = 3 \\ \Pr\{X(T_1 + (\lambda - 2)T) < L_p \& X(T_1 + (\lambda - 1)T) > L_f\} + \Pr\{X(T_1 + (\lambda - 3)T) < L_p \& L_p < X(T_1 + (\lambda - 2)T) < L_f \\ \& X(T_1 + (\lambda - 1)T) > L_f\} \times (1-p) + \sum_{n=2}^{\lambda-2} \Pr\{X(T_1 + (n - 2)T) < L_p \& L_p < X(T_1 + (n - 1)T) < L_f \& L_p < X(T_1 \\ + (\lambda - 2)T) < L_f \& X(T_1 + (\lambda - 1)T) > L_f\} \times (1-p)^{\lambda-n} + \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1 + (\lambda - 2)T) < L_f \\ \& X(T_1 + (\lambda - 1)T) > L_f\} \times (1-p)^{\lambda-1} & \lambda > 3 \end{cases} \quad (17)$$

where

$$\Pr\{X(T_1 + (\lambda - 2)T) < L_p \& X(T_1 + (\lambda - 1)T) > L_f\} \\ = \int_0^{L_p} f(u, \alpha(T_1 + (\lambda - 2)T), \beta)[1 - F(L_f - u, \alpha T, \beta)] du \quad (18)$$

$$\Pr\{X(T_1 + (\lambda - 3)T) < L_p \& L_p < X(T_1 + (\lambda - 2)T) \\ < L_f \& X(T_1 + (\lambda - 1)T) > L_f\} \\ = \int_0^{L_p} f(u, \alpha(T_1 + (\lambda - 3)T), \beta) \int_{L_p - u}^{L_f - u} f(v, \alpha T, \beta)[1 \\ - F(L_f - u - v, \alpha T, \beta)] dv du \quad (19)$$

$$\Pr\{X(T_1 + (n - 2)T) < L_p \& L_p < X(T_1 + (n - 1)T) \\ < L_f \& L_p < X(T_1 + (\lambda - 2)T) < L_f \& X(T_1 + (\lambda \\ - 1)T) > L_f\} \\ = \int_0^{L_p} f(u, \alpha(T_1 + (n - 2)T), \beta) \int_{L_p - u}^{L_f - u} f(v, \alpha T, \beta) \\ \times \int_0^{L_f - u - v} f(w, \alpha((\lambda - n - 1)T), \beta)[1 - F(L_f - u \\ - v - w, \alpha T, \beta)] dw dv du \quad (20)$$

4.3 Calculation of Average Availability

Availability is the most used key performance metric in maintenance support services. In this study, we use availability to measure the realized performance outcomes

and assume that the reward is directly linked to the availability. The average availability can be expressed as:

$$A = \frac{T_{\text{up}}}{T_{\text{up}} + T_{\text{down}}} \quad (21)$$

where T_{up} denotes the expected uptime per cycle, T_{down} denotes the expected downtime per cycle.

4.3.1 Derivation of Uptime

The expected uptime per cycle can be expressed as:

$$E(T_{\text{up}}) = \sum_{\lambda=1}^{\infty} (T_1 + (\lambda - 1)T) \times p_\lambda + \sum_{\lambda=1}^{\infty} (T_1 + (\lambda - 1)T) \times q_\lambda \quad (22)$$

4.3.2 Derivation of Downtime

The expected total downtime can be expressed as follow:

$$E(T_{\text{down}}) = E(\text{inspection time}) + E(\text{preventive repair time}) \\ + E(\text{corrective repair time}) \quad (23)$$

Let $E_{1,\lambda}(T_{\text{down}})$ represent the expected downtime when PM is performed perfectly at the λ^{th} inspection, $E_{2,\lambda}(T_{\text{down}})$ denote the expected downtime when CM is implemented at the λ^{th} inspection. The expected downtime is as follow:

$$E(T_{\text{down}}) = \sum_{i=1}^2 \sum_{\lambda=1}^{\infty} E_{i,\lambda}(T_{\text{down}}) \quad (24)$$

Figure 4 Perfect PM at the 3rd inspection.

Figure 5 CM at the 4th inspection.

The expected downtime when PM is performed perfectly at the λ^{th} inspection is given by

$$E_{1,\lambda}(T_{\text{down}}) = \begin{cases} (T_i + T_p) \times \Pr\{L_p < X(T_1) < L_f\} \times p & \lambda = 1 \\ (2T_i + T_p) \times \Pr\{X(T_1) < L_p \& L_p < X(T_1 + T) < L_f\} \times p + (2T_i + 2T_p) \times \Pr\{L_p < X(T_1) < L_f \\ \& L_p < X(T_1 + T) < L_f\} \times (1-p)p & \lambda = 2 \\ (\lambda T_i + T_p) \times \Pr\{X(T_1 + (\lambda-2)T) < L_p \& L_p < X(T_1 + (\lambda-1)T) < L_f\} \times p + \sum_{n=2}^{\lambda-1} (\lambda T_i + (\lambda \\ -n+1)T_p) \times \Pr\{X(T_1 + (n-2)T) < L_p \& L_p < X(T_1 + (n-1)T) < L_f \& L_p < X(T_1 + (\lambda \\ -1)T) < L_f\} \times (1-p)^{\lambda-n} p + (\lambda T_i + \lambda T_p) \times \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1 + (\lambda-1)T) \\ < L_f\} \times (1-p)^{\lambda-1} p & \lambda > 2 \end{cases} \quad (25)$$

The expected downtime when CM is implemented at the λ^{th} inspection is given by

$$E_{2,\lambda}(T_{\text{down}}) = \begin{cases} (T_i + T_f) \times \Pr\{X(T_1) > L_f\} & \lambda = 1 \\ (2T_i + T_f) \times \Pr\{X(T_1) < L_p \& X(T_1 + T) > L_f\} + (2T_i + T_p + T_f) \times \Pr\{L_p < X(T_1) < L_f \& X(T_1 + T) \\ > L_f\} \times (1-p) & \lambda = 2 \\ (3T_i + T_f) \times \Pr\{X(T_1 + T) < L_p \& X(T_1 + 2T) > L_f\} + (3T_i + T_p + T_f) \times \Pr\{X(T_1) < L_p \& L_p < X(T_1 \\ + T) < L_f \& X(T_1 + 2T) > L_f\} \times (1-p) + (3T_i + 2T_p + T_f) \times \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1 + T) \\ < L_f \& X(T_1 + 2T) > L_f\} \times (1-p)^2 & \lambda = 3 \\ (\lambda T_i + T_f) \times \Pr\{X(T_1 + (\lambda-2)T) < L_p \& X(T_1 + (\lambda-1)T) > L_f\} + (\lambda T_i + T_p + T_f) \times \Pr\{X(T_1 + (\lambda \\ -3)T) < L_p \& L_p < X(T_1 + (\lambda-2)T) < L_f \& X(T_1 + (\lambda-1)T) > L_f\} \times (1-p) + \sum_{n=2}^{\lambda-2} (\lambda T_i + (\lambda-n)T_p \\ + T_f) \times \Pr\{X(T_1 + (n-2)T) < L_p \& L_p < X(T_1 + (n-1)T) < L_f \& L_p < X(T_1 + (\lambda-2)T) < L_f \\ \& X(T_1 + (\lambda-1)T) > L_f\} \times (1-p)^{\lambda-n} + (\lambda T_i + (\lambda-1)T_p + T_f) \times \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1 \\ + (\lambda-2)T) < L_f \& X(T_1 + (\lambda-1)T) > L_f\} \times (1-p)^{\lambda-1} & \lambda > 3 \end{cases} \quad (26)$$

4.4 Calculation of Cost Rate

The derivation of the maintenance cost during the field operation is like that of the downtime. For example, the inspection time is substituted with the inspection cost, and

the PM/CM downtime is substituted with the corresponding PM/CM costs. Therefore, the expected maintenance cost when PM is performed perfectly at the λ^{th} inspection can be expressed as:

$$E_{1,\lambda}(\text{Cost}) = \begin{cases} (C_i + C_p) \times \Pr\{L_p < X(T_1) < L_f\} \times p & \lambda = 1 \\ (2C_i + C_p) \times \Pr\{X(T_1) < L_p \& L_p < X(T_1 + T) < L_f\} \times p + (2C_i + 2C_p) \times \Pr\{L_p < X(T_1) < L_f \& L_p \\ < X(T_1 + T) < L_f\} \times (1-p)p & \lambda = 2 \\ (\lambda C_i + C_p) \times \Pr\{X(T_1 + (\lambda-2)T) < L_p \& L_p < X(T_1 + (\lambda-1)T) < L_f\} \times p + \sum_{n=2}^{\lambda-1} (\lambda C_i + (\lambda-n+1)C_p) \\ \times \Pr\{X(T_1 + (n-2)T) < L_p \& L_p < X(T_1 + (n-1)T) < L_f \& L_p < X(T_1 + (\lambda-1)T) < L_f\} \times (1-p)^{\lambda-n} p \\ + (\lambda C_i + \lambda C_p) \times \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1 + (\lambda-1)T) < L_f\} \times (1-p)^{\lambda-1} p & \lambda > 2 \end{cases} \quad (27)$$

Likewise, the expected maintenance cost when a renewal is caused by a CM implemented at the λ^{th} inspection can be expressed as:

$$\begin{aligned}
E_{2,\lambda}(\text{Cost}) &= \begin{cases} (C_i + C_f) \times \Pr\{X(T_1) > L_f\} & \lambda = 1 \\
(2C_i + C_f) \times \Pr\{X(T_1) < L_p \& X(T_1 + T) > L_f\} + (2C_i + C_p + C_f) \times \Pr\{L_p < X(T_1) < L_f \& X(T_1 + T) > L_f\} \times (1 - p) & \lambda = 2 \\
(3C_i + C_f) \times \Pr\{X(T_1 + T) < L_p \& X(T_1 + 2T) > L_f\} + (3C_i + C_p + C_f) \times \Pr\{X(T_1) < L_p \& L_p < X(T_1 + T) < L_f \& X(T_1 + 2T) > L_f\} \times (1 - p) + (3C_i + 2C_p + C_f) \times \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1 + T) < L_f \& X(T_1 + 2T) > L_f\} \times (1 - p)^2 & \lambda = 3 \\
(\lambda C_i + C_f) \times \Pr\{X(T_1 + (\lambda - 2)T) < L_p \& X(T_1 + (\lambda - 1)T) > L_f\} + (\lambda C_i + C_p + C_f) \times \Pr\{X(T_1 + (\lambda - 3)T) < L_p \& L_p < X(T_1 + (\lambda - 2)T) < L_f \& X(T_1 + (\lambda - 1)T) > L_f\} \times (1 - p) + \sum_{n=2}^{\lambda-2} (\lambda C_i + (\lambda - n)C_p + C_f) \times \Pr\{X(T_1 + (n - 2)T) < L_p \& L_p < X(T_1 + (n - 1)T) < L_f \& L_p < X(T_1 + (\lambda - 2)T) < L_f \& X(T_1 + (\lambda - 1)T) > L_f\} \times (1 - p)^{\lambda-n} + (\lambda C_i + (\lambda - 1)C_p + C_f) \times \Pr\{L_p < X(T_1) < L_f \& L_p < X(T_1 + (\lambda - 2)T) < L_f \& X(T_1 + (\lambda - 1)T) > L_f\} \times (1 - p)^{\lambda-1} & \lambda > 3
\end{cases} \\
\end{aligned} \tag{28}$$

Similar to the Equation (24), the expected cost rate can be expressed as

$$EC = \frac{\sum_{i=1}^2 \sum_{\lambda=1}^{\infty} E_{i,\lambda}(\text{Cost})}{T_{\text{up}} + T_{\text{down}}} \tag{29}$$

5 Solution Algorithm

In this paper, we propose an optimization model of condition-based maintenance for gamma deteriorating systems under PBC. The objective of the model is to maximize the expected profit rate. Therefore, we need to find the optimal decision variables (i.e., first inspection interval, repeat inspection interval, and preventive maintenance threshold). From the equations deduced above, it is noted that the model is very difficult to be solved because the proposed model has three decision variables, and the expression is very complicated. If we use the traditional exact algorithms (such as, discrete algorithm), it will take a lot of time and the accuracy of the solution is relatively low. For this reason, the heuristic intelligence algorithm is adopted to solve the proposed model. Particle swarm optimization algorithm, as one of the most used heuristic intelligence algorithms, is used to solve the model in this paper [51,52,53]. The steps of PSO are explained as follows:

Solution procedure based on PSO algorithm

- Step 1:** Input model parameters (maintenance cost, maintenance time, gamma deteriorating process and revenue functions). Let $\varphi\{T_1, T, L_p\}$ be the vector of the three decision variables.
- Step 2:** Initialize the particle swarm. For each particle s ($s=1,2,\dots,S$), initialize its position $\varphi_s = \{\varphi_s^{T_1}, \varphi_s^T, \varphi_s^{L_p}\}$ and velocity $\nu_s = \{\nu_s^{T_1}, \nu_s^T, \nu_s^{L_p}\}$ randomly with $\varphi_s^{T_1} \in (0, \bar{T}_1)$, $\varphi_s^T \in (0, \bar{T})$, $\varphi_s^{L_p} \in (0, L_f)$ and $\nu_s^{T_1}, \nu_s^T, \nu_s^{L_p} \in [-1.5, 1.5]$.
- Step 3:** At each iteration time τ ($\tau=1, \dots, \tau_{\max}$), calculate the fitness value of particle s denoted by $EP(\varphi_s(\tau))$.
- Step 4:** Compare $EP(\varphi_s(\tau))$ with $EP(\varphi_s^{\text{best}}(\tau))$ where $\varphi_s^{\text{best}}(\tau)$ is the individual best position of particle s already found by time τ . If $EP(\varphi_s(\tau)) > EP(\varphi_s^{\text{best}}(\tau))$, update $\varphi_s^{\text{best}}(\tau)$ and $EP(\varphi_s^{\text{best}}(\tau))$ by $\varphi_s^{\text{best}}(\tau) = \varphi_s(\tau)$ and $EP(\varphi_s^{\text{best}}(\tau)) = EP(\varphi_s(\tau))$ respectively.
- Step 5:** Compare $EP(\varphi_s(\tau))$ with $EP(\varphi_g^{\text{best}}(\tau))$ where $\varphi_g^{\text{best}}(\tau)$ is the globally best position of all particles already found by time τ . If $EP(\varphi_s(\tau)) > EP(\varphi_g^{\text{best}}(\tau))$, update $\varphi_g^{\text{best}}(\tau)$ and $EP(\varphi_g^{\text{best}}(\tau))$ by $\varphi_g^{\text{best}}(\tau) = \varphi_s(\tau)$ and $EP(\varphi_g^{\text{best}}(\tau)) = EP(\varphi_s(\tau))$ respectively.
- Step 6:** Update the velocity and position of particle s at time $\tau+1$ by $\nu_s(\tau+1) = \nu_s(\tau) + m_1 r_1 (\varphi_s^{\text{best}}(\tau) - \varphi_s(\tau)) + m_2 r_2 (\varphi_g^{\text{best}}(\tau) - \varphi_s(\tau))$ and $\varphi_s(\tau+1) = \varphi_s(\tau) + \nu_s(\tau+1)$. m_1 and m_2 are the learning factors which demonstrate the ability of self-learning and group-learning. r_1 and r_2 are random numbers between 0 and 1.
- Step 7:** Check the termination condition of the algorithm. If the maximum iteration times $\tau_{\max}$ is reached or the convergence criteria is met, go to Step 8. Otherwise, go back to Step 3.
- Step 8:** Output the globally optimal position $\varphi_g^{\text{best}} = \{T_1^{\text{best}}, T^{\text{best}}, L_p^{\text{best}}\}$ and the corresponding fitness value $EP(\varphi_g^{\text{best}})$.
-

The flow chart of PSO algorithm is shown as Figure 6.

Figure 6 Flow chart of PSO algorithm

6 Numerical Example

In this section, a numerical example is provided to demonstrate the applicability of the proposed model. Firstly, we compare the cost rate, average availability, and profit rate between the proposed policy (i.e., profit maximization) and conventional policy (i.e., cost minimization). Then, a sensitivity analysis of different failure consequences' effect to optimization policy is investigated. Finally, a comparative analysis between first inspection model and traditional model (i.e., considering first inspection or not) is presented.

6.1 Comparison of Proposed Policy and Conventional Policy

The values of several model parameters used in this section are adopted from existing literature [42] with slightly modifications. Others are assumptions based on typical and reasonable values. The gamma deteriorating system's scale parameter and shape parameter are supposed to be $\alpha=1.8$, $\beta=1$ respectively, and the failure

maintenance threshold, L_f , is set to 50. The maintenance parameters include the probability of perfect preventive maintenance is listed in Table 2, and the PBC parameters is listed in Table 3.

Table 2 The maintenance parameters

T_i	T_p	T_f	C_i	C_p	C_f	p
0.2	4	6	4	40	200	0.99
		12			400	
		18			600	
		24			800	

Table 3 The PBC parameters

θ	π	$A_{\min}$
2	20	0.6

In Table 2, we selected different T_f and C_f so that investigating different failure consequences' effect to optimization policy.

Based on the existing models and solution algorithm, we use MATLAB software to solve the optimal first inspection interval, repeat inspection interval, and preventive maintenance threshold according to different optimization objectives. After substituting the parameters, results of the numerical example are summarized in Table 4.

It is noted from the calculation results in Table 4 that the profit rates from the proposed policy are better than conventional policy. It should be known that the improvement in profit rates is per unit time, the total profit will be greater over time. Besides, the proposed policy can also deliver better performance (availability). For example, when $C_i=4$, $C_p=40$, $C_f=800$, $T_i=0.2$, $T_p=4$, $T_f=6$, comparing with conventional policy, the proposed policy has increased profit by 3.25% and availability by 1.11%, while cost increased by 2.68%. It is dramatic that the cost under the proposed policy is slightly higher than the cost under conventional policy. However, it is reasonable that more frequent maintenance activities are required to keep high system performance under the proposed policy, while the objective of conventional policy is to minimum the cost.

6.2 Sensitivity Analysis

In this section, a sensitivity analysis is conducted in order to investigate the effect of different failure consequences on optimization policy.

Table 4 The optimization results under conventional policy and proposed policy

C_i	C_p	C_f	T_i	T_p	T_f	Conventional Policy						Proposed Policy					
						T_1	T	L_p	EC	A	EP	T_1	T	L_p	EC	A	EP
4	40	200	0.2	4	6	19.56	4.35	36.23	2.22	0.8069	3.92	20.64	3.77	39.07	2.27	0.8141	4.01
					12	19.56	4.35	36.23	2.22	0.8013	3.81	19.81	3.63	38.51	2.23	0.8037	3.85
					18	19.56	4.35	36.23	2.22	0.7957	3.70	19.33	3.51	38.19	2.23	0.7976	3.73
					24	19.56	4.35	36.23	2.22	0.7901	3.59	18.99	3.41	37.98	2.23	0.7932	3.63
4	40	400	0.2	4	6	18.54	3.99	35.45	2.35	0.8003	3.66	19.47	3.54	38.28	2.41	0.8089	3.76
					12	18.54	3.99	35.45	2.35	0.7971	3.59	19.09	3.44	38.04	2.38	0.8022	3.66
					18	18.54	3.99	35.45	2.35	0.7939	3.53	18.82	3.35	37.89	2.37	0.7973	3.58
					24	18.54	3.99	35.45	2.35	0.7907	3.46	18.60	3.27	37.78	2.36	0.7933	3.50
4	40	600	0.2	4	6	18.08	3.67	35.29	2.44	0.7969	3.49	18.90	3.38	37.93	2.51	0.8057	3.60
					12	18.08	3.67	35.29	2.44	0.7944	3.44	18.67	3.30	37.81	2.49	0.8006	3.53
					18	18.08	3.67	35.29	2.44	0.7919	3.39	18.48	3.22	37.72	2.47	0.7965	3.46
					24	18.08	3.67	35.29	2.44	0.7893	3.34	18.33	3.15	37.66	2.47	0.7929	3.39
4	40	800	0.2	4	6	17.79	3.38	35.34	2.52	0.7946	3.37	18.54	3.24	37.75	2.59	0.8034	3.48
					12	17.79	3.38	35.34	2.52	0.7924	3.33	18.38	3.17	37.68	2.57	0.7992	3.41
					18	17.79	3.38	35.34	2.52	0.7902	3.28	18.24	3.10	37.63	2.56	0.7956	3.35
					24	17.79	3.38	35.34	2.52	0.788	3.24	18.12	3.04	37.59	2.55	0.7924	3.30

Firstly, a sensitivity analysis of corrective maintenance cost, C_f , is performed. We adopted a method that changing one of the parameters and fixing the others. Specifically, we fix $C_i=4$, $C_p=40$, $T_i=0.2$, $T_p=4$, $T_f=6$, and change the C_f . The analysis data comes from Table 4 and the analysis results is shown in Figure 7. Where the percentage values represent the increase in profit.

Figure 7 Sensitivity analysis of corrective maintenance cost

It can be known from Figure 7 that the proposed policy can get more profit than conventional policy under different value of C_f . Also note that the profit will decrease as C_f increase. And more importantly, we observed that, with the increase of C_f , the increased profit

ratio from the proposed policy to conventional policy is greater. Therefore, for the proposed policy, the higher C_f , the more obvious the profit increase.

Then, we conduct a sensitivity analysis of time required for corrective maintenance, T_f . Similar to the sensitivity analysis of corrective maintenance cost, we fix $C_i=4$, $C_p=40$, $C_f=800$, $T_i=0.2$, $T_p=4$, and change the T_f . The analysis data comes from Table 4 too and the analysis results is shown in Figure 8.

Figure 8 Sensitivity analysis of time required for corrective maintenance

From Figure 8, it is noted that the proposed policy can get more profit than conventional policy under different

value of T_f . In addition, it can be noticed that the profit will decrease as T_f increase. We also observed that, with the increase of T_f , the increased profit ratio from profit maximization policy to cost minimization policy is lower. Therefore, for the proposed policy, the higher T_f , the less obvious the profit increase.

6.3 Comparison of Considering First Inspection Interval or Not

Different from the traditional models, the first inspection interval is considered in our model. In this section, we conduct a comparative analysis in order to investigate the effectiveness of the proposed model involving first

inspection interval.

We select a group of parameters and then calculate the optimal decision variables under the two conditions (i.e., considering first inspection interval or not). After that, the corresponding cost rate, availability, and profit are calculated. The results are summarized in Table 5. Then, the column comparison chart is made as Figure 9 shows, which comparing three aspects, namely expected cost rate (shown in Figure 9(a)), average availability (shown in Figure 9(b)), and expected profit rate (shown in Figure 9(c)). Where the percentage values represent the decrease in cost, increase in availability, and increase in profit, respectively.

Table 5 Comparison results of considering first inspection interval or not

C_i	C_p	C_f	T_i	T_p	T_f	Not considering first inspection interval					Considering first inspection interval					
						T	L_p	EC	A	EP	T_1	T	L_p	EC	A	EP
4	40	800	0.2	4	6	5.63	33.87	3.19	0.7780	2.37	18.54	3.24	37.75	2.59	0.8034	3.48
					12	5.43	34.05	3.17	0.7721	2.27	18.38	3.17	37.68	2.57	0.7992	3.41
					18	5.33	33.96	3.15	0.7668	2.19	18.24	3.10	37.63	2.56	0.7956	3.35
					24	5.20	34.03	3.14	0.7622	2.10	18.12	3.04	37.59	2.55	0.7924	3.30

Figure 9 Column comparison chart of considering first inspection interval or not

After observing the data in Table 5 and the column comparison chart in Figure 9, it can be concluded that our proposed model considering first inspection interval is better than the traditional model. Specifically, the proposed model can provide lower cost, higher availability, and higher profit under the same parameters. It shows the necessity of distinguishing first inspection interval and repeat inspection interval.

7 Conclusions and Future Research

This paper investigated the condition-based optimization

for gamma deteriorating systems under PBC. In this paper, the stepwise revenue function was adopted to link the cost and availability to profit, then a maintenance optimization model was established which taking profit as the optimization objective under PBC. In order to reduce the inspection frequency and the cost, first inspection interval was incorporated into the model formulation. In addition, the PSO algorithm was used to improve the speed and accuracy of the solution. A numerical example was given to demonstrate the effectiveness of the proposed model.

By comparison analysis, it can be proved that the proposed policy can increase profits and improve performance at certain cost. In sensitivity analysis, we observed the effect of different failure consequences on optimization policy. For the proposed policy, the higher C_f , the more obvious the profit increase; the higher T_f , the less obvious the profit increase. The final comparative study showed considering the first inspection interval can provide lower cost, higher availability, and higher profit.

This work can be extended along several interesting directions. Firstly, this paper investigates the system subject to gamma deteriorating process, it is also worth studying other deteriorating process, such as Wiener process and inverse Gaussian process, which may be more in line with the degradation laws of some components. In addition, the proposed model only considers the case of single component system, there are several cases worth considering, such as series-parallel system, cold standby system, and k -out-of- n system. These two directions mentioned above can be studied in the future, which can further expand the research scope of maintenance optimization under PBC.

8 Declaration

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Availability of data and materials

The datasets supporting the conclusions of this article are included within the article.

Authors' contributions

The author's contributions are as follows: XZ contributed the central idea, analyzed most of the data, and wrote the initial draft of the paper. QH contributed to carrying out additional analyses, interpreting the results, and reviewing the manuscript. LW contributed to refining the ideas and programming of the solving algorithm. JL and MS participated in the coordination of the study and reviewed the manuscript. All authors read and approved the final manuscript.

Competing interests

The authors declare no competing financial interests.

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